# Introduction to a high-performacne multi-period optimal power flow solver


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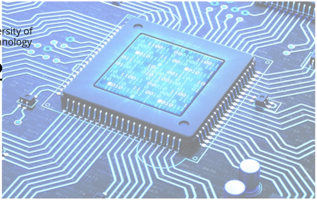
Hotel Scandic Lerkendal

Trondheim

NTNU | Norwegian University of Science and Technology

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# The presentation goal

**Purpose:** To discuss how to solve multi-period optimal power flow fast; **Large-Scale simulation with respect to time and space.**

**Phase I:** Background and Motivation

**Phase II:** Power Flow and Optimal Power Flow

**Phase III:** Solution Method

**Phase IV:** Speed up

**Phase V:** Future Work

| **Presentation Time:** 15-20 min |
|---|

Phase I:

**Background and Motivation**

# Electricity Grid

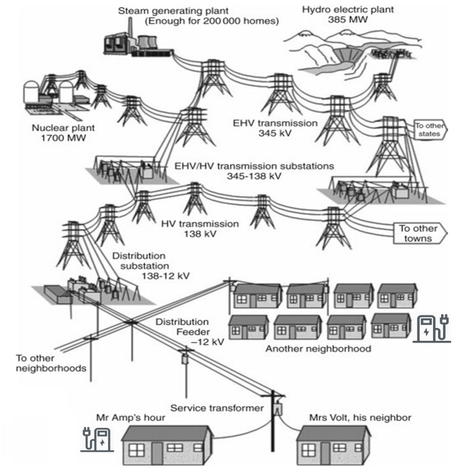


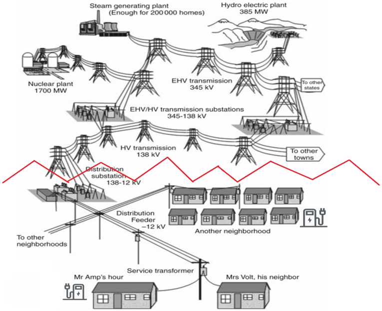

Figure 1: [www.sciencedirect.com/science/article/pii/B9781845697846500019 ]

# Sustainability/Green shift/$CO_2$ reduction

**For many reasons power electricity grid is facing decentralization**

Phasing out coal (carbon-heavy sources of production) and nuclear power plants

This chain between large power producers and consumers is weakened.



# Sustainability/Green shift/$CO_2$ reduction

**For many reasons power electricity grid is facing decentralization**

- Phasing out coal (carbon-heavy sources of production) and nuclear power plants
- Increase penetration of solar and wind production

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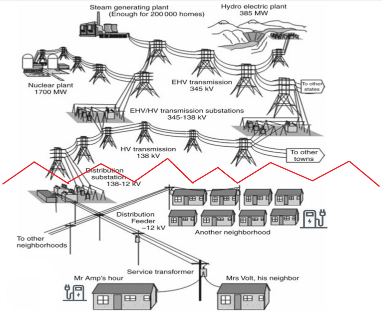


# Background: cont.

## Items

Green shift in electricity systems is needed for the reduction of $CO_2$ emissions

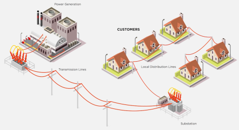



Figure 2: A Typical Power System [Rochester Gas & Electricity]

# Background: cont.

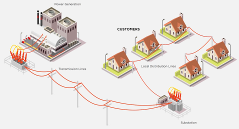
## Items

Green shift in electricity systems is needed for the reduction of $CO_2$ emissions

Integration of Distributed Energy Resources (DER) is a huge challenge

**Note**

DER includes Renewable Energy, Energy Storage, Electric Vehicles and Flexible Demand

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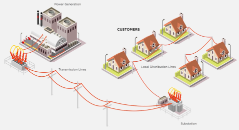
Grid companies must be able to analyse the impacts of DER

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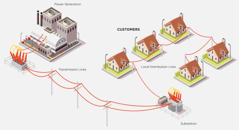
## Note

**Optimal Power Flow (OPF)**solvers are essential

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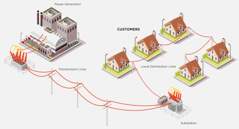
Grid companies must be able to analyse the impacts of DER

