MTRX5700

Experimental Robotics

Assignment 1

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1 Question 1

The following MATLAB code was used to generate the homogeneous transformation matrices required.

Listing 1: MTRX5700 A1Q1 MATLAB Code

```
close all
clear
clc

DEGREES = pi/180;
RADIANS = 180/pi;


fprintf('a)\n');
roll = 10*DEGREES; %alpha
pitch = 20*DEGREES; %beta
yaw = 30*DEGREES; %gamma

aPb = [1 2 3];
aRb = [cos(roll)*cos(pitch), cos(roll)*sin(pitch)*sin(yaw)-sin(roll)*cos(yaw), cos(roll)*sin(pitch)*cos(yaw)+sin(roll)*sin(yaw);
    sin(roll)*cos(pitch), sin(roll)*sin(pitch)*sin(yaw)+cos(roll)*cos(yaw), sin(roll)*sin(pitch)*cos(yaw)-cos(roll)*sin(yaw);
    -sin(pitch), cos(pitch)*sin(yaw), cos(pitch)*cos(yaw)];
%transpose(aPb)

aTb = [aRb transpose(aPb); 0 0 0 1]


fprintf('b)\n');

roll = 10*DEGREES; %alpha
pitch = 30*DEGREES; %beta
yaw = 30*DEGREES; %gamma

aPb = [3 0 0];
aRb = [cos(roll)*cos(pitch), cos(roll)*sin(pitch)*sin(yaw)-sin(roll)*cos(yaw), cos(roll)*sin(pitch)*cos(yaw)+sin(roll)*sin(yaw);
    sin(roll)*cos(pitch), sin(roll)*sin(pitch)*sin(yaw)+cos(roll)*cos(yaw), sin(roll)*sin(pitch)*cos(yaw)-cos(roll)*sin(yaw);
    -sin(pitch), cos(pitch)*sin(yaw), cos(pitch)*cos(yaw)];
%transpose(aPb)

aTb = [aRb transpose(aPb); 0 0 0 1]

fprintf('c)\n');

roll = 90*DEGREES; %alpha
pitch = 180*DEGREES; %beta
yaw = -90*DEGREES; %gamma

aPb = [0 0 1];
aRb = [cos(roll)*cos(pitch), cos(roll)*sin(pitch)*sin(yaw)-sin(roll)*cos(yaw), cos(roll)*sin(pitch)*cos(yaw)+sin(roll)*sin(yaw);
    sin(roll)*cos(pitch), sin(roll)*sin(pitch)*sin(yaw)+cos(roll)*cos(yaw), sin(roll)*sin(pitch)*cos(yaw)-cos(roll)*sin(yaw);
    -sin(pitch), cos(pitch)*sin(yaw), cos(pitch)*cos(yaw)];
%transpose(aPb)

aTb = [aRb transpose(aPb); 0 0 0 1]

roll = 90*DEGREES; %alpha
pitch = 180*DEGREES; %beta
yaw = 270*DEGREES; %gamma

aPb = [0 0 1];
aRb = [cos(roll)*cos(pitch), cos(roll)*sin(pitch)*sin(yaw)-sin(roll)*cos(yaw), cos(roll)*sin(pitch)*cos(yaw)+sin(roll)*sin(yaw);
    sin(roll)*cos(pitch), sin(roll)*sin(pitch)*sin(yaw)+cos(roll)*cos(yaw), sin(roll)*sin(pitch)*cos
```

```
        ↪ (yaw)-cos(roll)*sin(yaw);
    -sin(pitch), cos(pitch)*sin(yaw), cos(pitch)*cos(yaw)];
%transpose(aPb)

aTb = [aRb transpose(aPb); 0 0 0 1]
```

1.a

The homogeneous transformation matrix for
$\alpha = 10°$, $\beta = 20°$, $\gamma = 30°$, ${}^{A}P_B = \{1\ 2\ 3\}^T$
looks like:

$$A = \begin{pmatrix} 0.9254 & 0.0180 & 0.3785 & 1.0000 \\ 0.1632 & 0.8826 & -0.4410 & 2.0000 \\ -0.3420 & 0.4698 & 0.8138 & 3.0000 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

