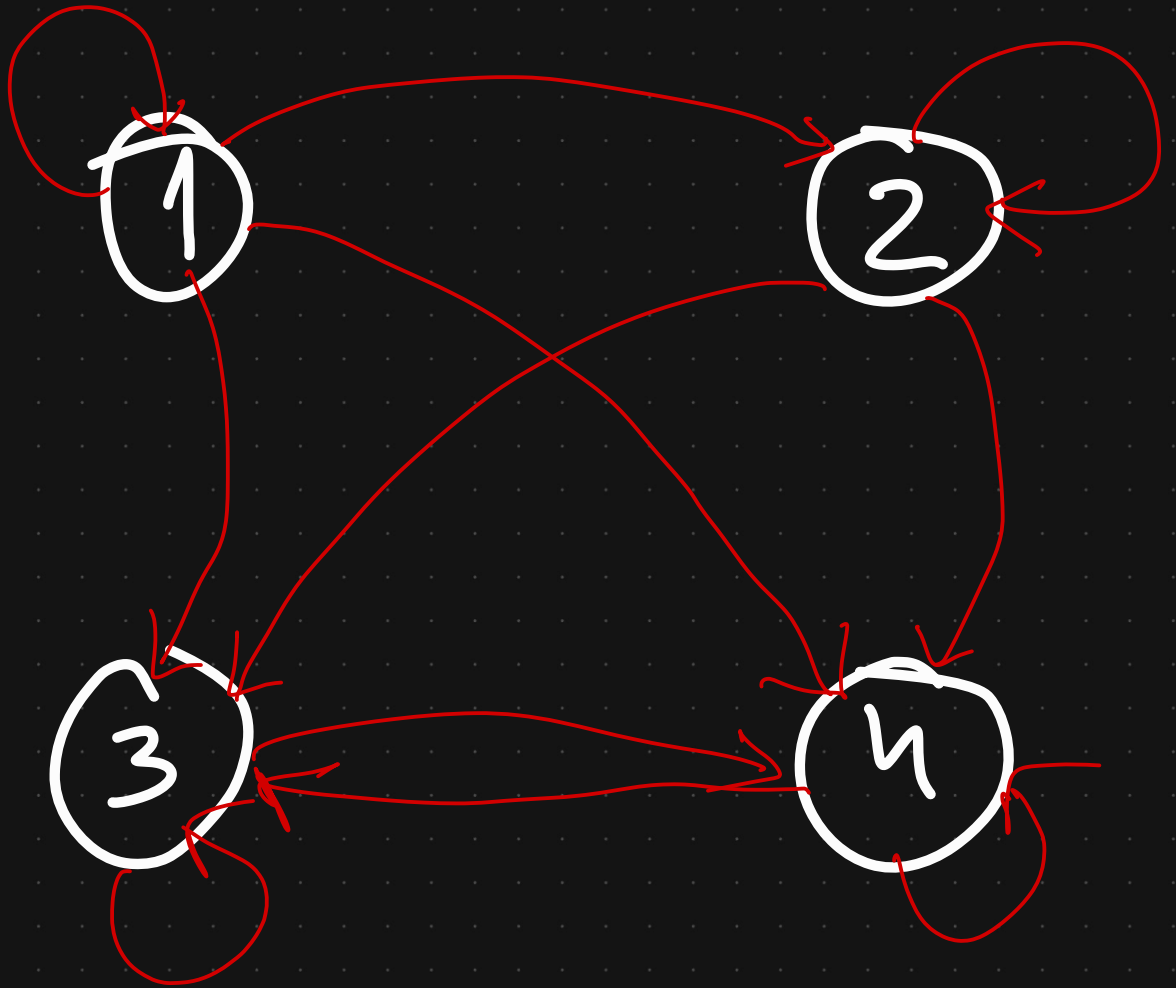


K_1

Period 2
 $\{1, 2, 3, 5\}$
 $\{4\}$
 } Not irreducible
 \rightarrow aperiodic

Period $K^n(x, x) > 0$; $n \geq 1$

K2

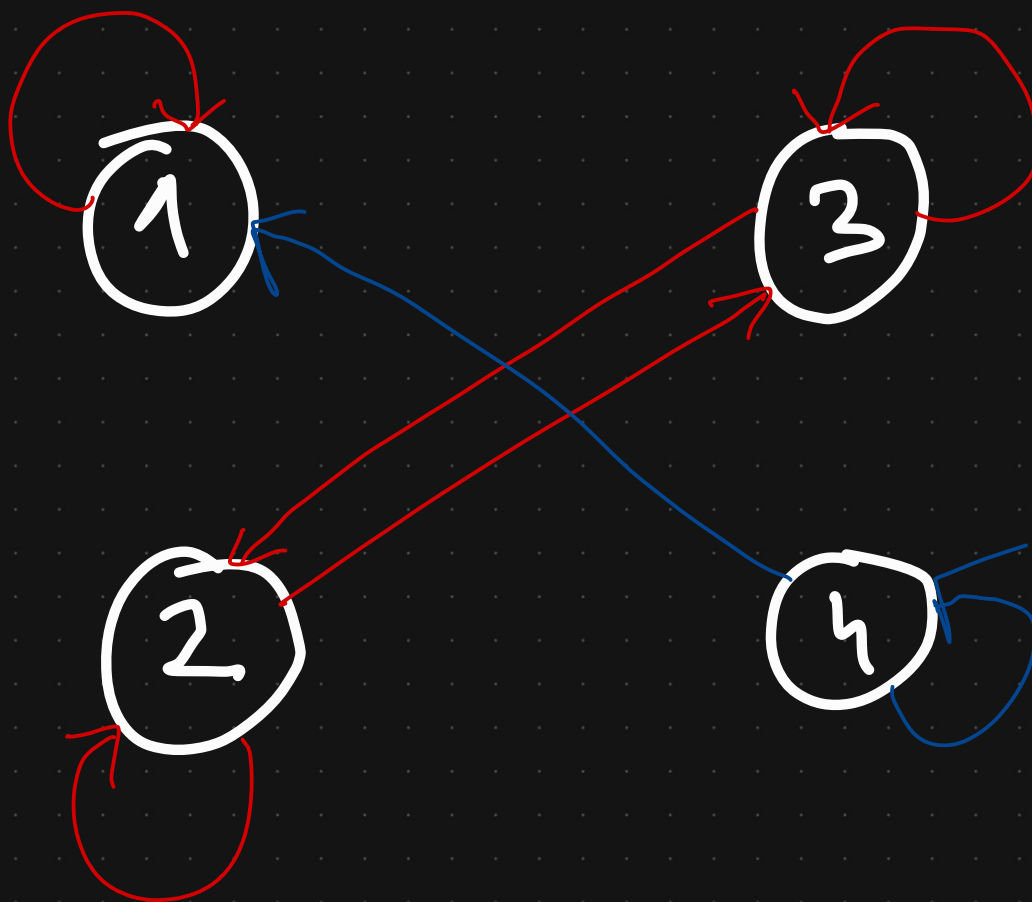


$\{2, 1\}$
 \downarrow
1
Aperiodic

$\{2, 2\}$
 \downarrow
1

$\{2, 3, 4\}$
 \downarrow \downarrow
2 2
not irreducible

K3



$\{1, 3\}$
 \downarrow
1

$\{2, 4\}$
 \downarrow
1

$\{2, 3\}$

not 2 irreducibles

$K \rightarrow$ Doubly stochastic, $P(T_x < \infty) = 1 \checkmark$
 $E[T_x] < \infty \checkmark$

$\pi(x) = \frac{1}{n} \cdot \frac{1}{x}$ Pos. Recc.



$$\begin{array}{c}
 A_0 \ A_1 \ A_2 \\
 \left[\begin{array}{ccc} 0.5 & 0 & 0.5 \\ 0 & 0.5 & \end{array} \right]
 \end{array}$$

