

Q1

The function then creates two matrices, A and B , which will be used to solve for the coefficients of the polynomial. The A matrix is a square matrix with dimensions $(n+1) \times (n+1)$, and the B matrix is a column vector with dimensions $(n+1) \times 1$. The elements of the A and B matrices are calculated using the x and y values of the given points.

The function then uses `numpy.linalg.solve` to solve the matrix equation $Ax = B$ and obtain the coefficients of the polynomial. The coefficients are stored in a `Polynomial` object, which is created using the `numpy` library.

Finally, the function displays the polynomial using the `Polynomial.show()` method, which takes the minimum and maximum x values of the given points as arguments. The function also plots the given points on a graph using the `matplotlib` library.

The function then returns the `Polynomial` object.

Q2

The first part of the function checks if the input is valid. The n input should be an integer, and should be nonnegative. If the input is invalid, the function raises an exception with a helpful error message.

Next, the function defines the interval $[0, \pi]$ and generates 100 equally spaced x values within the interval. The corresponding y values for these x values are obtained by evaluating the given function `fun` at each x value.

The function then creates a matrix A and a column vector b , which will be used to solve for the coefficients of the polynomial. The A matrix is a square matrix with dimensions $(n+1) \times (n+1)$, and the elements of the matrix are calculated using integration via the `quad` function from the. The b vector is a column vector with dimensions $(n+1) \times 1$, and the elements of the vector are also calculated using integration via the `quad` function. The function then uses `numpy.linalg.solve` to solve the matrix equation $Sx = b$ and obtain the coefficients of the polynomial.

Q3

It first constructs the polynomial `num` as a polynomial expression of the form $(x^{**2} - 1)**n$. This is done using the `Polynomial` class provided in the code.

It then takes the n th derivative of the polynomial using the `derivative()` method provided by the `Polynomial` class.

It then divides the resulting polynomial by the factor $2^{**n} * \text{math.factorial}(n)$ to obtain the nth Legendre polynomial.

Finally, the function returns the nth Legendre polynomial as a polynomial object