

NCERT Question 11.9.3.15

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Question 11.9.3.15 : Given a GP with $x(0) = 729$ and 7^{th} term 64, determine $s(6)$

Solution:

Parameter	Description	Value
$x(0)$	First Term	729
r	Common Ratio	
$x(n)$	$(n+1)^{th}$ Term	$x(0) r^n u(n)$
$x(6)$	7^{th} Term	64
$s(k)$	Sum of first $(k+1)$ terms	

TABLE 0
PARAMETER TABLE

$$X(z) = \frac{x(0)}{1 - rz^{-1}} \quad (1)$$

$$\text{ROC is } |z| > |r| \text{ as it is a GP.} \quad (2)$$

Sum to n terms of GP can be given as :

$$s(n) = x(n) * u(n) \quad (3)$$

$$\Rightarrow S(z) = X(z) U(z) \quad (4)$$

from Table 0 :

$$x(6) = x(0) r^6 \quad (5)$$

$$\Rightarrow 64 = 729 r^6 \quad (6)$$

$$\therefore r = \frac{2}{3} \quad (7)$$

using Table 0 and equation (1)

$$X(z) = \frac{729}{1 - \frac{2}{3}z^{-1}} \quad (8)$$

$$S(z) = \frac{729}{\left(1 - \frac{2}{3}z^{-1}\right)(1 - z^{-1})} \quad (9)$$

$$= 2187 \left(\frac{1}{1 - z^{-1}} - \frac{\frac{2}{3}}{1 - \frac{2}{3}z^{-1}} \right) \quad (10)$$

Using contour integration for inverse z transform,

$$s(6) = \frac{1}{2\pi j} \int S(z) z^5 dz \quad (11)$$

$$= \frac{1}{2\pi j} \left(\int \frac{2187z^6}{z-1} dz + \int \frac{1458z^6}{z - \frac{2}{3}} dz \right) \quad (12)$$

Solution of each of these integrals can be given by :

$$I = \frac{1}{(m-1)!} \lim_{z \rightarrow a} \frac{d^{m-1}}{dz^{m-1}} ((z-a)^m f(z)) \quad (13)$$

using equations (12) and (13):

$$\frac{1}{2\pi j} \left(\int \frac{2187z^6}{z-1} dz \right) = 2187 \quad (14)$$

$$\frac{1}{2\pi j} \left(\int \frac{1458z^6}{z - \frac{2}{3}} dz \right) = 128 \quad (15)$$

using equations (12), (14), (15):

$$s(6) = 2187 - 128 \quad (16)$$

$$= 2059 \quad (17)$$

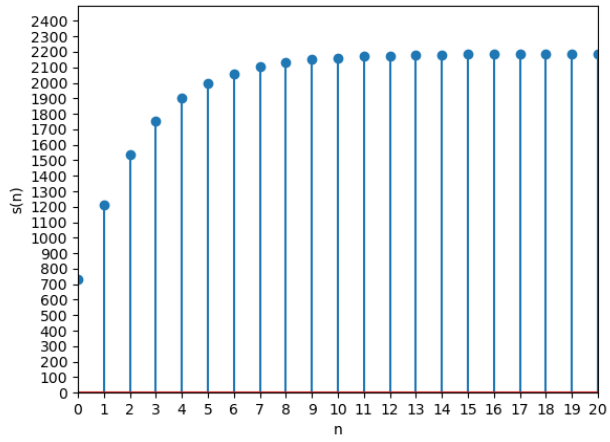


Fig. 0. Plot of $s(n)$

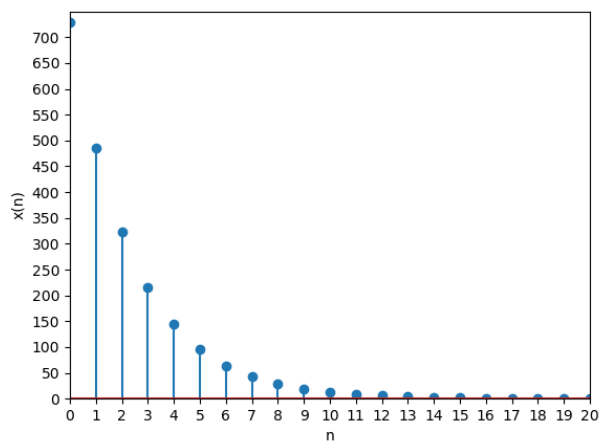


Fig. 0. Plot of $x(n)$