

# Introduction to a high-performance multi-period optimal power flow solver

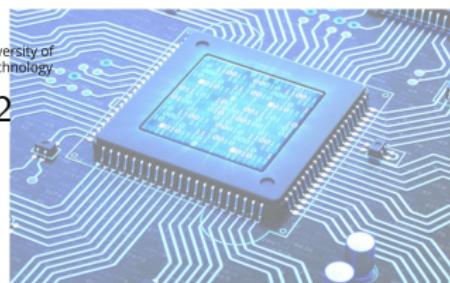
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Winter School Workshop 2022  
Hotel Scandic Lerkendal  
Trondheim



NTNU | Norwegian University of  
Science and Technology

March 3, 2022



Norwegian University of  
Science and Technology

# The presentation goal

**Purpose:** To discuss how to solve multi-period optimal power flow fast; **Large-Scale simulation with respect to time and space.**

**Phase I:** Background and Motivation

**Phase II:** Power Flow and Optimal Power Flow

**Phase III:** Solution Method

**Phase IV:** Speed up

**Phase V:** Future Work

**Presentation Time:** 15-20 min

## **Phase I: Background and Motivation**

# Electricity Grid

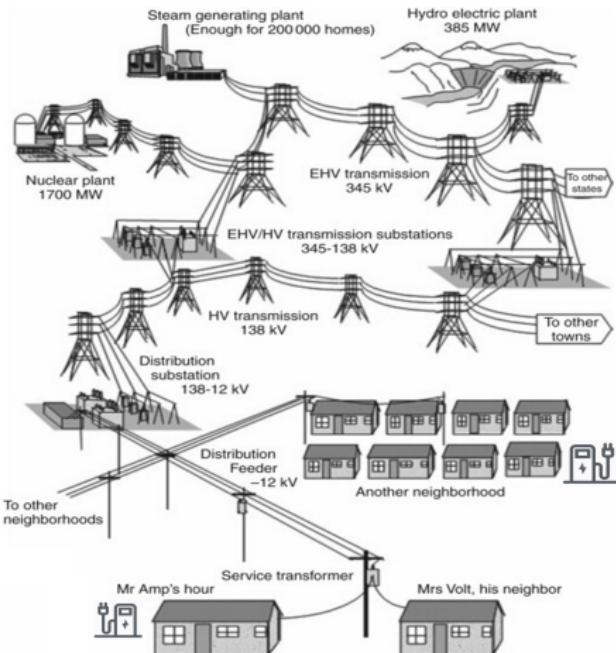


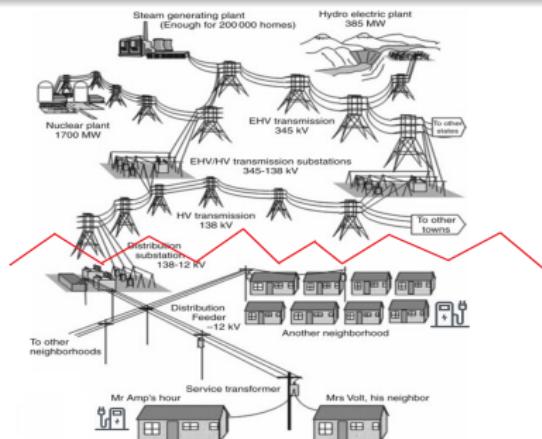
Figure 1: [www.sciencedirect.com/science/article/pii/B9781845697846500019 ]

## Sustainability/Green shift/CO<sub>2</sub> reduction

For many reasons power electricity grid is facing decentralization

Phasing out coal (carbon-heavy sources of production) and nuclear power plants

This chain between large power producers and consumers is weakened.



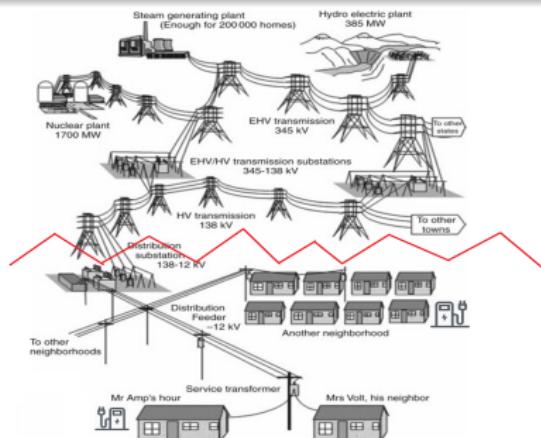
# Sustainability/Green shift/CO<sub>2</sub> reduction

For many reasons power electricity grid is facing decentralization

Phasing out coal (carbon-heavy sources of production) and nuclear power plants

Increase penetration of solar and wind production

This chain between large power producers and consumers is weakened.



