NCERT Question 10.5.2.5

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Question 10.5.2.5 : Find the number of terms in each of the following APs. Then express each term as x(n) and find the z transform and its ROC:
(*i*) 7, 13, 19, ... 205
(*ii*) 18, $15\frac{1}{2}$, 13, ... -47
Solution :

TABLE 0
PARAMETER TABLE

Parameter	Used to denote	Values
$x_i(n)$	n^{th} term of ith series $(i = (1, 2))$	$x_i(0) + nd_i$
$x_i(0)$	First term of ith AP	$x_1(0) = 7$ $x_2(0) = 18$
d_i	Commmon difference of ith AP	$d_1 = 6$ $d_2 = -2\frac{1}{2}$

The number of terms in the AP x(n) is given by :

$$\frac{x(n) - x(0)}{d} \qquad (1)$$

TABLE 0
ANSWER TABLE : NUMBER OF ELEMENTS

Paramter	AP	Value
n	7, 13, 19, ... 205	$\frac{205-7}{6} = 33$
n	18, $15\frac{1}{2}$, 13, ... $- 47$	$\frac{-47-18}{-2\frac{1}{2}} = 26$

Finding the z transform and ROC :

For an AP
$$x(n) = [x(0) + nd] u(n) \qquad (2)$$
$$= x(0) u(n) + dnu(n) \qquad (3)$$
$$z\{u(n)\} = U(z) = \frac{1}{1 - z^{-1}}, |z| > 1 \qquad (4)$$
$$z\{nu(n)\} = -z\frac{dU(z)}{dz} = \frac{z^{-1}}{(1 - z^{-1})^2}, |z| > 1 \qquad (5)$$

ROC is given by values of z for which :

$$|X(z)| = \sum_{n=-\infty}^{\infty} |x(n) z^{-n}| < \infty \qquad (6)$$

Using equations (7) and (8) in equation (6) :

$$z\{x(n)\} = X(z) = \frac{x(0)}{1 - z^{-1}} + d\frac{z^{-1}}{(1 - z^{-1})^2} \qquad (7)$$

(*i*)

$$x_1(n) = (7 + (n)6) u(n) \qquad (8)$$

$$x_1(n) = \begin{cases} 0 & \text{for } n < 0 \\ 7 + (n)6 & \text{for } n \geq 0 \end{cases} \qquad (9)$$

Using the values in Table 0 and equation (10) :

$$X_1(z) = \frac{7}{1 - z^{-1}} + \frac{6z^{-1}}{(1 - z^{-1})^2} \qquad (10)$$

$$= \frac{7 - z^{-1}}{(1 - z^{-1})^2} \qquad (11)$$

The ROC is $|z| > 1$ as it is an AP.

(*ii*)

$$x_2(n) = \left(18 + n\left(-2\frac{1}{2}\right)\right) u(n) \qquad (12)$$

$$x_2(n) = \begin{cases} 0 & \text{for } n < 0 \\ 18 + n\left(-2\frac{1}{2}\right) & \text{for } n \geq 0 \end{cases} \qquad (13)$$

Using the values in Table 0 and equation (10) :

$$X(z) = \frac{18}{1 - z^{-1}} - \left(2\frac{1}{2}\right)\frac{z^{-1}}{(1 - z^{-1})^2} \qquad (14)$$

$$= \frac{18 - \left(20\frac{1}{2}\right)z^{-1}}{(1 - z^{-1})^2} \qquad (15)$$

The ROC is $|z| > 1$ as it is an AP.

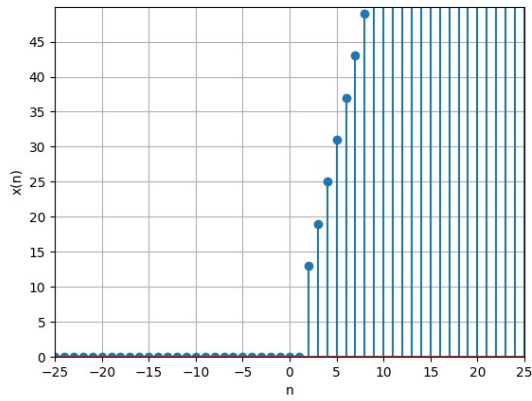The graphs for x(n) :
(*i*) The graph of $x_1(n)$ is :

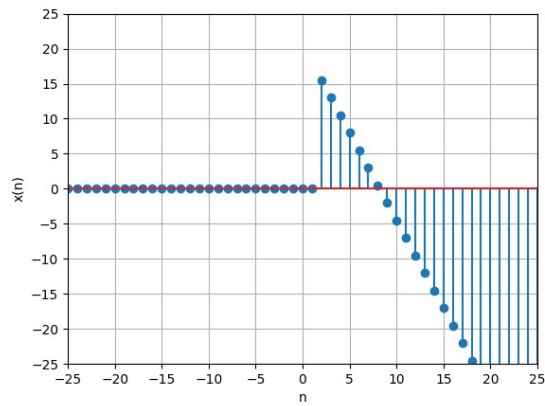Fig. 0. Plot of x(*n*)

(*ii*) The graph of $x_2(n)$ is :

Fig. 1. Plot of x(*n*)