1.b

The homogeneous transformation matrix for
$\alpha = 10°$, $\beta = 30°$, $\gamma = 30°$, ${}^{A}P_B = \{3\ 0\ 0\}^T$
looks like:

$$B = \begin{pmatrix} 0.8529 & 0.0958 & 0.5133 & 3.0000 \\ 0.1504 & 0.8963 & -0.4172 & 0 \\ -0.5000 & 0.4330 & 0.7500 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

1.c

The homogeneous transformation matrix for
$\alpha = 90°$, $\beta = 180°$, $\gamma =$ -90°, ${}^{A}P_B = \{0\ 0\ 1\}^T$
looks like:

$$C_1 = \begin{pmatrix} 0 & 0 & -1 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

In Comparison: The homogeneous transformation matrix for
$\alpha = 90°$, $\beta = 180°$, $\gamma = 270°$, ${}^{A}P_B = \{0\ 0\ 1\}^T$
looks like:

$$C_2 = \begin{pmatrix} 0 & 0 & -1 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Comparing these two we find that there is no difference between the two generated matrices.

2 Question 2

Listing 2: MTRX5700 A1Q1 MATLAB Code

```
close all
clear
clc

DEGREES = pi/180;
RADIANS = 180/pi;

R1 = [0.7500, -0.4330, -0.5000; 0.2165,0.8750, -0.4330; 0.6250, 0.2165, 0.7500]


determinantR1 = det(R1)
inverseR1 = inv(R1)
transposeR1 = transpose(R1)

beta1 = asin(-1*R1(3,1));
gamma1 = asin(R1(3,2)/cos(beta1));
alpha1 = acos(R1(1,1)/cos(beta1));

alpha1 = alpha1*RADIANS
beta1 = beta1*RADIANS
gamma1 = gamma1*RADIANS

R2 = [0.7725, -0.4460, -0.5150; 0.2165, 0.8750, -0.4330; 0.6000, 0.2078, 0.7200]
determinantR2 = det(R2)
inverseR2 = inv(R2)
transposeR2 = transpose(R2)

beta2 = asin(-1*R2(3,1));
gamma2 = asin(R2(3,2)/cos(beta2));
alpha2 = acos(R2(1,1)/cos(beta2));

alpha2 = alpha2*RADIANS
beta2 = beta2*RADIANS
gamma2 = gamma2*RADIANS

R3 = [0, 0, 1; 0.8660, 0.500, 0; -0.500, 0.8660, 0]
determinantR3 = det(R3)
inverseR3 = inv(R3)
transposeR3 = transpose(R3)

beta3 = asin(-1*R3(3,1));
gamma3 = asin(R3(3,2)/cos(beta3));
alpha3 = acos(R3(1,1)/cos(beta3));

alpha3 = alpha3*RADIANS
beta3 = beta3*RADIANS
gamma3 = gamma3*RADIANS

R4 = [-0.7500, -0.2165, -0.6250; 0.4330, -0.8750, -0.2165; 0.500, 0.4330, -0.7500]
determinantR4 = det(R4)
inverseR4 = inv(R4)
transposeR4 = transpose(R4)

beta4 = asin(-1*R4(3,1));
gamma4 = asin(R4(3,2)/cos(beta4));
alpha4 = acos(R4(1,1)/cos(beta4));

alpha4 = alpha4*RADIANS
beta4 = beta4*RADIANS
gamma4 = gamma4*RADIANS
```

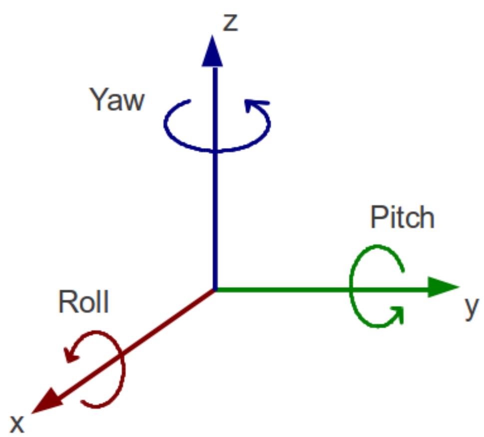

Figure 1: Roll Pitch and Yaw definitions (relative to axis names)

2.a Validity Check

We need to prove that the determinant of the matrix is equal to 1 and that the inverse of the matrix is equal to the transpose to prove validity.

