PHY208 Experiment 5

Kaustubh Singh

April 18, 2022

Verifying Malu's Law and Measuring the Specific Rotation of a Sugar Solution

1   Aim

We will verify Malus' Law by using a photodetector, and a laser. We will also measure the specific rotation of a sugar solution by using a half shade polarimeter.

2   Apparatus

For Malus' law

1. Laser, He-Ne 1.0 mW, 220 V AC

2. Optical profile bench

3. Polarizer

4. Photoelement

5. Digital multimeter

For measuring the specific rotation of a sugar solution

1. 100 cc beaker

2. Distilled water

3. Sugar

4. Laurent Half Shade Polarimeter

5. Sodium Lamp

6. Weight Measuring Device

3 Formulae

Malus' law states that the intensity of polarized light after passing through a polarizer, as a function of the angle of tilt with respect to the plane of polariztion of the incident light is given by

$$I = I_0 \cos^2 \theta \tag{1}$$

Here I_0 is the intensity of the incident light beam, and θ is the angle by which the polarizer is tilted from the angluar position when the intensity of the incident light is maximum.

The specific rotation of an optically active substance is given by

$$[\alpha]_\lambda^T = \frac{\alpha}{LC} \tag{2}$$

where α is the angle by which the plane of polarization rotates when it is passed through a tube of length L which has the optically active material at concentration C. Note that the specific rotation depends upon the wavelength of light λ, and the temprature T, so it is important that these are held constant during the experiment.

4 Theory

4.1 Malus' Law

Malus' law gives the intensity of polarized light after it has passed through a polarizer, as a function of the tilt of the polarizer with the plane of polarization of the incoming light beam.
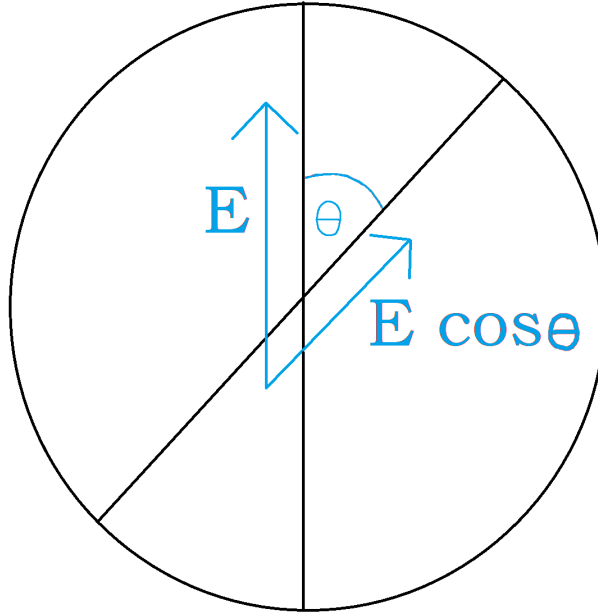
Figure 1 Intensity of Polarized Light after Passing Through a Polarizer

In Fig 1, we see an incoming electromagnetic wave with magnitude of electric field as E, has its magnitude decreased to $E\cos\theta$ after it has passed through the polarizer, when the plane of polarization of the incoming light is tilted at an angle of θ with respect to the axis of polarization. Since the intensity of light is proportional to the maximum magnitude of the electric field which it is composed of, we have

$$I_0 = kE^2$$

where k is a constant, and I_0 is the intensity of the incoming light.

After passing through the polarizer, the magnitude of the electric field becomes $E\cos\theta$, so the intensity becomes

$$I(\theta) = kE^2 \cos^2\theta$$

which after substitution gives Malus' law

$$I(\theta) = I_0 \cos^2\theta$$

4.2   Specific Rotation and Polarimeter

Certain soutions change the plane of polarization of polarized light as it passed through them. Sugar solution is one such example of a solution. The angle by which the plane of polarization of a given monochromatic light, at

3

a certain temprature rotates is proportional to the distance which the light covers inside the solution and the concentration of the solution.

