PHY208 Experiment 6

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Measuring the Wavelength of Light Using a Fresnel Mirror and Fresnel Bisprism

1 Aim

We will use the Fresnel Mirror and the Fresnel Biprism to measure the wavelength of light emitted by a laser.

2 Apparatus

1. Fresnel biprism
2. Prism table with holder
3. Fresnel mirror
4. Lens of focus 20 mm
5. Lens of focus 300 mm
6. Optical profile-bench
7. He- Ne Laser
8. Measuring tape

3 Formulae

The formula for the wavelength is the same for the Fresnel Mirror and the Fresnel Biprism as they both have similar underlying principles.

A schematic for the experimental setup is shown below

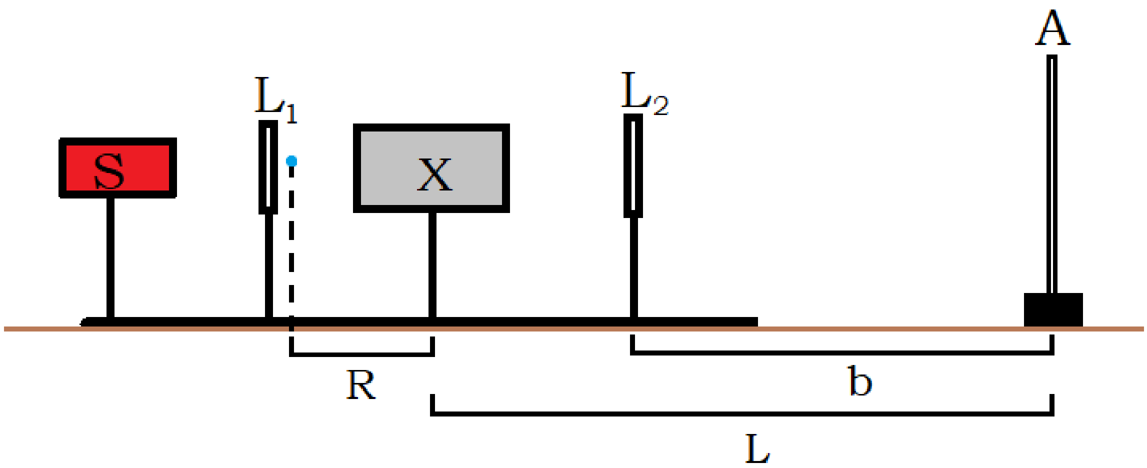

Figure 1 Schematic for the Experimental Setup of the Fresnel Mirror and Fresnel Biprism Experiment

S represents the laser

L_1 represents the first convex lens, which is used to make the light beam divergent

X represents the mirror or the biprism

R is the distance between the focus of L_1 and X

L_2 represents the second lens, which is used to find the distance between the virtual sources

A represents the screen

b represents the distance between L_2 and A

L represents the distance between X and L The fringe width p, for both the experiments is given by

$$p = \frac{\lambda(L+R)}{d} \tag{1}$$

where λ is the wavelenth of the monochromatic light which is used.

Here d is found experimentally as

$$d = \frac{Bf}{b-f} \tag{2}$$

where f is the focus of L_2 and B is the distance between the image of the virtual sources which is formed on the screen.

4 Theory

The Fresnel mirror and the Fresnel biprism reduce to a double slit interference problem. I will first obtain a formula for the fringe width in the double slit experiment, and then explain how it applies to the Fresnel mirror and biprism experiment.

