

# \* Tutorial Number - 3

Q-0)  $\rightarrow \frac{1}{D^2 - 7D + 6} \cdot e^{2x} =$

$\rightarrow \frac{1}{(D^2 - 7D + 6)} \cdot e^{2x} = \frac{1}{(D-6)(D-1)} \cdot e^{2x} \quad \text{--- (1)}$

Compare equation (1) with  $\frac{1}{\phi(D)} \cdot e^{ax}$ , we get

$a = 2$  and  $\phi(D) = (D-6)(D-1)$

$\therefore \phi(a) = (2-6)(2-1)$

$= -4 \neq 0$

$\therefore \frac{1}{(2^2 - 7D + 6)} \cdot e^{2x} = \frac{1}{\phi(a)} \cdot e^{ax}$

$= \boxed{\frac{-e^{2x}}{4}}$

$\rightarrow$  option 1.

Q-1)  $\frac{1}{D^2 - 5D + 6} \cdot 3e^{5x}$

$\rightarrow \frac{1}{D^2 - 5D + 6} \cdot 3e^{5x}$  compare it with  $\frac{1}{\phi(D)} \cdot e^{ax}$ , we get

$a = 5, \phi(D) = D^2 - 5D + 6 = \frac{5 \pm \sqrt{25 - 24}}{2} = \frac{5 \pm 1}{2} = 3 \text{ and } 2$

$= (D-3)(D-2)$

$\phi(a) = (5-3)(5-2)$

$= 6$

$\therefore \frac{1}{D^2 - 5D + 6} \cdot 3e^{5x} = 3 \times \frac{1}{6} \cdot e^{5x}$

$= \boxed{\frac{e^{5x}}{2}}$

$\rightarrow$  option 1.



$$Q-2) \frac{1}{(D^2-9)} (e^{3x} + 1)$$

$$\rightarrow \frac{1}{(D^2-9)} \cdot e^{3x} + \frac{1}{(D^2-9)} \cdot e^{0x} \text{ compare with } \frac{1}{\phi(D)} \cdot e^{ax}, \text{ we get.}$$

$$\therefore \phi(D) = D^2 - 9 \quad \text{and } a_1 = 3 \quad \text{and } a_2 = 0.$$

$$\therefore \phi(a_1) = \frac{(D+3)(D-3)}{(D+3)(D-3)} = 0$$

$$h = 1.$$

$$\phi(a_2) = (0)^2 - 9 = -9.$$

$$\frac{1}{(D^2-9)} \cdot e^{3x} + \frac{1}{(D^2-9)} \cdot e^{0x} = \left( \frac{x^1}{1!} \right) \frac{e^{3x}}{(3+3)} + \frac{1}{-9} \cdot e^{0x}.$$

$$= \boxed{\frac{x e^{3x}}{6} - \frac{1}{9}}$$

→ option 4.

$$Q-3) \frac{1}{(D^2+4D+3)} \cdot e^{-3x}$$

$$\rightarrow \frac{1}{(D^2+4D+3)} \cdot e^{-3x} \text{ compare with } \frac{1}{\phi(D)} \cdot e^{ax}, \text{ we get.}$$

$$a = -3, \quad \phi(D) = (D^2+4D+3) = (D+3)(D+1)$$

$$\therefore \frac{1}{(D+3)(D+1)} \cdot e^{-3x} = \left( \frac{x^1}{1!} \right) \frac{e^{-3x}}{(-3+1)} = \boxed{\frac{-x e^{-3x}}{2}}$$

→ option 2



Q-4)  $\frac{1}{(D-2)^3} \cdot (e^{2x} + 3^x)$

$\frac{1}{(D-2)^3} \cdot e^{2x} + \frac{1}{(D-2)^3} \cdot e^{x \log 3}$  compare with  $\frac{1}{\Phi(D)} \cdot e^{ax}$ , we get.

$\Phi(D) = (D-2)^3$

$\lambda_1 = 2$   $\lambda_2 = \log 3$   
 $h = 3$

$\therefore \frac{1}{(D-2)^3} \cdot e^{2x} + \frac{1}{(D-2)^3} \cdot e^{x \log 3} = \left[ \frac{x^3}{3!} \cdot \frac{e^{2x}}{1} + \frac{e^{x \log 3}}{[(\log 3) - 2]^3} \right]$   
 option 1.

