Kaustubh Kabra SE Comp-I, A, 20.

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* Tutorial Number - 6 (Application of DE)

1. A resistance of 50 ohms, an inductor of 2 henries and farad capacitor are all in series with an emf of 40 volts. Find the charge and current after circuit is closed at $t=0$ assuming that at that time the charge on the capacitor is 4 coloumb.

→ Given:- Resistance $(R) = 50\ \Omega$

Inductor $(L) = 2\ H$.

Capacitor $(C) = 1\ F$.

At $t=0$, $Q = 4$ coloumb, $I = 0$.

We know,

$$L\frac{d^2Q}{dt^2} + R\frac{dQ}{dt} + \frac{Q}{C} = E$$

Let $\frac{d}{dt} = D$

$$\therefore (2D^2 + 50D + 1)Q = 40.$$

$\phi(D) = 2D^2 + 50D + 1$ $\qquad f(t) = 40e^{0t}$

↳ roots $= \frac{-25 \pm \sqrt{623}}{2}$

$= -24.979,\ -0.002$

$$Q_c = C_1 . e^{-24.979t} + C_2 e^{-0.002t}$$

$$Q_P = \frac{1}{2D^2+50D+1} . 40.e^{0t}$$

$a=0,\ D \to 0.$

$$= \frac{1}{2(0)^2+50(0)+1} . 40$$

$$\boxed{\begin{aligned} Q &= C_1 . e^{-24.979t} + C_2 e^{-0.002t} + 40. \\ I &= -24.979C_1 e^{-24.979t} - 0.002C_2 e^{-0.002t} \end{aligned}}$$

Q-2. An electrical current circuit consist of an inductance 1 H, a resistance R of 20 Ω, and a condencer of capacitance C of 25 microfarads. If the differential equation of the electrical circuit is $L\frac{d^2Q}{dt^2}+R\frac{dQ}{dt}+\frac{Q}{C}=0$, find the current at time t given that at $t=0$, $q=0.025$ coloumbs, $\frac{dQ}{dt}=0$.

→ Given:- Inductance (L) = 1 H.

Resistance (R) = 20 Ω

Capacitance (C) = 25×10^{-6} F.

at $t=0$, $q=0.025$ coloumb, $\frac{dQ}{dt}=0$.

We know,

$$L\frac{d^2Q}{dt^2}+R\frac{dQ}{dt}+\frac{Q}{C}=0$$

Let $D=\frac{d}{dt}$ ∴ $\left(D^2+20D+\frac{1}{25\times10^{-6}}\right)Q=0$.

$$\phi(D)=D^2+20D+40000 \quad , \quad f(t)=0\,e^{0t}$$

↳ roots $\frac{-20\pm\sqrt{(-159600)}}{2}$

$$=\frac{-20\pm20\sqrt{399}}{2}=-10\pm10\sqrt{399}$$

$$Q_c=e^{-10t}\left[C_1.\cos(10\sqrt{399}\,t)+C_2.\sin(10\sqrt{399}\,t)\right]$$

$$Q_p=\frac{1}{D^2+20D+40000}.0$$

$$Q=Q_c+Q_p$$

$$\boxed{Q=e^{-10t}\left[C_1.\cos(10\sqrt{399}\,t)+C_2.\sin(10\sqrt{399}\,t)\right)}$$

$$I=-e^{-10t}.C_1\left[-10\cos(10\sqrt{399}\,t)+10\sqrt{399}\sin(10\sqrt{399}\,t)\right]$$
$$+e^{-10t}.C_2\left[-10\sin(10\sqrt{399}\,t)+10\sqrt{399}.\cos(10\sqrt{399}\,t)\right)$$

Q-3. The charge Q on the plate of condenser satisfies the DE $\frac{d^2Q}{dt^2} + \frac{Q}{LC} = \sin\left(\frac{t}{\sqrt{LC}}\right)$

Assuming that $\frac{1}{LC} = w^2$, find the charge Q at time t.

$\rightarrow$ Given :- charge Q satisfies $\frac{d^2Q}{dt^2} + \frac{Q}{LC} = \sin\left(\frac{t}{\sqrt{LC}}\right)$.

$$\Rightarrow \frac{d^2Q}{dt^2} + Q.w^2 = \sin(wt) \quad \ldots \left[\text{as } \frac{1}{LC} = w^2\right]$$

Let $\frac{d}{dt} = D$

$$\therefore \quad D^2Q + Qw^2 = \frac{E}{L}\sin(wt)$$

$$(D^2 + w^2)Q = \frac{E}{L}\sin(wt)$$

$\phi(D) = D^2 + w^2$, $f(t) = \frac{E}{L}.\sin(wt)$

$\hookrightarrow \pm i\sqrt{w}$

$$Q_C = C_1\cos(\sqrt{w}t) + C_2\sin(\sqrt{w}t)$$

$$Q_P = \frac{1}{D^2+w^2}\cdot\frac{E}{L}\sin(wt)$$

$a = w, \; r = 1$

$$= \left[\frac{-t}{2w}\right]^1 \cdot \frac{1}{1!}\cdot\sin\left(wt + \frac{\pi}{2}\right)\cdot\frac{E}{L}$$

$$Q_P = \frac{-E}{2wL}.t.\sin(wt)$$

$$\boxed{\begin{aligned} Q &= C_1\cos(\sqrt{w}t) + C_2\sin(\sqrt{w}t) - \frac{E}{2wL}.t\sin(wt) \\ Q &= C_1\cos\left(\frac{t}{\sqrt{\sqrt{LC}}}\right) + C_2\sin\left(\frac{t}{\sqrt{\sqrt{LC}}}\right) - \frac{E\sqrt{LC}}{2L}.t.\sin\left(\frac{t}{\sqrt{\sqrt{LC}}}\right)\end{aligned}}$$

Q-4. An inductance of 0.5 henries is connected in series with a resistor of $6\,\Omega$ and a capacitance of 0.02 F, a generator having alternative voltage $24 \sin 10t$, $t>0$. Find the charge and current at time t.