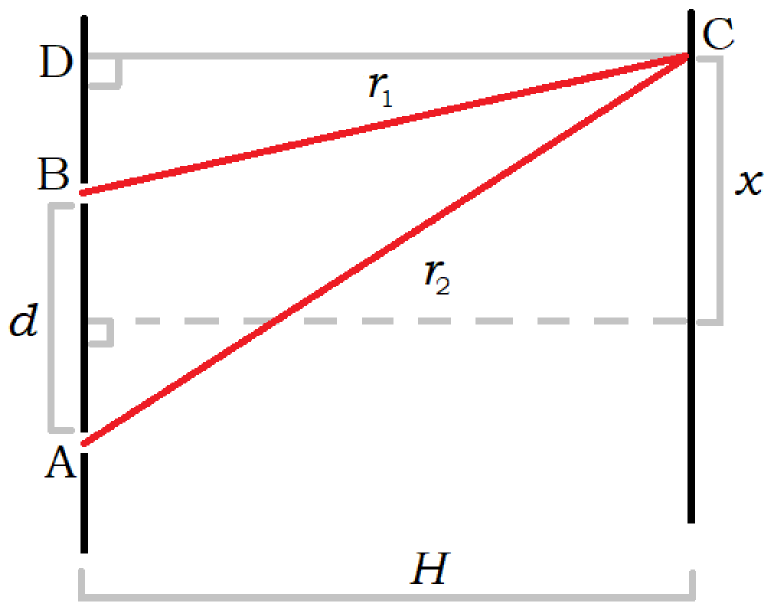

Figure 2 Double Slit Interference Setup

In Fig 2, note that x might be negative, depending upon whether the two interfering light beams intersect above the dotted horizontal line (which is when x would be positive), or below the dotted horizontal line (which is when x would be negative), while d, H, r_1 and r_2 are always positive. Also, the dotted line is the perpendicular bisector of AB

The path difference Δ, is given by $\Delta = |r_1 - r_2|$ Now applying the Pythagorean Theorem in the triangle DCB and triangle DCA, we get

$$r_1 = \sqrt{\left(x - \frac{d}{2}\right)^2 + H^2}$$

and

$$r_2 = \sqrt{\left(x + \frac{d}{2}\right)^2 + H^2}$$

The path difference becomes

$$\Delta = |r_1 - r_2| = H\left|\sqrt{\left(\frac{x}{H} - \frac{d}{2H}\right)^2 + 1} - \sqrt{\left(\frac{x}{H} + \frac{d}{2H}\right)^2 + 1}\right|$$

Expanding the squares, and neglecting the higher order terms in x/H and d/H (we are assuming that $x \ll H$ and $d \ll H$), we get

$$\Delta \approx H\left|\sqrt{1 - \frac{xd}{H^2}} - \sqrt{1 + \frac{xd}{H^2}}\right|$$

Since $xd/H^2 \ll 1$, we can neglect the higher order terms in the Taylor expansion of both the square roots.

$$\Delta \approx H\left|\left(1 - \frac{xd}{2H^2}\right) - \left(1 + \frac{xd}{2H^2}\right)\right|$$

Which yields

$$\Delta = \frac{xd}{H}$$

Now the phase difference would be given by $\delta = 2\pi\Delta/\lambda$ and the intensity of light at the screen as a function of the phase difference would be given by $I(\delta) = 4I_0^2\cos^2(\delta/2)$. The intensity would be maximum when $\delta/2 = n\pi$ where $n \in \mathbb{N} \cup \{0\}$.

Hence for maximum intensity, we have

$$\frac{\delta}{2} = \frac{1}{2}\frac{2\pi x_n d}{H\lambda} = n\pi$$

Which gives

$$x_n = \frac{nH\lambda}{d} \tag{3}$$

Where x_n represents the position of the n^{th} maximum. The fringe width would just be the difference

$$p := x_{n+1} - x_n = \frac{H\lambda}{d} \tag{4}$$

4.1 Fresnel Mirror

To see how the double slit interference relates to the Fresnel mirror experiment, consider the figure below.