2.a.i R_1

$$R_1 = \begin{pmatrix} 0.7500 & -0.4330 & -0.5000 \\ 0.2165 & 0.8750 & -0.4330 \\ 0.6250 & 0.2165 & 0.7500 \end{pmatrix}$$

$det(R_1) = 1.0000$

$$R_1^{-1} = \begin{pmatrix} 0.7500 & 0.2165 & 0.6250 \\ -0.4330 & 0.8750 & 0.2165 \\ -0.5000 & -0.4330 & 0.7500 \end{pmatrix}$$

$$R_1^T = \begin{pmatrix} 0.7500 & 0.2165 & 0.6250 \\ -0.4330 & 0.8750 & 0.2165 \\ -0.5000 & -0.4330 & 0.7500 \end{pmatrix}$$

$R_1^{-1} = R_1^T \therefore R_1$ is *valid*

2.a.ii R_2

$$R_2 = \begin{pmatrix} 0.7725 & -0.4460 & -0.5150 \\ 0.2165 & 0.8750 & -0.4330 \\ 0.6000 & 0.2078 & 0.7200 \end{pmatrix}$$

$det(R_2) = 0.9888$

$$R_2^{-1} = \begin{pmatrix} 0.7281 & 0.2165 & 0.6510 \\ -0.4204 & 0.8750 & 0.2255 \\ -0.4885 & -0.4330 & 0.7813 \end{pmatrix}$$

$$R_2^T = \begin{pmatrix} 0.7725 & 0.2165 & 0.6000 \\ -0.4460 & 0.8750 & 0.2078 \\ -0.5150 & -0.4330 & 0.7200 \end{pmatrix}$$

$R_2^{-1} \approxeq R_2^T \therefore R_2$ is *valid* within the limits of practical numerical applications.

2.a.iii R_3

$$R_3 = \begin{pmatrix} 0 & 0 & 1 \\ 0.8660 & 0.5000 & 0 \\ -0.5000 & 0.8660 & 0 \end{pmatrix}$$

$det(R_3) = 1$

$$R_3^{-1} = \begin{pmatrix} 0 & 0.8660 & -0.5000 \\ 0 & 0.5000 & 0.8660 \\ 1 & 0 & 0 \end{pmatrix}$$

$$R_3^T = \begin{pmatrix} 0 & 0.8660 & -0.5000 \\ 0 & 0.5000 & 0.8660 \\ 1 & 0 & 0 \end{pmatrix}$$

$R_3^{-1} = R_3^T \therefore R_3$ is *valid*

2.a.iv R_4

$$R_4 = \begin{pmatrix} -0.7500 & -0.2165 & -0.6250 \\ 0.4330 & -0.8750 & -0.2165 \\ 0.5000 & 0.4330 & -0.7500 \end{pmatrix}$$

$det(R_4) = -1$

$$R_4^{-1} = \begin{pmatrix} -0.7500 & 0.4330 & 0.5000 \\ -0.2165 & -0.8750 & 0.4330 \\ -0.6250 & -0.2165 & -0.7500 \end{pmatrix}$$

$$R_4^T = \begin{pmatrix} -0.7500 & 0.4330 & 0.5000 \\ -0.2165 & -0.8750 & 0.4330 \\ -0.6250 & -0.2165 & -0.7500 \end{pmatrix}$$

Although $R_4^{-1} = R_4^T, det(R_4) \neq 1 \therefore R_4$ is *invalid*

2.b Roll/Pitch/Yaw Angles

Roll Angle is α, Pitch Angle is β, Yaw Angle is γ.
Assumption: The believability of the angles can be determined purely mathematically. In reality these angles may not necessarily be believable depending on the application at hand. However, we cannot account for this without further information about the system.

2.b.i R_1

The following values are believable
$\alpha_1 = 16.1021°$
$\beta_1 = -36.6822°$
$\gamma_1 = 16.1016°$

2.b.ii R_2

The following values are believable
$\alpha_2 = 15.0665°$
$\beta_2 = -36.8699°$
$\gamma_2 = 15.0552°$

2.b.iii R_3

The following values are believable
$\alpha_3 = 90°$
$\beta_3 = 30°$
$\gamma_3 = 89.5611°$

2.b.iv R_4

The following values are not believable as R_4 is an invalid rotational matrix
$\alpha_4 = 150°$
$\beta_4 = -30°$
$\gamma_4 = 29.9990°$

2.c Angle Estimation

For a matrix such as R_2 we can adjust our matrix values slightly (making them less accurate i.e. to fewer decimal values) to still get a reasonable estimation of our angles. We can realise this by seeing that the determinant is so close to 1. R_4, however, cannot give us a reasonable estimate for the angles as the determinant is too far from the required value of 1.