The inegration of DER in smart grids calls for **much more sophisticated solvers** for OPF

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# Challenges in the planning and operation of the grid

**Planning:** Optimizing the right type, size and timing of new grid investments

- Local generation (e.g. PV) and increased load (e.g. EVs) can be located in areas where the grid is weak
- Energy storage and demand flexibility are alternatives to grid reinforcements

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- Energy storage and demand flexibility are alternatives to grid reinforcements

**Operation:**Optimize the use of controllable assets such as energy storage and flexible demand to secure, reliable and economic operation of the distribution grids. This means:

- Making the right use of Demand Response
- being able to value the use of end-user flexibility for local or system-wide grid services
- Simulating and optimizing the grids in the presence of **future local markets for energy and flexibility**

# Limitations of traditional grid operation and planning

## Notes

Classical single-period OPF does not offer a possibility for optimal operational scheduling of storage and flexible demand

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We therefore aim to develop the foundations for a new generation of Multi-Period OPF (MPOPF) solvers

i. Solves the OPF problem over several coupled time-steps

ii. Computation time is an issue when using both commercial or free optimization solvers

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MPOPF is an extremely challenging scientific task:

i. Nonlinearity
ii. Large-scale problem with respect with to time and space
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i. Nonlinearity
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## Hardware is reaching its limit with respect to CPU clock speed

# Solution:

## High-Performance Solver [1-2]

Algorithmic design tailored to the conventional OPF algorithms speed-up the solution proposal

Prototype model shows convincing results for real-sized system with distributed renewables, storages and EVs

i. A high-performance and memory-efficient sparse algorithm

ii. Utilizing the structure of the underlying mathematical formulation

1. S. Zaferanlouei, H. Farahmand, V. V. Vadlamudi, M. Korpås,"BATTPOWER Toolbox: Memory-Efficient and High-Performance Multi-Period AC Optimal Power Flow Solver", IEEE Transactions on Power Systems, Jan. 16th, 2021.

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Optimal utilization of **stored energy** and **flexibility** where and when it creates the highest value for the system


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Can be used for grid planning, grid operation and local markets

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# Power System— Today

# Power System— Future

Phase II:

**Power Flow and Optimal Power Flow**

# Power Flow Equations

## Source of Non-linearity

Nonlinear relationship between the voltage phasors and the power injections

Power Line with conductance G and susceptance B

Node i

$S_i = P_i + jQ_i$
$= V_i.\bar{I}_i$

Node j

$S_j = P_j + jQ_j$
$= V_j.\bar{I}_j$

$$Y = G + jB \; and \; G = \frac{R}{R^2 + X^2}, B = \frac{-X}{R^2 + X^2}$$

Where R and X are resistance and reactance respectively

$$\begin{aligned} P_i + jQ_i &= V_i.\bar{I}_i \\ &= V_i.(\bar{Y}_i.\bar{V}) \\ &= V_i.(G_i - jB_i)\bar{V} \\ |V_i|^2 &= V_i.\bar{V}_i \end{aligned}$$

"Power Flow Equations"

"Load Flow Equations"

Y admittance matrix

G and B conductance and susceptance matrices

$\mathrm{Y} = \mathrm{G} + j\mathrm{B}$

> Coupled quadratics in complex voltage phasors

# Power Flow Equations in Different Coordinates

The bus injection model

Power Line with conductance G and susceptance B

Node i: $S_i = P_i + jQ_i = V_i.\bar{I}_i$

Node j: $S_j = P_j + jQ_j = V_j.\bar{I}_j$

$$Y = G + jB \text{ and } G = \frac{R}{R^2 + X^2}, B = \frac{-X}{R^2 + X^2}$$

Where R and X are resistance and reactance respectively

| Polar Voltage Coordinates | $V_i = \lvert V_i \rvert \angle \theta_i \quad \theta_1 = 0$ |
|---|---|

$$P_i = |V_i| \sum_{j=1}^{N} |V_j| (\boldsymbol{G}_{ij} cos(\theta i - \theta j) + \boldsymbol{B}_{ij} sin(\theta i - \theta j))$$

$$Q_i = |V_i| \sum_{j=1}^{N} |V_j| (\boldsymbol{G}_{ij} sin(\theta i - \theta j) - \boldsymbol{B}_{ij} cos(\theta i - \theta j))$$

