# * Mathematics-3 (M3) - Assignment Number - 1

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Q-1. Solve $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = e^{e^x}$ (Nov. 2018)

→ Let $\frac{d}{dx} = D$,

$$\therefore D^2y + 3Dy + 2y = e^{e^x}$$

$$(D^2 + 3D + 2)y = e^{e^x}$$

$$\therefore \phi(D) = D^2 + 3D + 2 \qquad , f(x) = e^{e^x}$$
$$= (D+2)(D+1)$$

$$\therefore y_c = C_1 e^{-x} + C_2 e^{-2x}$$

$$y_p = \frac{1}{\phi(D)} \cdot f(x)$$

$$= \frac{1}{(D+2)(D+1)} \cdot e^{e^x} = \frac{1}{D+2}\left[\frac{1}{D+1} \cdot e^{e^x}\right]$$

$$= \frac{1}{D+2}\left[ e^{-x} \int e^{x} . e^{e^{x}} . dx \right]$$

$$= \frac{1}{D+2}\left[ e^{-x} . e^{e^{x}} \right]$$

$$= e^{-2x} \int e^{2x} . e^{-x} . e^{e^{x}} . dx$$

$$y_p = e^{-2x} . e^{e^{x}}$$

$$y = y_c + y_p .$$

$$\boxed{y = C_1 e^{-x} + C_2 e^{-2x} + e^{-2x} . e^{e^{x}}}$$

Q-2. Solve $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = xe^{-x} . \cos x$ (May 2017)

→ Let $\frac{d}{dx} = D$,

$$\therefore (D^2 + 2D + 1)y = x . e^{-x} . \cos x$$

$$\phi(D) = D^2 + 2D + 1$$
$$= (D+1)^2$$

$$\therefore y_c = (c_1 + xc_2)e^{-x} \quad \text{— (1)}$$

$$f(x) = x . e^{-x} . \cos x = e^{-x}(x . \cos x).$$

$$\therefore y_p = \frac{1}{\phi(D)} . f(x) = \frac{1}{(D+1)^2} . e^{-x} . (x . \cos x)$$

Compare with $\frac{1}{\phi(D+a)} \cdot e^{+ax} \cdot V = \frac{1}{\phi(D)} \cdot e^{ax} \cdot V$

$$\therefore y_p = \frac{1}{(D+1)^2} \cdot e^{-x} \cdot (x \cos x)$$

$\because a = -1$, $V = x \cos x$

$$= \frac{1}{(D+1-1)^2} \cdot e^{-x} (x \cdot \cos x)$$

$$= e^{-x} \left[ \frac{1}{D^2} \cdot (x \cdot \cos x) \right]$$

Here, $\phi(D) = D^2$, $V = \cos x$.

$$\therefore y_p = e^{-x} \left[ x - \frac{\phi'(D)}{\phi(D)} \right] \frac{1}{\phi(D)} \cdot \cos x$$

$$= e^{-x} \left[ x - \frac{2D}{D^2} \right] \left[ \frac{1}{D^2} \cdot \cos x \right]$$

$\to a = 1, D^2 \to -1$

$$= e^{-x} \left[ x - \frac{2}{D} \right] \left[ \frac{\cos x}{-1} \right]$$

$$= e^{-x} \left[ -x \cos x + 2 \int \cos x \right] \quad \because [1/D = \int \cos x]$$

$$y_p = e^{-x} [2 \sin x - x \cos x] \quad \text{—— } (2)$$

$$\therefore y = y_c + y_p$$

$$\boxed{y = e^{-x} [(c_1 + x c_2) + 2 \sin x - x \cos x]} \text{ — from (1) and (2)}$$

Q-3. Solve $(D^2 - 2D + 2)y = e^x . \tan x$ (Nov. 2017)

→ Here, $\phi(D) = D^2 - 2D + 2$ , $f(x) = e^x . \tan x$.

roots $= \frac{2 \pm \sqrt{4-8}}{2}$

$= 1 \pm i$

$\therefore y_c = e^x [c_1 \cos x + c_2 \sin x]$ — ①

In MVP, $y_c = c_1 y_1 + c_2 y_2$.

$\therefore y_1 = e^x . \cos x$ , $y_2 = e^x . \sin x$.

