

GATE ME 30

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Question GATE ME 30 :

The figure shows a block of mass $m = 20$ kg attached to a pair of identical linear springs, each having a spring constant $k = 1000$ N/m. The block oscillates on a frictionless horizontal surface. Assuming free vibrations, the time taken by the block to complete ten oscillations is _____ seconds . (Rounded off to two decimal places) Take $\pi = 3.14$. (GATE ME 2023)



Solution:

Parameter	Description	Value
k_i	spring constant	1000 N/m
m	mass of block	20Kg
k	Equivalent spring constant	$k_1 + k_2$ (parallel)
ω_n	Natural frequency	$\sqrt{\frac{k}{m}}$
T	Time period of an oscillation	$\frac{2\pi}{\omega_n}$
x	Displacement of block	
a	Acceleration of block	$\frac{d^2x}{dt^2}$
F	Force on block	
A	Amplitude of oscillation	$x(0)$

TABLE 0
PARAMETER TABLE

$$F = ma \quad (1)$$

$$F = -kx \quad (2)$$

$$\Rightarrow ma + kx = 0 \quad (3)$$

$$\therefore m \frac{d^2x}{dt^2} + kx = 0 \quad (4)$$

The Laplace transform of the terms is ,

$$x \xleftrightarrow{\mathcal{L}} X(s) \quad (5)$$

$$\frac{d^2x}{dt^2} \xleftrightarrow{\mathcal{L}} s^2X(s) - sx(0) - \dot{x}(0) \quad (6)$$

Using equation (5) and (6) in equation (4),

$$m(s^2X(s) - sx(0) - \dot{x}(0)) + kX(s) = 0 \quad (7)$$

$$ms^2X(s) - msA + kX(s) = 0 \quad (8)$$

$$X(s) = \frac{msA}{ms^2 + k} \quad (9)$$

$$= \frac{sA}{s^2 + \frac{k}{m}} \quad (10)$$

The inverse Laplace transform of such terms is given by,

$$\frac{s}{s^2 + a^2} \xleftrightarrow{\mathcal{L}^{-1}} \cos(at) u(t) \quad (11)$$

\therefore the inverse Laplace of (10) is,

$$x(t) = A \cos\left(\sqrt{\frac{k}{m}}t\right) \quad (12)$$

From equation (12) and Table 0 ,the time to complete one oscillation is,

$$T_n = \frac{2\pi}{\sqrt{\frac{k}{m}}} \quad (13)$$

$$= \frac{\pi}{5} \quad (14)$$

\therefore the time required for 10 oscillations is ,

$$10T_n = 2\pi \quad (15)$$

$$= 6.28s \quad (16)$$

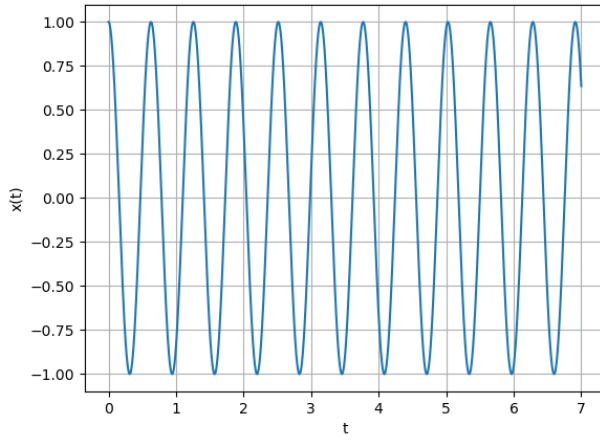


Fig. 0. Plot of $x(t)$