3 Question 3

Listing 3: MTRX5700 A1Q1 MATLAB Code

```
close all
clear
clc

DEGREES = pi/180;
RADIANS = 180/pi;

R1 = [0.7500, -0.4330, -0.5000; 0.2165,0.8750, -0.4330; 0.6250, 0.2165, 0.7500]


determinantR1 = det(R1)
inverseR1 = inv(R1)
transposeR1 = transpose(R1)

beta1 = asin(-1*R1(3,1));
gamma1 = asin(R1(3,2)/cos(beta1));
alpha1 = acos(R1(1,1)/cos(beta1));

alpha1 = alpha1*RADIANS
beta1 = beta1*RADIANS
gamma1 = gamma1*RADIANS

R2 = [0.7725, -0.4460, -0.5150; 0.2165, 0.8750, -0.4330; 0.6000, 0.2078, 0.7200]
determinantR2 = det(R2)
inverseR2 = inv(R2)
transposeR2 = transpose(R2)

beta2 = asin(-1*R2(3,1));
gamma2 = asin(R2(3,2)/cos(beta2));
alpha2 = acos(R2(1,1)/cos(beta2));

alpha2 = alpha2*RADIANS
beta2 = beta2*RADIANS
gamma2 = gamma2*RADIANS

R3 = [0, 0, 1; 0.8660, 0.500, 0; -0.500, 0.8660, 0]
determinantR3 = det(R3)
inverseR3 = inv(R3)
transposeR3 = transpose(R3)

beta3 = asin(-1*R3(3,1));
gamma3 = asin(R3(3,2)/cos(beta3));
alpha3 = acos(R3(1,1)/cos(beta3));

alpha3 = alpha3*RADIANS
beta3 = beta3*RADIANS
gamma3 = gamma3*RADIANS

R4 = [-0.7500, -0.2165, -0.6250; 0.4330, -0.8750, -0.2165; 0.500, 0.4330, -0.7500]
determinantR4 = det(R4)
inverseR4 = inv(R4)
transposeR4 = transpose(R4)

beta4 = asin(-1*R4(3,1));
gamma4 = asin(R4(3,2)/cos(beta4));
alpha4 = acos(R4(1,1)/cos(beta4));

alpha4 = alpha4*RADIANS
beta4 = beta4*RADIANS
gamma4 = gamma4*RADIANS
```

DH Notation

i	θ	**d**	**r**	α
1	θ_1	67	100	$\pi/2$
2	θ_2	0	250	0
3	θ_2	0	0	$\pi/2$
4	θ_4	0	250	$-\pi/2$
5	θ_5	0	0	$-\pi/2$
6	θ_6	0	245	0

Table 1: D-H Variable Notation

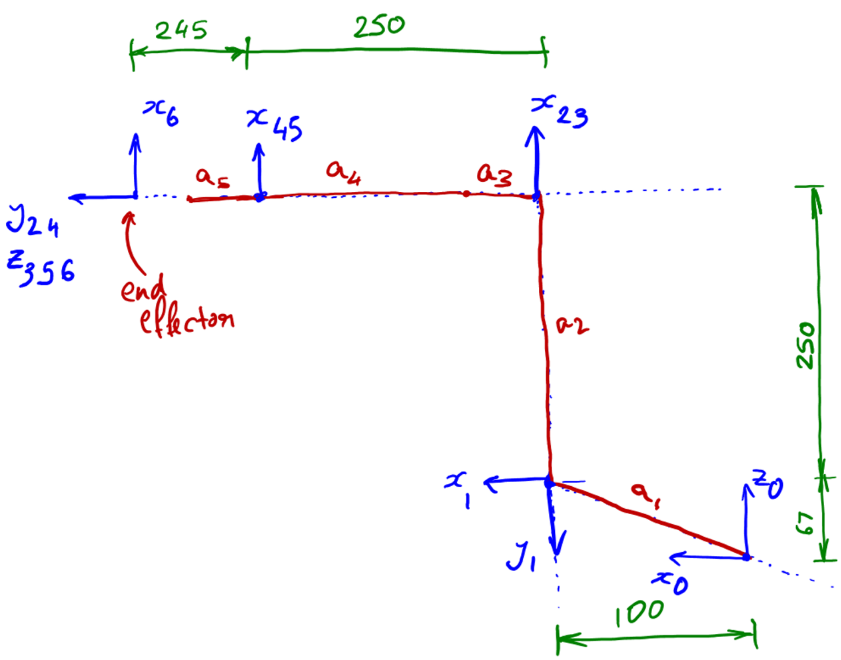

Figure 2: D-H Standard for Forward Kinematics Coordinate Systems

Resulting Transformation Matrix

The resulting transformation matrix can be represented as:

$$
{}^0T_6 = \begin{pmatrix} n_x & s_x & a_x & p_x \\ n_y & s_y & a_y & p_y \\ n_z & s_z & a_z & p_z \\ 0 & 0 & 0 & 1 \end{pmatrix} = {}^0A_1 {}^1A_2 {}^2A_3 {}^3A_4 {}^4A_5 {}^5A_6
$$

$\mathbf{n}_x =- \sin\theta_6(\cos\theta_4 \sin\theta_1 - \sin\theta_4(\cos\theta_1 \sin\theta_2 \sin\theta_3 - \cos\theta_1 \cos\theta_2 \cos\theta_3)) - \cos\theta_6(\cos\theta_5(\sin\theta_1 \sin\theta_4 + \cos\theta_4(\cos\theta_1 \sin\theta_2 \sin\theta_3 - \cos\theta_1 \cos\theta_2 \cos\theta_3)) + \sin\theta_5(\cos\theta_1 \cos\theta_2 \sin\theta_3 + \cos\theta_1 \cos\theta_3 \sin\theta_2))$

$\mathbf{s}_x =\sin\theta_6(\cos\theta_5(\sin\theta_1 \sin\theta_4 + \cos\theta_4(\cos\theta_1 \sin\theta_2 \sin\theta_3 - \cos\theta_1 \cos\theta_2 \cos\theta_3)) + \sin\theta_5(\cos\theta_1 \cos\theta_2 \sin\theta_3 + \cos\theta_1 \cos\theta_3 \sin\theta_2)) - \cos\theta_6(\cos\theta_4 \sin\theta_1 - \sin\theta_4(\cos\theta_1 \sin\theta_2 \sin\theta_3 - \cos\theta_1 \cos\theta_2 \cos\theta_3))$

$\mathbf{a}_x =\sin\theta_5(\sin\theta_1 \sin\theta_4 + \cos\theta_4(\cos\theta_1 \sin\theta_2 \sin\theta_3 - \cos\theta_1 \cos\theta_2 \cos\theta_3)) - \cos\theta_5(\cos\theta_1 \cos\theta_2 \sin\theta_3 + \cos\theta_1 \cos\theta_3 \sin\theta_2)$

$\mathbf{p}_x =100\cos\theta_1 + 250\cos\theta_1 \cos\theta_2 - 250\sin\theta_1 \sin\theta_4 - 250\sin\theta_6(\cos\theta_4 \sin\theta_1 - \sin\theta_4(\cos\theta_1 \sin\theta_2 \sin\theta_3 - \cos\theta_1 \cos\theta_2 \cos\theta_3)) - 250\cos\theta_6(\cos\theta_5(\sin\theta_1 \sin\theta_4 + \cos\theta_4(\cos\theta_1 \sin\theta_2 \sin\theta_3 - \cos\theta_1 \cos\theta_2 \cos\theta_3)) + \sin\theta_5(\cos\theta_1 \cos\theta_2 \sin\theta_3 + \cos\theta_1 \cos\theta_3 \sin\theta_2)) - 250\cos\theta_4(\cos\theta_1 \sin\theta_2 \sin\theta_3 - \cos\theta_1 \cos\theta_2 \cos\theta_3)$

$\mathbf{n}_y =\sin\theta_6(\cos\theta_1 \cos\theta_4 + \sin\theta_4(\sin\theta_1 \sin\theta_2 \sin\theta_3 - \cos\theta_2 \cos\theta_3 \sin\theta_1)) + \cos\theta_6(\cos\theta_5(\cos\theta_1 \sin\theta_4 - \cos\theta_4(\sin\theta_1 \sin\theta_2 \sin\theta_3 - \cos\theta_2 \cos\theta_3 \sin\theta_1)) - \sin\theta_5(\cos\theta_2 \sin\theta_1 \sin\theta_3 + \cos\theta_3 \sin\theta_1 \sin\theta_2))$

