

* Discrete Mathematics (DM) - Assignment Number - 6

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Q-1. What is Monoid? Show that the algebraic structure $(A, +)$ is a monoid, where A is set of integers and $+$ is a binary operation given addition of two integers.

→ Monoid:-

A monoid is a semi-group $(A, *)$ that has an identity element.

Example:- $(A, +)$ is algebraic system, where

A is set of integers and $(+)$ is binary operation.

Let $A = \{ \dots, 0, 1, 2, 3, \dots \}$

Therefore checking properties:-

① Associative Property:-

The operation $(+)$ is an associative property as,
 $(a+b)+c = a+(b+c)$

② Closure Property:-

Operation $(+)$ is a closure as the sum of two integer numbers is integer.

③ Identity Property :- There is identity element in set A , The element 0 is an identity element, that is the operation $(+)$.

$(A, +)$ satisfy all three properties

Therefore,

$(A, +)$ is a Monoid.

Hence Proved.

Q - 2. Define the following :-

- ① Ring
- ② Field
- ③ Integral Domain.

→ 1. Ring -

A ring R is said to be a ring with unit element if there exists an element, denoted by the symbol 1 such that $a \cdot 1 = 1 \cdot a = a$, $a \in R$.

A ring R is said to be a commutative ring if $a \cdot b = b \cdot a$ for all $a, b \in R$.

Example: - 1. The integers with the usual addition and multiplication operations. These form a commutative ring in which multiplication is commutative.

2. The integers modulo m with addition and multiplication modulo m .

2. Field :-

A field is a set F which is closed under two operations $(+)$ and $(*)$ such that

① F is an abelian group under $(+)$.

② $F - \{0\}$ is an abelian group under $(*)$.

Example:-

1. $(\mathbb{Q}, +)$
2. $(\mathbb{R}, *)$
3. $(\mathbb{Z}, +)$

3. Integral Domain:-

A non-trivial ring with unity is said to be an integral domain if it is commutative and contains no divisor of zero.

Example:-

1. $(\mathbb{Z}, +)$
2. $(\mathbb{R}, +)$
3. $(\mathbb{Q}, +)$