

MTRX5700
EXPERIMENTAL ROBOTICS

Assignment 1

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1 Question 1

The following MATLAB code was used to generate the homogeneous transformation matrices required.

Listing 1: MTRX5700 A1Q1 MATLAB Code

```
1 close all
2 clear
3 clc
4
5 DEGREES = pi/180;
6 RADIANS = 180/pi;
7
8
9 fprintf('a)\n');
10 roll = 10*DEGREES; %alpha
11 pitch = 20*DEGREES; %beta
12 yaw = 30*DEGREES; %gamma
13
14 aPb = [1 2 3];
15 aRb = [cos(roll)*cos(pitch), cos(roll)*sin(pitch)*sin(yaw)-sin(roll)*cos(yaw), cos(roll)*sin(pitch)*
    ↪ cos(yaw)+sin(roll)*sin(yaw);
16     sin(roll)*cos(pitch), sin(roll)*sin(pitch)*sin(yaw)+cos(roll)*cos(yaw), sin(roll)*sin(pitch)*cos
    ↪ (yaw)-cos(roll)*sin(yaw);
17     -sin(pitch), cos(pitch)*sin(yaw), cos(pitch)*cos(yaw)];
18 %transpose(aPb)
19
20 aTb = [aRb transpose(aPb); 0 0 0 1]
21
22
23 fprintf('b)\n');
24
25 roll = 10*DEGREES; %alpha
26 pitch = 30*DEGREES; %beta
27 yaw = 30*DEGREES; %gamma
28
29 aPb = [3 0 0];
30 aRb = [cos(roll)*cos(pitch), cos(roll)*sin(pitch)*sin(yaw)-sin(roll)*cos(yaw), cos(roll)*sin(pitch)*
    ↪ cos(yaw)+sin(roll)*sin(yaw);
31     sin(roll)*cos(pitch), sin(roll)*sin(pitch)*sin(yaw)+cos(roll)*cos(yaw), sin(roll)*sin(pitch)*cos
    ↪ (yaw)-cos(roll)*sin(yaw);
32     -sin(pitch), cos(pitch)*sin(yaw), cos(pitch)*cos(yaw)];
33 %transpose(aPb)
34
35 aTb = [aRb transpose(aPb); 0 0 0 1]
36
37 fprintf('c)\n');
38
39 roll = 90*DEGREES; %alpha
40 pitch = 180*DEGREES; %beta
41 yaw = -90*DEGREES; %gamma
42
43 aPb = [0 0 1];
44 aRb = [cos(roll)*cos(pitch), cos(roll)*sin(pitch)*sin(yaw)-sin(roll)*cos(yaw), cos(roll)*sin(pitch)*
    ↪ cos(yaw)+sin(roll)*sin(yaw);
45     sin(roll)*cos(pitch), sin(roll)*sin(pitch)*sin(yaw)+cos(roll)*cos(yaw), sin(roll)*sin(pitch)*cos
    ↪ (yaw)-cos(roll)*sin(yaw);
46     -sin(pitch), cos(pitch)*sin(yaw), cos(pitch)*cos(yaw)];
47 %transpose(aPb)
48
49 aTb = [aRb transpose(aPb); 0 0 0 1]
50
51 roll = 90*DEGREES; %alpha
52 pitch = 180*DEGREES; %beta
53 yaw = 270*DEGREES; %gamma
54
55 aPb = [0 0 1];
56 aRb = [cos(roll)*cos(pitch), cos(roll)*sin(pitch)*sin(yaw)-sin(roll)*cos(yaw), cos(roll)*sin(pitch)*
    ↪ cos(yaw)+sin(roll)*sin(yaw);
57     sin(roll)*cos(pitch), sin(roll)*sin(pitch)*sin(yaw)+cos(roll)*cos(yaw), sin(roll)*sin(pitch)*cos
```

```

        ↪ (yaw)-cos(roll)*sin(yaw);
58     -sin(pitch), cos(pitch)*sin(yaw), cos(pitch)*cos(yaw)];
59     %transpose(aPb)
60
61     aTb = [aRb transpose(aPb); 0 0 0 1]