$$\alpha \propto LC$$

Where the variables are the same as those in Eq 2. The constant of proportionality is called the specific rotation of that solution. It is the angle by which the plane of polarization of the monochromatic light rotates when it passed through unit length of that solution when it has unit concentration. Mathematically,

$$\alpha = [\alpha]_\lambda^T LC$$

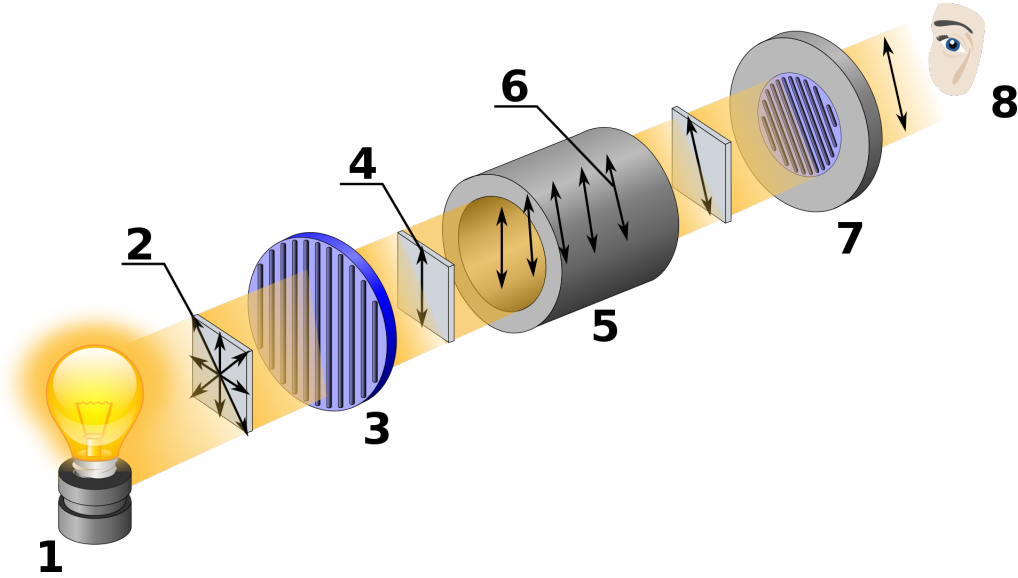A polarimeter is an instrument which is used to measure the specific rotation of a solution.

Figure 2 Schematic of a Polarimeter

As can be seen above, there are two polarizers in a polarimeter. The first one (labeled 3) is just called the polarizer, and the second one (labeled 7) is called the analyser. The optically active solution is placed in between the polarizer and the analyser. The solution would tilt the plane of polarization of the light that passed through the polarizer. This setup can be used to measure the angle by which the plane of polarization has been tilted (by tilting the analyser until the intensity of observed light is maximum). In a Laurent Half Shade Polarimeter, the polarizer (labeled 3 in Fig 2) is half shaded, that is it behaves as if it is composed of two polarizers which are half circles, and have their axis of polarization perpendicular to each other, as shown below.
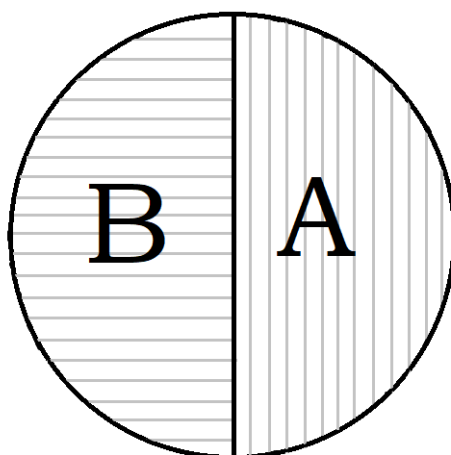
Figure 3 Laurent Half Shade

In Fig 3, the half labeled A has its axis of polarization vertically oriented, and the axis of polarization of the half labeled B is horizontally oriented.

As a result of this peculiar arrangement, the intensity of the light coming from the two halves would usually be different. However, when the axis of the analyser is at an angle which is an integer multiple of 45°with respect to the line adjoining the two halves of the half shade, the intensity from both the halves would be equal. There would be 4 such distinct angular positions, as shown below.
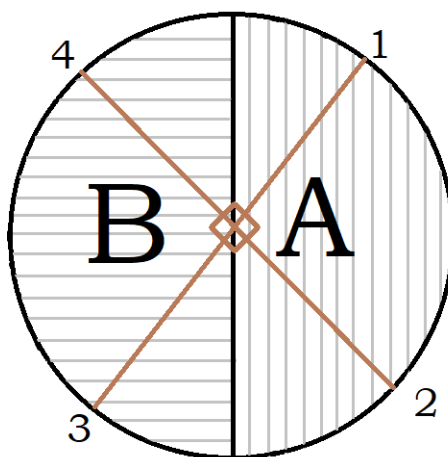

Figure 4 Laurent Half Shade

Note that when the concentration of the optically active substance is changed, the brown cross would rotate. It was noticed that at two of these angular

positions (180°apart), the light observed was more intense than that at the other two angular positions. Taking measurments at the less intense angular positions was found to be easier. The half shades are useful as it is easier to compare the intensity of two halves than to tell when the intensity of a uniformly bright circle is maximum.

