Q-1. A fair coin is tossed 600 times. Using normal distribution, find the probability of getting : i) Number of heads less than 270.
ii) Number of heads between 280 to 360.

→ A fair coin tossing 600 times results into head or tail which is $P = 0.5$

Let x = Number of heads in 600 tosses

$$x \rightarrow B(600, 0.5)$$

$$\mu = E(x) = np = 600 \times 0.5 = 300$$

$$\sigma^2 = Var(x) = npq = 600 \times 0.5 \times 0.5 = 150$$

i) P(Number of heads less than 270)

$$P(x < 270) = P\left(\frac{x-\mu}{\sigma} < \frac{270-30}{\sqrt{150}}\right) P\left(\frac{x-np}{\sqrt{npq}} < \frac{270-300}{\sqrt{150}}\right)$$

$$= P(z < -2.4495)$$

$$= P(z > 2.4495)$$

$$\therefore \boxed{P(x < 270) = 0.0071428}$$

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ii) P(Number of heads are between 280 and 350)

$$P(280 < x < 350) = P\left(\frac{280-300}{\sqrt{150}} < z < \frac{350-300}{\sqrt{150}}\right)$$

Using $z = \frac{x-\mu}{\sigma} \rightarrow N(0,1)$, we get

$$P \simeq P(-1.633 < z < 4.0823) = B = 1 - A - C.$$

$$= 1 - P(z < -1.633) - P(z > 4.0823)$$

$$= 1 - P(z > 1.633) - P(z > 4.0823)$$

$$= 1 - 0.51551 - 0.000022518$$

$$\boxed{\therefore P(280 < x < 350) = 0.4845}$$

Q-2. In a certain examination test, 2000 students in a subject of statistics. Average marks obtained were 50% with standard deviation 5%. How many student do you expect to obtain more than 60% of marks supposing that marks are distributed normally. [Nov. 2017]

→ Given:- $\mu = 0.5$, $\sigma = 0.05$

$x_1 = 0.6$ $\quad z_1 = \frac{0.6-0.5}{0.05} = 2.$

∴ Corresponding of $z = 2$ is 0.4772

$$\therefore P(x \geq 6) = P(z \geq 2) = 0.5 - 0.4772$$

$$= 0.0228.$$

∴ Number of student expected to get more than 60% marks $= 0.0228 \times 2000$

$\boxed{= 48 \text{ students approx}}$

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Q-3. Assuming that the diameters of 1000 brass plugs takes consecutively from machine form a normal distribution with mean 0.7515 cm and standard deviation 0.0020 cm. How many of the plugs are likely to be approved if the accetable diameter is 0.752 ± 0.004?

→ Given :- $\sigma = 0.0020$ cm

$\mu = 0.7515$

$x_1 = 0.752 + 0.004 = 0.756$

$x_2 = 0.752 - 0.004 = 0.748$

Using $z = \frac{x-\mu}{\sigma}$, we have.

$$Z_1 = \frac{x_1 - \mu}{\sigma} = \frac{0.756 - 0.7515}{0.0020} = 2.25$$

$$Z_2 = \frac{x_2 - \mu}{\sigma} = \frac{0.748 - 0.7515}{0.0020} = -1.75$$

A_1 corresponding to $(Z_1 = 2.25) = 0.4878$

A_2 corresponding to $(Z_2 = -1.75) = 0.4099$

$$P(0.740 < x < 0.756) = A_1 + A_2$$
$$= 0.4878 + 0.4099$$
$$= 0.9477.$$

∴ Number of plugs likely to be approved $= 1000 \times 0.9477$

$= \boxed{948 \text{ approx.}}$

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Q-4. In a certain factory turning out razor blades, there is a small chance of 1/500 for any blade to be defective. The blades are supplied in a packet of 10. Use possion distribution to calculate the approximate number of packets containing no defective and two defective blades, in a consignment of 10000 packets. [May 2018]

→ Here, $P = 0.002$, $n = 10$, $z = np = 0.02$

$$P(\text{no - defective}) = P(r=0) = \frac{e^{-0.02}(0.02)^0}{0!} = \frac{1}{e^{0.02}}$$
$$= 0.9801$$

$$P(\text{2 - defective}) = P(r=2) = \frac{e^{-0.02}(0.02)^2}{2} = 0.0001960$$

∴ Number of packets containing no defective blades in consignment of 10000 packets

$$= 10000 \times 0.9801 = \boxed{9802 \text{ packets}}$$

∴ Number of packets containing two defective blades in consignment of 10000 packets

$$= 10000 \times 0.0001960$$
$$= 2 \text{ packets (approx).}$$

Q - 5. A manufacturer of cotter pins knows that 2% of his product is defective. If he sells cotter pins in boxes of 100 pins and garantees that not more than 5 pins will be defective in a box. Find the approximate probability that a box will fail to meet the gauranteed quality.

→ Here, $n = 100$

$p \rightarrow$ the probability of defective pins $= \frac{2}{100} = 0.02$.

$z =$ mean number of defective pins in a box

$z = np = 100 \times 0.02 = 2$.

Since p is small, we can use poission distribution.

$$P(r) = \frac{e^{-z} z^{r}}{r!}$$
$$= \frac{e^{-2} (2)^{r}}{r!}$$

$$P(r > 5) = 1 - P(r \leq 5)$$
$$= 1 - \sum_{r=0}^{5} \frac{e^{-2} (2)^{r}}{r!} = 1 - e^{-2} \sum_{r=0}^{5} \frac{(2)^{r}}{r!}$$
$$= \boxed{0.0165}$$

∴ Probability that a box will fail $= 0.0165$.