Total No. of Questions—8] [Total No. of Printed Pages—5

Seat No.	

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S.E. (Comp. & IT) (Second Semester) EXAMINATION, 2019

ENGINEERING MATHEMATICS—III

(2015 PATTERN)

Time : Two Hours **Maximum Marks : 50**

***N.B.* :—** (*i*) Neat diagrams must be drawn wherever necessary.

(*ii*) Figures to the right indicate full marks.

(*iii*) Use of electronic pocket calculator is allowed.

(*iv*) Assume suitable data, if necessary.

1. (*a*) Solve any *two* differential equations : [8]

(*i*) $\dfrac{d^2y}{dx^2} + 7\dfrac{dy}{dx} - 2y = e^{4x} \cosh 2x$

(*ii*) $x^2 \dfrac{d^2y}{dx^2} - 3x\dfrac{dy}{dx} + 3y = x^2 \sin(\log x)$

(*iii*) $\dfrac{d^2y}{dx^2} - 8\dfrac{dy}{dx} + 16y = \dfrac{e^{4x}}{x^6}$, by using the method of variation of parameters.

(*b*) Solve the integral equation : [4]

$$\int_0^\infty f(x) \sin \lambda x \; dx = \begin{cases} 1-\lambda, & 0 \le \lambda \le 1 \\ 0, & \lambda > 1 \end{cases}$$

P.T.O.

Or

2. (*a*) A capacitor of 10^{-3} farads and inductor of (0.4) henries are connected in series with an applied emf 20 volts in an electrical circuit. Find the current and charge at any time t. [4]

(*b*) Solve any *one* of the following : [4]

(*i*) Obtain $z[k\,e^{-k}],\ k \geq 0$

(*ii*) Obtain $z^{-1}\left[\frac{8z}{(z-1)(z-2)}\right], |z| > 2, k \geq 0$.

(*c*) Solve the difference equation : [4]

$$y_{k+1} + \frac{1}{2} y_k = \left(\frac{1}{2}\right)^k$$

where $y_0 = 0,\ k \geq 0$.

3. (*a*) The first three moments of a distribution about the value 2 are 1, 16 and –40. Find the first three central moments, standard deviation and β_1. [4]

(*b*) Fit a straight line of the form X = aY + b to the following data by the least square method : [4]

X	**Y**
2	2
5	3
8	4
11	5
17	7
20	8

(*c*) On an average, there are 2 printing mistakes on a page of a book. Using Poisson distribution, find the probability that a randomly selected page from the book has at least one printing mistake. [4]

Or

4. (*a*) 200 students appeared for an examination. Average marks were 50% with standard deviation 5%. How many students are expected to score at least 60% marks assuming that marks are normally distributed. [Given : Z = 2, A = 0.4772]. [4]

(*b*) On an average, a box containing 10 articles is likely to have 2 defectives. If we consider a consignment of 100 boxes, how many of them are expected to have at the most one defective ? [4]

(*c*) Find the regression equation of Y on X for a bivariate data with the following details. $n = 25,\ \sum_{i=1}^{n} x_i = 75,\ \sum_{i=1}^{n} y_i = 100,$ $\sum_{i=1}^{n} x_i^2 = 250,\ \sum_{i=1}^{n} y_i^2 = 500,\ \sum_{i=1}^{n} x_i y_i = 325.$ [4]

5. (*a*) Find the directional dervative of $\phi(x, y, z) = xy^3 + yz^3$ at the point (2, –1, 1) in the direction of vector $\bar{i} + 2\bar{j} + 2\bar{k}$. [4]

(*b*) Show that $\bar{F} = (x^2 - yz)\bar{i} + (y^2 - zx)\bar{j} + (z^2 - xy)\bar{k}$ is irrotational. Hence find the scalar potential ϕ such that $\bar{F} = \nabla\phi$. [4]

(*c*) Evaluate $\oint_C \bar{F} \cdot d\bar{r}$ where $\bar{F} = \sin z\, \bar{i} + \cos x\, \bar{j} + \sin y\, \bar{k}$ and C is the boundary of the rectangle $0 \leq x \leq \pi$ and $0 \leq y \leq 1$ and $z = 3$. [5]

Or

6. (*a*) Show that (any *one*) : [4]

(*i*) $\nabla \cdot \left[r \nabla \left(\frac{1}{rn} \right) \right] = \frac{n(n-2)}{r^{n+1}}$

(*ii*) $\nabla^2 f(r) = f''(r) + \frac{2}{r} f'(r)$.

(*b*) Find the directional derivative of $\phi = xy^2 + yz^3$ at (1, –1, 1) towards the point (2, 1, –1). [4]

(*c*) If : [5]

$$\bar{F} = (2xy + 3z^2)\bar{i} + (x^2 + 4yz)\bar{j} + (2y^2 + 6xz)\bar{k}$$

Evaluate :

$$\int_C \bar{F} \cdot d\bar{r}$$

where C is the curve $x = t,\ y = t^2,\ z = t^3$ joining the points (0, 0, 0) and (1, 1, 1).

7. (*a*) Determine the analytic function $f(z) = u + iv$ if $u = 4xy - 3x + 2$. [4]

(*b*) Find the bilinear tranformation which maps the point $z = i, -1, 1$ into the point $w = 0, 1\ \infty$. [4]

(*c*) Evaluate :

$$\int_C \frac{3z+4}{(z-1)(z-2)}\,dz,$$

where C is the circle $|z-1| = \frac{3}{2}$. [5]

Or

8. (*a*) Determine the analytic function $f(z) = u + iv$ if $u = x^2 - y^2 - 2xy - 2x - y - 1$. [4]

(*b*) Under the transformation $w = \frac{1}{z}$, find the image of $|z - 3i| = 3$. [4]

(*c*) Evaluate :

$$\int_C \frac{z\,dz}{(z-1)(z-3)}$$

where C is the circle $|z| = \frac{3}{2}$. [5]