## Background: cont.

## Items

Green shift in electricity systems is needed for the reduction of CO<sub>2</sub> emissions



Figure 2: A Typical Power System [Rochester Gas & Electricity]

# Background: cont.

## Items

Green shift in electricity systems is needed for the reduction of CO<sub>2</sub> emissions

Integration of Distributed Energy Resources (DER) is a huge challenge

### Note

DER includes Renewable Energy, Energy Storage, Electric Vehicles and Flexible Demand



Figure 2: A Typical Power System [Rochester Gas & Electricity]

## Background: cont.

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Grid companies must be able to analyse the impacts of DER

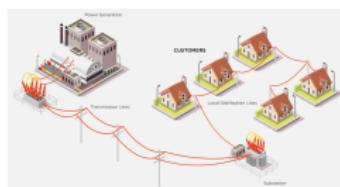


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## Background: cont.

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Integration of Distributed Energy Resources (DER) is a huge challenge

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## Note

Optimal Power Flow (OPF) solvers are essential



Figure 2: A Typical Power System [Rochester Gas & Electric] 

# Background: cont.

## Items

Green shift in electricity systems is needed for the reduction of CO<sub>2</sub> emissions

Integration of Distributed Energy Resources (DER) is a huge challenge

Grid companies must be able to analyse the impacts of DER

The integration of DER in smart grids calls for **much more sophisticated solvers** for OPF



Figure 2: A Typical Power System [Rochester Gas & Electricity]

# Challenges in the planning and operation of the grid

**Planning:** Optimizing the right type, size and timing of new grid investments

Local generation (e.g. PV) and increased load (e.g. EVs) can be located in areas where the grid is weak

Energy storage and demand flexibility are alternatives to grid reinforcements

# Challenges in the planning and operation of the grid

**Planning:** Optimizing the right type, size and timing of new grid investments

Local generation (e.g. PV) and increased load (e.g. EVs) can be located in areas where the grid is weak

Energy storage and demand flexibility are alternatives to grid reinforcements

**Operation:** Optimize the use of controllable assets such as energy storage and flexible demand to secure, reliable and economic operation of the distribution grids. This means:

Making the right use of Demand Response

being able to value the use of end-user flexibility for local or system-wide grid services

Simulating and optimizing the grids in the presence of **future local markets for energy and flexibility**

# Limitations of traditional grid operation and planning

## Notes

Classical single-period OPF does not offer a possibility for optimal operational scheduling of storage and flexible demand



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We therefore aim to develop the foundations for a new generation of Multi-Period OPF (MPOPF) solvers

- i. Solves the OPF problem over several coupled time-steps
- ii. Computation time is an issue when using both commercial or free optimization solvers



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- i. Nonlinearity
- ii. Large-scale problem with respect with to time and space
- iii. Involves stochastic generations and load



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- ii. Large-scale problem with respect with to time and space
- iii. Involves stochastic generations and load

Hardware is reaching its limit with respect to CPU clock speed



# Solution:

## High-Performance Solver [1-2]

Algorithmic design tailored to the conventional OPF algorithms speed-up the solution proposal

Prototype model shows convincing results for real-sized system with distributed renewables, storages and EVs

- i. A high-performance and memory-efficient sparse algorithm
- ii. Utilizing the structure of the underlying mathematical formulation

1. S. Zaferanlouei, H. Farahmand, V. V. Vadlamudi, M. Korpås, "BATTPOWER Toolbox: Memory-Efficient and High-Performance Multi-Period AC Optimal Power Flow Solver", IEEE Transactions on Power Systems, Jan. 16th, 2021.