| Rectangular Voltage Coordinates | $V_i = V_{di} + jV_{qi} \quad V_{q1} = 0$ |
|---|---|

$$P_i = V_{di}(\boldsymbol{G}_{ij} V_{dj} - \boldsymbol{B}_{ij} V_{qj}) + V_{qi}(\boldsymbol{B}_{ij} V_{dj} + \boldsymbol{G}_{ij} V_{qj})$$

$$Q_i = -V_{di}(\boldsymbol{B}_{ij} V_{dj} + \boldsymbol{G}_{ij} V_{qj}) + V_{qi}(\boldsymbol{G}_{ij} V_{dj} - \boldsymbol{B}_{ij} V_{qj})$$

$V_{di} = Re(V_i) \quad V_{qi} = Im(V_i)$ $\qquad N =$ number of Nodes

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## Power Flow

PF equations are a set of nonlinear algebraic equations which can be solved with Newton-Raphson, Gauss–Seidel, Fast-decoupled-load-flow method and etc.

# Single period ACOPF - problem formulation

## Single Period AC Optimal Power Flow (ACOPF)

We are controlling some variables in OPF to minimise system costs with respect to constraints.

| | | |
|---|---|---|
| Objective Function | $\min_{\mathrm{x}} f(\mathrm{x})$ | |
| Equality Constraints<br>(Balance)<br>Power Flow | $\text{s.t. } \mathrm{g}(\mathrm{x}) = \begin{bmatrix} \widetilde{\mathrm{g}}(\mathrm{x}) \\ \overline{\mathrm{g}}(\mathrm{x}) \end{bmatrix} = 0$ | $\in \mathbb{R}^{n_{gx} \times 1}$ |
| Inequality constraints<br>(line and transformer)<br>Operational Constraints | $\mathrm{h}(\mathrm{x}) = \begin{bmatrix} \widetilde{\mathrm{h}}(\mathrm{x}) \\ \overline{\mathrm{h}}(\mathrm{x}) \end{bmatrix} \leq 0$ | $\in \mathbb{R}^{n_{hx} \times 1}$ |
| Vector of Variables | $\mathrm{x} = \begin{bmatrix} \boldsymbol{\Theta} & \boldsymbol{\mathcal{V}} & \boldsymbol{\mathcal{P}}^{\mathrm{g}} & \boldsymbol{\mathcal{Q}}^{\mathrm{g}} \end{bmatrix}^{\top}$ | $\in \mathbb{R}^{n_x \times 1}$ |

## Limitation of Power Flow

- PF is only a set of static equations which provides status of a system for time: $t = t_s$
- It does not include generation constraints and operational constraints

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- OPF is an optimisation problem which optimises status of a system for a SINGLE time: $t = t_s$.
- Although it includes the operational constraints for single time, it will not include operation of DERs and generators over a time horizon.
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## Our suggested approach: multi-period ACOPF

Solves the OPF problem over several time-steps at once useful formulation for systems with Energy Storage and Shiftable Loads (e.g. Evs)

## Advantages of MultiPeriod ACOPF

> Integrating dependent time power system's components such as stationary ESS, EV, generators ramp rate, and so on.

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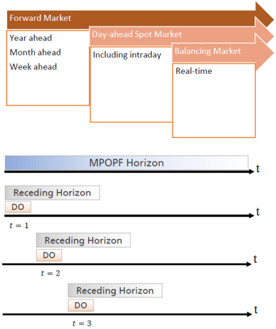

## Advantages of MultiPeriod ACOPF

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## Disadvantage of MultiPeriod ACOPF

The problem grows very large as the number of time-steps is increased, which may lead to an intractable solution.

