NCERT Question 11.9.3.15

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Question 11.9.3.15 :

Given a GP with a = 729 and 7th term 64, find S_7.

Solution:

Parameter	Description	Value
x(0)	First Term	729
r	Common Ratio	
x(n)	$(n+1)^{th}$ Term	$x(0)\, r^n u(n)$
x(6)	7th Term	64
y(k)	Sum of first $(k+1)$ terms	

TABLE 0
PARAMETER TABLE

from Table 0 :

$$x(6) = x(0)\, r^6 \quad (1)$$

$$\implies 64 = 729 r^6 \quad (2)$$

$$\therefore r = \frac{2}{3} \quad (3)$$

using Table 0 and equation (3)

$$X(z) = \frac{729}{1 - \frac{2}{3}z^{-1}}, |z| > \frac{2}{3} \quad (4)$$

using Table 0 and equation (4)

$$Y(z) = \frac{729}{\left(1 - \frac{2}{3}z^{-1}\right)\left(1 - z^{-1}\right)} \quad (5)$$

$$= 2187 \left(\frac{1}{1 - z^{-1}} - \frac{\frac{2}{3}}{1 - \frac{2}{3}z^{-1}} \right), |z| > 1 \quad (6)$$

Using contour integration for inverse z transform,

$$y(6) = \frac{1}{2\pi j} \oint Y(z)\, z^5 dz \quad (7)$$

Using equation (6) in (7) :

$$y(6) = \frac{1}{2\pi j}\left(\oint \frac{2187 z^6}{z-1}dz - \oint \frac{1458 z^6}{z - \frac{2}{3}}dz \right) \quad (8)$$

$$\frac{1}{2\pi j}\left(\oint \frac{2187 z^6}{z-1}dz \right) = 2187 \quad (9)$$

$$\frac{1}{2\pi j}\left(\oint \frac{1458 z^6}{z - \frac{2}{3}}dz \right) = 128 \quad (10)$$

using equations (9) and (10) in (8):
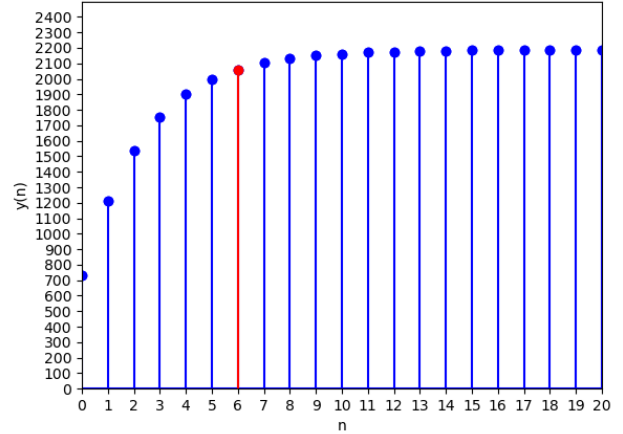
$$y(6) = 2187 - 128 \quad (11)$$
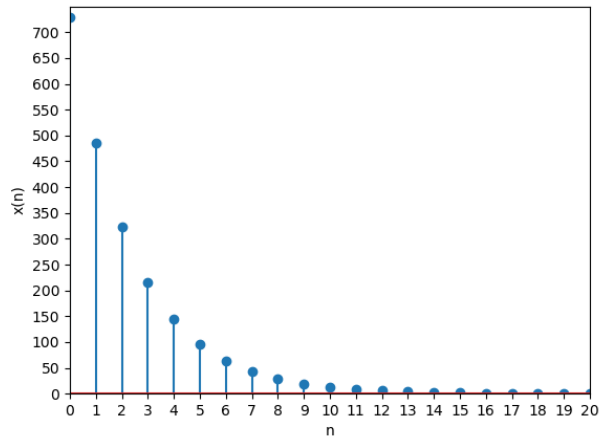
$$= 2059 \quad (12)$$

Fig. 0. Plot of $y(n)$

Fig. 0.   Plot of $x(n)$