GATE 2022 EE 39

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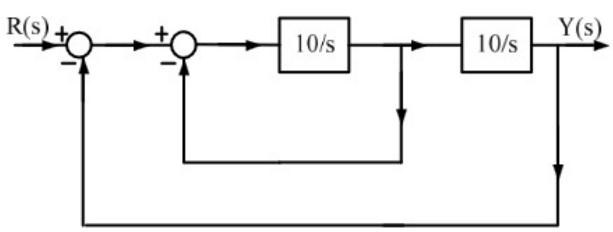
Question GATE 22 EE 39 :
The damping ratio and undamped natural frequency of a closed loop system as shown in the figure, are denoted as ζ and ω_n, respectively. The values of ζ and ω_n are



1) $\zeta = 0.5$ and $\omega_n = 10$ rad/s
2) $\zeta = 0.1$ and $\omega_n = 10$ rad/s
3) $\zeta = 0.707$ and $\omega_n = 10$ rad/s
4) $\zeta = 0.707$ and $\omega_n = 100$ rad/s

(GATE EE 2022)

Solution:

We will use Mason's Gain Formula to

Parameter	Description	Values
m	load of system	
k	stiffness of system	
ω_n	Natural frequency	$\sqrt{\frac{k}{m}}$
ζ	Damping ratio	$\frac{c}{2m\omega_n}$
y(t)	Output of system	
x(t)	Input to the system	
c	Damping coefficient	
T(s)	Transfer function of system	$\frac{Y(s)}{R(s)}$

TABLE 4
PARAMETER TABLE

calculate the transfer function of this system. First converting the given diagram to a signal flow graph : Mason's Gain Formula is given

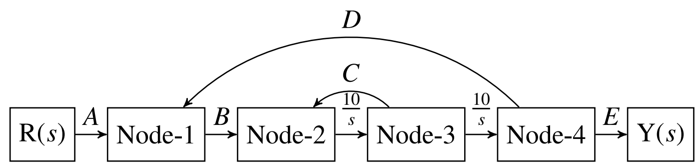


Fig. 4. Signal Flow Diagram

by :

$$H(s) = \sum_{i=1}^{N} \left(\frac{P_i \Delta_i}{\Delta} \right) \tag{1}$$

This signal flow graph has only one

Parameter	Description
N	Number of forward paths
L	Number of loops
P_k	Forward path gain of k^{th} path
Δ_k	Associated path factor
Δ	Determinant of the graph

TABLE 4
PARAMETER TABLE - MASON'S GAIN LAW

Parameter	Formula
Δ	$1 + \sum_{k=1}^{L} \left((-1)^k \text{Product of gain of groups of k isolated loops} \right)$
Δ_k	Δ part of graph that is not touching k^{th} forward path

TABLE 4
FORMULA TABLE - MASON'S GAIN LAW

forward path whose gain is given by :

$$P_1 = \frac{10}{s}\frac{10}{s} \tag{2}$$

$$= \frac{100}{s^2} \tag{3}$$

The loop gain for loop between Node-2 and Node-3 is :

$$L_1 = \frac{10}{s}(-1) \tag{4}$$
$$= -\frac{10}{s} \tag{5}$$

The loop gain for loop between Node-1 and Node-4 is :

$$L_1 = \frac{10}{s}\frac{10}{s}(-1) \tag{6}$$
$$= -\frac{100}{s^2} \tag{7}$$

Using Table 4, Δ is :

$$\Delta = 1 - \left(-\frac{10}{s} - \frac{100}{s^2}\right) \tag{8}$$
$$= 1 + \frac{10}{s} + \frac{100}{s^2} \tag{9}$$

There are no two isolated loops available. Hence all further terms will b zero.
As both the loops are in contact with the only forward path,

$$\Delta_1 = 1 \tag{10}$$

Using equation (1) :

$$H(s) = \frac{\frac{100}{s^2}}{1 + \frac{10}{s} + \frac{100}{s^2}} \tag{11}$$
$$= \frac{100}{s^2 + 10s + 100} \tag{12}$$

Referring to Table 4, the general equation of the damping system is second order and can be written as :

$$m\ddot{y}(t) + c\dot{y}(t) + ky(t) = x(t) \tag{13}$$

Take the Laplace transform and solve for $\frac{Y(s)}{X(s)}$:

$$\frac{Y(s)}{X(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \tag{14}$$
$$\implies H(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \tag{15}$$

Comparing equations (12) and (15) ,

$$\omega_n^2 = 100 \tag{16}$$
$$\implies \omega_n = 10 \text{ rad/s} \tag{17}$$
$$2\zeta\omega_n = 10 \tag{18}$$
$$\implies \zeta = 0.5 \tag{19}$$

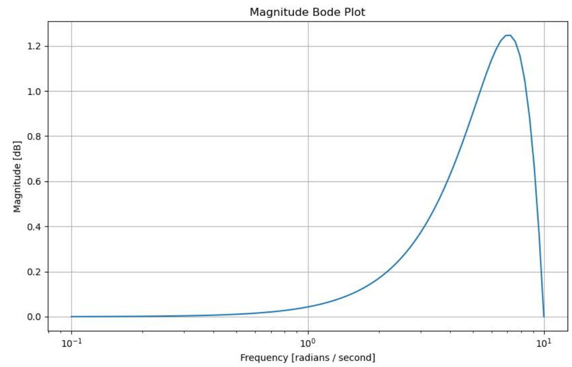


Fig. 4. Magnitude plot

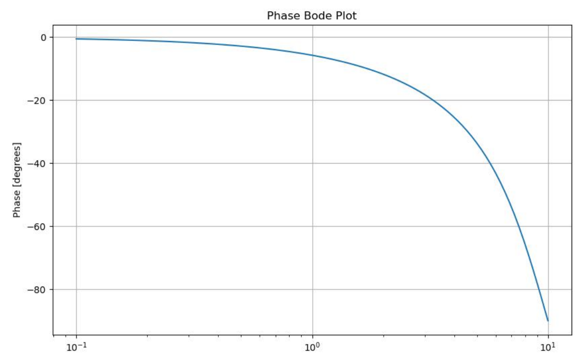


Fig. 4. Phase plot