GATE ME 30

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Question GATE ME 30 :
The figure shows a block of mass m = 20 kg attached to a pair of identical linear springs, each having a spring constant k = 1000 N/m. The block oscillates on a frictionless horizontal surface. Assuming free vibrations, the time taken by the block to complete ten oscillations is ____ seconds . (Rounded off to two decimal places) Take $\pi = 3.14$.
(GATE ME 2023)

Solution:

Derivation for natural frequency ω_n:

Parameter	Description	Value
k	spring constant	1000 N/m
m	mass of block	20Kg
k_{eq}	Equivalent spring constant	$k_1 + k_2$ (parallel)
ω_n	Natural frequency	$\sqrt{\frac{k_{eq}}{m}}$
T	Time period of an oscillation	$\frac{2\pi}{\omega_n}$
x	Displacement of block	
a	Acceleration of block	
F	Force on block	

TABLE 0
PARAMETER TABLE

$$F = ma \tag{1}$$
$$F = -kx \text{ using Hooke's Law} \tag{2}$$
$$\implies ma = -kx \tag{3}$$
$$\therefore m\frac{d^2x}{dt^2} = -kx \tag{4}$$

The derivative of x has x in it's equation. So we can assume x to be of the form :

$$x = Ce^{\alpha t} \tag{5}$$
$$\text{Let } \frac{d^2x}{dt^2} = -\frac{k}{m}x = -\omega^2 x \tag{6}$$

Using equations (5) and (6),

$$C\alpha^2 e^{\alpha t} = -\omega^2 x \tag{7}$$
$$\implies \alpha^2 = -\omega^2 \tag{8}$$
$$\therefore \alpha = \pm\iota\omega \tag{9}$$

Using equations (5) and (9), there are two solutions x_1 and x_2 for x ,

$$x_1 = Ce^{-\iota\omega t} \tag{10}$$
$$x_2 = Ce^{\iota\omega t} \tag{11}$$

The value of x is real. Hence the general solution can be written as the linear combination of x_1 and x_2

$$x(t) = C_1 e^{\iota\omega t} + C_2 e^{-\iota\omega t} \tag{12}$$

as x always has a real value,

$$\therefore C_1 - C_2 = 0 \text{ to cancel imaginary term} \tag{13}$$
$$\implies x = (A)\cos(\omega t) \text{ A is a constant} \tag{14}$$

Observe in equation (14), the time period is $\frac{2\pi}{\omega}$

$\therefore\ \omega$ is the natural frequency of the system.
From equation (6)

$$\omega = \sqrt{\frac{k}{m}} \tag{15}$$

using table Table 0 ,

$$k_{eq} = 2000 \tag{16}$$
$$\omega_n = 10 rad/s \tag{17}$$

The time required to complete 10 oscillations using (16) and (17) is

$$10T = 10\frac{2\pi}{\omega_n} \tag{18}$$
$$= 2\pi \tag{19}$$
$$= 6.28 \tag{20}$$