Q-5)  $\frac{1}{D^5 - D} \cdot 12 e^x$

$\rightarrow \frac{1}{D^5 - D} \cdot 12 e^x$  compare with  $\frac{1}{\Phi(D)} \cdot e^{ax}$ , we get

$\lambda = 1, \Phi(D) = D(D^4 - 1) = D(D^2 + 1)(D^2 - 1)$   
 $= D(D^2 + 1)(D + 1)(D - 1)$   
 $\lambda = 1$

$= 12 \left( \frac{x^1}{1!} \right) \cdot \frac{e^x}{(1)(1^2 + 1)(1 + 1)}$

$= \frac{12 x e^x}{4}$

$= \boxed{3 x e^x}$   
 $\rightarrow D$



$$Q-6) \frac{1}{(D^2+1)(D-1)} \cdot e^x$$

$\rightarrow \frac{1}{(D^2+1)(D-1)} \cdot e^x$  compare with  $\frac{1}{\phi(D)} \cdot e^{ax}$ , we get.

$$a = 1, \quad \phi(D) = (D^2+1) \{ (D-1) \}$$

$$\therefore \underline{Q} = \left( \frac{x^1}{1!} \right) \cdot \frac{e^x}{(1^2+1)}$$

$$= \boxed{\frac{x e^x}{2}} \rightarrow (C)$$

$$Q-7) \frac{1}{D^2-4D+4} \cdot \sin 2x$$

$$\rightarrow a = 2 \quad \therefore \text{sub } D^2 = -4$$

$$\therefore \frac{1}{(-4)-4D+4} \cdot \sin 2x$$

$$= \frac{-1}{4D} \cdot \sin 2x$$

$$= \frac{-D}{4D^2} \cdot \sin 2x$$

$$= \frac{-D}{4(-4)} \cdot \sin 2x$$

$$= \frac{D \sin 2x}{16}$$

$$= \frac{2 \cos 2x}{16}$$

$$= \boxed{\frac{\cos 2x}{8}} \rightarrow (B)$$



Q-8)  $\frac{1}{D^3 + D} \cdot \cos x$

$\rightarrow a = 1, \therefore \text{sub } D^2 = -1$

$\frac{1}{D(D^2 + 1)} \cdot \cos x$

As  $D^2 + 1 = 0$  when  $D^2 = -1$

$\therefore$

$\frac{1}{D(D^2 + 1)} \cdot \cos x = \cancel{\left(\frac{-x}{2(1)}\right)^1} \cdot \cancel{\frac{1}{1!}} \cdot \cancel{\cos\left(x + \frac{\pi}{2}\right)}$

$= \frac{D}{(D^2 + 1)D^2} \cdot \cos x$

$= \frac{1}{D^2 + 1} \left[ \frac{D}{D^2} \cdot \frac{1}{1!} \cdot \cos x \right]$

$= \frac{1}{D^2 + 1} \left[ \frac{D \cdot \cos x}{-1} \right]$

$= \frac{-\sin x}{D^2 + 1}$

$= \left(\frac{-x}{2(1)}\right)^1 \cdot \frac{1}{1!} \cdot \sin\left(x + \frac{\pi}{2}\right)$

$= \frac{-x}{2} \cdot \left[ \sin x \cdot \cos\left(\frac{\pi}{2}\right) + \cos x \cdot \sin\left(\frac{\pi}{2}\right) \right]$

$= \frac{-x \cos x}{2}$

$\rightarrow \textcircled{D}$



$$Q-9) \frac{1}{D^2+1} \cdot \sin x$$

$$\rightarrow \lambda = 1, D^2 \rightarrow -1, \therefore \lambda = 1$$

$$\begin{aligned} \therefore \frac{1}{(D^2+1)} \cdot \sin x &= \left( \frac{-x}{2(1)} \right)^1 \cdot \frac{1}{1!} \cdot \sin\left(x + \frac{\pi}{2}\right) \\ &= \frac{-x}{2} \cdot \left[ \sin x \cdot \cos\left(\frac{\pi}{2}\right) + \cos x \cdot \sin\left(\frac{\pi}{2}\right) \right] \\ &= \boxed{\frac{-x \cos x}{2}} \end{aligned}$$

A.

$$Q-10) \frac{1}{D^3+9D} \cdot \sin 3x$$

$$\begin{aligned} \rightarrow \frac{1}{D(D^2+9)} \cdot \sin 3x &= \frac{1}{(D^2+9)} \left[ \frac{D}{D^2} \cdot \sin 3x \right] \\ &= \frac{D}{(D^2+9)} \left[ \frac{1}{D^2} \cdot \sin 3x \right] \end{aligned}$$