# MultiPeriod ACOPF-problem formulation

| | |
|---|---|
| Objective Function | $\min_{\mathsf{X}} F(\mathsf{X})$ |
| Equality Constraints (Balance) Power Flow | $\text{s.t.}\;\; G(\mathsf{X}) = \left[\widetilde{G}(\mathsf{X})\;\overline{G}(\mathsf{X})\;\overline{G}^{s}(\mathsf{X})\right]^{\top} = 0 \in \mathbb{R}^{N_g \times 1}$ |
| Inequality constraints (line and transformer) Operational Constraints | $H(\mathsf{X}) = \left[\widetilde{H}(\mathsf{X})\quad \overline{H}(\mathsf{X})\right]^{\top} \leq 0 \in \mathbb{R}^{N_h \times 1}$ |
| Vectors of Constraints | $\widetilde{\mathsf{G}}(\mathsf{X}) = \left[\widetilde{\mathsf{g}}(\mathsf{x}_1)\;\widetilde{\mathsf{g}}(\mathsf{x}_2)\;\ldots\;\widetilde{\mathsf{g}}(\mathsf{x}_T)\right]^{\top}$ <br> $\overline{\mathsf{G}}(\mathsf{X}) = \left[\overline{\mathsf{g}}(\mathsf{x}_1)\;\overline{\mathsf{g}}(\mathsf{x}_2)\;\ldots\;\overline{\mathsf{g}}(\mathsf{x}_T)\right]^{\top}$ <br> $\overline{\mathsf{G}}^{s}(\mathsf{X}) = \left[\overline{\mathsf{g}}^{s}(\boldsymbol{\tau}_1)\;\overline{\mathsf{g}}^{s}(\boldsymbol{\tau}_2)\;\ldots\;\overline{\mathsf{g}}^{s}(\boldsymbol{\tau}_T)\right]^{\top}$ <br> $\widetilde{\mathsf{H}}(\mathsf{X}) = \left[\widetilde{\mathsf{h}}(\mathsf{x}_1)\;\widetilde{\mathsf{h}}(\mathsf{x}_2)\;\ldots\;\widetilde{\mathsf{h}}(\mathsf{x}_T)\right]^{\top}$ <br> $\overline{\mathsf{H}}(\mathsf{X}) = \left[\overline{\mathsf{h}}(\mathsf{x}_1)\;\overline{\mathsf{h}}(\mathsf{x}_2)\;\ldots\;\overline{\mathsf{h}}(\mathsf{x}_T)\right]^{\top}$ |

Phase III:
**Solution Methods**

# Nonlinear Programming (NLP)

**How do we solve Optimal Power Flow?**

I. Interior Point Method

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I. Interior Point Method

II. Gradient Decent Method

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I. Interior Point Method
II. Gradient Decent Method
III. Heuristic Methods

# Nonlinear Programming (NLP)

**How do we solve Optimal Power Flow?**

I. Interior Point Method
II. Gradient Decent Method
III. Heuristic Methods

**Interior Point Method**

The focus of this study is the interior point method solution

**Step(1): Applying Slack variables and the barrier term:**

$$\min_X \left[ F(X) - \gamma \sum_{i=1}^{N_h} ln(z_i) \right]$$

$$\text{s.t. } G(X) = 0$$

$$H(X) + Z = 0$$

$$Z \geq 0$$

slack variables "Z" convert inequality constraints to equality constraints.

**Step (2): Calculate and form the Lagrangian of Barrier subproblem**

$$\mathcal{L}^{\gamma}(\mathrm{X}, \mathrm{Z}, \boldsymbol{\lambda}, \boldsymbol{\mu}) = f(\mathrm{X}) + \boldsymbol{\lambda}^{\top}\mathrm{G}(\mathrm{X}) + \boldsymbol{\mu}^{\top}(\mathrm{H}(\mathrm{X}) + \mathrm{Z}) - \gamma \sum_{i=1}^{N_g} ln(z_i)$$

