GATE ME 30

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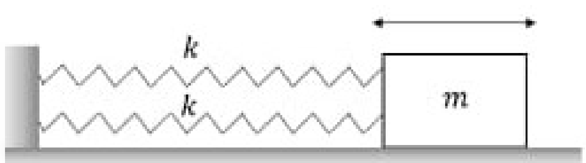
Question GATE ME 30 :
The figure shows a block of mass m = 20 kg attached to a pair of identical linear springs, each having a spring constant k = 1000 N/m. The block oscillates on a frictionless horizontal surface. Assuming free vibrations, the time taken by the block to complete ten oscillations is ____ seconds . (Rounded off to two decimal places) Take $\pi = 3.14$.
(GATE ME 2023)



Solution:

Parameter	Description	Value
k_i	spring constant	1000 N/m
m	mass of block	20Kg
k	Equivalent spring constant	$k_1 + k_2$ (parallel)
ω_n	Natural frequency	$\sqrt{\frac{k}{m}}$
T	Time period of an oscillation	$\frac{2\pi}{\omega_n}$
x	Displacement of block	
a	Acceleration of block	$\frac{d^2x}{dt^2}$
F	Force on block	
A	Amplitude of oscillation	x(0)

TABLE 0
Parameter Table

$$F = ma \tag{1}$$

$$F = -kx \tag{2}$$

$$\implies ma + kx = 0 \tag{3}$$

$$\therefore m\frac{d^2x}{dt^2} + kx = 0 \tag{4}$$

The Laplace transform of the terms is ,

$$\frac{d^2x}{dt^2} \overset{\mathcal{L}}{\longleftrightarrow} s^2X(s) - sx(0) - \dot{x}(0) \tag{5}$$

$$x \overset{\mathcal{L}}{\longleftrightarrow} X(s) \tag{6}$$

Using equation (5) and (6) in equation (4),

$$m\left(s^2X(s) - sx(0) - \dot{x}(0)\right) + kX(s) = 0 \tag{7}$$

$$ms^2X(s) - msA + m(0) + kX(s) = 0 \tag{8}$$

$$X(s) = \frac{msA}{ms^2 + k} \tag{9}$$

$$\implies X(s) = \frac{sA}{s^2 + \frac{k}{m}} \tag{10}$$

The inverse Laplace transform of such terms is given by,

$$\frac{s}{s^2 + a^2} \overset{\mathcal{L}^-}{\longleftrightarrow} cos(at)\,u(t) \tag{11}$$

$\therefore$ the inverse Laplace of (10) is,

$$x(t) = Acos\left(\sqrt{\frac{k}{m}}t\right) \tag{12}$$

From equation (12) and Table 0 ,the time to complete one oscillation is,

$$T_n = \frac{2\pi}{\sqrt{\frac{k}{m}}} \tag{13}$$

$$= \frac{\pi}{5} \tag{14}$$

$\therefore$ the time required for 10 oscillations is ,

$$10T_n = 2\pi \tag{15}$$

$$= 6.28s \tag{16}$$

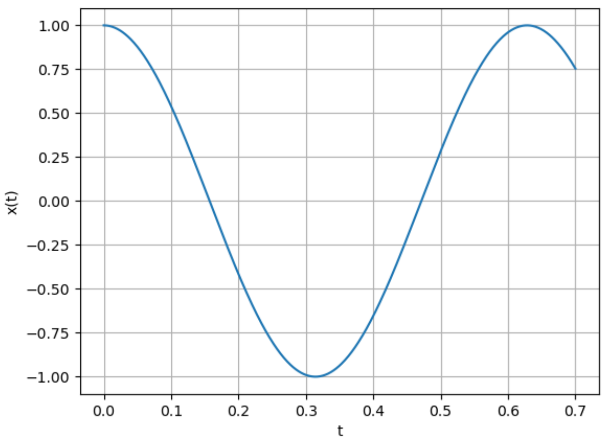


Fig. 0. Plot of $x(t)$