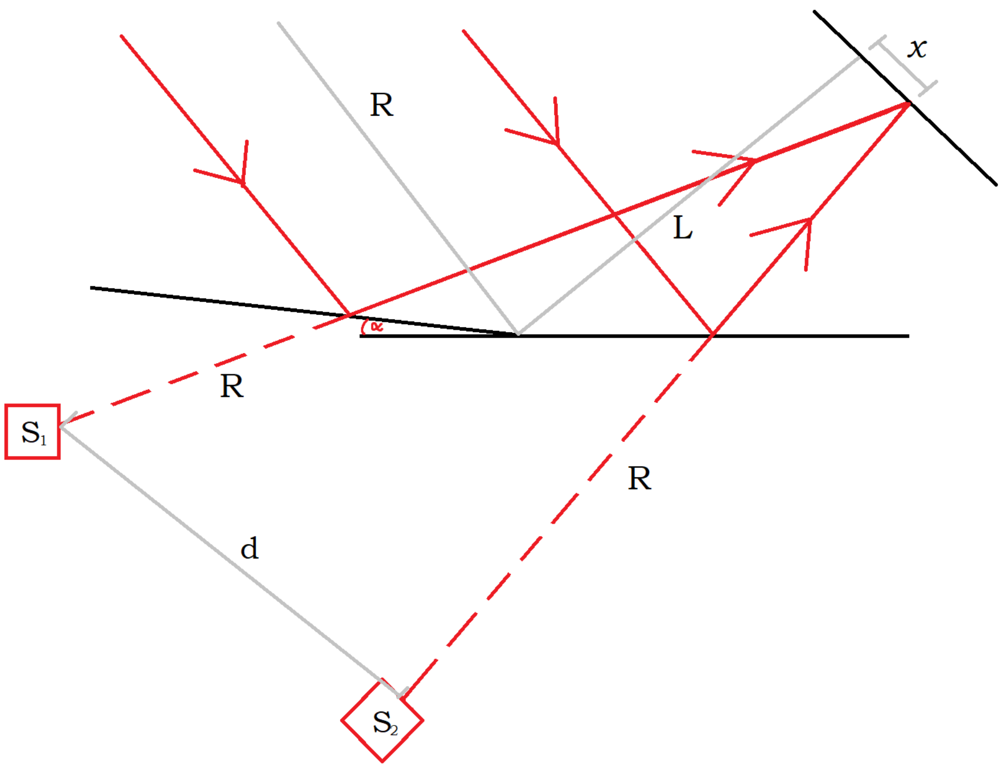

Figure 3 Schematic of Interference from Fresnel's Mirrors

In Fig 3, we see two light rays coming from the first lens and striking two different mirrors inclined at an angle α. Note that the two light rays appear to be parallel in Fig 3, but they actually should be diverging outwards. After hitting the mirror they reflect and intersect each other at the screen. R is the same as that in Fig. 1.

I have extended the reflected rays backwards to show an equivalent description, where there is no mirror, and the light beams come from two sources S_1 and S_2 (which are each placed at a distance R from their respective points of reflection), instead of coming from a single source and being reflected by the mirror. The sources S_1 and S_2 are often referred to as 'virtual sources'. Now we can see how this reduces to the double slit interference, as we have two beams emerging from two light sources, and interfering at the screen.

Now Eq. 4 would hold here, and we would have

$$p = \frac{(L+R)\lambda}{d} \tag{5}$$

We have assumed that the beam is not too thick. Here, p is the fringe width.

With some geometry and using the fact that the angle of incidence is equal to the angle of reflection, it can be shown that

$$d = 2R\sin\alpha \tag{6}$$

where α is the anlge of the mirror. However, we can't find the α directly, as it is too small. To find d experimentally, we use a second convex lens, as shown in Fig. 1, which magnifies the image of the two incident sources. Using the lens forumla, we get

$$\frac{1}{g} + \frac{1}{b} = \frac{1}{f}$$

where g is the distance from the virtual sources to the second lens, b is the same as in Fig. 1, and f is the focus of the second lens. The magnification formula gives

$$\frac{g}{b} = \frac{d}{B}$$

where d is the distance between the imaginary sources, and B is the distnace between the image of the imaginary sources on the screen. Solving for d gives

$$d = \frac{Bf}{b-f}$$

4.2 Fresnel Biprism

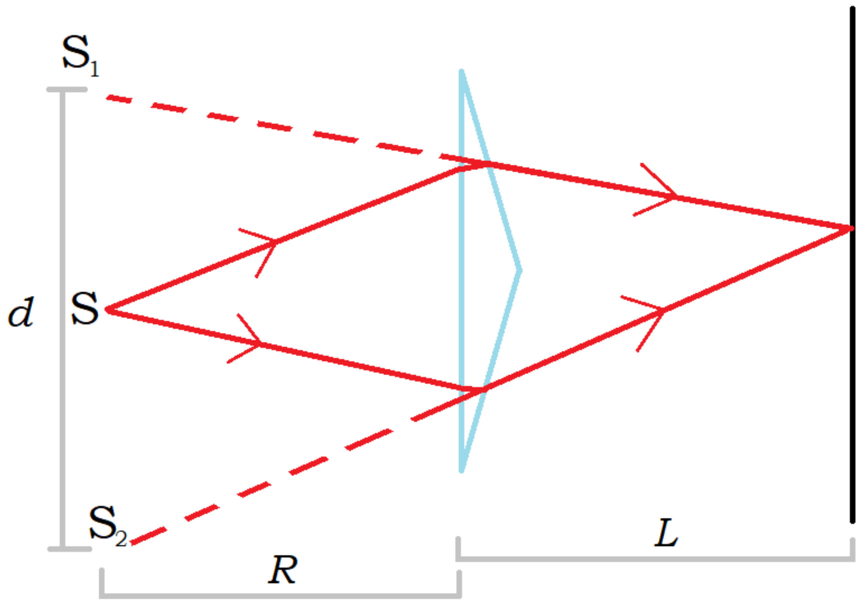

Figure 4 Schematic of Interference from Fresnel's Birprism

In Fig. 4, S represents the focus of the first lens. We can see that the Fresnel Biprism too reduces to double slit interference with the two virtual sources S_1 and S_2. The calculation for the fringe width would be performed in a manner similar to that in section 4.1, and we would obtain