$\therefore y_1' = e^x \cos x - e^x \sin x$ , $y_2' = e^x \sin x + e^x \cos x$

$\therefore W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix}$

$= \begin{vmatrix} e^x \cos x & e^x \sin x \\ e^x \cos x - e^x \sin x & e^x \sin x + e^x \cos x \end{vmatrix}$

$= e^{2x} ( \cancel{\sin x . \cos x} + \cos^2 x - \cancel{\sin x . \cos x} + \sin^2 x )$

$= e^{2x} ( \cos^2 x + \sin^2 x )$

$= e^{2x} (1)$

$W = e^{2x}$ — ②

$u = \int \frac{-y_2}{W} . f(x) . dx = \int - \frac{\cancel{e^x} . \sin x}{\cancel{e^{2x}}} . \cancel{e^x} . \tan x$

$u = - [\log(\sec x + \tan x) - \sin x]$ — ③

$$v = \int \frac{y_1}{W} \cdot f(x) \cdot dx = \int \frac{\cancel{e^x}. \cos x}{e^{2x}} . e^x . \tan x . dx$$
$$= \int \sin x . dx$$
$$= -\cos x.$$

Using MVP, $y_P = u y_1 + v y_2$

$$= [\sin x - \log(\sec x + \tan x)] e^x . \cos x + [-\cos x] e^x . \sin x$$

$$y_P = -e^x . \cos x . \log(\sec x + \tan x) \quad — \quad (4)$$

$$y = y_c + y_P$$

$$\therefore y = e^x [c_1 \cos x + c_2 \sin x - \cos x . \log(\sec x + \tan x)].$$

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Q-4. $x^2 . \frac{d^2y}{dx^2} + (-x) . \frac{dy}{dx} + 4y = \cos(\log x) + x \sin(\log x)$ [Nov. 2018]

$\rightarrow$ Let $D, \frac{d}{dz}$, $x = e^z$, $z = \log x$.

$$x . \frac{dy}{dx} = Dy \quad , \quad x^2 \frac{d^2y}{dx^2} = D(D-1)y$$

Given equation becomes,

$$D(D-1)y - Dy + 4y = \cos(z) + e^z \sin(z)$$

$(D^2 - 2D + 4)y = \cos(z) + e^z . \sin(z)$ , $y$ depends on $z$.

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$$\phi(D) = D^2 - 2D + 4 \quad , f(z) = \cos z + e^z . \sin z$$
$$\rightarrow 1 \pm \sqrt{3}\, i$$

$$\therefore y_c = e^z (C_1 \cos\sqrt{3}z + C_2 \sin\sqrt{3}z)$$

$$y_p = \frac{1}{\phi(D)} . f(z) = \frac{1}{(D^2-2D+4)} . [\cos z + e^z \sin z]$$

$$= \frac{1}{(D^2-2D+4)} . \cos z + \frac{1}{(D^2-2D+4)} . e^z \sin z.$$

$a = 1, D^2 = -1$ $\quad$ $a = 1, V = \sin z., D \rightarrow D+1$

$$= -\frac{1}{2D-3} . \cos z + e^z \left[ \frac{1}{D^2+3} . \sin z \right]$$

$$= -\frac{(2D+3)}{4D^2-9} . \cos z + e^z \left[ \frac{1}{D^2+3} . \sin z \right]$$

$a = 1, D^2 \rightarrow -1$ $\quad$ $a = 1, D^2 \rightarrow -1$

$$= \frac{(2D+3)\cos z}{13} + \frac{e^z \sin z}{2}$$

$$\therefore y_p = \frac{3\cos(\log x) - 2\sin(\log x)}{13} + \frac{x \sin(\log x)}{2} \quad [\because z = \log x]$$

$$\therefore y = y_c + y_p$$

$$\boxed{y = x[C_1 \cos(\sqrt{3}\log x) + C_2 \sin(\sqrt{3}\log x)] + \frac{x \sin \log x}{2} + \frac{3\cos(\log x)}{13} - \frac{2\sin}{13}}$$

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Q-5. Solve the following simultaneously $\frac{dy}{dt} + x = \sin t$, $\frac{dx}{dt} + y = \cos t$

→ Let $\frac{d}{dt} = D$ $\therefore$ $Dy + x = \sin t$ — ①

$Dx + y = \cos t$ — ②

Differentiate eq. 2, we get

$D^2x + Dy = D\cos t$ — ③

③ - ①, we get

$D^2x - x = -2\sin t$

$(D^2 - 1)x = -2\sin t$

$\phi(D) = D^2 - 1 = (D+1)(D-1)$, $f(t) = -2\sin t$.

$\therefore x_c = C_1 e^{-t} + C_2 e^{t}$ — ④

$x_p = \frac{1}{\phi(D)} \cdot f(t)$

$= \frac{1}{D^2-1} \cdot (-2\sin t)$

$a = 1, D^2 \to -1$

$x_p = \sin t$ — ⑤

$\boxed{x = x_c + x_p = C_1 e^{-t} + C_2 e^{t} + \sin t}$ — ⑥

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Subtitute ⑥ in ②, we get

$$y = \cancel{\cos t} + C_1 e^{-t} - C_2 e^{t} - \cancel{\cos t}$$

$$\boxed{y = C_1 e^{-t} - C_2 e^{t}}$$