```

1.a

The homogeneous transformation matrix for
 $\alpha = 10^\circ, \beta = 20^\circ, \gamma = 30^\circ, {}^A P_B = \{1 \ 2 \ 3\}^T$
looks like:

$$A = \begin{pmatrix} 0.9254 & 0.0180 & 0.3785 & 1.0000 \\ 0.1632 & 0.8826 & -0.4410 & 2.0000 \\ -0.3420 & 0.4698 & 0.8138 & 3.0000 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

1.b

The homogeneous transformation matrix for
 $\alpha = 10^\circ, \beta = 30^\circ, \gamma = 30^\circ, {}^A P_B = \{3 \ 0 \ 0\}^T$
looks like:

$$B = \begin{pmatrix} 0.8529 & 0.0958 & 0.5133 & 3.0000 \\ 0.1504 & 0.8963 & -0.4172 & 0 \\ -0.5000 & 0.4330 & 0.7500 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

1.c

The homogeneous transformation matrix for
 $\alpha = 90^\circ, \beta = 180^\circ, \gamma = -90^\circ, {}^A P_B = \{0 \ 0 \ 1\}^T$
looks like:

$$C_1 = \begin{pmatrix} 0 & 0 & -1 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

In Comparison: The homogeneous transformation matrix for
 $\alpha = 90^\circ, \beta = 180^\circ, \gamma = 270^\circ, {}^A P_B = \{0 \ 0 \ 1\}^T$
looks like:

$$C_2 = \begin{pmatrix} 0 & 0 & -1 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Comparing these two we find that there is no difference between the two generated matrices.

2 Question 2

Listing 2: MTRX5700 A1Q1 MATLAB Code

```
1 close all
2 clear
3 clc
4
5 DEGREES = pi/180;
6 RADIANS = 180/pi;
7
8 R1 = [0.7500, -0.4330, -0.5000; 0.2165, 0.8750, -0.4330; 0.6250, 0.2165, 0.7500]
9
10
11 determinantR1 = det(R1)
12 inverseR1 = inv(R1)
13 transposeR1 = transpose(R1)
14
15 beta1 = asin(-1*R1(3,1));
16 gamma1 = asin(R1(3,2)/cos(beta1));
17 alpha1 = acos(R1(1,1)/cos(beta1));
18
19 alpha1 = alpha1*RADIANS
20 beta1 = beta1*RADIANS
21 gamma1 = gamma1*RADIANS
22
23 R2 = [0.7725, -0.4460, -0.5150; 0.2165, 0.8750, -0.4330; 0.6000, 0.2078, 0.7200]
24 determinantR2 = det(R2)
25 inverseR2 = inv(R2)
26 transposeR2 = transpose(R2)
27
28 beta2 = asin(-1*R2(3,1));
29 gamma2 = asin(R2(3,2)/cos(beta2));
30 alpha2 = acos(R2(1,1)/cos(beta2));
31
32 alpha2 = alpha2*RADIANS
33 beta2 = beta2*RADIANS
34 gamma2 = gamma2*RADIANS
35
36 R3 = [0, 0, 1; 0.8660, 0.500, 0; -0.500, 0.8660, 0]
37 determinantR3 = det(R3)
38 inverseR3 = inv(R3)
39 transposeR3 = transpose(R3)
40
41 beta3 = asin(-1*R3(3,1));
42 gamma3 = asin(R3(3,2)/cos(beta3));
43 alpha3 = acos(R3(1,1)/cos(beta3));
44
45 alpha3 = alpha3*RADIANS
46 beta3 = beta3*RADIANS
47 gamma3 = gamma3*RADIANS
48
49 R4 = [-0.7500, -0.2165, -0.6250; 0.4330, -0.8750, -0.2165; 0.500, 0.4330, -0.7500]
50 determinantR4 = det(R4)
51 inverseR4 = inv(R4)
52 transposeR4 = transpose(R4)
53
54 beta4 = asin(-1*R4(3,1));
55 gamma4 = asin(R4(3,2)/cos(beta4));
56 alpha4 = acos(R4(1,1)/cos(beta4));
57
58 alpha4 = alpha4*RADIANS
59 beta4 = beta4*RADIANS
60 gamma4 = gamma4*RADIANS
```

