CHAPTER 14 :OSCILLATIONS

One mark questions

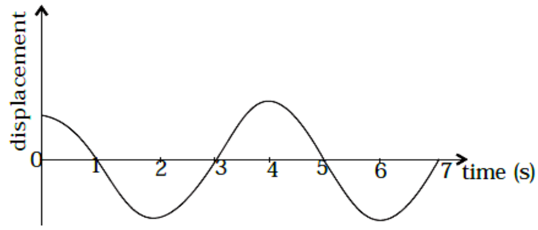
1. Define frequency of periodic motion.
2. Give an example for a non-simple harmonic periodic motion.
3. What is the SI unit of frequency?
4. Give the relation between period and frequency of periodic motion.
5. What is the mean position(or equilibrium position) of an oscillating body?
6. Define the phase of particle in oscillatory motion.
7. What is the net external force acting on the body at its equilibrium position?
8. Where will be the force acting on a particle executing SHMmaximum?
9. Define amplitude of simple harmonic motion.
10. What is the SI unit of angular frequency?
11. Give the relation between angular frequency and frequency.
12. Mention the relation between angular frequency and period.
13. Write the relation between m, ω and k, where the terms have usual meaning.
14. Give the expression ofthe force law (Hooke's law)fora particle executingSHM.
15. Mention the expression for velocity of a particle executing SHM.
16. What is the phase difference between velocity and displacement of a particle executing SHM?
17. What is the phase difference between acceleration and displacement of a particle executing SHM?
18. Write the relation between velocity amplitude 'v', the displacement amplitude 'A' and the angular frequency 'ω' of SHM.
19. Give the expression for acceleration of a particle executing SHM.
20. Give the expression for kinetic energy of the particle executing SHM at mean position.
21. What is the minimum value of kinetic energy of a particle executing SHM?
22. Does the total mechanical energy of a harmonic oscillator depend on time?
23. When will the motion of a simple pendulum be simple harmonic?
24. The time period of simple pendulum is T, What is the time period when mass of the bob is doubled?
25. How does the time period of a simple pendulum vary when it is taken from equator to poles?
26. What happens to the time period of a simple pendulum when it is taken from earth to the moon?
27. How does the time period of simple pendulum vary with its length?
28. How is the time period of the pendulum affected when it is taken to hills or in to mines?
29. What is the cause for damped oscillations?
30. What happens to the mechanical energy of the particle executing damped oscillations?
31. Whether the amplitude increase or decrease or remains same in damped oscillations?
32. What is resonance?
33. What is the frequency of oscillation of a simple pendulum mounted in a cabinthat is freely falling under gravity?
34. What is the condition for resonance?

Two marks questions

1. What is periodic motion? Give an example.
2. Define period of periodic motion.State its SI unit.
3. On an average a human heart is found to beat 75 times in a minute. Calculate its frequency and period.
4. A particle takes 32s to make 20 oscillations. Calculate time period and frequency.
5. Which of the followingfunctions of time represent (a) periodic and(b) non-periodic motion? Also give the period foreach in case of periodic motion [$\omega$is anypositive constant].

   (a) $\sin 3\omega t$ (b) $\sin\omega t + \cos\omega t$ (c) $\sin\omega t + \cos 2\omega t + \sin 4\omega t$

