

NCERT Question 11.9.3.15

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Question 11.9.3.15 :

Given a GP with $a = 729$ and 7^{th} term 64, find S_7 .

Solution:

| Parameter | Description | Value |
|-----------|----------------------------|----------------|
| $x(0)$ | First Term | 729 |
| r | Common Ratio | |
| $x(n)$ | $(n+1)^{th}$ Term | $x(0)r^n u(n)$ |
| $x(6)$ | 7^{th} Term | 64 |
| $y(k)$ | Sum of first $(k+1)$ terms | |

TABLE 0
PARAMETER TABLE

$$X(z) = \frac{x(0)}{1 - rz^{-1}}, |z| > |r| \quad (1)$$

Sum to n terms of GP can be given as :

$$y(n) = x(n) * u(n) \quad (2)$$

$$\Rightarrow Y(z) = X(z) U(z) \quad (3)$$

from Table 0 :

$$x(6) = x(0)r^6 \quad (4)$$

$$\Rightarrow 64 = 729r^6 \quad (5)$$

$$\therefore r = \frac{2}{3} \quad (6)$$

using Table 0 and equation (1)

$$X(z) = \frac{729}{1 - \frac{2}{3}z^{-1}}, |z| > \frac{2}{3} \quad (7)$$

using Table 0, equation (3) and equation (7)

$$Y(z) = \frac{729}{\left(1 - \frac{2}{3}z^{-1}\right)(1 - z^{-1})} \quad (8)$$

$$= 2187 \left(\frac{1}{1 - z^{-1}} - \frac{\frac{2}{3}}{1 - \frac{2}{3}z^{-1}} \right), |z| > 1 \quad (9)$$

Using contour integration for inverse z transform,

$$y(6) = \frac{1}{2\pi j} \oint Y(z) z^5 dz \quad (10)$$

Using equation (9) in (10) :

$$y(6) = \frac{1}{2\pi j} \left(\oint \frac{2187z^6}{z-1} dz + \oint \frac{1458z^6}{z-\frac{2}{3}} dz \right) \quad (11)$$

Solution of each of these integrals can be given by :

$$I = \frac{1}{(m-1)!} \lim_{z \rightarrow a} \frac{d^{m-1}}{dz^{m-1}} ((z-a)^m f(z)) \quad (12)$$

where m is the number of times pole is repeated.

using equations (11) and (12):

$$\frac{1}{2\pi j} \left(\oint \frac{2187z^6}{z-1} dz \right) = 2187 \quad (13)$$

$$\frac{1}{2\pi j} \left(\oint \frac{1458z^6}{z-\frac{2}{3}} dz \right) = 128 \quad (14)$$

using equations (13) and (14) in (11):

$$y(6) = 2187 - 128 \quad (15)$$

$$= 2059 \quad (16)$$

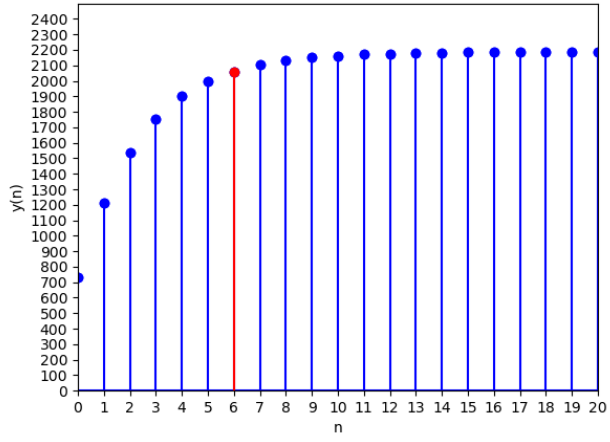


Fig. 0. Plot of $y(n)$

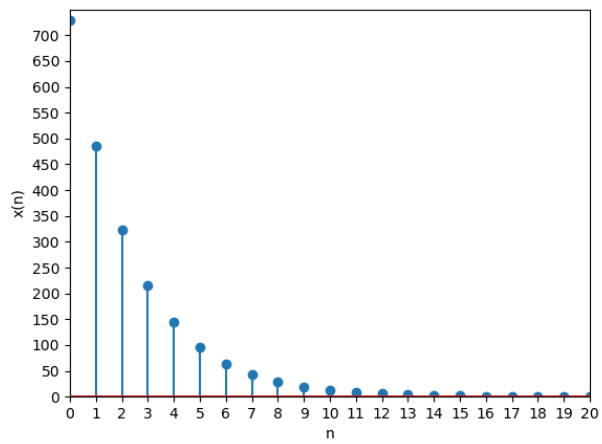


Fig. 0. Plot of $x(n)$