NCERT Question 10.5.2.5

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Question 10.5.2.5 : Find the number of terms in each of the following APs :

(i) 7, 13, 19, ... 205

(ii) 18, $15\frac{1}{2}$, 13, ... -47

Solution :

(i) The n^{th} term of the Arithmetic progression is given as a + (n-1)*d where a is the first term and d is the common difference.

The common difference of the AP is given by the difference between successive terms.

Common difference (d) = 13 - 7 = 6
First term (a) = 7

If 205 is the *nth* term of the series, we have :

$$205 = 7 + (n-1) * 6$$
$$\implies 198 = (n-1) * 6$$
$$\implies 33 = n - 1$$
$$\implies n = 34$$

Answer : There are 34 elements in the series.

(ii) The n^{th} term of the Arithmetic progression is given as a + (n-1)*d where a is the first term and d is the common difference.

The common difference of the AP is given by the difference between successive terms.

Common difference (d) = $15\frac{1}{2}$ - 18 = -$2\frac{1}{2}$
First term (a) = 18

If -47 is the *nth* term of the series, we have :

$$-47 = 18 + (n-1) * \left(-2\left(\frac{1}{2}\right)\right)$$
$$\implies -65 = (n-1) * \left(-2\left(\frac{1}{2}\right)\right)$$
$$\implies 26 = n - 1$$
$$\implies n = 27$$

Answer : There are 27 elements in the series.

Question : Express the n^{th} term in each case as x(n) and find its z transform.

Solution :

(i) The n^{th} term of the Arithmetic progression (T_n) is given as a + (n-1)*d where a is the first term and d is the common difference.

$\therefore T_n = 7 + (n-1) * 6$
$x(n) = 7 + (n-1) * 6$

The Z transformation for x[n] is given by :

$$X(z) = \left[\sum_{n=-\infty}^{\infty} x[n]z^{-n}\right]$$

However, x[n] cannot be summed from $-\infty$ to zero as the number of terms cannot be negative due to which the n^{th} will not be defined for this range of n.
So, we will modify x[n] by multiplying it with unit step function(u(t)) so we have the value as zero for n¡1.

$$\begin{aligned} X(z) &= \sum_{n=-\infty}^{\infty} x[n]u(n)z^{-n} && (1)\\ &= \sum_{n=1}^{\infty} x[n]z^{-n} \\ &= \sum_{n=1}^{\infty}(a + (n-1) * d)z^{-n} \\ &= \left[\sum_{n=1}^{\infty} a * z^{-n}\right] + \left[\sum_{n=1}^{\infty}(nd) * z^{-n}\right] - \left[\sum_{n=1}^{\infty} d * z^{-n}\right] \\ &= \left[\sum_{n=1}^{\infty} 7 * z^{-n}\right] + \left[\sum_{n=1}^{\infty}(6n) * z^{-n}\right] - \left[\sum_{n=1}^{\infty} 6 * z^{-n}\right] \\ &= \left[\sum_{n=1}^{\infty} z^{-n}\right] + \left[\sum_{n=1}^{\infty}(6n) * z^{-n}\right] \\ &= \frac{1}{z-1} + 6 * \left[\sum_{n=1}^{\infty}(n) * z^{-n}\right] && (2) \end{aligned}$$

Calculating the integral in the above expression :

$$S = \left[\sum_{n=1}^{\infty}(n) * z^{-n}\right]$$
$$S = z^{-1} + 2z^{-2} + 3z^{-3} + ... \tag{3}$$
$$S * z^{-1} = z^{-2} + 2z^{-3} + 3z{-}4 + ... \tag{4}$$

Subtracting (3) and (4)

$$\begin{aligned} S(1 - z^{-1}) &= z^{-1} + z^{-2} + z^{-3} + ... \\ &= \frac{1}{(z-1)*(1-z^{-1})} \\ &= \frac{1}{z - 2 + z^{-1}} \\ \therefore S &= \frac{z}{(z-1)^2} \end{aligned} \tag{5}$$

Using equations 2 and 5 :

$$\begin{aligned} X(z) &= \frac{1}{z-1} + \frac{6z}{(z-1)^2} \\ &= \frac{7z-1}{(z-1)^2} \end{aligned}$$

Answer : The z transformation for x(n) where x(n) is the n^{th} term of the AP is $\frac{7z-1}{(z-1)^2}$.

(ii) The n^{th} term of the Arithmetic progression (T_n) is given as a + (n-1)*d where a is the first term and d is the common difference.

$\therefore T_n = 18 + (n-1) * \left(-2\frac{1}{2}\right)$

$x(n) = 18 + (n-1) * \left(-2\frac{1}{2}\right)$

The Z transformation for x[n] is given by :

$$X(z) = \left[\sum_{n=-\infty}^{\infty} x[n]z^{-n}\right]$$

However, x[n] cannot be summed from $-\infty$ to zero as the number of terms cannot be negative due to which the n^{th} will not be defined for this range of n.
So, we will modify x[n] by multiplying it with unit step function(u(t)) so we have the value as zero for n¡1.

$$\begin{aligned} X(z) &= \sum_{n=-\infty}^{\infty} x[n]u(n)z^{-n} \qquad (6) \\ &= \sum_{n=1}^{\infty} x[n]z^{-n} \\ &= \sum_{n=1}^{\infty}(a + (n-1)*d)z^{-n} \\ &= \left[\sum_{n=1}^{\infty} a * z^{-n}\right] + \left[\sum_{n=1}^{\infty}(nd) * z^{-n}\right] - \left[\sum_{n=1}^{\infty} d * z^{-n}\right] \\ &= \left[\sum_{n=1}^{\infty} 18z^{-n}\right] + \left[\sum_{n=1}^{\infty}(n)\left(-2\frac{1}{2}\right)z^{-n}\right] - \left[\sum_{n=1}^{\infty}\left(-2\frac{1}{2}\right)z^{-n}\right] \\ &= \left[\sum_{n=1}^{\infty}\left(20\frac{1}{2}\right)z^{-n}\right] - \left[\sum_{n=1}^{\infty}\left(n\left(2\frac{1}{2}\right)\right) * z^{-n}\right] \\ &= \left(20\frac{1}{2}\right)\left[\sum_{n=1}^{\infty} z^{-n}\right] - \left[\sum_{n=1}^{\infty}\left(n\left(2\frac{1}{2}\right)\right) * z^{-n}\right] \\ &= \left(20\frac{1}{2}\right)\frac{1}{z-1} - \left(2\frac{1}{2}\right) * \left[\sum_{n=1}^{\infty}(n) * z^{-n}\right] \\ &= \left(20\frac{1}{2}\right)\frac{1}{z-1} - \left(2\frac{1}{2}\right) * S \qquad (7) \end{aligned}$$

Using equation (5) and (7) :

$$\begin{aligned} X(z) &= \left(20\frac{1}{2}\right)\frac{1}{z-1} - \left(2\frac{1}{2}\right)\frac{z}{(z-1)^2} \\ &= \frac{18z - \left(20\frac{1}{2}\right)}{(z-1)^2} \end{aligned}$$

Answer : The z transformation for x(n) where x(n) is the n^{th} term of the AP is $\frac{18z - \left(20\frac{1}{2}\right)}{(z-1)^2}$.