

Number Systems

(Decimal, Binary, Octal, Hexadecimal)

NUMBER SYSTEM

➤ A number system defines how a number can be represented using distinct symbols. A number can be represented differently in different systems.

➤ Many number systems were introduced with the passage of time, like:

- Decimal number system
- Binary number system
- Octal number system
- Hexadecimal number system

Decimal Number (base or radix 10)

We use the decimal numbering system as a default, so when you see a number 56 your assumption is that its base or radix is 10 or $(56)_{10}$ which is “56 base 10”.

Each digit is weighted based on its position in the sequence (power of 10) from the Least Significant Digit (LSD, power of 0) to the Most Significant Digit (MSD, highest power).

Each digit must be less than 10 (0 to 9)

- For example $(2375.46)_{10}$ is evaluated as:

	MSD						LSD
Digit notation	d_3	d_2	d_1	d_0	.	d_{-1}	d_{-2}
Digit	2	3	7	5	.	4	6
Value	10^3	10^2	10^1	10^0		10^{-1}	10^{-2}
Results=Value*Digit	2000	300	70	5		0.4	0.06

$$\begin{aligned}(2375.46)_{10} &= 2 \times 10^3 + 3 \times 10^2 + 7 \times 10^1 + 5 \times 10^0 + 4 \times 10^{-1} + 6 \times 10^{-2} \\ &= 2000 + 300 + 70 + 5 + 0.4 + 0.06\end{aligned}$$

Binary Number (base or radix 2)

- Digital and computer technology is based on the binary number system, since the foundation is based on a transistor, which only has two states: on or off.
- Each digit of the number is called a bit
- An 8-bit group - a Byte
- An 4-bit group - a nibble
- Each bit is weighted based on its position in the sequence (powers of 2) from the Least Significant Bit (LSB) to the Most Significant Bit (MSB).
- Each bit must be less than 2 which means it has to be either 0 or 1.

For example $(1010.11)_2$ is evaluated as:

	MSB			LSB		
Digit notation	b_3	b_2	b_1	b_0	b_{-1}	b_{-2}
Digit	1	0	1	0	1	1
Value	2^3	2^2	2^1	2^0	2^{-1}	2^{-2}
Results=Value*Digit	8	0	2	0	0.5	0.25

$$(1010.11)_2 = 8 + 0 + 2 + 0 + 0.5 + 0.25 = (10.75)_{10}$$

Octal Number System

Each Octal number can be represented using only 3 bits, with each group of bits having a distinct values between 000 (for 0) and 111(for 7). The equivalent binary number of Octal number are as given below –

Octal Digit Value	Binary Equivalent
0	000
1	001
2	010
3	011
4	100
5	101
6	110
7	111

- The octal number system consists of 8 digits i.e. 0 to 7 with the base 8.
- 1 octal digit is equivalent to 3 bits.

Examples:

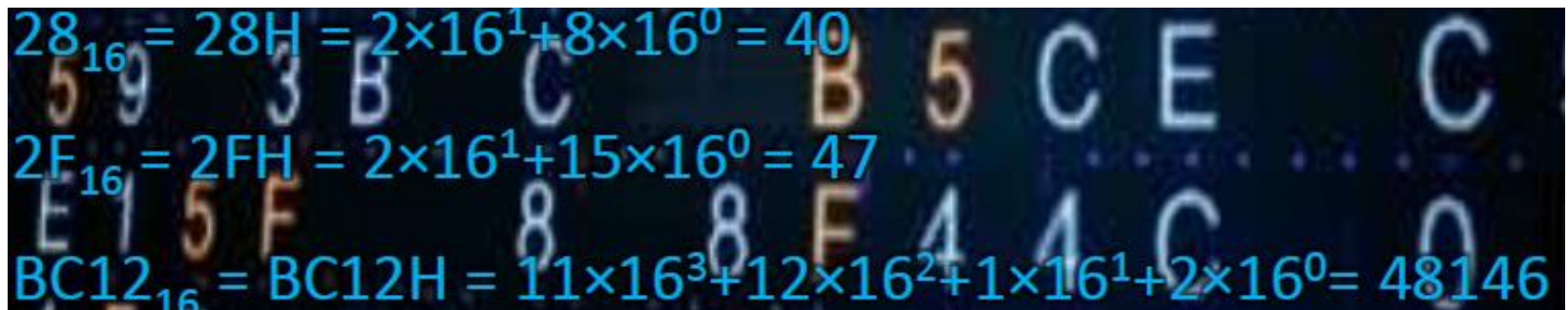
$$27_8 = 2 \times 8^1 + 7 \times 8^0 = 16 + 7 = 23$$

$$30_8 = 3 \times 8^1 + 0 \times 8^0 = 24$$

$$4307_8 = 4 \times 8^3 + 3 \times 8^2 + 0 \times 8^1 + 7 \times 8^0 = 2247$$

Hexadecimal Number system

- The prefix 'hexa' stands for 6 and the prefix 'deci' stands for 10.
- This number system contains 16 systems and therefore has the base 16.
- It uses the digits (0-9) and (A-F).
- Numbers are 0, 1, 2...8, 9, A, B, C, D, E, F. B is 11, E is 14.
- 1 Hex digit is equal to 4 bits.
- Each place value in a hexadecimal number is a power of 16.
- Examples:



The image displays three examples of hexadecimal conversions, each with a blue text overlay on a dark background featuring floating hexadecimal digits. The examples are:

- $28_{16} = 28H = 2 \times 16^1 + 8 \times 16^0 = 40$
- $2F_{16} = 2FH = 2 \times 16^1 + 15 \times 16^0 = 47$
- $BC12_{16} = BC12H = 11 \times 16^3 + 12 \times 16^2 + 1 \times 16^1 + 2 \times 16^0 = 48146$