GATE 2021 EC 5

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Question GATE 21 EC 5 :
Consider two 16-point sequences $x[n]$ and $h[n]$. Let the linear convolution of $x[n]$ and $h[n]$ be denoted by $y[n]$, while $z[n]$ denotes the 16-point inverse discrete Fourier transform ($IDFT$) of the product of the 16-point DFTs of $x(n)$ and $h[n]$. The values of k for which $z[k] = y[k]$ are

1) $k = 0, 1, 2, 3, ..., 15$
2) $k = 0$
3) $k = 15$
4) $k = 0$ and $k = 15$

(GATE EC 2021)

Solution:

We can write $z[n]$ as ,

Parameter	Description
$x[n]$	Given 16 point sequence
$h[n]$	Given 16 point sequence
$y[n]$	Linear convolution of $x[n]$ and $h[n]$
$z[n]$	IDFT of products of DFTs of $h[n]$ and $x[n]$
N	number of terms (16)

TABLE 4
PARAMETER TABLE

$$z[n] = IDFT[X[f]H[f]] \tag{1}$$

We know that product in frequency domain is convolution in time domain. However, $z[n]$ is not linear convolution of $x[n]$ and $h[n]$ in the time domain due to periodicity of DFT. Point-wise multiplication in the frequency domain (product of DFTs) doesn't translate directly to convolution in the time domain due to periodicity and potential aliasing.
$z[n]$ is the circular convolution of $x[n]$ and $h[n]$. Using the formula for circular convolution,

$$z[n] = \sum_{m=0}^{N-1} x[n]\, h[(n-m)\, mod\,(N)] \tag{2}$$

$y[n]$ is a linear convolution of $x[n]$ and $h[n]$.

$$y[n] = \sum_{m=-\infty}^{\infty} x(m)\, h[n-m] \tag{3}$$

Each term of $y[n]$ will be sum of products of terms of $x[n]$ and $h[n]$. The number of terms in each summation will go from 1 for $n = 1$ to 16 for $n = 15$.
$z[n]$ is expressed as sum of 15 terms for all permissible values of n.
Thus,$z[k] = y[k]$ can be possible for only $k = 15$.
For $k = 15$,

$$y[15] = x[0]h[15] + x[1]h[14] + ...x[15]h[0] \tag{4}$$

$$z[15] = x[0]h[(15)\, mod\,(16)] + x[1]h[(14)\, mod\,(16) \tag{5}$$

$$= x[0]h[15] + x[1]h[14] + ...x[15]h[0] \tag{6}$$

Thus , $z[15] = y[15]$.
Graphically, let

$$x = [1, 2, 3, 4, ..., 16] \tag{7}$$

$$h = [1, 1, 1, 1, ...1] \tag{8}$$

Then the plot for z and y is as shown bellow.

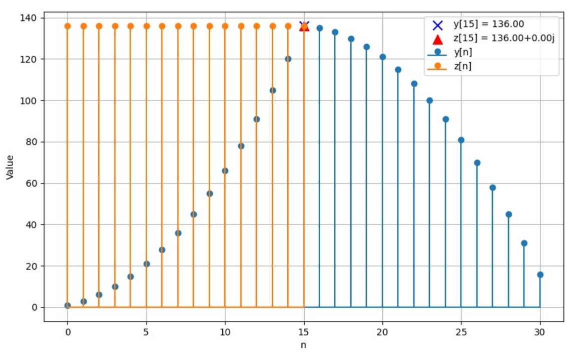

Fig. 4. Plot of y $[n]$ and z $[n]$