Q-1. Find a root of the equation $\cos x = x e^{x}$ using the bisection method at the end of sixth iteration.

→ $f(x) = \cos x = x e^{x}$

$f(0) = 1$, $f(1) = -2.18$

Roots lies between 0 and 1.

$$z_1 = \frac{0+1}{2} = 0.5$$

$f(z_1) = 0.05$, roots between 0.5 and 1.

$$z_2 = \frac{0.5+1}{2} = 0.75$$

$f(z_2) = -0.86$, roots between 0.5 and 0.75.

$$Z_3 = \frac{0.5 + 0.75}{2} = 0.625$$

$f(Z_3) = -0.36$, roots between 0.5 and 0.625

$$Z_4 = \frac{0.5 + 0.625}{2} = 0.5625$$

$f(Z_4) = -0.14$, roots between 0.5 and 0.5625

$$Z_5 = \frac{0.5 + 0.5625}{2} = 0.5312$$

$f(Z_5) = -0.041$, roots between 0.5 and 0.5312.

$$Z_6 = \frac{0.5 + 0.5312}{2} = 0.5156$$

$\therefore$ Approximation to the root is $\boxed{0.5156}$

Q-2. Find a real root of equation $x^3 - 2x - 5 = 0$ using secant methods correctly to three decimal places.

$\rightarrow$ $f(x) = x^3 - 2x - 5 = 0$

$f(1) = -7$, $f(2) = -1$, $f(3) = 16$.

$$x_2 = x_1 - \left[\frac{x_1 - x_0}{f_1 - f_0}\right] \cdot f_1 \qquad \begin{array}{l} x_0 = 2 \\ x_1 = 3 \end{array}$$

$$x_2 = 3 - \left[\frac{3-2}{16-[-1]}\right] \cdot 16$$

$$\boxed{x_2 = 2.058823}$$

$$f(x_2) = -0.390799$$

$$x_3 = x_2 - \left[\frac{x_2 - x_1}{f_2 - f_1}\right] \cdot f_2$$

$$\boxed{x_3 = 2.081203}$$

$$f(x_3) = 0.147204$$

$$x_4 = x_3 - \left[\frac{x_3 - x_2}{f_3 - f_2}\right] \cdot f_3$$

$$\boxed{x_4 = 2.094824}$$

$$f(x_4) = 0.003042$$

$$x_5 = x_4 - \left[\frac{x_4 - x_3}{f_4 - f_3}\right] \cdot f_4$$

$$\boxed{x_5 = 2.094549}$$

Root is 2.094 correct to three decimal places.

Q - 3. Find a real root of $2x - \log_{10}^{x} = 7$ correct to fore decimal places using iteration method.

→ $f(x) = 2x - \log_{10}^{x} - 7.$

$f(3) = -1.4471$, $f(4) = 0.398$

∴ roots between 3 and 4.

$$x = \frac{1}{2}(\log_{10}^{x} + 7) = \phi(x) \quad — \quad ①$$

$$d\,\cancel{x} = \frac{1}{2}\left(\frac{1}{x} \cdot \log_{10}^{e}\right) = \phi'(x) \quad — \quad ②$$

and $|\phi'(x)| < 1$ in interval $(3, 4)$.

$f(4) < |f(3)|$, the root is near 4, Hence iteration method can be applied.

$x_0 = 3.6$, $x_1 = \phi(x_0) = \frac{1}{2} \log_{10}^{3.6} + 7 = 3.77815$

$x_2 = \phi(x_1) = 3.78863$

$x_3 = \phi(x_2) = 3.788924$

$x_4 = \phi(x_3) = 3.78927.$

Hence, x_3 and x_4 are almost equal desired root is $\boxed{3.78927}$.

Q-4. Find a positive real root of $x \log_{10}^{x} = 1.2$, Using bisection method at the end of 5^{th} iteration.

$\longrightarrow$ $f(x) = x \log_{10}^{x} - 1.2$

$f(2) = -0.598$, $f(3) = 0.231$

$\therefore$ Roots lies between 2 and 3.

$$Z_1 = \frac{2+3}{2} = 2.5$$

$f(z_1) = 0.205$, root between 2.5 and 3.

$$Z_2 = \frac{2.5+3}{2} = 2.75$$

$f(z_2) = -0.008$, roots between 2.5 and 2.75

$$Z_3 = \frac{2.5+2.75}{2} = 2.625$$

$f(z_3) = -0.099$, roots between 2.5 and 2.625.

$$Z_4 = \frac{2.5+2.625}{2} = 2.5625$$

$f(z_4) = -0.047$, roots between 2.5 and 2.5625

$$Z_5 = \frac{2.5+2.5625}{2} = 2.53125$$

$\therefore$ Desired root is $\boxed{2.53125}$

Q-5. Find by Newton Raphson method $3x - \cos x - 1 = 0$ corred to four decimal places

$\longrightarrow$ $f(x) = 3x - \cos x - 1$

$f'(x) = 3 + \sin x$

$f(0) = -1 - 1 = -2$, $f(1) = 1.4597$.

$\therefore$ Roots lies between 0 and 1.

$f''(x) = \cos x$, $[0, 1]$

$x_0 = 0.6$, $f(x_0) \cdot f''(x_0) > 0$.

By NR formula

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 0.5 - \left[\frac{0.5 - e^{-0.5}}{1 + e^{-0.5}}\right]$$

$$\boxed{x_1 = 0.56631}$$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} \Rightarrow \boxed{x_2 = 0.56714}$$

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} \Rightarrow \boxed{x_3 = 0.56714}$$

As x_2 and x_3 are equal, Hence decimal root is $\boxed{0.5671}$ corrected upto four decimal places