$\lambda = 3, D^2 \rightarrow -9$

$$= \frac{D}{D^2+9} \left[ \frac{\sin 3x}{-9} \right]$$

$$= \frac{D \sin 3x}{-9(D^2+9)}$$

$$= \frac{3 \cos 3x}{-9(D^2+9)}$$

$$= \frac{-1}{3} \left[ \frac{\cos 3x}{D^2+9} \right]$$

$$= \frac{-1}{3} \left[ \left( \frac{-x}{2(3)} \right)^1 \cdot \frac{1}{1!} \cdot \cos\left(3x + \frac{\pi}{2}\right) \right]$$

$$= \frac{-1}{3} \left[ \frac{-x}{6} \cdot \left\{ \cos(3x) \cos \frac{\pi}{2} - \sin(3x) \sin\left(\frac{\pi}{2}\right) \right\} \right]$$



$$= \frac{-1}{3} \left[ \frac{-x}{6} \cdot \{-\sin(3x)\} \right]$$

$$= \boxed{\frac{-2 \sin(3x)}{18}} \rightarrow \textcircled{B}$$

Q-11)  $\frac{1}{D^4 + 10D^2 + 9} (\sin 2x + \cos 4x)$

$$\rightarrow \frac{1}{[(D^2)^2 + 10D^2 + 9]} (\sin 2x) + \frac{1}{[(D^2)^2 + 10D^2 + 9]} (\cos 4x)$$

$$= \frac{1}{(D^2+9)(D^2+1)} \cdot \sin 2x + \frac{\cos 4x}{(D^2+9)(D^2+1)}$$

$\lambda=2, D^2 \rightarrow -4$        $\lambda=4, D^2 \rightarrow -16$

$$= \frac{1}{(-4+9)(-4+1)} \cdot \sin 2x + \frac{\cos 4x}{(-16+9)(-16+1)}$$

$$= \boxed{\frac{-\sin 2x}{15} + \frac{\cos 4x}{105}} \rightarrow \textcircled{B}$$

Q-12)  $\frac{1}{D^2 - 2D + 5} \cdot 10 \sin x$

$$\rightarrow \lambda=1, D^2 \rightarrow -1$$

$$\therefore \frac{1}{-1-2D+5} \cdot 10 \sin x = \frac{10 \sin x}{-2(2+D)}$$

$$= \frac{-5 \sin x (D+2)}{D^2 - 4}$$

$$= \frac{-5 (D+2) \sin x}{-5}$$

$$= \boxed{\cos x + 2 \sin x} \rightarrow \textcircled{A}$$



Q-13)  $\frac{1}{D^4 - m^4} \cdot \cos mx$

$\rightarrow \frac{1}{(D^2 - m^2)(D^2 + m^2)} \cdot \cos mx$

$= \frac{1}{(D^2 + m^2)} \cdot \left[ \frac{1}{(D^2 - m^2)} \cdot \cos mx \right]$

$a = m, D^2 \rightarrow -m^2$   
 $= \frac{1}{(D^2 + m^2)} \cdot \left[ \frac{\cos mx}{-2m^2} \right]$

$= -\frac{1}{2m^2} \left[ \frac{1}{(D^2 + m^2)} \cdot \cos mx \right]$

$= -\frac{1}{2m^2} \left[ \left( \frac{-x}{2(m)} \right) \cdot \frac{1}{1!} \cdot \cos \left( mx + \frac{\pi}{2} \right) \right]$

$= -\frac{1}{2m^2} \left[ \frac{-x}{2m} \cdot \left[ \cos mx \cdot \cos \left( \frac{\pi}{2} \right) - \sin(mx) \cdot \sin \left( \frac{\pi}{2} \right) \right] \right]$

$= -\frac{1}{2m^2} \left[ \frac{-x}{2m} \left[ -\sin(mx) \right] \right]$

$= \boxed{\frac{-x \sin(mx)}{4m^3}}$

$\rightarrow \textcircled{B}$



Q-14.  $\frac{1}{D^3-4D} \cdot 2 \cosh(2x)$

$$\rightarrow \frac{1}{D(D+2)(D-2)} \cdot 2 \left[ \frac{e^{2x} + e^{-2x}}{2} \right]$$

$$= \frac{1}{D(D+2)} \left[ \frac{1}{(D-2)} \cdot e^{2x} \right] + \frac{1}{D(D-2)} \left[ \frac{1}{(D+2)} \cdot e^{-2x} \right]$$

$$= \therefore \frac{1}{2(2+2)} \left[ \frac{x}{1!} e^{2x} \right] + \frac{1}{(-2)(-2-2)} \left[ \frac{x}{1!} e^{-2x} \right]$$