**Step (3): Calculate the KKT[1] of the Lagrangian**

| | |
|---|---|
| Step (3) $KKT_X$ | $\mathcal{L}^{\gamma}_{\mathrm{X}}(\mathrm{X}, \mathrm{Z}, \boldsymbol{\lambda}, \boldsymbol{\mu}) = f_{\mathrm{X}} + \boldsymbol{\lambda}^{\top}\mathrm{G}_{\mathrm{X}} + \boldsymbol{\mu}^{\top}\mathrm{H}_{\mathrm{X}} = 0$ |
| Step (3) $KKT_Z$ | $\mathcal{L}^{\gamma}_{\mathrm{Z}}(\mathrm{X}, \mathrm{Z}, \boldsymbol{\lambda}, \boldsymbol{\mu}) = \boldsymbol{\mu}^{\top} - \gamma \mathrm{e}^{\top}\mathrm{diag}(\mathrm{Z})^{-1} = 0$ |
| Step (3) $KKT_{\lambda}$ | $\mathcal{L}^{\gamma}_{\boldsymbol{\lambda}}(\mathrm{X}, \mathrm{Z}, \boldsymbol{\lambda}, \boldsymbol{\mu}) = \mathrm{G}^{\top}(\mathrm{X}) = 0$ |
| Step (3) $KKT_{\mu}$ | $\mathcal{L}^{\gamma}_{\boldsymbol{\mu}}(\mathrm{X}, \mathrm{Z}, \boldsymbol{\lambda}, \boldsymbol{\mu}) = \mathrm{H}^{\top}(\mathrm{X}) + \mathrm{Z}^{\top} = 0$ |

[1] Karush–Kuhn–Tucker conditions

Nonlinear Algebraic Equations

$$\boldsymbol{\Omega}(X, Z, \boldsymbol{\lambda}, \boldsymbol{\mu}) = \begin{bmatrix} f_X + \boldsymbol{\lambda}^\top G_X + \boldsymbol{\mu}^\top H_X \\ \mathrm{diag}(Z)\boldsymbol{\mu}^\top - \gamma e^\top \\ G^\top(X) \\ H^\top(X) + Z^\top \end{bmatrix} = 0$$

*S.t.*

$$Z > 0$$

$$\boldsymbol{\mu} > 0$$

**Step (4): Apply Newton Raphson Method**

$$[\boldsymbol{\Omega}_X \ \boldsymbol{\Omega}_Z \ \boldsymbol{\Omega}_\lambda \ \boldsymbol{\Omega}_\mu]^k [\Delta X \ \Delta Z \ \Delta \boldsymbol{\lambda} \ \Delta \boldsymbol{\mu}]^{\top k} = -\boldsymbol{\Omega}(X, Z, \boldsymbol{\lambda}, \boldsymbol{\mu})^k$$

Step (5): Inverse Jacobian of Newton Raphson

$$\begin{bmatrix} M & G_X^\top \\ G_X & 0 \end{bmatrix}^k \begin{bmatrix} \Delta X \\ \Delta \boldsymbol{\lambda} \end{bmatrix}^k = \begin{bmatrix} -N \\ -G(X) \end{bmatrix}^k$$

$M \in \mathbb{R}^{N_x \times N_x}$ and $N \in \mathbb{R}^{N_x \times 1}$ are defined as:

$$\begin{aligned} M &= \mathcal{L}_{XX}^{\gamma} + H_X^\top \mathrm{diag}(Z)^{-1}\mathrm{diag}(\boldsymbol{\mu})H_X \\ N &= f_X^\top + G_X^\top \boldsymbol{\lambda} + H_X^\top \boldsymbol{\mu} + H_X^\top \mathrm{diag}(Z)^{-1}(\gamma e + \mathrm{diag}(\boldsymbol{\mu})H(X)) \\ \mathcal{L}_{XX}^{\gamma} &= f_{XX} + G_{XX}(\boldsymbol{\lambda}) + H_{XX}(\boldsymbol{\mu}) \end{aligned}$$

# Iterations

## Successive iterations in Interior Point Method

i. **Function Evaluations** calculation of $G_X = \frac{\partial G}{\partial X}$, $H_X = \frac{\partial H}{\partial X}$, $F_X = \frac{\partial F}{\partial X}$, $G_{XX} = \frac{\partial}{\partial X}(G_X^\top \boldsymbol{\lambda})$, $H_{XX} = \frac{\partial}{\partial X}(H_X^\top \boldsymbol{\lambda})$, $F_{XX} = \frac{\partial}{\partial X}(F_X^\top)$ in order to form coefficient matrix and right hand side of

$$\begin{bmatrix} M & G_X^\top \\ G_X & 0 \end{bmatrix}^k \begin{bmatrix} \Delta X \\ \Delta \boldsymbol{\lambda} \end{bmatrix}^k = \begin{bmatrix} -N \\ -G(X) \end{bmatrix}^k$$