$$p = \frac{(L+R)\lambda}{d}$$

where d would be given by

$$d = \frac{Bf}{b-f}$$

All the variables in the above equations are the same as those in section 4.1. Note that there is one difference in the Fresnel Miror and Biprism as in the case of the Fresnel Biprism, we have

$$d = 2R\beta(\mu - 1) \tag{7}$$

where μ is the refractive index of the prism with respect to air (for the wavelength of light used), and $\beta = (\pi - \alpha)/2$ where α is the angle of prism (α is very close to 180°).

Note that in both the cases, d varies linearly with R if the other variables (like the angle of mirror) are held constant.

5 Observation Tables

The focus of the first lens (L_1 in Fig. 1) was 2cm.

The focus of the second lens (L_2 in Fig. 1) was 30cm.

The variable names in the following tables are the same as those in Fig. 1. p stands for the fringe width, and B stands for the distance between the image of the imaginary sources on the screen.

5.1 Fresnel Mirror

S.no.	p in cm	R in cm	L in cm	B in cm	b in cm
1	0.3539	14.2	481.2	1.2	460.6
2	0.0988	25.2	363	2.7	353.4
3	0.1338	25.2	363	1.8	356
4	0.1242	25.2	363	2.0	352.6
5	0.1153	33.6	352.2	2.0	344.6
6	0.1489	26.6	352.4	1.5	341.6

5.2 Fresnel Biprism

S.no.	p in cm	R in cm	L in cm	B in cm	b in cm
1	0.1052	24.6	360	2.55	348.8
2	0.0787	29.2	330.6	2.9	322.8
3	0.0780	30.4	361.6	3.25	355.6
4	0.0856	28.2	361.6	3.3	356.0
5	0.0957	26.0	361.6	2.75	352.0
6	0.1071	22.8	361.6	2.25	338.2
7	0.1312	18.6	361.6	1.9	343.8
8	0.1755	15.0	361.6	1.25	340.0

6 Calculations

The wavelength of a He- Ne laser is 632.8 nm.

6.1 Fresnel Mirror

S.no.	d in cm	λ in nm
1	0.084	597.23
2	0.250	637.45
3	0.166	570.92
4	0.186	595.05
5	0.191	569.98
6	0.144	567.38

Taking the average of all the λ values, we get

$$\lambda_{\text{mirror}} = 589.67 \text{ nm}$$

6.2 Fresnel Biprism

S.no.	d in cm	λ in nm
1	0.240	656.37
2	0.297	649.92
3	0.299	595.84
4	0.304	666.88
5	0.256	632.60
6	0.219	610.21
7	0.182	626.82
8	0.121	563.72

Taking the average of all the λ values, we get

$$\lambda_{\text{biprism}} = 625.30 \text{ nm}$$

7 Error Analysis

The error in a function $f(x_1, x_2, \ldots, x_k)$ where $x_1, x_2, \ldots, x_k$ are independent variables with errors $\Delta x_1, \Delta x_2, \ldots, \Delta x_k$ respectively has been estimated as

$$\Delta f = \sqrt{\sum_{i=1}^{N} \left(\frac{\partial f}{\partial x_i} \Delta x_i \right)^2} \tag{8}$$

Here I am assuming that all errors are independent and random.

Using Eq. 8, the formula for the error in λ is

$$\Delta\lambda = \frac{1}{L+R}\sqrt{(d\Delta P)^2 + (p\Delta d)^2 + \left(\frac{pd}{L+R}\right)^2 (\Delta L^2 + \Delta R^2)}$$

ΔL, Δb and ΔR were taken to be 1 cm

ΔB was taken to be 0.1 cm. The errors in Δp and Δd were calculated using Eq. 8.

