GATE 2022 EE 28

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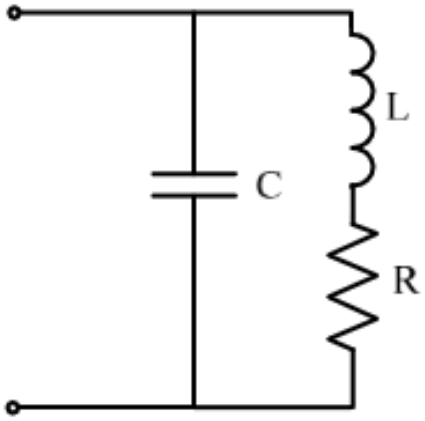
Question GATE 22 EE 28 :
The network shown below has a resonant frequency of 150 kHz and a bandwidth of 600 Hz. The Q-factor of the network is ____
(GATE EE 2022)



Solution:
In the state of resonance ,

Parameter	Description	Values
f_0	Resonant Frequency	
ω_0	Resonant angular frequency	$2\pi f_0$
R	Given resistance	
C	Given capacitance	
L	Given inductancce	
V	Voltage applied	
I_0	Peak current	$\frac{V}{R}$

TABLE 0
Parameter Table

$$\omega_0 = \frac{1}{\sqrt{LC}} \tag{1}$$

$$f_0 = \frac{1}{2\pi\sqrt{LC}} \tag{2}$$

Bandwidth is range of frequencies where power is $\geq$ maximum power.
Hence it is the range of frequencies between the two points where power is half.

$$\text{Power is half} \implies I = \frac{I_0}{\sqrt{2}} \tag{3}$$

Current at any point is given by ,

$$\frac{V}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}} = \frac{V}{R\sqrt{2}} \tag{4}$$

Solving the above equation, we will get two solution ω_1 and ω_2 which will satisfy

$$\left(\omega L - \frac{1}{\omega C}\right) = \pm R \tag{5}$$

$$\implies \omega_2 - \omega_1 = \frac{R}{L} \tag{6}$$

$$\therefore \text{Bandwidth} = \frac{R}{L} \tag{7}$$

$$Q \text{ factor} = 2\pi \frac{\text{Peak energy stored}}{\text{Energy disspated in one cycle}} \tag{8}$$

$$= \frac{\frac{1}{2}LI^2}{\frac{1}{2}I^2R\frac{1}{f_0}} \tag{9}$$

$$= \frac{\omega_0}{\frac{R}{L}} \tag{10}$$

$$\tag{11}$$

Using equations (1) and (7),

$$Q = \frac{150000}{600} \qquad (12)$$

$$= 250 \qquad (13)$$