NCERT Question 10.5.2.5

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Question 10.5.2.5 : Find the number of terms in each of the following APs. Then express each term as x(n) and find the z transform and its ROC:
(*i*) 7, 13, 19, ... 205
(*ii*) 18, $15\frac{1}{2}$, 13, ... -47

Solution :

Parameter	Used to denote	Values
$x_i(n)$	n^{th} term of i^{th} series ($i = (1,2)$)	$x_i(0) + nd_i$
$x_i(0)$	First term of i^{th} AP	$x_1(0) = 7$ $x_2(0) = 18$
d_i	Commmon difference of i^{th} AP	$d_1 = 6$ $d_2 = -2\frac{1}{2}$

TABLE 0
Parameter Table

The number of terms in the AP x(n) is given by :

$$\frac{x(n) - x(0)}{d} + 1 \tag{1}$$

(*i*)

$x_1(n) = 205$

Using (1), Number of terms in the AP is :

$$\frac{205 - 7}{6} + 1 = 34 \tag{2}$$

$$\tag{3}$$

$\therefore$ There are 34 elements in the series.
(*ii*)

$x_2(n) = -47$

Using (1), Number of terms in the AP is :

$$\frac{-47 - 18}{-2\frac{1}{2}} + 1 = 27 \tag{4}$$

$\therefore$ There are 27 elements in the series.
Finding the z transform and ROC :

$$\text{For an AP } x(n) = [x(0) + nd]\,u(n) \tag{5}$$

$$= x(0)\,u(n) + dnu(n) \tag{6}$$

$$z\{u(n)\} = U(z) = \frac{1}{1 - z^{-1}}, |z| > 1 \tag{7}$$

$$z\{nu(n)\} = -z\frac{dU(z)}{dz} = \frac{z^{-1}}{(1 - z^{-1})^2}, |z| > 1 \tag{8}$$

ROC is given by values of z for which :

$$|X(z)| = \sum_{n=-\infty}^{\infty} |x(n)\,z^{-n}| < \infty \tag{9}$$

Using equations (7) and (8) in equation (6) :

$$z\{x(n)\} = X(z) = \frac{x(0)}{1 - z^{-1}} + d\frac{z^{-1}}{(1 - z^{-1})^2} \tag{10}$$

(*i*)

$x_1(n) = (7 + (n)\,6)\,u(n)$

$$x_1(n) = \begin{cases} 0 & \text{for } n < 0 \\ 7 + (n)\,6 & \text{for } n \geq 0 \end{cases} \tag{11}$$

Using the values in Table 0 and equation (10) :

$$X_1(z) = \frac{7}{1 - z^{-1}} + \frac{6z^{-1}}{(1 - z^{-1})^2} \tag{12}$$

$$= \frac{7 - z^{-1}}{(1 - z^{-1})^2} \tag{13}$$

The ROC is $|z| > 1$ as it is an AP.
(*ii*)

$x_2(n) = \left(18 + n\left(-2\frac{1}{2}\right)\right)u(n)$

$$x_2(n) = \begin{cases} 0 & \text{for } n < 0 \\ 18 + n\left(-2\frac{1}{2}\right) & \text{for } n \geq 0 \end{cases} \tag{14}$$

Using the values in Table 0 and equation (10) :

$$X_2(z) = \frac{18}{1 - z^{-1}} - \left(2\frac{1}{2}\right)\frac{z^{-1}}{(1 - z^{-1})^2} \tag{15}$$

$$= \frac{18 - \left(20\frac{1}{2}\right)z^{-1}}{(1 - z^{-1})^2} \tag{16}$$

The ROC is $|z| > 1$ as it is an AP.

The graphs for x(n) :

(i) The graph of $x_1(n)$ is :

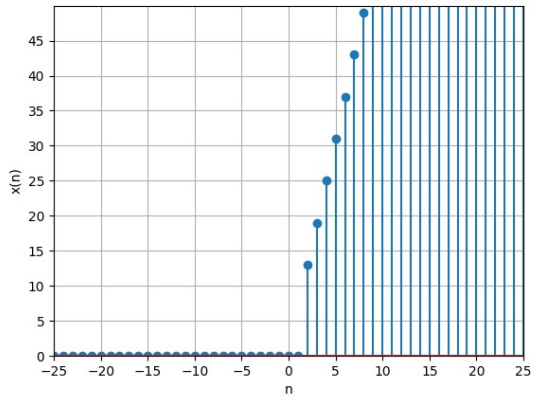

Fig. 0. Plot of x(n)

(ii) The graph of $x_2(n)$ is :

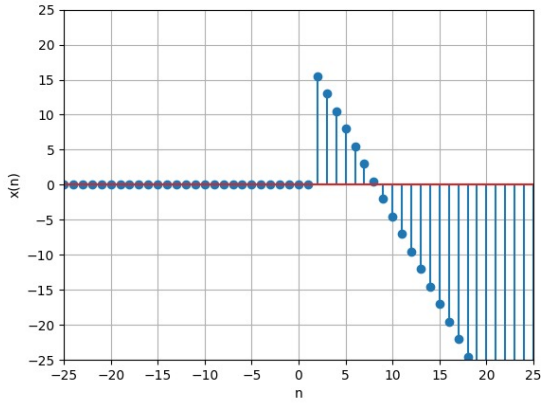

Fig. 1. Plot of x(n)