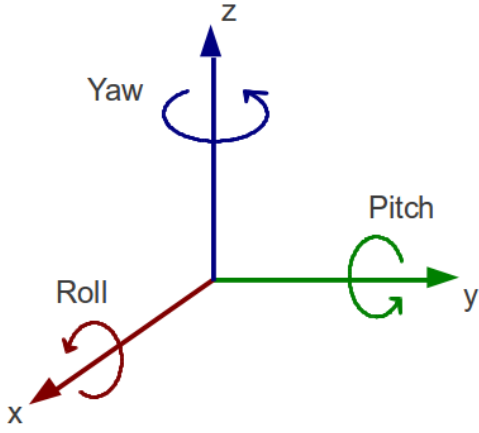


Figure 1: Roll Pitch and Yaw definitions (relative to axis names)

2.a Validity Check

We need to prove that the determinant of the matrix is equal to 1 and that the inverse of the matrix is equal to the transpose to prove validity.

2.a.i R_1

$$R_1 = \begin{pmatrix} 0.7500 & -0.4330 & -0.5000 \\ 0.2165 & 0.8750 & -0.4330 \\ 0.6250 & 0.2165 & 0.7500 \end{pmatrix}$$

$$\det(R_1) = 1.0000$$

$$R_1^{-1} = \begin{pmatrix} 0.7500 & 0.2165 & 0.6250 \\ -0.4330 & 0.8750 & 0.2165 \\ -0.5000 & -0.4330 & 0.7500 \end{pmatrix}$$

$$R_1^T = \begin{pmatrix} 0.7500 & 0.2165 & 0.6250 \\ -0.4330 & 0.8750 & 0.2165 \\ -0.5000 & -0.4330 & 0.7500 \end{pmatrix}$$

$$R_1^{-1} = R_1^T \therefore R_1 \text{ is valid}$$

2.a.ii R_2

$$R_2 = \begin{pmatrix} 0.7725 & -0.4460 & -0.5150 \\ 0.2165 & 0.8750 & -0.4330 \\ 0.6000 & 0.2078 & 0.7200 \end{pmatrix}$$

$$\det(R_2) = 0.9888$$

$$R_2^{-1} = \begin{pmatrix} 0.7281 & 0.2165 & 0.6510 \\ -0.4204 & 0.8750 & 0.2255 \\ -0.4885 & -0.4330 & 0.7813 \end{pmatrix}$$

$$R_2^T = \begin{pmatrix} 0.7725 & 0.2165 & 0.6000 \\ -0.4460 & 0.8750 & 0.2078 \\ -0.5150 & -0.4330 & 0.7200 \end{pmatrix}$$

$R_2^{-1} \cong R_2^T \therefore R_2$ is valid within the limits of practical numerical applications.

2.a.iii R_3

$$R_3 = \begin{pmatrix} 0 & 0 & 1 \\ 0.8660 & 0.5000 & 0 \\ -0.5000 & 0.8660 & 0 \end{pmatrix}$$

$$\det(R_3) = 1$$

$$R_3^{-1} = \begin{pmatrix} 0 & 0.8660 & -0.5000 \\ 0 & 0.5000 & 0.8660 \\ 1 & 0 & 0 \end{pmatrix}$$

$$R_3^T = \begin{pmatrix} 0 & 0.8660 & -0.5000 \\ 0 & 0.5000 & 0.8660 \\ 1 & 0 & 0 \end{pmatrix}$$

$$R_3^{-1} = R_3^T \therefore R_3 \text{ is } \textit{valid}$$

2.a.iv R_4

$$R_4 = \begin{pmatrix} -0.7500 & -0.2165 & -0.6250 \\ 0.4330 & -0.8750 & -0.2165 \\ 0.5000 & 0.4330 & -0.7500 \end{pmatrix}$$