# Iterations

## Successive iterations in Interior Point Method

i. Function Evaluations

ii. **Linear Algebraic Solver** Calculate the Inverse $\begin{bmatrix} M & G_X^\top \\ G_X & 0 \end{bmatrix}^k$ in

$$\begin{bmatrix} M & G_X^\top \\ G_X & 0 \end{bmatrix}^k \begin{bmatrix} \Delta X \\ \Delta \lambda \end{bmatrix}^k = \begin{bmatrix} -N \\ -G(X) \end{bmatrix}^k$$

# Iterations

## Successive iterations in Interior Point Method

i. Function Evaluations

ii. Linear Algebraic Solver

iii. Miscellaneous Computational time for other components of IP such as step control and step update: $X^{k+1} = X^k + \Delta X$

# Iterations

## Successive iterations in Interior Point Method

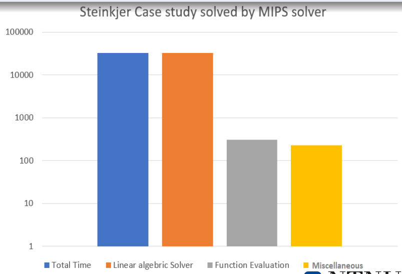
i. Function Evaluations

ii. Linear Algebraic Solver

iii. Miscellaneous

iv. Bottleneck of IP:

| | |
|---|---|
| • BUS | 974 |
| • Branch | 1023 |
| • GEN HYDRO | 1 |
| • PCC (66kV feeder) | 1 |
| • BATTERY | 200 |
| Varibles for N= 720 | 1837440 |



Phase IV:

**Speed-up**

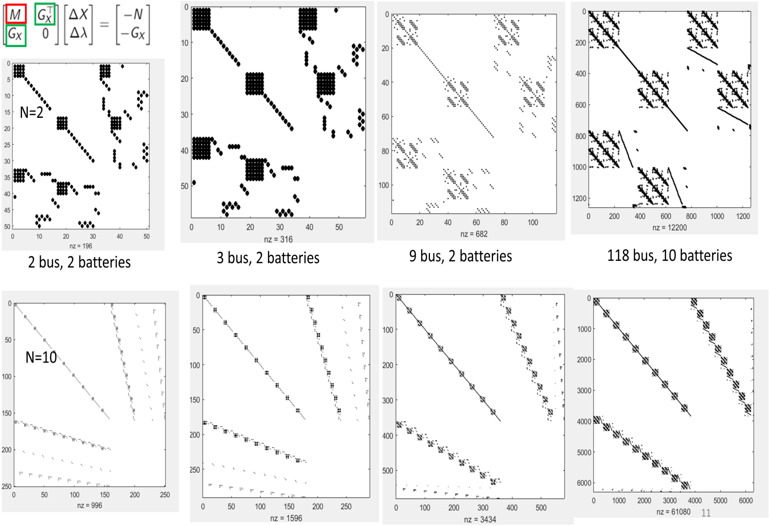



Figure 3: Sparse structure of the Newton-Raphson Jacobian

# Connectivity Matrices

| | |
|---|---|
| a term in pf equation: | $\begin{bmatrix} \mathrm{C}_g \end{bmatrix}_{n_b \times n_g} \begin{bmatrix} \mathrm{P}_g \end{bmatrix}_{n_g \times 1}$ |
| example: | $\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}_{n_b \times n_g} \begin{bmatrix} \mathrm{P}_{g1} \\ \mathrm{P}_{g2} \\ \mathrm{P}_{g3} \\ \mathrm{P}_{g4} \\ \mathrm{P}_{g5} \end{bmatrix}_{n_g \times 1} = \begin{bmatrix} 0 \\ \mathrm{P}_{g2} \\ 0 \\ \mathrm{P}_{g1} \\ 0 \\ \mathrm{P}_{g3} \\ 0 \\ 0 \\ \mathrm{P}_{g4} \\ \mathrm{P}_{g5} \end{bmatrix}_{n_b \times 1}$ |

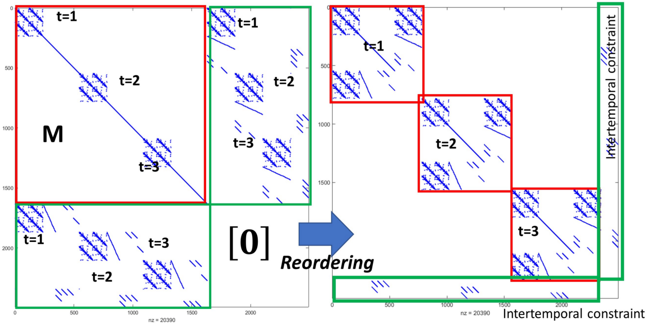



Figure 4: Structure of Jacobian of the Newton-Raphson's algorithm before and after reordering.