7.1 Fresnel Mirror

S.no.	λ in nm
1	597.23±0.03
2	637.45±.10
3	570.92±0.07
4	595.05±0.06
5	569.98±0.04
6	567.38±0.05

$$\lambda_{\text{mirror}} = 589.67 \pm 0.03 \text{ nm}$$

7.2 Fresnel Biprism

S.no.	λ in nm
1	656.37±0.03
2	649.92±0.06
3	595.84±0.05
4	666.88±0.04
5	632.60±0.05
6	610.21±0.05
7	626.82±0.04
8	563.72±0.03

$$\lambda_{\text{biprism}} = 625.30 \pm 0.02 \text{ nm}$$

The variation of d with R (when L is held constant) is shown below. If our derivation of Eq. 7 is correct, we should expect a linear variation.

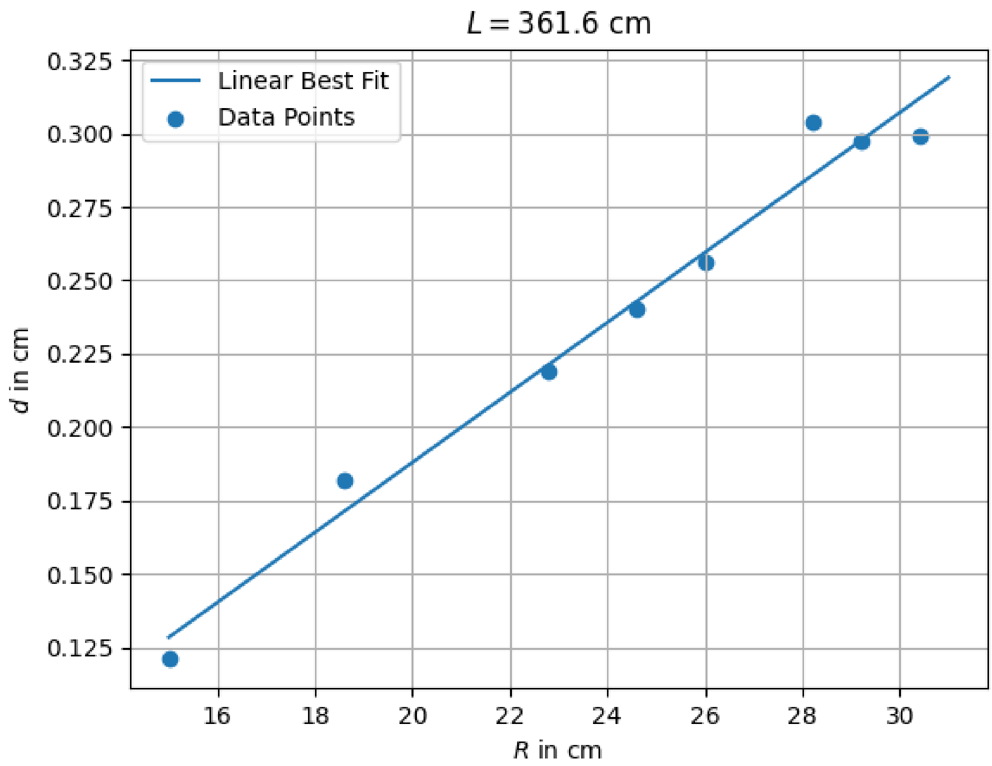

Figure 5 Variation of d with R when L is held constant at 361.6 cm for the Fresnel Biprism

8 Precautions and Sources of Errors

1. Make sure that the height of the first lens is adjusted so that the beam hits the bimirror right at the intersection of the two mirrors. (For the biprism, it should hit the corner of the biprism)

2. The measuring tape should be taut.

3. Do not tilt the screen while taking measurments as that can increase the fringe width which is seen on the screen.

4. Do not move the screen while taking measurments.

5. Adjust the second lens so that the image of the virutal sources is as clear as possible.

9 Conclusion

We can see that the measured value of wavelength is close to the actual value (632.8 nm) for the Fresnel Biprism experiment. The same can't be said for the Fresnel Mirror. Note that the forumla used by me to calculate the fringe width (and hence the wavelength), which is shown in Eq. 1, includes a factor of $L+R$ in the numerator. This is not the case in the forumla given to us by the lab manual. In the lab manual, they have taken the equation of fringe width to be $p = \lambda L/d$ (using different variable names) here, L is the distnace from the mirror (or biprism) to screen, and d is the distance between the imaginary sources, p is the fringe width, and R is the distance between the focus of the first lens and the mirror (or the biprism). When we use the formula for wavelength given to us in the lab manual, the average of the measured values of λ for the mirror and the biprism become 629.64 nm and 668.29 nm respectively. The measured wavelength while using their forumla is closer to the actual wavelength for the Fresnel Mirror experiment, but is further off for the Fresnel biprism experiment.

