Total No. of Questions—8] [Total No. of Printed Pages—5

Seat No.	

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S.E. (COMP/IT) (II Semester) EXAMINATION, 2018

ENGINEERING MATHEMATICS–III

(2015 PATTERN)

Time : Two Hours **Maximum Marks : 50**

N.B. :— (*i*) Neat diagrams must be drawn wherever necessary.

(*ii*) Figures to the right indicate full marks.

(*iii*) Your answers will be valued as a whole.

(*iv*) Use of electronic pocket calculator is allowed.

(*v*) Assume suitable data, if necessary.

1. (*a*) Solve any *two* : [8]

(*i*) $(D^2 + 4)y = e^x + x^2$

(*ii*) $(D^2 + 6D + 9)y = x^{-4}e^{-3x}$

(*iii*) $x^2 \dfrac{d^2y}{dx^2} + 3x\dfrac{dy}{dz} + y = \dfrac{1}{x}\sin(\log x)$.

(*b*) Solve the integral equation : [4]

$$\int_0^\infty f(x)\sin \lambda x \, d\lambda = \begin{cases} 1-\lambda, & 0 \le \lambda \le 1 \\ 0, & \lambda \ge 1 \end{cases}$$

P.T.O.

2. (*a*) An uncharged condenser of capacity C charged by applying an emf of value E sin $\frac{t}{\sqrt{LC}}$ through leads of inductance L and of negligible resistance. The charge Q on the plate of condenser satisfies the diff. equation $\frac{d^2Q}{dt^2}+\frac{Q}{LC}=\frac{E}{L}\sin\frac{t}{\sqrt{LC}}$. Find the charge at any time t. [4]

(*b*) Solve any *one* of the following : [4]

(*i*) Find $z\left\{\cos\left(\frac{k\pi}{2}+\frac{\pi}{4}\right)\right\},\ k \geq 0.$

(*ii*) Using inversion integral method, find :

$$z^{-1}\left\{\frac{z}{(z-1)(z-2)}\right\}$$

(*c*) Solve the following difference equation $y_{k+2} - 5y_{k+1} + 6y_k = u_k$ with $y_0 = 0$, $y_1 = 1$ and $u_k = 1$ for $k \geq 0$. [4]

3. (*a*) The first four moments of a distribution about the value 5 are –4, 22, –117 and 560 respectively. Find the moments about the mean. Also calcualte β_1 and β_2. [4]

(*b*) Fit a straight line of the form $y = ax + b$ to the following data by the least square method : [4]

x	0	6	8	10	14	16	18	20
y	3	12	15	18	24	27	30	33

(*c*) There is a small probability of $\frac{1}{1000}$ for any computer produced to be defective. Determine in a sample of 2000 computers, the probability that there are : [4]

(*i*) no defectives, and

(*ii*) 2 defectives.

Or

4. (*a*) The lifetime of an article has a normal distribution with mean 400 hours and standard deviation 50 hours. Assuming normal distribution, find the expected number of articles out of 2000 whose lifetime lies between 335 hours to 465 hours. (Given : Z = 1.3, A = 0.4032). [4]

(*b*) On an average 20% of the workers in an industry suffer with a certain diseases. If 12 workers are chosen from the industry, find the probability that : [4]

(*i*) Exactly 2 workers suffer from the diseases,

(*ii*) At least one worker suffers from the disease.

(*c*) Obtain the line of regression of y on x for the following data. Also, estimate the value of y for $x = 10$. [4]

x	2	4	5	6	8	11
y	18	12	10	8	7	5

5. (*a*) Find the directional derivative of the function $\phi = x^2yz^3$ at $(2, 1, -1)$ towards the point $(1, -1, 1)$. [4]

(*b*) Show that $\overline{F} = (y \cos z)\overline{i} + (x \cos z)\overline{j} - xy \sin z\overline{k}$ is irrotational. Find scalar of ϕ such that $\overline{F} = \nabla\phi$. [4]

(*c*) Evaluate $\int_C \overline{F}.d\overline{r}$ for $\overline{F} = 3x^2 i + (2xz - y)\overline{j} + z\overline{k}$ along the curve $x = t^2$, $y = t$, $z = t^3$ from $t = 0$, $t = 1$. [5]

Or

6. (*a*) Show that (any *one*) : [4]

(*i*) $\nabla \times (\overline{r}r^3) = 0$

(*ii*) $\nabla^2 (r^2 \log r) = 5 + 6 \log r$.

(*b*) Find the directional derivative of the function $\phi = x^2y + xyz + z^3$ at $(1, 2, -1)$ along the direction $8\overline{i} + 8\overline{j}$. [4]

(*c*) Find the work done in moving a particle along the circle $x^2 + y^2 = 4$ under the field of force $\overline{F} = x\overline{i} + y^2\overline{j}$. [5]

7. (*a*) Find an analytic function $f(z) = u + iv$ where $v = 4x^3y - 4xy^3$. [4]

(*b*) Find a Bilinear Transformation $w = f(z)$ which transforms the points $z = \infty, i, 0$ on z-plane to the points $w = 0, i, \infty$ on w-plane respectively. [4]

(*c*) Use Residue theorem, to evaluate the integral

$$\oint_C \left[\frac{4z-1}{z^2 - z - 6}\right] dz$$

where c is a closed curve $|z| = 4$. [5]

Or

8. (*a*) Find an analytic function $f(z) = u(r, \theta) + iv(r, \theta)$ where $v = r^n \sin(n\theta)$. [4]

(*b*) Find the mapping of $y^2 = 2y - x^2$ on z-plane through the transformation $w = \frac{2}{z}$ on w-plane. [4]

(*c*) Evaluate the integral

$$\oint_C \left[\frac{\sin \pi z^2 + 6z}{(z-1)\,(z+2)}\right] dz$$

where C is a closed curve $|z| = 3$. [5]