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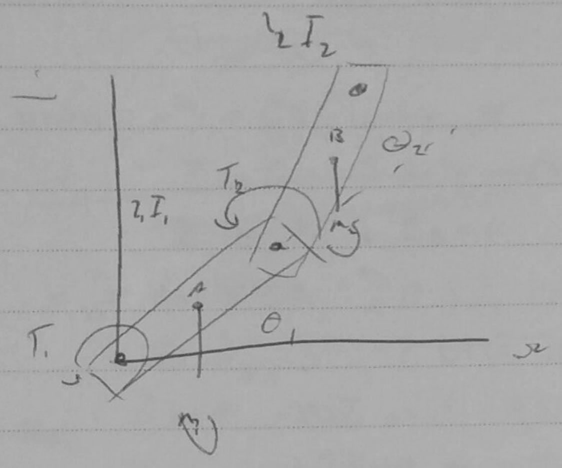

$l_1 = 1m$

$l_2 = 0.75m$

$m_1 = 5\,kg$

$m_2 = 4\,kg$

$I_1 = 0.4\,kg\,m^2$

$I_2 = 0.2\,kg\,m^2$

$$x_B = l_1 C_1 + 0.5 l_2 C_{12}$$

$$\Rightarrow \dot{x}_B = -l_1 S_1 \dot{\theta}_1 - 0.5 l_2 S_{12}(\dot{\theta}_1 + \dot{\theta}_2)$$

$$y_B = l_1 S_1 + 0.5 l_2 S_{12}$$

$$\Rightarrow \dot{y}_B = l_1 C_1 \dot{\theta}_1 + 0.5 l_2 C_{12}(\dot{\theta}_1 + \dot{\theta}_2)$$

$$V_B^2 = \dot{x}_B^2 + \dot{y}_B^2$$

$$= \dot{\theta}_1^2 (l_1^2 + 0.25 l_2^2 + l_1 l_2 C_2) + \dot{\theta}_2^2 (0.25 l_2^2) + \dot{\theta}_1 \dot{\theta}_2 (0.5 l_2^2 + l_1 l_2 C_2)$$

$$K = K_1 + K_2 = \left[\frac{1}{2} I_A \dot{\theta}_1^2\right] + \left[\frac{1}{2} I_B (\dot{\theta}_1 + \dot{\theta}_2)^2 + \frac{1}{2} m_2 V_B^2\right]$$

$$P = m_1 g \frac{l_1}{2} S_1 + m_2 g \left(l_1 S_1 + \frac{l_2}{2} S_{12}\right)$$

$\therefore$ ~~E-K~~

$$\therefore L = K - P$$

$$= \frac{1}{2} I_A \dot{\theta}_1^2 + \frac{1}{2} I_B (\dot{\theta}_1 + \dot{\theta}_2)^2 + \frac{1}{2} m_2 \Big[\dot{\theta}_1^2 (l_1^2 + 0.25 l_2^2 + l_1 l_2 C_2) + \dot{\theta}_2^2 (0.25 l_2^2) + \dot{\theta}_1 \dot{\theta}_2 (0.5 l_2^2 + l_1 l_2 C_2)\Big]$$

$$- m_1 g \frac{l_1}{2} S_1 + m_2 g \left(l_1 S_1 + \frac{l_2}{2} S_{12}\right)$$

$$\cancel{= \dot{\theta}_2^2 \left(\frac{1}{2} I_A + \frac{1}{2}\right.}$$

$$T_i = \frac{\partial}{\partial t}\left(\frac{\partial L}{\partial \dot{\theta}_i}\right) - \frac{\partial L}{\partial \theta_i} \qquad f_i = \frac{\partial}{\partial t}\left(\frac{\partial L}{\partial \dot{x}_i}\right) - \frac{\partial L}{\partial x_i}$$

$$K = \frac{1}{2} I_A \dot{\theta}_1^2 + \left[\frac{1}{2} I_B (\dot{\theta}_1 + \dot{\theta}_2)^2 + \frac{1}{2} m_2 V_B^2\right]$$

$$= \frac{1}{2} I_A \dot{\theta}_1^2 + \frac{1}{2} I_B \dot{\theta}_1^2 + I_B \dot{\theta}_1 \dot{\theta}_2 + \frac{1}{2} I_B \dot{\theta}_2^2$$

$$+ \frac{1}{2} m_2 \left[\dot{\theta}_1^2 (l_1^2 + 0.25 l_2^2 + l_1 l_2 C_2) + \dot{\theta}_2^2 (0.25 l_2^2)\right.$$

$$\left. + \dot{\theta}_1 \dot{\theta}_2 (0.5 l_2^2 + l_1 l_2 C_2)\right]$$

$$\cancel{= \dot{\theta}_1^2 \left(\frac{1}{2} I_A + \frac{1}{2} I_B\right) + \dot{\theta}_1 \dot{\theta}_2 (I_B) + \dot{\theta}_2^2 \left(\frac{1}{2} I_B\right)}$$