5 Observation Tables

5.1 Malus' Law

Angle	Current	Angle	Current	Angle	Current	Angle	Current
0	13.9	90	6.3	180	13.7	280	3.3
5	14.4	95	4.7	185	14.3	285	1.9
10	14.7	100	3.1	190	14.7	290	0.9
15	14.9	105	1.8	195	14.9	295	0.3
20	15.1	110	0.9	200	15.0	300	0
25	15.2	115	0.2	210	15.1	305	0
30	15.3	120	0	220	15.2	310	0.5
35	15.3	125	0.1	225	15.1	315	1.3
40	15.3	130	0.6	230	15.0	320	2.1
45	15.2	135	1.5	235	14.8	325	3.6
50	15	140	2.6	240	14.0	330	65.1
55	14.7	145	3.9	245	13.4	335	6.7
60	14.4	150	5.5	250	12.2	340	8.4
65	13.7	155	7.0	255	10.8	345	10.1
70	12.8	160	8.4	260	9.2	350	11.7
75	11.3	165	9.9	265	7.7	355	13.1
80	9.8	170	11.5	270	6.1		
85	8.0	175	12.8	275	4.7		

The angles are in degrees and the current is in mA. The minimum of 0 mA current occurs at 120°.

5.2 Specific Rotation

These measurments are the compilation of readings taken over a week, which is the reason for the irregular number of readings for some concentrations. 10 MSR (1°) corresponded to 9 VSR. Hence the least count is 0.1°.

The temprature of the room in which the experiment was conducted was 28°C. The length of the polarimeter tube was 20 ± 0.1 cm. The least count of

the beaker in which the sugar was mixed was 1 cc. The error while weighing the sugar was 0.05 g. A sodium lamp was used as the light source.

5.2.1 Distilled Water

S.no.	Distilled Water			
	Position 1		Position 2	
	MSR	VSR	MSR	VSR
1	150	1	331	4
2	149	3	330	2
3	149	2	330	0
4	150	1	331	2
5	150	8	330	0
6	151	2	328	7
7	149	1	331	3
8	150	1	329	0

5.2.2 0.0199 g/cc

S.no.	0.0199 g/cc Concentration			
	Position 1		Position 2	
	MSR	VSR	MSR	VSR
1	152	5	332	7
2	152	4	333	4
3	153	6	333	2
4	151	5	332	8
5	152	2	333	3

5.2.3 0.04 g/cc

S.no.	0.04 g/cc Concentration			
	Position 1		Position 2	
	MSR	VSR	MSR	VSR
1	155	5	335	2
2	155	5	335	7
3	154	8	335	0
4	155	2	335	3
5	156	0	335	7

5.2.4   0.08 g/cc

S.no.	0.04 g/cc Concentration			
	Position 1		Position 2	
	MSR	VSR	MSR	VSR
1	160	0	340	3
2	160	0	339	0
3	160	1	339	0
4	160	0	339	3
5	159	6	340	0
6	159	2	339	0
7	159	3	339	6
8	159	0	339	4
9	159	5	339	4

5.2.5   0.099 g/cc

S.no.	0.099 g/cc Concentration			
	Position 1		Position 2	
	MSR	VSR	MSR	VSR
1	163	6	342	3
2	162	1	342	3
3	162	2	342	0
4	161	7	342	3
5	162	0	342	4

5.2.6   0.16 g/cc

S.no.	0.099 g/cc Concentration			
	Position 1		Position 2	
	MSR	VSR	MSR	VSR
1	168	1	348	0
2	167	5	349	1
3	167	3	348	5
4	167	3	348	5
5	168	1	348	6
6	169	0	348	4
7	169	6	348	3
8	169	3	348	5
9	169	0	348	4