## Jacobian of Newton-Raphson

$$\begin{bmatrix} M & G_{\mathbf{X}}^{\top} \\ G_{\mathbf{X}} & 0 \end{bmatrix} \begin{bmatrix} \Delta X \\ \Delta \lambda \end{bmatrix} = \begin{bmatrix} -N \\ -G(X) \end{bmatrix}$$

The solution is published in Papers I and II of this thesis.

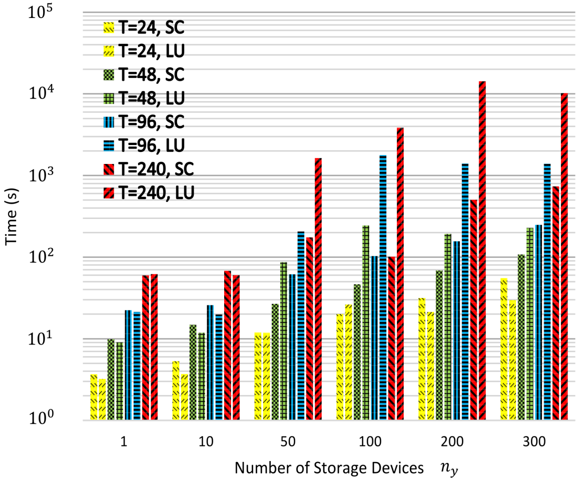



Figure 5: Total time ($\text{TotalTime} = \text{No.of Iter.} \times \text{TimePerIter}$) for solution of the linear KKT systems, Case: IEEE 118.

# Analytical Derivatives

Table 1: Total time ($\text{TotalTime} = \text{No.of Iter.} \times \text{TimePerIter}$) elapsed to calculate: 1) Analytical (hand-coded) derivatives, and 2) Numerical derivatives

| | | | | Analytical | | | Numerical | | |
|---|---|---|---|---|---|---|---|---|---|
| Case | $T$ | $n_y$ | iter | $F_{\mathbf{X}}$(s) | $G_{\mathbf{X}}+H_{\mathbf{X}}$(s) | $\mathcal{L}^{\gamma}_{\mathbf{XX}}$(s) | $F_{\mathbf{X}}$(s) | $G_{\mathbf{X}}+H_{\mathbf{X}}$(s) | $\mathcal{L}^{\gamma}_{\mathbf{XX}}$(s) |
| Case9 | 2 | 5 | 13 | 0.03 | 0.13 | 0.14 | 0.43 | 0.98 | 140.07 |
| Case9 | 10 | 5 | 23 | 0.08 | 0.36 | 0.37 | 11.32 | 30.62 | 22815.29 |
| IEEE30 | 2 | 5 | 12 | 0.04 | 0.25 | 0.18 | 1.01 | 2.16 | 682.70 |
| IEEE30 | 10 | 5 | 16 | 0.05 | 0.24 | 0.25 | 16.73 | 49.12 | 79712.78 |
| IEEE118 | 2 | 5 | 22 | 0.04 | 0.19 | 0.20 | 7.41 | 18.07 | 24557.09 |
| IEEE118 | 10 | 5 | 37 | 0.09 | 0.62 | 0.82 | 158.21[1] | 572.09[1] | 4599735[1] |
| Pegase 1354 | 2 | 5 | 23 | 0.05 | 0.61 | 0.78 | 85.54[1] | 496.18[1] | 7185888[1] |
| Pegase 1354 | 10 | 5 | 33 | 0.10 | 3.77 | 5.15 | 588.06[1] | 3530[1] | 51550941[1] |

[1] Estimated total time: The time elapsed for one iteration multiplied to the iteration that would take to converge

## Test Cases

Test cases of Case9, IEEE30, IEEE118, and PEGASE1354 are part of open source MATPOWER library.

Phase V:

**Future Works**

Thank you for your attention!

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