NCERT Question 10.5.2.5

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Question 10.5.2.5 : Find the number of terms in each of the following APs. Then express each term as x(n) and find the z transform and its ROC:
(*i*) 7, 13, 19, ... 205
(*ii*) 18, $15\frac{1}{2}$, 13, ... -47

Solution :

Parameter	Used to denote	Values
$x_i(n)$	n^{th} term of i^{th} series ($i = (1,2)$)	$x_i(0) + nd_i$
$x_i(0)$	First term of i^{th} AP	$x_1(0) = 7$ $x_2(0) = 18$
d_i	Commmon difference of i^{th} AP	$d_1 = 6$ $d_2 = -2\frac{1}{2}$

TABLE 0
Parameter Table

The number of terms in the AP x(n) is given by:

$$\frac{x(n) - x(0)}{d} \tag{1}$$

$$X_i(z) = \frac{x_i(0)}{1 - z^{-1}} + d_i \frac{z^{-1}}{(1 - z^{-1})^2} \text{ , for i=1,2} \tag{2}$$

(*i*)

$$x_1(n) = (7 + (n)\,6)\,u(n) \tag{3}$$

Using the values in Table 0 and equation (1),

$$k_1 = \frac{205 - 7}{6} = 33 \tag{4}$$

Using the values in Table 0 and equation (7) :

$$X_1(z) = \frac{7 - z^{-1}}{(1 - z^{-1})^2} \tag{5}$$

The ROC is $|z| > 1$ as it is an AP.
(*ii*)

$$x_2(n) = \left(18 + n\left(-2\frac{1}{2}\right)\right)u(n) \tag{6}$$

Using the values in Table 0 and equation (1),

$$k_2 = \frac{-47 - 18}{-2\frac{1}{2}} = 26 \tag{7}$$

Using the values in Table 0 and equation (7) :

$$X_2(z) = \frac{18 - \left(20\frac{1}{2}\right)z^{-1}}{(1 - z^{-1})^2} \tag{8}$$

The ROC is $|z| > 1$ as it is an AP.

The graph of $x_1(n)$ is :

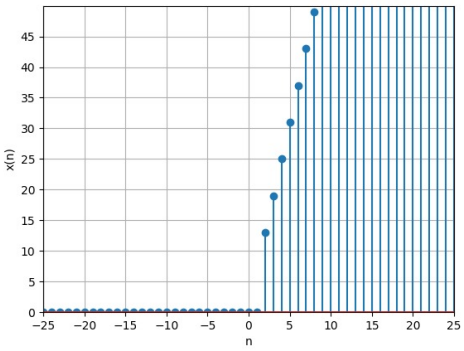

Fig. 0. Plot of $x_1(n)$

The graph of $x_2(n)$ is :

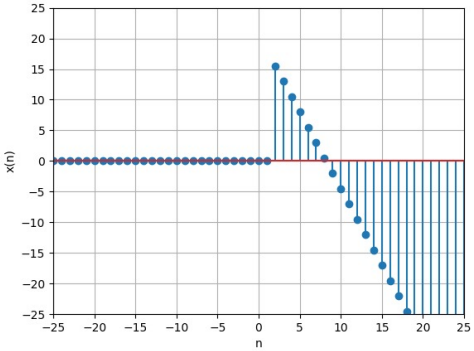

Fig. 1. Plot of $x_2(n)$