Considering link L1 and link L2, take end point of link L2 as $P(x, y)$

$x = L1\cos\theta_1 + L2\cos(\theta_1 + \theta_2)$

$\dot{x} = (-L1sin\theta_1 - L2\sin(\theta_1 + \theta_2))\dot{\theta_1} - (L2sin(\theta_1 + \theta_2))\dot{\theta_2}$

$y = L1\sin\theta_1 + L2\sin(\theta_1 + \theta_2)$

$\dot{y} = (L1cos\theta_1 - L2\cos(\theta_1 + \theta_2))\dot{\theta_1} - (L2cos(\theta_1 + \theta_2))\dot{\theta_2}$

$[\begin{matrix}\dot{x}\\ \dot{y}\end{matrix}] = [\begin{matrix}-L1S_1 - L2S_{12} & -L2S_{12}\\ L1C_1 + L2C_{12} & L2C_{12}\end{matrix}][\begin{matrix}\dot{\theta}_1\\ \dot{\theta}_2\end{matrix}]$

$[\begin{matrix}\dot{x}\\ \dot{y}\end{matrix}] = [\,J_1\,][\begin{matrix}\dot{\theta}_1\\ \dot{\theta}_2\end{matrix}]$, where $J_1\ is\ jacobian$ L1 & L2

$J_1 = [\begin{matrix}-L1S_1 - L2S_{12} & -L2S_{12}\\ L1C_1 + L2C_{12} & L2C_{12}\end{matrix}]$

$\det(J_1) = -(L1S_1 + L2S_{12})(L2C_{12}) + (L2S_{12})(L1C_1 + L2C_{12})$

$\det(J_1) = L1L2S_2$

Same as above,

$\det(J_2) = L2L3S_3$

Finding singularities

Singularities occurs when $\det(J) = 0$

A

$\det(J_1) = 0$

$L1L2S_2 = 0$

L1=0.25, L2=0.25

Then $sin\theta_2 = 0$

$-\frac{2\pi}{3} \le \theta_2 \le \frac{2\pi}{3},\ \theta_2 = 0$

B

$\det(J_2) = 0$

$L2L3S_3 = 0$

L2=0.25, L3=0.315

Then $sin\theta_3 = 0$

$-\frac{\pi}{2} \le \theta_3 \le \frac{\pi}{2},\ \theta_3 = 0$

So for the whole system singularities occurs according to the intersection of results of **A** & **B**,

$$\boldsymbol{\theta_2 = 0}\ \&\ \boldsymbol{\theta_3 = 0}$$