$$\det(R_4) = -1$$

$$R_4^{-1} = \begin{pmatrix} -0.7500 & 0.4330 & 0.5000 \\ -0.2165 & -0.8750 & 0.4330 \\ -0.6250 & -0.2165 & -0.7500 \end{pmatrix}$$

$$R_4^T = \begin{pmatrix} -0.7500 & 0.4330 & 0.5000 \\ -0.2165 & -0.8750 & 0.4330 \\ -0.6250 & -0.2165 & -0.7500 \end{pmatrix}$$

$$\text{Although } R_4^{-1} = R_4^T, \det(R_4) \neq 1 \therefore R_4 \text{ is } \textit{invalid}$$

2.b Roll/Pitch/Yaw Angles

Roll Angle is α , Pitch Angle is β , Yaw Angle is γ .

Assumption: The believability of the angles can be determined purely mathematically. In reality these angles may not necessarily be believable depending on the application at hand. However, we cannot account for this without further information about the system.

2.b.i R_1

The following values are believable

$$\alpha_1 = 16.1021^\circ$$

$$\beta_1 = -36.6822^\circ$$

$$\gamma_1 = 16.1016^\circ$$

2.b.ii R_2

The following values are believable

$$\alpha_2 = 15.0665^\circ$$

$$\beta_2 = -36.8699^\circ$$

$$\gamma_2 = 15.0552^\circ$$

2.b.iii R_3

The following values are believable

$$\alpha_3 = 90^\circ$$

$$\beta_3 = 30^\circ$$

$$\gamma_3 = 89.5611^\circ$$

2.b.iv R_4

The following values are not believable as R_4 is an invalid rotational matrix

$$\alpha_4 = 150^\circ$$

$$\beta_4 = -30^\circ$$

$$\gamma_4 = 29.9990^\circ$$

2.c Angle Estimation

For a matrix such as R_2 we can adjust our matrix values slightly (making them less accurate i.e. to fewer decimal values) to still get a reasonable estimation of our angles. We can realise this by seeing that the determinant is so close to 1. R_4 , however, cannot give us a reasonable estimate for the angles as the determinant is too far from the required value of 1.

3 Question 3

Listing 3: MTRX5700 A1Q1 MATLAB Code

```
1 close all
2 clear
3 clc
4
5 DEGREES = pi/180;
6 RADIANS = 180/pi;
7
8 R1 = [0.7500, -0.4330, -0.5000; 0.2165, 0.8750, -0.4330; 0.6250, 0.2165, 0.7500]
9
10
11 determinantR1 = det(R1)
12 inverseR1 = inv(R1)
13 transposeR1 = transpose(R1)
14
15 beta1 = asin(-1*R1(3,1));
16 gamma1 = asin(R1(3,2)/cos(beta1));
17 alpha1 = acos(R1(1,1)/cos(beta1));
18
19 alpha1 = alpha1*RADIANS
20 beta1 = beta1*RADIANS
21 gamma1 = gamma1*RADIANS
22
23 R2 = [0.7725, -0.4460, -0.5150; 0.2165, 0.8750, -0.4330; 0.6000, 0.2078, 0.7200]
24 determinantR2 = det(R2)
25 inverseR2 = inv(R2)
26 transposeR2 = transpose(R2)
27
28 beta2 = asin(-1*R2(3,1));
29 gamma2 = asin(R2(3,2)/cos(beta2));
30 alpha2 = acos(R2(1,1)/cos(beta2));
31
32 alpha2 = alpha2*RADIANS
33 beta2 = beta2*RADIANS
34 gamma2 = gamma2*RADIANS
35
36 R3 = [0, 0, 1; 0.8660, 0.500, 0; -0.500, 0.8660, 0]
37 determinantR3 = det(R3)
38 inverseR3 = inv(R3)
39 transposeR3 = transpose(R3)
40
41 beta3 = asin(-1*R3(3,1));
42 gamma3 = asin(R3(3,2)/cos(beta3));
43 alpha3 = acos(R3(1,1)/cos(beta3));
44
45 alpha3 = alpha3*RADIANS
46 beta3 = beta3*RADIANS
47 gamma3 = gamma3*RADIANS
48
49 R4 = [-0.7500, -0.2165, -0.6250; 0.4330, -0.8750, -0.2165; 0.500, 0.4330, -0.7500]
50 determinantR4 = det(R4)
51 inverseR4 = inv(R4)
52 transposeR4 = transpose(R4)
53
54 beta4 = asin(-1*R4(3,1));
55 gamma4 = asin(R4(3,2)/cos(beta4));
56 alpha4 = acos(R4(1,1)/cos(beta4));
57
58 alpha4 = alpha4*RADIANS
59 beta4 = beta4*RADIANS
60 gamma4 = gamma4*RADIANS
```


