1 Theoretical problems

1. Show that there exists $x \in \Phi$ so that for all $i \in I$ we have $s(x)_i > 0$: If we take into consideration that $s(x) = b - Ax$ holds, for which exists $x \in \Phi$ (explained in b)) that $(Ax)_i < b_i$, we can easily derive that $b_i - Ax_i > 0$, thus for all $i \in I$ we have $(s(x))_i > 0$.

2. Show that the analytic center optimization problem is equivalent to a strictly con- vex optimization problem: The solution set of system $Ax \leq b$ of m inequalities with n variables (A is m $\times$ n matrix) is strictly convex because solution set is in polyhedral bounded by linear inequalities. If we consider (we can describe this polyhedral with system):

$$\alpha_1 x_1 \leq b, \alpha_2 x_2 \leq b$$

using convex condition with $t \in [0, 1]$ we have:

$$t\alpha_1 x + (1-t)\alpha_2 y \leq tb + (1-t)b = b$$

Thus, satisfying our convex condition. If we formulate analytical center as:

$$\text{argmin} \left\{ -\sum_i^n \log\left(b_i - a_i^T x\right) \right\}$$

Here we are trying to put implicit constraint that $b_i - \alpha_i x > 0, i \in I$, which means that polyhedron defined by linear inequalites needs to be bounded. Having in mind that, we can say that our barrier is bounded, thus strictly convex and after that we can conclude that analytic center function is strictly convex because $\log(x) \in \mathbf{R}^+$as proven. With this fulfilled we can also easily impose that $s(x)_i > 0$ as mentioned in a).

3. Show that the analytic center is unique:
Analytic center definition:

$$\prod_{i \in I} s(x)_i$$

We can convert this to easily to $s(x)_i = \log\left(b_i - a_i^T x\right)$, thus:

$$s(x) = \sum_i^n \log\left(b_i - a_i^T x\right)$$

Since $s(x)$ is strictly convex our center is stationary point lying in the polyhedron $x^* \in H$.

4. If we take into consideration from previous task that we can write equation of analytic centre as:

$$\text{argmin} \left\{ -\sum_i^n \log\left(b_i - a_i^T x\right) \right\}$$

we can formulate our problem as: $\text{argmin}\left\{-\log\left(x_1\right)-\log\left(x_2\right)-\log\left(1-ax_1-x_2\right)\right\}$, s.t. $x_1 \geq 0; x_2 \geq 0; a \in \mathbf{R}^{+}$If we approach to solution via derivatives:

$$\frac{\partial}{\partial x_1}=\frac{ax_1-1+ax_1+x_2}{x_1\left(1-ax_1-x_2\right)}$$
$$\frac{\partial}{\partial x_2}=\frac{ax_1+2x_2-1}{x_2\left(1-ax_1-x_2\right)}$$

Thus: $ax_1+2x_2-1=0, 2ax_1+x_2-1=0 \rightarrow x_1=\frac{1}{3a}, x_2=\frac{1}{3}$

5. If we take into consideration from 2 nd task that we can write equation of analytic centre as:

$$\text{argmin}\left\{-\sum_{i}^{n}\log\left(b_i-a_i^T x\right)\right\}$$

we can formulate our problem as:

$$\text{argmin}\left\{-\log\left(x_1\right)-\log\left(x_2\right)-\log\left(1-x_1-x_2\right)-\log\left(1-x_1-x_2\right)\right\},$$

s.t. $x_1 \geq 0; x_2 \geq 0$ If we approach to solution via derivatives:

$$\frac{\partial}{\partial x_1}=\frac{3x_1+x_2-1}{x_1\left(-x_1-x_2+1\right)}$$
$$\frac{\partial}{\partial x_2}=\frac{x_1+3x_2-1}{x_2\left(-x_1-x_2+1\right)}$$