6 Calculations

6.1 Malus' Law

The normalized graph of the table in 5.1 is given below. It was shifted so that the minimum corresponded with the 90°mark.
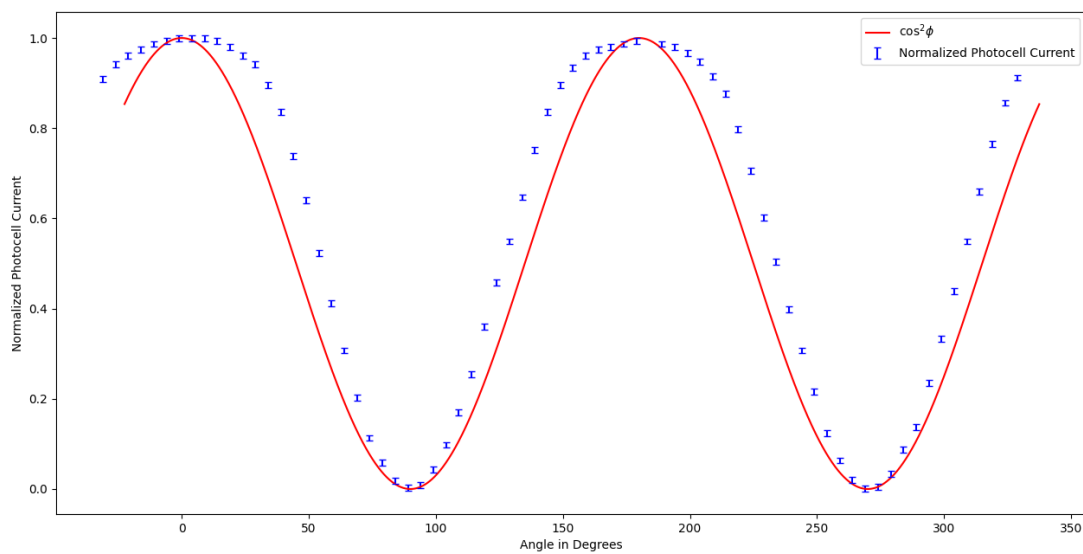

Figure 5 Normalized Current vs the Angle

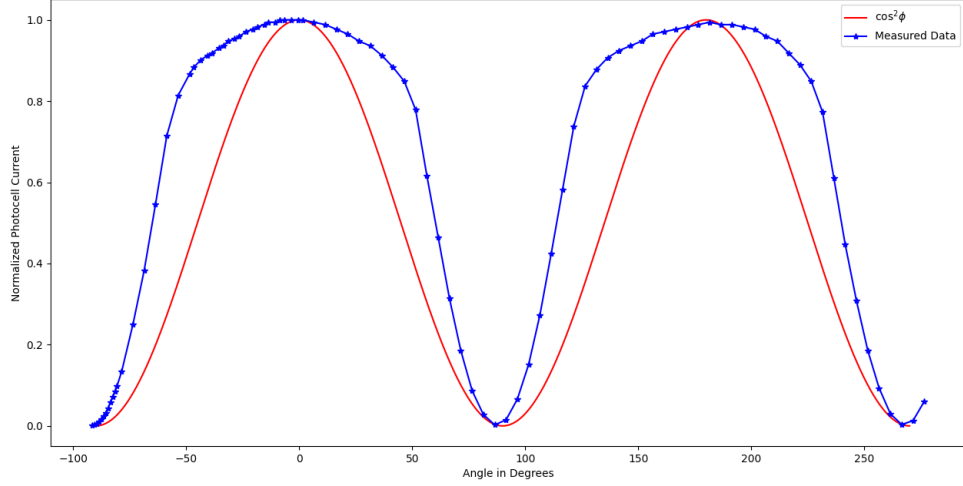The graph below shows the same graph, but when the first polarizer was removed.

Figure 6 Normalized Current vs the Angle Without the First Polarizer

Clearly having the first polarizer makes a difference.Also note that the expected graph lies well outside the error bars in Fig 5, and more so in Fig 6. This is suggestive of some significant systemic error in my measurments.

6.2   Specific Rotation of Sugar Solution

The average of the readings in section 5.2 for all concentrations is given below.

Concentration	Position 1 (in degrees)	Position 2 (in degreees)
Distilled Water	149.987	330.225
0.0199 g/cc	152.440	333.080
0.04 g/cc	155.400	335.380
0.08 g/cc	159.633	339.444
0.099 g/cc	162.320	342.260
0.16 g/cc	168.356	348.478

The angle of rotation for $L = 20$ cm, for each concentration is given below.