DH Notation

i	θ	d	r	α
1	θ_1	67	100	$\pi/2$
2	θ_2	0	250	0
3	θ_2	0	0	$\pi/2$
4	θ_4	0	250	$-\pi/2$
5	θ_5	0	0	$-\pi/2$
6	θ_6	0	245	0

Table 1: D-H Variable Notation

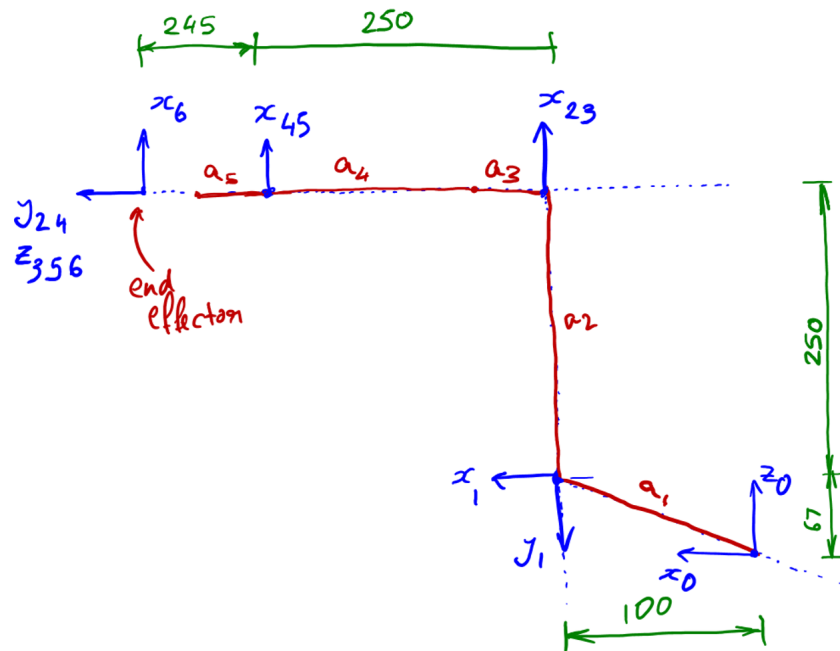


Figure 2: D-H Standard for Forward Kinematics Coordinate Systems

Resulting Transformation Matrix

The resulting transformation matrix can be represented as:

$${}^0T_6 = \begin{pmatrix} n_x & s_x & a_x & p_x \\ n_y & s_y & a_y & p_y \\ n_z & s_z & a_z & p_z \\ 0 & 0 & 0 & 1 \end{pmatrix} = {}^0A_1 {}^1A_2 {}^2A_3 {}^3A_4 {}^4A_5 {}^5A_6$$

$$\mathbf{n}_x = -\sin \theta_6 (\cos \theta_4 \sin \theta_1 - \sin \theta_4 (\cos \theta_1 \sin \theta_2 \sin \theta_3 - \cos \theta_1 \cos \theta_2 \cos \theta_3)) - \cos \theta_6 (\cos \theta_5 (\sin \theta_1 \sin \theta_4 + \cos \theta_4 (\cos \theta_1 \sin \theta_2 \sin \theta_3 - \cos \theta_1 \cos \theta_2 \cos \theta_3)) + \sin \theta_5 (\cos \theta_1 \cos \theta_2 \sin \theta_3 + \cos \theta_1 \cos \theta_3 \sin \theta_2))$$