Thus: $3x_1+x_2-1=0, x_1+3x_2-1=0 \rightarrow x_1=\frac{1}{4}, x_2=\frac{1}{4}$

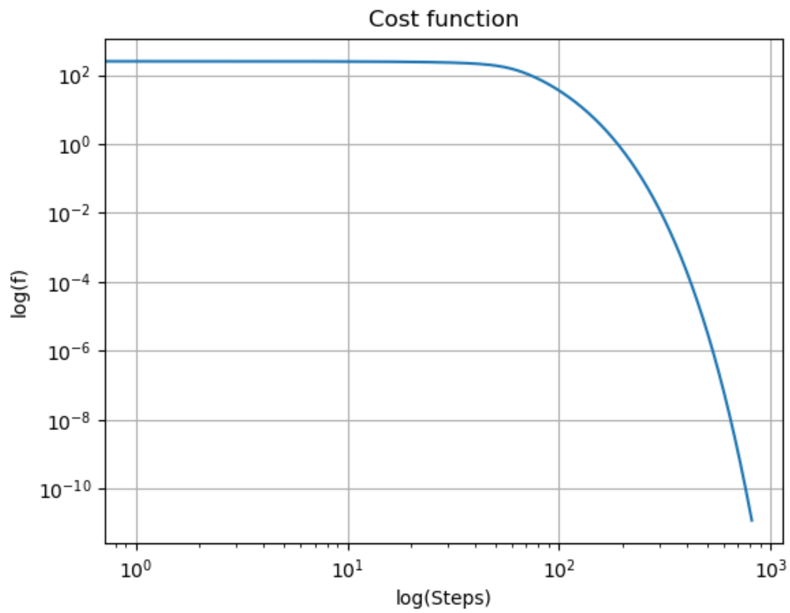

Figure 1: Cost function minimization

2 Program that runs in the interior

Using conventional Excel LP solver,it is noticeable that most of the variables are set to 0, as presented in Table 1. Using Python implementation for the given problem, we can see that implemented algorithm is working. In Figure 1, cost function is presented during optimization iterations. It is visible that cost function converges with final value of $f_{min} \approx 1.22e^{-11}$. In Figure **??**, performance of the algorithm through iteration is presented. It is confirmed that invariant iterations are satisfied and that weak duality hypothesis is converging to 0. Having that in mind, implementation doesn’t give correct results.

Variable	Value
Potatoes (x_1)	0
Bread (x_2)	6.23
Milk (x_3)	0
Eggs (x_4)	0
Yoghurt (x_5)	0
Veg. oil (x_6)	0.5884
Beef ((x_7)	0
Strawberries (x_8)	0
$min.f(x)$	148.831

Table 1: Optimization results from a Excel LP solver

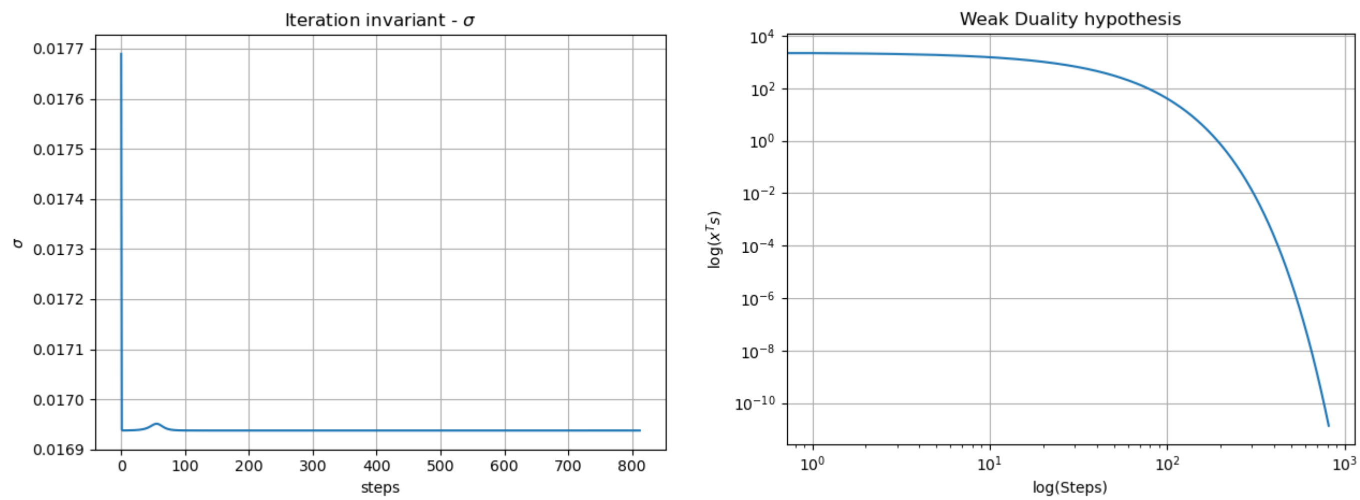

Figure 2: Evaluation of performance on implemented IPM