$\lambda = 2, h = 1$                        $\lambda = -2, h = 1$

$$= \frac{x e^{2x}}{8} + \frac{x e^{-2x}}{8}$$

$$= \frac{x}{4} \cdot \left[ \frac{e^{2x} + e^{-2x}}{2} \right]$$

$$= \boxed{\frac{x}{4} \cdot \cosh(2x)} \rightarrow \textcircled{C}$$



$$Q-15 \quad \frac{1}{D^2+6D-9} \cdot \sinh(3x)$$

$$\rightarrow \frac{1}{D^2+6D-9} \cdot \left[ \frac{e^{3x} - e^{-3x}}{2} \right]$$

$$= \frac{1}{2} \left[ \frac{1}{(D^2+6D-9)} \cdot e^{3x} \right] - \left[ \frac{1}{(D^2+6D-9)} \cdot e^{-3x} \right]$$

$$= \frac{1}{2} \left[ \frac{1}{8+18-9} \cdot e^{3x} \right] - \left[ \frac{1}{8-18-9} \cdot e^{-3x} \right]$$

$$= \frac{1}{2} \left[ \frac{e^{3x} + e^{-3x}}{18} \right]$$

$$= \frac{1}{18} \left[ \frac{e^{3x} + e^{-3x}}{2} \right]$$

$$= \boxed{\frac{\cosh(3x)}{18}} \rightarrow \textcircled{A}$$

$$Q-16) \quad \frac{1}{D^3+8} (x^4 + 2x + 1)$$

$$\rightarrow \frac{1}{(D+2)(D^2-2D+4)} (x^4 + 2x + 1)$$

$$= \frac{1}{8(1 + \frac{D^3}{8})} (x^4 + 2x + 1)$$

$$t = \frac{D^3}{8}, \quad \frac{1}{1+t} = 1 - t + t^2 - t^3 + t^4 - \dots$$

$$\therefore = \frac{1}{8} \left[ (1) - \left( \frac{D^3}{8} \right) + \left( \frac{D^3}{8} \right)^2 - \left( \frac{D^3}{8} \right)^3 + \left( \frac{D^3}{8} \right)^4 \right] (x^4 + 2x + 1)$$



$$= \frac{1}{8} \left[ 1 - \frac{D^3}{8} + \frac{D^6}{8^4} - \frac{D^9}{8^3} \right] (x^4 + 2x + 1)$$

$$= \frac{1}{8} \left[ (x^4 + 2x + 1) - \frac{(24x)}{8} \right]$$

$$= \frac{1}{8} [x^4 - x + 1] \rightarrow \textcircled{D}$$

Q-17)  $\frac{1}{(D^4 + D^2 + 1)} \cdot (53x^2 + 17)$

$\rightarrow \frac{1}{[1 + (D^4 + D^2)]} \cdot (53x^2 + 17)$

$t = D^4 + D^2 \quad \therefore \frac{1}{1+t} = 1 - t + t^2 - \dots$

$$= [1 - (D^4 + D^2) + (D^4 + D^2)^2] (53x^2 + 17)$$

$$= [1 - \cancel{D^4} - D^2 + D^8 + \cancel{D^4} + 2D^6] (53x^2 + 17)$$

$$= [D^8 + 2D^6 - D^2 + 1] (53x^2 + 17)$$

$$= [-106 + 53x^2 + 17]$$

$$= \boxed{53x^2 - 89} \rightarrow \textcircled{B}$$

Q-18)  $\frac{1}{(D^2 - D + 1)} \cdot (3x^2 - 1)$

$\rightarrow t = D^2 - D \quad , \quad \frac{1}{1+t} = 1 - t + t^2 - \dots$

$$= [1 - (D^2 - D) + (D^2 - D)^2] (3x^2 - 1)$$

$$= [1 - \cancel{D^2} + D + D^4 + \cancel{D^2} - 2D^3] (3x^2 - 1)$$

$$= [D^4 - 2D^3 + D + 1] (3x^2 - 1)$$

$$= [6x + 3x^2 - 1]$$

$$= \boxed{3x^2 + 6x - 1} \rightarrow \textcircled{C}$$



Q-19)  $\frac{1}{D^2-1} \cdot x^3$

$\rightarrow \frac{-1}{(1-D^2)} \cdot x^3$

$\therefore t = D^2, \frac{1}{1-t} = 1+t+t^2+t^3+\dots$

$\therefore = -1 \left[ 1 + D^2 + (D^2)^2 + (D^2)^3 \right] x^3$

$= -1 \left[ 1 + D^2 + D^4 + D^6 \right] x^3$

$= - \left[ x^3 + 6x \right]$

$= \boxed{-x^3 - 6x}$

$\rightarrow \textcircled{D}$



# Tutorial three

Total points 40/40



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✓ Q zero \*

2/2

$$\frac{1}{D^2 - 7D + 6} e^{2x} =$$

$$\frac{-e^{2x}}{4}$$

☒ Option 1



$$\frac{e^{2x}}{4}$$

☐ Option 2