$$\mathbf{s}_x = \sin \theta_6 (\cos \theta_5 (\sin \theta_1 \sin \theta_4 + \cos \theta_4 (\cos \theta_1 \sin \theta_2 \sin \theta_3 - \cos \theta_1 \cos \theta_2 \cos \theta_3)) + \sin \theta_5 (\cos \theta_1 \cos \theta_2 \sin \theta_3 + \cos \theta_1 \cos \theta_3 \sin \theta_2)) - \cos \theta_6 (\cos \theta_4 \sin \theta_1 - \sin \theta_4 (\cos \theta_1 \sin \theta_2 \sin \theta_3 - \cos \theta_1 \cos \theta_2 \cos \theta_3))$$

$$\mathbf{a}_x = \sin \theta_5 (\sin \theta_1 \sin \theta_4 + \cos \theta_4 (\cos \theta_1 \sin \theta_2 \sin \theta_3 - \cos \theta_1 \cos \theta_2 \cos \theta_3)) - \cos \theta_5 (\cos \theta_1 \cos \theta_2 \sin \theta_3 + \cos \theta_1 \cos \theta_3 \sin \theta_2)$$

$$\mathbf{p}_x = 100 \cos \theta_1 + 250 \cos \theta_1 \cos \theta_2 - 250 \sin \theta_1 \sin \theta_4 - 245 \sin \theta_6 (\cos \theta_4 \sin \theta_1 - \sin \theta_4 (\cos \theta_1 \sin \theta_2 \sin \theta_3 - \cos \theta_1 \cos \theta_2 \cos \theta_3)) - 245 \cos \theta_6 (\cos \theta_5 (\sin \theta_1 \sin \theta_4 + \cos \theta_4 (\cos \theta_1 \sin \theta_2 \sin \theta_3 - \cos \theta_1 \cos \theta_2 \cos \theta_3)) + \sin \theta_5 (\cos \theta_1 \cos \theta_2 \sin \theta_3 + \cos \theta_1 \cos \theta_3 \sin \theta_2)) - 250 \cos \theta_4 (\cos \theta_1 \sin \theta_2 \sin \theta_3 - \cos \theta_1 \cos \theta_2 \cos \theta_3)$$

$$\mathbf{n}_y = \sin \theta_6 (\cos \theta_1 \cos \theta_4 + \sin \theta_4 (\sin \theta_1 \sin \theta_2 \sin \theta_3 - \cos \theta_2 \cos \theta_3 \sin \theta_1)) + \cos \theta_6 (\cos \theta_5 (\cos \theta_1 \sin \theta_4 - \cos \theta_4 (\sin \theta_1 \sin \theta_2 \sin \theta_3 - \cos \theta_2 \cos \theta_3 \sin \theta_1)) - \sin \theta_5 (\cos \theta_2 \sin \theta_1 \sin \theta_3 + \cos \theta_3 \sin \theta_1 \sin \theta_2))$$

$$\mathbf{s}_y = \cos \theta_6 (\cos \theta_1 \cos \theta_4 + \sin \theta_4 (\sin \theta_1 \sin \theta_2 \sin \theta_3 - \cos \theta_2 \cos \theta_3 \sin \theta_1)) - \sin \theta_6 (\cos \theta_5 (\cos \theta_1 \sin \theta_4 - \cos \theta_4 (\sin \theta_1 \sin \theta_2 \sin \theta_3 - \cos \theta_2 \cos \theta_3 \sin \theta_1)) - \sin \theta_5 (\cos \theta_2 \sin \theta_1 \sin \theta_3 + \cos \theta_3 \sin \theta_1 \sin \theta_2))$$

