Q-1. Find value of y for $x = 0.5$ for the following set of value of x and y using Newton's forward differeke formula.

x	0	1	2	3	4
y	1	5	25	100	250

→

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
0	y_1				
		$5-1=4$			
1	5		$20-4=16$		
		$25-5=20$		$55-16=39$	
2	25		$75-20=55$		$20-35=-15$
		$100-25=75$		$75-55=20$	
3	100		$150-75=75$		
		$250-100=150$			
4	250				

$h=1$, $x=0.5$, $x_0=0$, $y_0=1$.

$$u = \frac{x-x_0}{h} = \frac{0.5-0}{1} = 0.5.$$

$$\therefore y = y_0 + u\Delta y_0 + \frac{u(u-1)}{2!}\Delta^2 y_0 + \frac{u(u-1)(u-2)}{3!}\Delta^3 y_0 + \cdots$$

$$\therefore y = 1 + 0.5\times 4 + \frac{0.5(0.5-1)}{2}\times 16 + \frac{0.5(0.5-1)(0.5-2)}{6}\times 39$$
$$+ \frac{0.5(0.5-1)(0.5-2)(0.5-3)}{24}\times(-19)$$

$$\boxed{y = 4.1796}$$

$\therefore$ At $x=0.5$, y is 4.1796.

Q-2. A curve is drawn to pass through the points given by the following table.

x	1	1.5	2	2.5	3	3.5	4
y	2	2.4	2.7	2.8	3	2.6	2.1

Estimate the area bounded by the curve the x-axis and ordinates of $x=1$ and $x=4$.

→ Area A is given by $A = \int_1^4 y \cdot dx$

Employing Simpson's $\frac{1}{3}$rd rule, $h = 0.5$

$$A = \frac{0.5}{3}\left[(2+2.1) + 4(2.4 + 2.8 + 2.6) + 2(2.7+3)\right]$$

$$= \frac{0.5}{3}\left[(4.1) + 4(7.8) + 2(5.7)\right]$$

$$= 7.8.$$

$\therefore$ **Area = 7.8**

Q-3. Solve the equation :- $\frac{dy}{dx} = y - x.$

by second order Range-Kutta method subject to order, $y(0) = 2$, and calculate y at $x = 0.2$ taking $h = 0.1$

→ $f(x,y) = y - x$, $x_0 = 0$, $y_0 = 2$, $h = 0.1$

Step ①:- $k_1 = h f(x_0, y_0) = h(y_0 - x_0) = 0.2$

$k_2 = h f(x_0 + h, y_0 + h) = h(y_0 + h - x_0 - h) = 0.21$

$k = \frac{1}{2}(k_1 + k_2) = 0.205$

$y_{x=0.1} = y_1 = y_0 + k = 2.205$

Step ② for y at $x=0.2$, $x_0=0.1$, $y_0=2.205$, $h=0.1$.

$$k_1 = h(x_0, y_0) = 0.2105$$

$$k_2 = 0.22155$$

$$k = \frac{1}{2}(k_1 + k_2) = 0.216025$$

$$\therefore y_{x=0.2} = y_0 + k = 2.421025$$

$$\boxed{y_{x=0.2} = 2.421025}$$

Q-4. Using forth order Runge-Kutta method evaluate the value of y when $x=1.1$ given that

$$\frac{dy}{dx} + \frac{y}{x} = \frac{1}{x^2}, \quad y(1) = 1.$$

$\rightarrow$ $f(x,y) = \frac{1-xy}{x^2}$

$f(x_0, y_0) = 0.$

$k_1 = h f(x_0, y_0) = 0.1(0) = 0$

$k_2 = h f[(x_0+h)\ (y_0+h)] = 0.00454.$

$k_3 = -0.00432.$

$$k_4 = -0.00788$$

$$k = \frac{1}{6}[k_1 + 2k_2 + 2k_3 + k_4]$$

$$k = -0.0042667$$

$$\therefore y_{x=1.1} = 1 + (-0.0042667)$$

$$\boxed{y_{x=1.1} = 0.9957}$$

Q-5. Forth order Runge-Kutta method, $\frac{dy}{dx} = x^2 + y^2$, $y(0) = 0$, Estimate $y(0.2)$, $y(0$

$\longrightarrow$ $f(x, y) = x^2 + y^2$, $h = 0.2$

$$k_1 = f(x_0, y_0) = 0.$$

$$k_2 = 0.01$$

$$k_3 = 0.01$$

$$k_4 = 0.04$$

$$k = \frac{1}{6}[k_1 + 2k_2 + 2k_3 + k_4]$$

$$k = 0.015$$

$$y_{x=0.2} = 0 + k = 0.015$$

Second iteration

$$x_1 = 0.2 \quad , y_1 = 0.002667.$$

$$k_1 = 0.04$$

$$k_2 = 0.09004$$

$$k_3 = 0.090136$$

$$k_4 = 0.160428$$

$$k = 0.093346$$

$$\therefore \boxed{y_{x=0.4} = 0.02136\ 0224.}$$