

#### Q1

The code defines a function `fun(x)` that returns the sine of  $x^2$  and another function `derivative(x)` that returns the derivative of `fun(x)` using the analytical formula. It then defines a function `ForwardFinitDifferenceApp(x, h = 0.01)` that calculates the forward finite difference approximation of the derivative of `fun(x)` using a small step size  $h$ .

The code also defines a function `visualise()` that generates a plot to compare the actual derivative and the forward finite difference approximation of the derivative for values of  $x$  ranging from 0 to 1. The plot is generated using the Matplotlib library and displays two lines, one representing the actual derivative and the other representing the approximation using the forward finite difference method.

Finally, the code checks if it is being executed as the main program and calls the `visualise()` function.

#### Q2

The code defines a function `fun(x)` that returns the sine of  $x^2$  and another function `derivative(x)` that returns the derivative of `fun(x)` using the analytical formula. It then defines a function `FinitDifferenceApp(x, h=0.01, type='f')` that calculates the finite difference approximation of the derivative of `fun(x)` using a small step size  $h$  and the specified type of finite difference scheme, which can be 'f' for forward difference, 'b' for backward difference, or 'c' for central difference.

The code also defines a function `visualise()` that generates a plot to compare the absolute error of the finite difference approximation and the actual derivative for values of  $x$  ranging from 0 to 1 using the different types of finite difference schemes. The plot is generated using the Matplotlib library and displays three lines, one representing the absolute error of the forward difference, another representing the absolute error of the backward difference, and the third representing the absolute error of the central difference scheme.

Finally, the code checks if it is being executed as the main program and calls the `visualise()` function.

#### Q4

The main function of the code is `visualize`, which takes two arguments  $a$  and  $b$  - the limits of integration, and performs the following steps:

Define a list `x_axis` to store the number of intervals used for approximation.

Define a list `y_axis` to store the computed area for each value of  $M$  (the number of intervals).

Compute the actual area under the curve using the function `fun_integral`.

For each value of  $M$  from 1 to 100, compute the area under the curve using the trapezoidal rule with  $M$  intervals.

Append the value of  $M$  to `x_axis`, and the computed area to `y_axis`.

Plot the values in `x_axis` and `y_axis` on a graph, with  $M$  on the x-axis and the computed area on the y-axis.

Also plot a horizontal line for the actual area, to compare with the computed values.

The function `fun` computes the function  $2x \cdot \exp(x^2)$ , which is the function whose integral is approximated using the trapezoidal rule. The function `fun_integral` computes the integral of `fun` using the exact formula  $\exp(x^2)$ .

The graph generated by the code shows that as the number of intervals used for approximation is increased, the computed area approaches the actual area under the curve. This is expected, as the trapezoidal rule becomes more accurate with more intervals.