$\{S\}$
 \downarrow
 2 period

$$\begin{aligned}
 E[T_1] &= 1 \cdot \frac{1}{2} \cdot \frac{1}{2} \\
 &\quad + 2 \cdot \frac{1}{2} \cdot \frac{1}{2} \\
 &\quad + 3 \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \\
 &\quad + \dots \\
 &= \sum_{n=1}^{+\infty} \frac{n}{2^{2n}} < \infty
 \end{aligned}$$

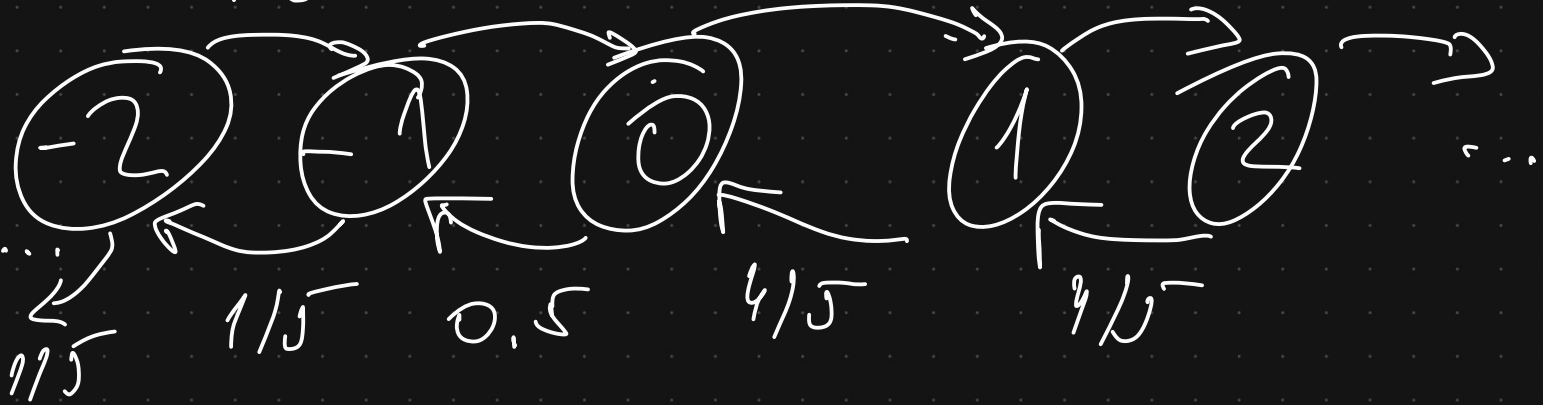
$$P(T_1 < \infty) = 1$$

$$\begin{aligned}
 P(T_1 < \infty) &= \frac{1}{2} \cdot \frac{1}{2} + \frac{1}{2} \cdot \frac{1}{2} + \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \\
 &\quad + \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} + \dots \\
 &= \frac{1}{4} + \frac{1}{4} + \frac{1}{16} + \frac{1}{64} + \dots \\
 &= \sum_{n=1}^{\infty} \frac{1}{2^{2n}} = \frac{1}{4} + \frac{1}{2^4} + \frac{1}{2^6} < \infty
 \end{aligned}$$

K5

$$\begin{bmatrix} -2 & -1 & 0 & 1 \\ 0 & 4/5 & 0 & 0 \\ 1/5 & 0 & 1/5 & 0 \\ 0 & 0.5 & 0 & 1/5 \end{bmatrix}$$

$$\begin{matrix} 4/5 & 4/5 & 0.5 & 1/5 \end{matrix}$$



Approx

$$\begin{bmatrix} -2 & -1 & 0 & 1 & 2 \\ -2 & 1/5 & 4/5 & 0 & 0 & 0 \\ -1 & 1/5 & 0 & 1/5 & 0 & 0 \\ 0 & 0 & 0.5 & 0 & 0.5 & 0 \\ 1 & 0 & 0 & 4/5 & 0 & 1/5 \\ 2 & 0 & 0 & 0 & 4/5 & 1/5 \end{bmatrix}$$

$$E(T_1) = + \frac{1}{5} \cdot \frac{4}{5} \cdot 2 + \frac{1}{5} \cdot \frac{1}{5} \cdot \frac{4}{5} \cdot 3$$