NCERT Question 11.9.3.15

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Question 11.9.3.15 : Given a GP with x_0 = 729 and 7^{th} term 64, determine s(6)

Solution:

Parameter	**Description**	**Value**
x(0)	First Term	729
r	Common Ratio	
x(n)	$(n+1)^{th}$ Term	$x(0)\,r^n u(n)$
x(6)	7^{th} Term	64
s(k)	Sum of first $(k+1)$ terms	

TABLE 0
Parameter Table

$$X(z) = \frac{x(0)}{1 - rz^{-1}} \tag{1}$$

$$\text{ROC is } |z| > |r| \text{ as it is a GP.} \tag{2}$$

Sum to n terms of GP can be given as :

$$s(n) = x(n) * u(n) \tag{3}$$

$$\implies S(Z) = X(Z)\,U(Z) \tag{4}$$

from Table 0 :

$$x(6) = x(0)\,r^6 \tag{5}$$

$$\implies 64 = 729r^6 \tag{6}$$

$$\therefore r = \frac{2}{3} \tag{7}$$

using Table 0 and equation (1)

$$X(z) = \frac{729}{1 - \frac{2}{3}z^{-1}} \tag{8}$$

using Table 0, equation (4) and equation (8)

$$S(z) = \frac{729}{\left(1 - \frac{2}{3}z^{-1}\right)\left(1 - z^{-1}\right)} \tag{9}$$

$$= 2187\left(\frac{1}{1 - z^{-1}} - \frac{\frac{2}{3}}{1 - \frac{2}{3}z^{-1}}\right) \tag{10}$$

Using contour integration for inverse Z transform,

$$s(6) = \frac{1}{2\pi j}\int S(z)\,z^5 dz \tag{11}$$

$$= \frac{1}{2\pi j}\left(\int \frac{2187z^6}{z-1}dz + \int \frac{1458z^6}{z-\frac{2}{3}}dz\right) \tag{12}$$

Solution of each of these integrals can be given by :

$$I = \frac{1}{(m-1)!}\lim_{z\to a}\frac{d^{m-1}}{dz^{m-1}}\left((z-a)^m f(z)\right) \tag{13}$$

using equations (12) and (13):

$$\frac{1}{2\pi j}\left(\int \frac{2187z^6}{z-1}dz\right) = \lim_{z\to 1}\left((z-1)\frac{2187z^6}{z-1}\right) \tag{14}$$

$$= \lim_{z\to 1}\left(2187z^6\right) \tag{15}$$

$$= 2187 \tag{16}$$

$$\frac{1}{2\pi j}\left(\int \frac{1458z^6}{z-\frac{2}{3}}dz\right) = \lim_{z\to \frac{2}{3}}\left(\left(z-\frac{2}{3}\right)\frac{1458z^6}{z-\frac{2}{3}}\right) \tag{17}$$

$$= \lim_{z\to \frac{2}{3}}\left(1458z^6\right) \tag{18}$$

$$= 128 \tag{19}$$

using equations (12), (16), (19):

$$s(6) = 2187 - 128 \tag{20}$$
$$= 2059 \tag{21}$$

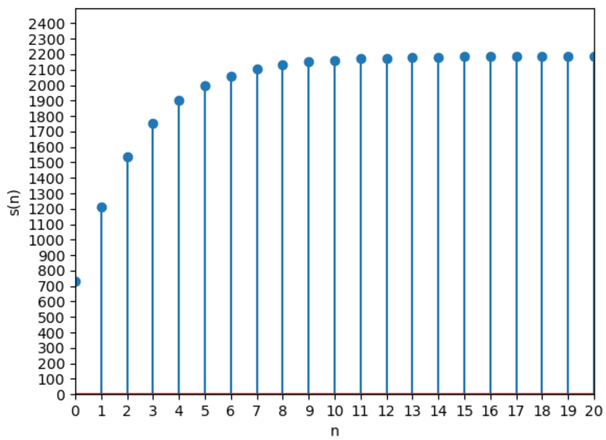

Fig. 0. Plot of $s(n)$

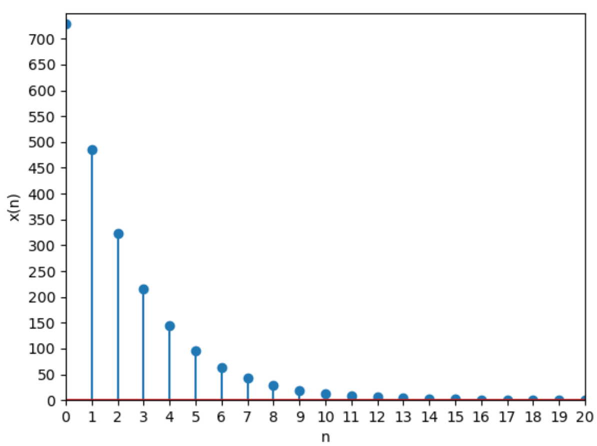

Fig. 0. Plot of $x(n)$