$$\cancel{+ \dot{\theta}_1^2 [}$$

$$= \dot{\theta}_1^2 \left[\frac{1}{2} I_A + \frac{1}{2} I_B + \frac{1}{2} m_2 (l_1^2 + 0.25 l_2^2 + l_1 l_2 C_2)\right]$$

$$+ \dot{\theta}_1 \dot{\theta}_2 \left[I_B + \frac{1}{2} m_2 (0.5 l_2^2 + l_1 l_2 C_2)\right]$$

$$+ \dot{\theta}_2^2 \left[\frac{1}{2} I_B\right]$$

$$P = m_1 g \frac{l_1}{2} S_1 + m_2 g \left(l_1 S_1 + \frac{l_2}{2} S_{12}\right)$$

$$L = K - P$$

$$= \dot{\theta}_1^2 \left[\frac{1}{2} I_A + \frac{1}{2} I_B + \frac{1}{2} m_2 (l_1^2 + 0.25 l_2^2 + l_1 l_2 C_2)\right]$$

$$+ \dot{\theta}_1 \dot{\theta}_2 \left[I_B + \frac{1}{2} m_2 (0.5 l_2^2 + l_1 l_2 C_2)\right]$$

$$+ \dot{\theta}_2^2 \left[\frac{1}{2} I_B\right] - m_1 g \frac{l_1}{2} S_1 - m_2 g \left(l_1 S_1 + \frac{l_2}{2} S_{12}\right)$$

$$T_i = \frac{\partial}{\partial t} \frac{\partial L}{\partial \dot{\theta}_i} - \frac{\partial L}{\partial \theta_i}$$

$$\frac{\partial L}{\partial \theta_1} = -m_1 g \frac{l_1}{2} C_1 - m_2 g l_1 C_1 - m_2 g \frac{l_2}{2} C_{12}$$

$$\frac{\partial L}{\partial \dot{\theta}_1} = \dot{\theta}_1 \left[I_A + I_B + m_2 \left(l_1^2 + 0.25 l_2^2 + l_1 l_2 C_2 \right) \right]$$
$$+ \dot{\theta}_2 \left[I_B + \frac{1}{2} m_2 \left(0.5 l_2^2 + l_1 l_2 C_2 \right) \right]$$

$$\frac{\partial}{\partial t} \frac{\partial L}{\partial \dot{\theta}_1} = \ddot{\theta}_1 \left[I_A + I_B + m_2 \left(l_1^2 + 0.25 l_2^2 + l_1 l_2 C_2 \right) \right]$$
$$- m_2 l_1 l_2 S_2 \dot{\theta}_1 \dot{\theta}_2$$
$$+ \ddot{\theta}_2 \left[I_B + \frac{1}{2} m_2 \left(0.5 l_2^2 + l_1 l_2 C_2 \right) \right]$$
$$- m_2 l_1 l_2 S_2 \dot{\theta}_2^2$$

$$\therefore T_1 = \ddot{\theta}_1 \left[I_A + I_B + m_2 \left(l_1^2 + 0.25 l_2^2 + l_1 l_2 C_2 \right) \right]$$
$$+ \ddot{\theta}_2 \left[I_B + \frac{1}{2} m_2 \left(0.5 l_2^2 + l_1 l_2 C_2 \right) \right]$$
$$- m_2 l_1 l_2 S_2 \dot{\theta}_1 \dot{\theta}_2 - m_2 l_1 l_2 S_2 \dot{\theta}_2^2$$
$$+ m_1 g \frac{l_1}{2} C_1 + m_2 g l_1 C_1 + m_2 g \frac{l_2}{2} C_{12}$$

$$\cancel{\frac{\partial L}{\partial \theta_2} = \frac{1}{2} m_1 \dot{\theta}_1^2 l_1 l_2 S_2}$$

$$\frac{\partial L}{\partial \theta_2} = -\frac{1}{2} m_2 l_1 l_2 S_2 \dot{\theta}_1^2 - \frac{1}{2} m_2 l_1 l_2 S_2 \dot{\theta}_1 \dot{\theta}_2 - m_2 g \frac{l_2}{2} C_{12}$$

$$\frac{\partial L}{\partial \dot{\theta}_2} = \dot{\theta}_1 \left[I_B + \frac{1}{2} m_2 \left(0.5 l_2^2 + l_1 l_2 C_2 \right) \right]$$

$$+ \dot{\theta}_2 \left[I_B \right]$$

$$\frac{\partial}{\partial t} \frac{\partial L}{\partial \dot{\theta}_2} = \ddot{\theta}_1 \left[I_B + \frac{1}{2} m_2 \left(0.5 l_2^2 + l_1 l_2 C_2 \right) \right]$$

