NCERT Question 10.5.2.5

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Question 10.5.2.5 : Find the number of terms in each of the following APs. Then express each term as x(n) and find the z transform and its ROC:
(i) 7, 13, 19, ... 205
(ii) 18, $15\frac{1}{2}$, 13, ... -47

Solution : (i)

Parameter	Used to denote	Values
$x_i(n)$	n^{th} term of i^{th} series $(i = (1, 2))$	$x_i(0) + nd_i$
$x_1(0)$	First term of 1^{st} AP	7
$x_2(0)$	First term of 2^{nd} AP	18
d_1	Common difference of 1^{st} AP	6
d_2	Common difference of 2^{nd}	$-2\frac{1}{2}$

TABLE 0
Parameter Table

$x_1(n) = x_1(0) + nd_1$
If 205 is the nth term of the series, we have :

$$205 = 7 + (n)6 \tag{1}$$
$$\implies 198 = 6n \tag{2}$$
$$\implies 33 = n \tag{3}$$

$\therefore$ There are 34 elements in the series.

(ii)
$x_2(0) = x_2(0) + nd_2$
If -47 is the n^{th} term of the series, we have :

$$-47 = 18 + (n)\left(-2\left(\frac{1}{2}\right)\right) \tag{4}$$
$$\implies -65 = (n)\left(-2\left(\frac{1}{2}\right)\right) \tag{5}$$
$$\implies n = 26 \tag{6}$$

$\therefore$ There are 27 elements in the series.

Finding the z transform :
(i)

$x_1(n) = (7 + (n)6)u(n)$

$$x_1(n) = \begin{cases} 0 & \text{for } n < 0 \\ 7 + (n)6 & \text{for } n \geq 0 \end{cases} \tag{7}$$

$$X_1(z) = \sum_{n=-\infty}^{\infty} x_1(n)u(n)z^{-n} \tag{8}$$
$$= \sum_{n=0}^{\infty} (x_1(0) + nd_1)z^{-n} \tag{9}$$
$$= \left[\sum_{n=0}^{\infty} x_1(0)z^{-n}\right] + \left[\sum_{n=0}^{\infty} nd_1 z^{-n}\right] \tag{10}$$
$$\text{Using } U(z) = \frac{1}{1 - z^{-1}} \tag{11}$$
$$\text{The z transform of nu}(n) \text{ is -z}\frac{dU(z)}{dz} \tag{12}$$
$$\therefore \sum_{n=0}^{\infty} nz^{-n} = \frac{z^{-1}}{(1 - z^{-1})^2} \tag{13}$$

Using equations (10) and (13) :

$$X_1(z) = \frac{7}{1 - z^{-1}} + \frac{6z^{-1}}{(1 - z^{-1})^2} \tag{14}$$
$$= \frac{7 - z^{-1}}{(1 - z^{-1})^2} \tag{15}$$

(ii)

$$x_2(n) = \left(18 + n\left(-2\frac{1}{2}\right)\right)u(n) \tag{16}$$

$$x_2(n) = \begin{cases} 0 & \text{for } n < 0 \\ 18 + n\left(-2\frac{1}{2}\right) & \text{for } n \geq 0 \end{cases} \tag{17}$$

$$X(z) = \sum_{n=0}^{\infty} x_2(n)u(n)z^{-n} \tag{18}$$
$$= \sum_{n=0}^{\infty} (x_2(0) + nd_2)z^{-n} \tag{19}$$
$$= x_2(0)\sum_{n=0}^{\infty} z^{-n} + d_2\sum_{n=0}^{\infty} nz^{-n} \tag{20}$$

Using equation (13) and U(z),

$$X(z) = \frac{18}{1 - z^{-1}} - \left(2\frac{1}{2}\right)\frac{z^{-1}}{(1 - z^{-1})^2} \tag{21}$$

$$= \frac{18 - \left(20\frac{1}{2}\right)z^{-1}}{(1 - z^{-1})^2} \tag{22}$$

The graph of x(n) and the ROC of X(z) :
(i) The graph of $x_1(n)$ is :

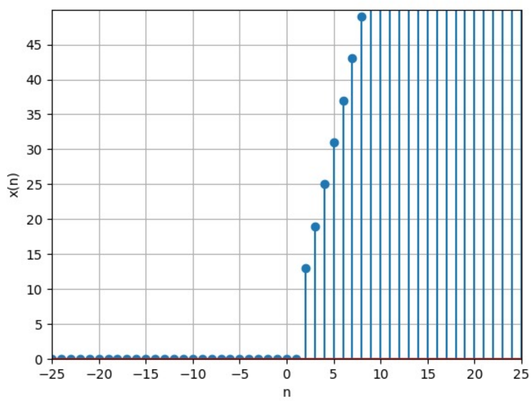

Fig. 0. Plot of x(n)

$$|X(z)| = \sum_{n=-\infty}^{\infty} |x(n) z^{-n}| < \infty \tag{23}$$

By equation (10) :
For an infinite GP with ratio z^{-1} to converge, we must have $|z^{-1}| < 1$ .Hence, for $X_1(z)$ to converge, we must have $|z^{-1}| < 1$ or $|z| > 1$.

(ii) The graph of $x_2(n)$ is :

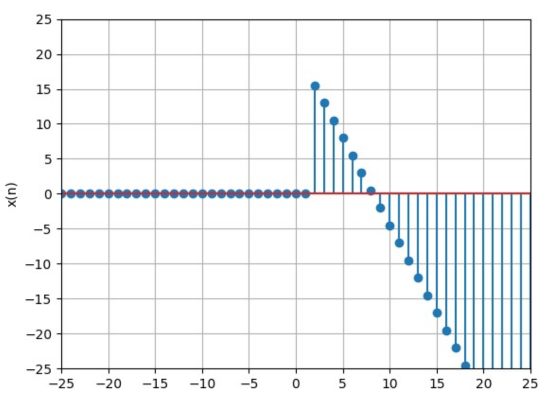

Fig. 1. Plot of x(n)

By equation (20) and (23) :
For an infinite GP with ratio z^{-1} to converge, we must have $|z^{-1}| < 1$ .Hence, for $X_1(z)$ to converge, we must have $|z^{-1}| < 1$ or $|z| > 1$.