2. S. Zaferanlouei, et al., "BATTPOWER Application: Large-Scale Integration of EVs in an Active Distribution Grid —A Norwegian Case Study", Under review in the journal of EPSR

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### Benefits

Optimal utilization of **stored energy** and **flexibility** where and when it creates the highest value for the system

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Optimal utilization of **stored energy** and **flexibility** where and when it creates the highest value for the system

Can be used for grid planning, grid operation and local markets

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# Power System— Today

# Power System— Future

Phase II:  
**Power Flow and Optimal Power Flow**

# Power Flow Equations

## Source of Non-linearity

Nonlinear relationship between the voltage phasors and the power injections

Power Line with conductance G and susceptance B

$$S_i = P_i + jQ_i = V_i \cdot \bar{I}_i$$
$$S_j = P_j + jQ_j = V_j \cdot \bar{I}_j$$
$$Y = G + jB \text{ and } G = \frac{R}{R^2 + X^2}, B = \frac{-X}{R^2 + X^2}$$

Where R and X are resistance and reactance respectively

$$\begin{aligned} P_i + jQ_i &= V_i \cdot \bar{I}_i \\ &= V_i \cdot (\bar{Y}_i \cdot \bar{V}) \\ &= V_i \cdot (G_i - jB_i) \bar{V} \\ |V_i|^2 &= V_i \cdot \bar{V}_i \end{aligned}$$

"Power Flow Equations"  
"Load Flow Equations"

Y admittance matrix

G and B conductance and susceptance matrices

$$Y = G + jB$$

> Coupled quadratics in complex voltage phasors

# Power Flow Equations in Different Coordinates

The **bus injection** model

Power Line with conductance G and susceptance B	
Node i $S_i = P_i + jQ_i$ $= V_i \cdot \bar{I}_i$	Node j $S_j = P_j + jQ_j$ $= V_j \cdot \bar{I}_j$
$Y = G + jB$ and $G = \frac{R}{R^2 + X^2}$ , $B = \frac{-X}{R^2 + X^2}$ Where R and X are resistance and reactance respectively	
Polar Voltage Coordinates	$V_i =  V_i  \angle \theta_i \quad \theta_1 = 0$
$P_i =  V_i  \sum_{j=1}^N  V_j  (G_{ij} \cos(\theta_i - \theta_j) + B_{ij} \sin(\theta_i - \theta_j))$	
$Q_i =  V_i  \sum_{j=1}^N  V_j  (G_{ij} \sin(\theta_i - \theta_j) - B_{ij} \cos(\theta_i - \theta_j))$	
Rectangular Voltage Coordinates	$V_i = V_{di} + jV_{qi} \quad V_{q1} = 0$
$P_i = V_{di} (G_{ij} V_{dj} - B_{ij} V_{qj}) + V_{qi} (B_{ij} V_{dj} + G_{ij} V_{qj})$	
$Q_i = -V_{di} (B_{ij} V_{dj} + G_{ij} V_{qj}) + V_{qi} (G_{ij} V_{dj} - B_{ij} V_{qj})$	
$V_{di} = \text{Re}(V_i) \quad V_{qi} = \text{Im}(V_i)$	$N = \text{number of Nodes}$

# Power Flow Equations in Different Coordinates

The **bus injection** model

Power Line with conductance G and susceptance B	
Node i $S_i = P_i + jQ_i$ $= V_i \cdot I_i$	Node j $S_j = P_j + jQ_j$ $= V_j \cdot I_j$
$Y = G + jB$ and $G = \frac{R}{R^2 + X^2}$ , $B = \frac{-X}{R^2 + X^2}$	Where R and X are resistance and reactance respectively
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## Power Flow

PF equations are a set of nonlinear algebraic equations which can be solved with Newton-Raphson, Gauss-Seidel, Fast-decoupled-load-flow method and etc.

# Single period ACOPF - problem formulation

## Single Period AC Optimal Power Flow (ACOPF)

We are controlling some variables in OPF to minimise system costs with respect to constraints.

Objective Function	$\min_x f(x)$
Equality Constraints (Balance) Power Flow	s.t. $g(x) = \begin{bmatrix} \tilde{g}(x) \\ \bar{g}(x) \end{bmatrix} = 0 \in \mathbb{R}^{n_{gx} \times 1}$
Inequality constraints (line and transformer)	$h(x) = \begin{bmatrix} \tilde{h}(x) \\ \bar{h}(x) \end{bmatrix} \leq 0 \in \mathbb{R}^{n_{hx} \times 1}$
Operational Constraints	
Vector of Variables	$x = [\Theta \ \mathcal{V} \ \mathcal{P}^g \ \mathcal{Q}^g]^\top \in \mathbb{R}^{n_x \times 1}$

## Limitation of Power Flow

- PF is only a set of static equations which provides status of a system for time:  $t = t_s$
- It does not include generation constraints and operational constraints

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## Limitation of Single Period AC Optimal Power Flow

- OPF is an optimisation problem which optimises status of a system for a SINGLE time:  $t = t_s$ .
- Although it includes the operational constraints for single time, it will not include operation of DERs and generators over a time horizon.
- It is not capable of integrating storage devices and EVs.