$$- \frac{1}{2} m_2 l_1 l_2 S_2 \dot{\theta}_1 \dot{\theta}_2$$

$$+ \ddot{\theta}_2 \left[I_B \right]$$

$$\cancel{+ \ddot{\theta}_2 \left[I_B \right]}$$

$$T_2 = \ddot{\theta}_1 \left[I_B + \frac{1}{2} m_2 \left(0.5 l_2^2 + l_1 l_2 C_2 \right) \right] + \ddot{\theta}_2 \left[I_B \right]$$

$$- \frac{1}{2} m_2 l_1 l_2 S_2 \dot{\theta}_1 \dot{\theta}_2 + \frac{1}{2} m_2 l_1 l_2 S_2 \dot{\theta}_1^2$$

$$+ \frac{1}{2} m_2 l_1 l_2 S_2 \dot{\theta}_1 \dot{\theta}_2 + m_2 g \frac{l_2}{2} C_{12}$$

$$T_1 = \ddot{\theta}_1 \left[I_A + I_B + m_2 \left(l_1^2 + 0.25\, l_2^2 + l_1 l_2 c_2 \right) \right]$$

$$+ \ddot{\theta}_2 \left[I_B + \tfrac{1}{2} m_2 \left(0.5\, l_2^2 + l_1 l_2 c_2 \right) \right]$$

$$+ - m_2 l_1 l_2 s_2 \dot{\theta}_1 \dot{\theta}_2 - m_2 l_1 l_2 s_2 \dot{\theta}_2^2$$

$$+ m g \frac{l_1}{2} c_1 + m_2 g l_1 c_1 + m_2 g \frac{l_2}{2} c_{12}$$

$$T_2 = \ddot{\theta}_1 \left[I_B + \tfrac{1}{2} m_2 \left(0.5\, l_2^2 + l_1 l_2 c_2 \right) \right] + \ddot{\theta}_2 \left[I_B \right]$$

$$\cancel{- \tfrac{1}{2} m_2 l_1 l_2 s_2 \dot{\theta}_1 \dot{\theta}_2} + \tfrac{1}{2} m_2 l_1 l_2 s_2 \dot{\theta}_1^2$$

$$\cancel{+ \tfrac{1}{2} m_2 l_1 l_2 s_2 \dot{\theta}_1 \dot{\theta}_2} + m_2 g \frac{l_2}{2} c_{12}$$

$$\therefore T_1 = \ddot{\theta}_1 \left[5.1625 + 3 \cos \theta_2 \right] + \ddot{\theta}_2 \left[0.7625 + 1.5 \cos \theta_2 \right]$$

$$- \dot{\theta}_1 \dot{\theta}_2 \left[3 \sin \theta_2 \right] - \dot{\theta}_2^2 \left[3 \sin \theta_2 \right]$$

$$+ 6.5\, g \cos \theta_1 + 1.5\, g \cos(\theta_1 + \theta_2)$$

$$\therefore T_2 = \ddot{\theta}_1 \left[0.7625 + 1.5 \cos \theta_2 \right] + \ddot{\theta}_2 \left[0.2 \right]$$

$$\cancel{- \dot{\theta}_1 \dot{\theta}_2 \left[1.5 \sin \theta_2 \right]} + \dot{\theta}_1^2 \left[1.5 \sin \theta_2 \right] + 1.5\, g \cos(\theta_1 + \theta_2)$$

where g = acceleration due to gravity $\approx 9.81\, m s^{-2}$

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$$T_1 = \ddot{\theta}_1\left[5.1625 + 3\cos\theta_2\right] + \ddot{\theta}_2\left[0.7625 + 1.5\cos\theta_2\right]$$

$$- \dot{\theta}_1\dot{\theta}_2\left[3\sin\theta_2\right] - \dot{\theta}_2^{\,2}\left[3\sin\theta_2\right]$$

$$+ 6.5g\cos\theta_1 + 1.5g\cos(\theta_1+\theta_2)$$

$$\therefore \ddot{\theta}_1 = \frac{\left(T_1 - \ddot{\theta}_2\left[0.7625 + 1.5\cos\theta_2\right] + \dot{\theta}_1\dot{\theta}_2\left[3\sin\theta_2\right] + \dot{\theta}_2^{\,2}\left[3\sin\theta_2\right] - 6.5g\cos\theta_1 + 1.5g\cos(\theta_1+\theta_2)\right)}{5.1625 + 3\cos\theta_2}$$

$\longrightarrow$ (1)

$$T_2 = \ddot{\theta}_1\left[0.7625 + 1.5\cos\theta_2\right] + \ddot{\theta}_2\left[0.2\right]$$

$$+ \dot{\theta}_1^{\,2}\left[1.5\sin\theta_2\right] + 1.5g\cos(\theta_1+\theta_2)$$

$$\therefore \ddot{\theta}_2 = \frac{\left(T_2 - \ddot{\theta}_1\left[0.7625 + 1.5\cos\theta_2\right] - \dot{\theta}_1^{\,2}\left[1.5\sin\theta_2\right] - 1.5g\cos(\theta_1+\theta_2)\right)}{0.2}$$

$\longrightarrow$ (2)