Concentration (in g/cc)	Angle of Rotation (in degrees)
0.0199	2.654
0.04	5.284
0.08	9.433
0.099	12.184
0.16	18.310

The graph of concentration vs angle of rotation per unit cm is shown below.
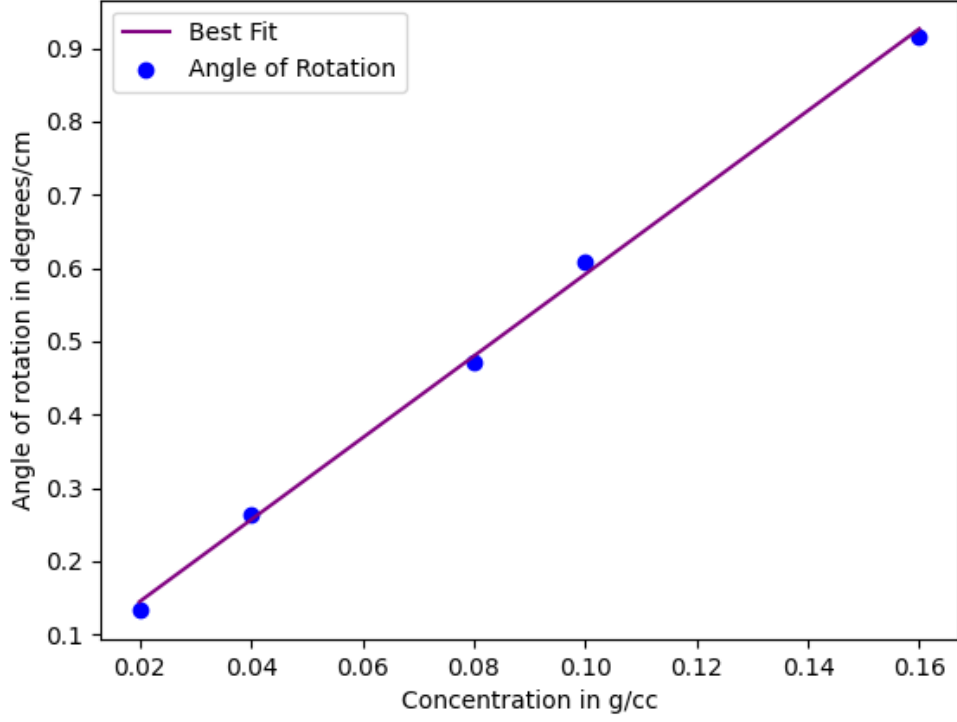

Figure 7 Concentration vs Angle of Rotation/Length

The slope of the best fit line was $5.579°/\text{cm/g/cc}$ with a standard deviation of $0.017°\text{cm}^2/\text{g}$.

Hence the specific rotation of the sugar solution was found to be
$$5.579°\text{cm}^2/\text{g}$$

7 Error Analysis

The error in the angle of rotation for a particular concentration is given by

$$\Delta\alpha = \frac{\gamma}{2}\sqrt{\frac{2}{9} + \frac{2}{N}}$$

where N is the number of measurments taken for that concentration, and γ is the least count.

Concentration (in g/cc)	Angle of Rotation (in degrees)	Percentage Error
0.0199	2.654 ± 0.040	1.51%
0.04	5.284±0.040	0.76%
0.08	9.433±0.034	0.36%
0.099	12.184±0.040	0.28%
0.16	18.310±0.034	0.18%

8   Precautions and Sources of Errors

8.1   Malus' Law

1. Make sure that there is no bright source of light close to the photodetector.

2. Do not touch the laser or the photodetector while making measurments

3. Do not look directly into the laser beam.

4. The μA readings of the photodetector is scaled down by 10. Multiply the μA readings by 10 to get the correct readings.

5. Make sure you add both the polarizers since having the frist polarizer does make a difference in the graph obtained.

6. Do not move any of the polarizers or the photodetector while making measurments as the intensity varies with the position of the polarizers and the photodetector.

8.2   Specific Rotation

1. Take readings when both the halves are equally bright and the intensity is relatively less (there are 4 angles where both halves are equally bright, and two of them (180°apart) are a little dimmer than the other two). This is because it seems to be easier to differentiate a change in intensity for that case.

2. Clean the beakers thoroughly before mixing sugar.

3. Use distilled water.

4. The capillary effect in the beakers might give readings slightly higher

than the actual ones.

5. Make sure that the sugar is mixed with water completely before performing the experiment.

9   Conclusion

We have measured the specific rotation of a sugar solution and verified Malus' law. However, we noticed that the deviation from an actual cosine squared curve is quiet large in the Malu's law measurments. Since the random error is not comparable to the observed erros, we can conclude that there area some serious systemic errors in my measurments for the Malus' law experiment.