→ Given:- Inductance $(L) = 0.5$ H.
Resistor $(R) = 6\,\Omega$
Capacitor $(C) = 0.02$ F
Voltage $(E) = 24 \sin(10t)$

We know,

$$L \frac{d^2Q}{dt^2} + R\frac{dQ}{dt} + \frac{Q}{C} = E.$$

Let $\frac{d}{dt} = D$

$$\therefore \quad 0.5 D^2 Q + 6DQ + \frac{Q}{0.02} = 24 \sin(10t)$$

$$(D^2 + 12D + 100)Q = 48 \sin(10t)$$

$\phi(D) = D^2 + 12D + 100$, $f(t) = 48 \sin(10t)$
↳ roots → $-6 \pm 8i$.

$$Q_c = e^{-6t}[c_1 \cos(8t) + c_2 \sin(8t)]$$

$$Q_P = \frac{1}{D^2+12D+100} . 48 \sin(10t)$$

$a = 10,\ D^2 = -100$

$$= \frac{1}{\cancel{-100} + 12D \cancel{+100}} . 48 \sin(10t)$$

$$= \frac{1}{\cancel{12}D} . \overset{4}{\cancel{48}} \sin(10t)$$

$$= 4 \int \sin(10t)$$

$$Q_P = -\frac{2 \cos(10t)}{5}$$

$Q = Q_c + Q_p$

$$\boxed{Q = e^{-6t}[c_1 \cos(8t) + c_2 \sin(8t)] - \frac{2}{5} \cdot \cos(10t)}$$

$$\text{current} = \frac{dQ}{dt} = \frac{d}{dt}\left[e^{-6t} \cdot c_1 \cos(8t) + e^{-6t} \cdot c_2 \cdot \sin(8t) - \frac{2}{5} \cdot \cos(10t)\right]$$

$$= e^{-6t} \cdot c_1 \cdot \{-\sin(8t)\} \cdot 8 + e^{-6t} \cdot (-6) \cdot c_1 \cos(8t)$$
$$+ e^{-6t} \cdot c_2 \cdot \{\cos(8t)\} \cdot 8 + e^{-6t} \cdot (-6) \cdot c_2 \cdot \sin(8t)$$
$$- \frac{2}{\cancel{5}_1} \{-\sin(10t) \cdot \cancel{10}^2$$

$$\boxed{\begin{aligned} I = \frac{dQ}{dt} &= c_1 e^{-6t}[-8 \cdot \sin(8t) - 6\cos(8t)] \\ &+ c_2 e^{-6t}[8\cos(8t) - 6\sin(8t)] + 4\sin(10t) \end{aligned}}$$

Q-5. An emf of 200 volts is in series with a resistor of 10 Ω, a one henry inductor and 0.02 farad capacitor. At $t = 0$, $Q = I = 0$, Find charge Q and current I at any time t.

→ Given :- Emf (E) = 200 V.

Resistance (R) = 10 Ω

Inductance (L) = 1 H.

Capacitance (C) = 0.02 F

at $T = 0$, $Q = I = 0$

We know,

$$L \frac{d^2Q}{dt^2} + R \frac{dQ}{dt} + \frac{Q}{C} = E$$

Let $\frac{d}{dt} = D$

$$\left(D^2 + 10D + \frac{1}{0.02}\right) Q = 200$$

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$\phi(D) = D^2 + 10D + 50$, $f(t) = 200.$

$\hookrightarrow$ roots $-5 \pm 5i.$

$$Q_c = e^{-5t}(C_1 . \cos(5t) + C_2 . \sin(5t))$$

$$Q_p = \frac{1}{D^2+10D+50} . 200 e^{0t}$$

$a = 0,$

$$= \frac{1}{(0)^2+10(0)+50} . 200$$

$$Q_p = 4$$

$$Q_r = Q_c + Q_p$$

$$Q = e^{-5t}(C_1 \cos(5t) + C_2 . \sin(5t)) + 4.$$

$$I = \frac{dQ}{dt} = e^{-5t} . C_1 [-5 . \cos(5t) - 5\sin(5t)] + e^{-5t} . C_2 [5\sin(5t) + 5\cos(5t)]$$

At $T=0, Q=0$

$$\therefore 0 = e^0 [C_1 \cos(0) + C_2 . \sin(0)] + 4$$

$$0 = C_1 + 4$$

$$\boxed{C_1 = -4}$$

At $T=0, I=0$

$$\therefore 0 = e^0 . C_1 [-5 . \cos(0) - 5\sin(0)] + e^0 . C_2 [5\sin(0) + 5\cos(0)]$$

$$0 = C_1[-5] + C_2[+5]$$

$$0 = +20 + 5C_2$$

$$\boxed{C_2 = -4}$$

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