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## Our suggested approach: multi-period ACOPF

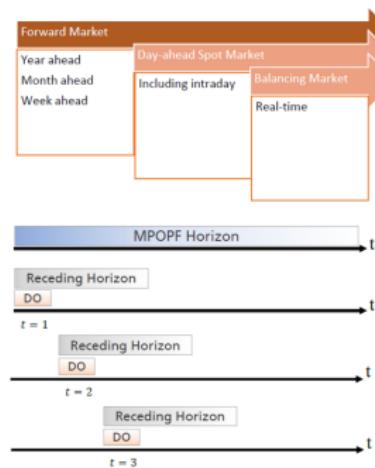
Solves the OPF problem over several time-steps at once useful formulation for systems with Energy Storage and Shiftable Loads (e.g. EVs)

## Advantages of MultiPeriod ACOPF

- > Integrating dependent time power system's components such as stationary ESS, EV, generators ramp rate, and so on.

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## Disadvantage of MultiPeriod ACOPF

The problem grows very large as the number of time-steps is increased, which may lead to an intractable solution.

# MultiPeriod ACOPF-problem formulation

Objective Function	$\min_X F(X)$
Equality Constraints (Balance) Power Flow	s.t. $G(X) = [\tilde{G}(X) \ \bar{G}(X) \ \bar{G}^s(X)]^\top = 0 \in \mathbb{R}^{N_g \times 1}$
Inequality constraints (line and transformer)	$H(X) = [\tilde{H}(X) \ \bar{H}(X)]^\top \leq 0 \in \mathbb{R}^{N_h \times 1}$
Operational Constraints	$\tilde{G}(X) = [\tilde{g}(x_1) \ \tilde{g}(x_2) \ \dots \ \tilde{g}(x_T)]^\top$ $\bar{G}(X) = [\bar{g}(x_1) \ \bar{g}(x_2) \ \dots \ \bar{g}(x_T)]^\top$ $\bar{G}^s(X) = [\bar{g}^s(\tau_1) \ \bar{g}^s(\tau_2) \ \dots \ \bar{g}^s(\tau_T)]^\top$ $\tilde{H}(X) = [\tilde{h}(x_1) \ \tilde{h}(x_2) \ \dots \ \tilde{h}(x_T)]^\top$ $\bar{H}(X) = [\bar{h}(x_1) \ \bar{h}(x_2) \ \dots \ \bar{h}(x_T)]^\top$
Vectors of Constraints	

## Phase III: **Solution Methods**

# Nonlinear Programming (NLP)

How do we solve Optimal Power Flow?

## I. Interior Point Method

# Nonlinear Programming (NLP)

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- II. Gradient Decent Method

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- III. Heuristic Methods

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How do we solve Optimal Power Flow?

- I. Interior Point Method
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## Interior Point Method

The focus of this study is the interior point method solution

### Step(1): Applying Slack variables and the barrier term:

$$\min_X \left[ F(X) - \gamma \sum_{i=1}^{N_h} \ln(z_i) \right]$$

$$\text{s.t. } G(X) = 0$$

$$H(X) + Z = 0$$

$$Z \geq 0$$

slack variables "Z" convert inequality constraints to equality constraints.

Step (2): Calculate and form the Lagrangian of Barrier subproblem

$$\mathcal{L}^\gamma(X, Z, \lambda, \mu) = f(X) + \lambda^\top G(X) + \mu^\top (H(X) + Z) - \gamma \sum_{i=1}^{N_g} \ln(z_i)$$

Step (3): Calculate the KKT<sup>1</sup> of the Lagrangian

Step (3)  
 $KKT_X$   $\mathcal{L}_X^\gamma(X, Z, \lambda, \mu) = f_X + \lambda^\top G_X + \mu^\top H_X = 0$

Step (3)  
 $KKT_Z$   $\mathcal{L}_Z^\gamma(X, Z, \lambda, \mu) = \mu^\top - \gamma e^\top \text{diag}(Z)^{-1} = 0$

Step (3)  
 $KKT_\lambda$   $\mathcal{L}_\lambda^\gamma(X, Z, \lambda, \mu) = G^\top(X) = 0$