To check if my equation was correct, I varied R while keeping L constant and tried to find how the fringe width varied in the Fresnel Biprism experiment (I couldn't do the same for the Fresnel Mirror experiment since the fringe width depends upon d, and d depends upon the angle of the mirror, which we have to adjust most of the times in order to see the interference pattern) If my equation is correct, we should expect p to vary as $X_1/R + X_2$ (while we keep L constant) where X_1 and X_2 are constants, since

$$p = \frac{\lambda(L+R)}{d}$$

and

$$d = 2R\beta(\mu - 1)$$

by Eq. 7. Substituting, we get

$$p(R) = \frac{\lambda}{2\beta(\mu-1)}\left(\frac{L}{R} + 1\right)$$

Here λ, μ, β are the same constants as discussed in section 4.2, and we are keeping L (the biprism to screen distance) constant. If the lab manual is correct, the fringe width should vary as $\lambda X_3/R$ where $X_3 = \lambda L/(2\beta(\mu-1))$. I then performed a best fit on the p vs R data, with the functions $X_1/R + X_2$ and X_3/R, to see which of the two functions fit the data best. The graph is shown below.

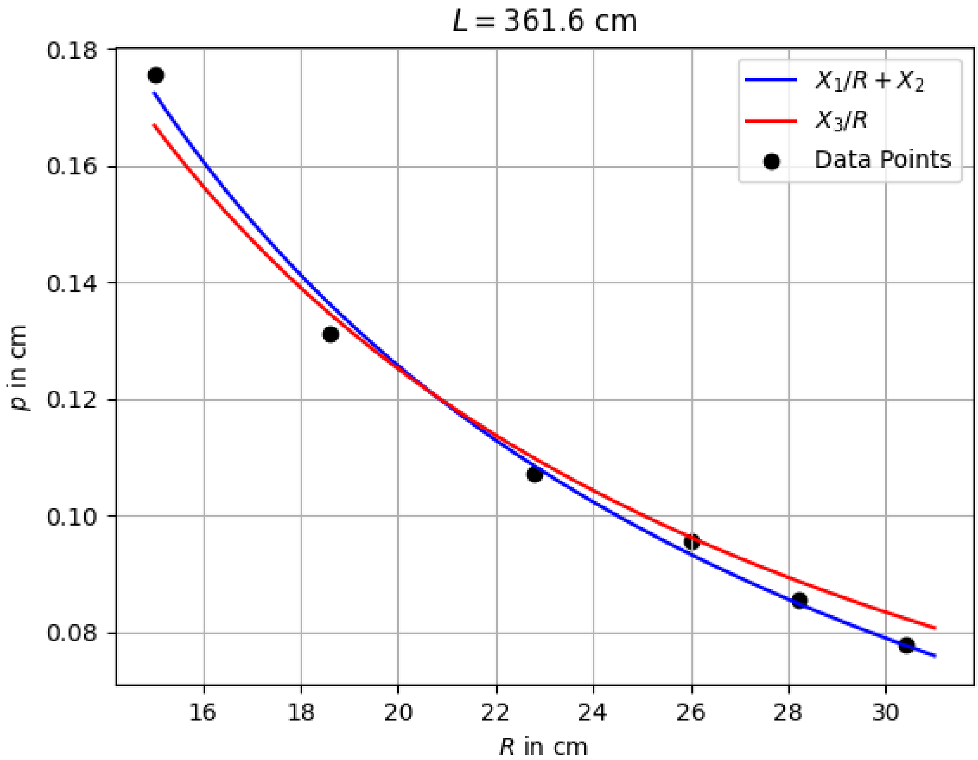

Figure 6 Variation of p with R when L is held constant at 361.6 cm for the Fresnel Biprism

We can see that the $X_1/R + X_2$ curve fits better than the X_3/R one, this is also evidenced by the standard deviation from the data points, which is lower for the former. The better agreement with the data points could just be a result of the fact that there are more free parameters in $X_1/R+X_2$ than there are in X_3/R. To increase my confidence in my equation, I should take more readings and see if the data matches with the $X_1/R+X_2$ function or the X_3/R function. However, the fringe widths become too small if R increases, and it gets difficult to distinguish the maximas. This would suggest that for the biprism, my equation is correct. However, for the Fresnel mirror, it would seem as though my equation is incorrect and the one in the manual is correct. As of yet, I have not been able to figure out why this is the case.