$$\mathbf{a}_y = -\sin \theta_5 (\cos \theta_1 \sin \theta_4 - \cos \theta_4 (\sin \theta_1 \sin \theta_2 \sin \theta_3 - \cos \theta_2 \cos \theta_3 \sin \theta_1)) - \cos \theta_5 (\cos \theta_2 \sin \theta_1 \sin \theta_3 + \cos \theta_3 \sin \theta_1 \sin \theta_2)$$

$$\mathbf{p}_y = 100 \sin \theta_1 + 250 \cos \theta_2 \sin \theta_1 + 250 \cos \theta_1 \sin \theta_4 + 245 \sin \theta_6 (\cos \theta_1 \cos \theta_4 + \sin \theta_4 (\sin \theta_1 \sin \theta_2 \sin \theta_3 - \cos \theta_2 \cos \theta_3 \sin \theta_1)) - 250 \cos \theta_4 (\sin \theta_1 \sin \theta_2 \sin \theta_3 - \cos \theta_2 \cos \theta_3 \sin \theta_1) + 245 \cos \theta_6 (\cos \theta_5 (\cos \theta_1 \sin \theta_4 - \cos \theta_4 (\sin \theta_1 \sin \theta_2 \sin \theta_3 - \cos \theta_2 \cos \theta_3 \sin \theta_1)) - \sin \theta_5 (\cos \theta_2 \sin \theta_1 \sin \theta_3 + \cos \theta_3 \sin \theta_1 \sin \theta_2))$$

$$\mathbf{n}_z = \cos \theta_6 (\sin \theta_5 (\cos \theta_2 \cos \theta_3 - \sin \theta_2 \sin \theta_3) + \cos \theta_4 \cos \theta_5 (\cos \theta_2 \sin \theta_3 + \cos \theta_3 \sin \theta_2)) - \sin \theta_4 \sin \theta_6 (\cos \theta_2 \sin \theta_3 + \cos \theta_3 \sin \theta_2)$$

$$\mathbf{s}_z = -\sin \theta_6 (\sin \theta_5 (\cos \theta_2 \cos \theta_3 - \sin \theta_2 \sin \theta_3) + \cos \theta_4 \cos \theta_5 (\cos \theta_2 \sin \theta_3 + \cos \theta_3 \sin \theta_2)) - \cos \theta_6 \sin \theta_4 (\cos \theta_2 \sin \theta_3 + \cos \theta_3 \sin \theta_2)$$

$$\mathbf{a}_z = \cos \theta_5 (\cos \theta_2 \cos \theta_3 - \sin \theta_2 \sin \theta_3) - \cos \theta_4 \sin \theta_5 (\cos \theta_2 \sin \theta_3 + \cos \theta_3 \sin \theta_2)$$

$$\mathbf{p}_z = 250 \sin \theta_2 + 245 \cos \theta_6 (\sin \theta_5 (\cos \theta_2 \cos \theta_3 - \sin \theta_2 \sin \theta_3) + \cos \theta_4 \cos \theta_5 (\cos \theta_2 \sin \theta_3 + \cos \theta_3 \sin \theta_2)) + 250 \cos \theta_4 (\cos \theta_2 \sin \theta_3 + \cos \theta_3 \sin \theta_2) - 245 \sin \theta_4 \sin \theta_6 (\cos \theta_2 \sin \theta_3 + \cos \theta_3 \sin \theta_2) + 67$$

4 Question 4

4.a

4.b

4.c

5 Question 5

5.a

5.b

5.c

6 Question 6

6.a

6.b

```
sa2 =  
-133.57970015587177746184941710562 -56.676681927833899035216072316143  
sa3 =  
166.90301822803787842663334478947 13.096981771962121573366655210527  
sa5 =  
-123.32331807216610408729072583881 -46.420299844128225660657381049333  
sb2 =  
-121.68117590050949105729393864272 -45.154934789134486545751233766901  
sb3 =  
166.52624111137500451154270487582 13.473758888624995488457295124181  
sb5 =  
-134.84506521086551657675556438805 -58.318824099490512065212859512228
```

6.c

7 Question 7

7.a Qualitative Analysis of Block Stacking Method

7.b CODE

8 Question 8

8.a

8.b

8.c