Step (3)  
 $KKT_\mu$   $\mathcal{L}_\mu^\gamma(X, Z, \lambda, \mu) = H^\top(X) + Z^\top = 0$

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<sup>1</sup>Karush–Kuhn–Tucker conditions

Nonlinear  
Algebraic  
Equations

$$\Omega(X, Z, \lambda, \mu) = \begin{bmatrix} f_X + \lambda^T G_X + \mu^T H_X \\ \text{diag}(Z)\mu^T - \gamma e^T \\ G^T(X) \\ H^T(X) + Z^T \end{bmatrix} = 0$$

S.t.

$$Z > 0$$

$$\mu > 0$$

Step (4): Apply Newton Raphson Method

$$[\Omega_X \ \Omega_Z \ \Omega_\lambda \ \Omega_\mu]^k [\Delta X \ \Delta Z \ \Delta \lambda \ \Delta \mu]^{+k} = -\Omega(X, Z, \lambda, \mu)^k$$

Step (5):  
Inverse Jacobian  
of Newton Raphson

$$\begin{bmatrix} M & G_X^\top \\ G_X & 0 \end{bmatrix}^k \begin{bmatrix} \Delta X \\ \Delta \lambda \end{bmatrix}^k = \begin{bmatrix} -N \\ -G(X) \end{bmatrix}$$

$M \in \mathbb{R}^{N_x \times N_x}$  and  $N \in \mathbb{R}^{N_x \times 1}$  are defined as:

$$M = \mathcal{L}_{xx}^\gamma + H_x^\top \text{diag}(Z)^{-1} \text{diag}(\mu) H_x$$

$$N = f_x^\top + G_x^\top \lambda + H_x^\top \mu + H_x^\top \text{diag}(Z)^{-1} (\gamma e + \text{diag}(\mu) H(x))$$

$$\mathcal{L}_{xx}^\gamma = f_{xx} + G_{xx}(\lambda) + H_{xx}(\mu)$$



# Iterations

## Successive iterations in Interior Point Method

- i. Function Evaluations calculation of  $G_X = \frac{\partial G}{\partial X}$ ,  $H_X = \frac{\partial H}{\partial X}$ ,  
 $F_X = \frac{\partial F}{\partial X}$ ,  $G_{XX} = \frac{\partial}{\partial X}(G_X^\top \lambda)$ ,  $H_{XX} = \frac{\partial}{\partial X}(H_X^\top \lambda)$ ,  $F_{XX} = \frac{\partial}{\partial X}(F_X^\top)$   
in order to form coefficient matrix and right hand side of

$$\begin{bmatrix} M & G_X^\top \\ G_X & 0 \end{bmatrix}^k \begin{bmatrix} \Delta X \\ \Delta \lambda \end{bmatrix}^k = \begin{bmatrix} -N \\ -G(X) \end{bmatrix}^k$$

# Iterations

## Successive iterations in Interior Point Method

### i. Function Evaluations

ii. **Linear Algebraic Solver** Calculate the Inverse  $\begin{bmatrix} M & G_X^\top \\ G_X & 0 \end{bmatrix}^k$  in

$$\begin{bmatrix} M & G_X^\top \\ G_X & 0 \end{bmatrix}^k \begin{bmatrix} \Delta X \\ \Delta \lambda \end{bmatrix}^k = \begin{bmatrix} -N \\ -G(X) \end{bmatrix}^k$$

## Successive iterations in Interior Point Method

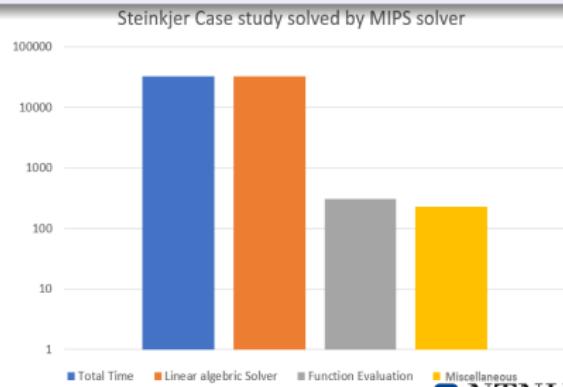
- i. Function Evaluations
- ii. Linear Algebraic Solver
- iii. Miscellaneous Computational time for other components of IP such as step control and step update:  $X^{k+1} = X^k + \Delta X$