$\mathbf{s}_y =\cos\theta_6(\cos\theta_1 \cos\theta_4 + \sin\theta_4(\sin\theta_1 \sin\theta_2 \sin\theta_3 - \cos\theta_2 \cos\theta_3 \sin\theta_1)) - \sin\theta_6(\cos\theta_5(\cos\theta_1 \sin\theta_4 - \cos\theta_4(\sin\theta_1 \sin\theta_2 \sin\theta_3 - \cos\theta_2 \cos\theta_3 \sin\theta_1)) - \sin\theta_5(\cos\theta_2 \sin\theta_1 \sin\theta_3 + \cos\theta_3 \sin\theta_1 \sin\theta_2))$

$\mathbf{a}_y =- \sin\theta_5(\cos\theta_1 \sin\theta_4 - \cos\theta_4(\sin\theta_1 \sin\theta_2 \sin\theta_3 - \cos\theta_2 \cos\theta_3 \sin\theta_1)) - \cos\theta_5(\cos\theta_2 \sin\theta_1 \sin\theta_3 + \cos\theta_3 \sin\theta_1 \sin\theta_2)$

$\mathbf{p}_y =100\sin\theta_1 + 250\cos\theta_2 \sin\theta_1 + 250\cos\theta_1 \sin\theta_4 + 250\sin\theta_6(\cos\theta_1 \cos\theta_4 + \sin\theta_4(\sin\theta_1 \sin\theta_2 \sin\theta_3 - \cos\theta_2 \cos\theta_3 \sin\theta_1)) - 250\cos\theta_4(\sin\theta_1 \sin\theta_2 \sin\theta_3 - \cos\theta_2 \cos\theta_3 \sin\theta_1) + 250\cos\theta_6(\cos\theta_5(\cos\theta_1 \sin\theta_4 - \cos\theta_4(\sin\theta_1 \sin\theta_2 \sin\theta_3 - \cos\theta_2 \cos\theta_3 \sin\theta_1)) - \sin\theta_5(\cos\theta_2 \sin\theta_1 \sin\theta_3 + \cos\theta_3 \sin\theta_1 \sin\theta_2))$

$\mathbf{n}_z =\cos\theta_6(\sin\theta_5(\cos\theta_2 \cos\theta_3 - \sin\theta_2 \sin\theta_3) + \cos\theta_4 \cos\theta_5(\cos\theta_2 \sin\theta_3 + \cos\theta_3 \sin\theta_2)) - \sin\theta_4 \sin\theta_6(\cos\theta_2 \sin\theta_3 + \cos\theta_3 \sin\theta_2)$

$\mathbf{s}_z =- \sin\theta_6(\sin\theta_5(\cos\theta_2 \cos\theta_3 - \sin\theta_2 \sin\theta_3) + \cos\theta_4 \cos\theta_5(\cos\theta_2 \sin\theta_3 + \cos\theta_3 \sin\theta_2)) - \cos\theta_6 \sin\theta_4(\cos\theta_2 \sin\theta_3 + \cos\theta_3 \sin\theta_2)$

$\mathbf{a}_z =\cos\theta_5(\cos\theta_2 \cos\theta_3 - \sin\theta_2 \sin\theta_3) - \cos\theta_4 \sin\theta_5(\cos\theta_2 \sin\theta_3 + \cos\theta_3 \sin\theta_2)$

$\mathbf{p}_z =250\sin\theta_2 + 250\cos\theta_6(\sin\theta_5(\cos\theta_2 \cos\theta_3 - \sin\theta_2 \sin\theta_3) + \cos\theta_4 \cos\theta_5(\cos\theta_2 \sin\theta_3 + \cos\theta_3 \sin\theta_2)) + 250\cos\theta_4(\cos\theta_2 \sin\theta_3 + \cos\theta_3 \sin\theta_2) - 250\sin\theta_4 \sin\theta_6(\cos\theta_2 \sin\theta_3 + \cos\theta_3 \sin\theta_2) + 67$

4 Question 4

4.a

4.b

4.c

5 Question 5

5.a

5.b

5.c

6 Question 6

6.a

6.b

6.c

7 Question 7

7.a

7.b

7.c

8 Question 8

8.a

8.b

8.c