# Iterations

## Successive iterations in Interior Point Method

- i. Function Evaluations
- ii. Linear Algebraic Solver
- iii. Miscellaneous
- iv. Bottleneck of IP:

• BUS	974
• Branch	1023
• GEN HYDRO	1
• PCC (66kV feeder)	1
• BATTERY	200
<u>Variables for N= 720</u>	1837440



## Phase IV: **Speed-up**

$$\begin{bmatrix} M & G_X \\ G_X & 0 \end{bmatrix} \begin{bmatrix} \Delta X \\ \Delta \lambda \end{bmatrix} = \begin{bmatrix} -N \\ -G_X \end{bmatrix}$$

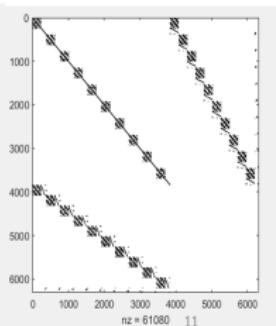
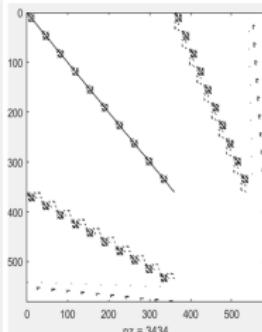
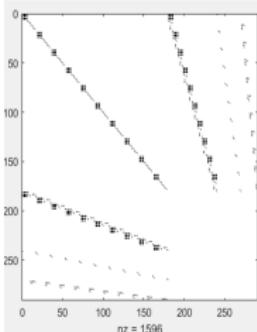
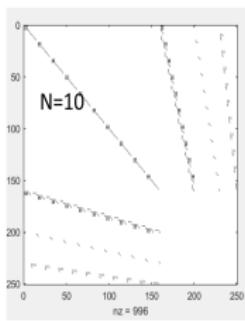
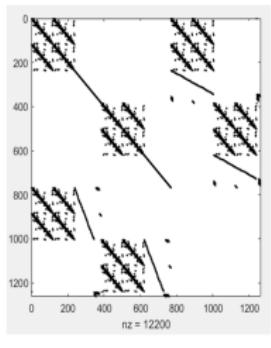
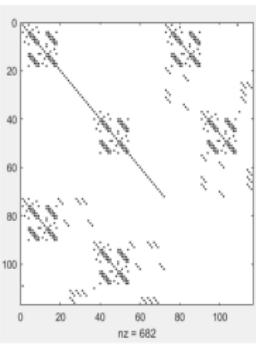
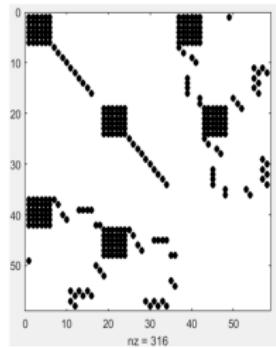
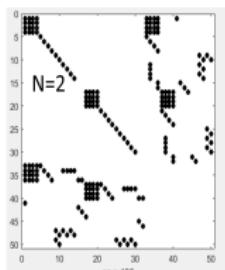
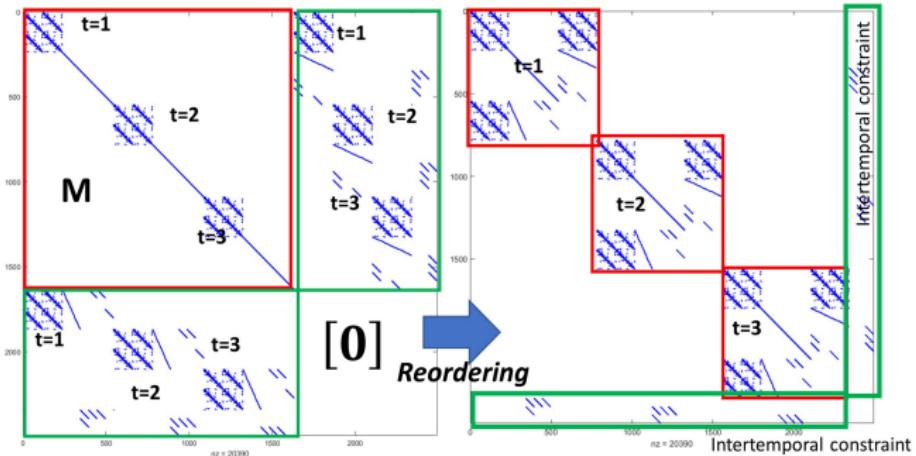


Figure 3: Sparse structure of the Newton-Raphson Jacobian

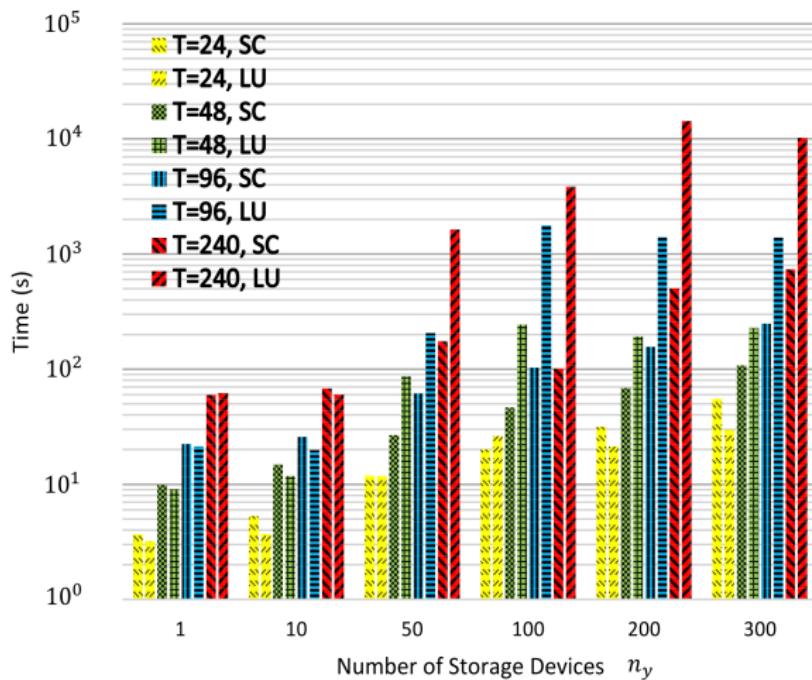


**Figure 4:** Structure of Jacobian of the Newton-Raphson's algorithm before and after reordering.

## Jacobian of Newton-Raphson

$$\begin{bmatrix} M & G_x^\top \\ G_x & 0 \end{bmatrix} \begin{bmatrix} \Delta x \\ \Delta \lambda \end{bmatrix} = \begin{bmatrix} -N \\ -G(x) \end{bmatrix}$$

The solution is published in Papers I and II of this thesis.



**Figure 5:** Total time ( $\text{TotalTime} = \text{No.of Iter.} \times \text{TimePerIter}$ ) for solution of the linear KKT systems, Case: IEEE 118.

# Analytical Derivatives

Table 1: Total time (TotalTime = No.of Iter.  $\times$  TimePerIter) elapsed to calculate: 1) Analytical (hand-coded) derivatives, and 2) Numerical derivatives

Case	T	$n_y$	iter	Analytical			Numerical		
				$F_X(s)$	$G_X + H_X(s)$	$\mathcal{L}_{XX}^\gamma(s)$	$F_X(s)$	$G_X + H_X(s)$	$\mathcal{L}_{XX}^\gamma(s)$
Case9	2	5	13	0.03	0.13	0.14	0.43	0.98	140.07
Case9	10	5	23	0.08	0.36	0.37	11.32	30.62	22815.29
IEEE30	2	5	12	0.04	0.25	0.18	1.01	2.16	682.70
IEEE30	10	5	16	0.05	0.24	0.25	16.73	49.12	79712.78
IEEE118	2	5	22	0.04	0.19	0.20	7.41	18.07	24557.09
IEEE118	10	5	37	0.09	0.62	0.82	158.21 <sup>1</sup>	572.09 <sup>1</sup>	4599735 <sup>1</sup>
Pegase 1354	2	5	23	0.05	0.61	0.78	85.54 <sup>1</sup>	496.18 <sup>1</sup>	7185888 <sup>1</sup>
Pegase 1354	10	5	33	0.10	3.77	5.15	588.06 <sup>1</sup>	3530 <sup>1</sup>	51550941 <sup>1</sup>

<sup>1</sup> Estimated total time: The time elapsed for one iteration multiplied to the iteration that would take to converge

## Test Cases

Test cases of Case9, IEEE30, IEEE118, and PEGASE1354 are part of open source MATPOWER library.

## Phase V: **Future Works**

Thank you for your attention!