   (d) $3\cos(\pi/4 - 2\omega t)$ (e) $\sin^2\omega t$ (f) $\sin^3\omega t$ (g)$e^{-\omega t}$ (h) $\log(\omega t)$
6. Give the expression for the displacement of a particle executing simple harmonic motion.
7. Plot the graph of $x(t)$ as a function of time for the motion represented by the equation $x(t) = A\cos(\omega t + \phi)$,Where A, $\omega$and $\phi$are constants.
8. The displacement of a particle executing SHM is given by $x(t) = A\cos(\omega t + \phi)$. Writethe expression for velocity and acceleration.
9. Does the function '$\sin\omega t - \cos\omega t$' represent SHM?Find its (i) period and (ii) phase angle.
10. Mention the expression for kinetic energy of a particle executing SHM. Explain the terms.
11. Where is the kinetic energy of a particle executing SHM (i) minimumand (ii) maximum?
12. Give the expression for the time period of a simple harmonic oscillator (spring system).
13. Draw the free-body diagram of the simple pendulum showing the forces acting on the bob.
14. Mention the expression for potential energy of a particle executing SHM. Explain the terms.
15. Where is the potential energy of a particle executing SHM (i) minimum and (ii) maximum?
16. Give the expression for total mechanical energy of a particle executing SHM.
17. Mention the expression for time period of simple pendulum. Explain the terms.
18. What is the length of a simple pendulum, which ticks seconds?
19. Write the expression for time period of oscillations of loaded spring. Explain the terms.
20. What are free oscillations? Give an example.
21. What are damped oscillations? Give an example.
22. What are forced or driven oscillations? Give an example.
23. What are the two basic characteristics of a simple harmonicmotion?
24. What is the ratio of maximum acceleration to the maximumvelocity of a simple harmonic oscillator?
25. What is the ratio between the distance travelled by the oscillatorin one time period and amplitude?
26. The displacement-time curve for a particle executing S.H.M. is given. (i) What is the time period of S.H.M? (ii) What is the phase of the particle at t = 2s?



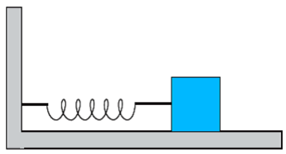
27. Give the expression for damping force. Explain the terms.
28. Draw the displacement time graph for damped oscillations.
29. Write the expression for angular frequency of damped oscillator. Explain the terms.
30. Give the expression for total mechanical energy of the damped oscillator. Explain the terms.
31. A man with a wristwatch on his hand falls from the top of a tower. Does thewatch give correct time during the free fall?And why?
32. Give the graphical representation ofthe variation of potential energy, kinetic energy and the total energy as functions of position x for a linear harmonic oscillator with amplitude A.

Four/Five marks questions

1. State force law definition of simple harmonic motion and hence obtain the expression for angular frequency.
2. Plot the velocity-time graph and acceleration-time graph of a particle executing SHM whose displacement is given by $x(t) = A\cos(\omega t)$.
   Also give the expressions for the velocity and acceleration.
3. Arrive at the expression for time period of oscillation of a mass attached to a spring.
4. Explain simple harmonic motion with reference to uniform circular motion with the help of a diagram.
5. Show that in simple harmonic motion, the acceleration is directly proportional to its displacement at the given instant of time.
6. Derive the expression for the kinetic energy and potential energy of a harmonic oscillator.
7. Arrive at an expression for the time period of simple pendulum.
8. Discuss the effect of damping force on a system (a mass attached to a spring)executing SHM in a viscous medium and obtain an expression for displacement of the damped oscillator.

Four/Five marksProblems

1. A particle executing S.H.M. has a maximum speed of 30 cm/s and a maximum acceleration of 60 cm/s^2. Calculate the period of the oscillation.
2. A particle oscillates with SHM according to the equation:$x = 5\cos(2\pi t + \pi/4)$ metre.
   At t = 1.5 s, calculate the (a) displacement, (b) speed and (c) acceleration of the body.
3. A 5 kg collar is attached to a spring of spring constant 500 Nm^{-1}. It slides without friction over a horizontal rod.The collar is displaced from its equilibrium position by 10.0 cm and released. Calculate (a) the period of oscillation, (b) the maximum speed and (c) maximum acceleration of the collar.
4. A spring balance has a scale that reads from 0 to 50 kg. The length of the scale is 20cm. A body suspended from this balance, when displaced and released, oscillateswith a period of 0.6 s. What is the weight of the body?
5. A spring having a spring constant of 1200 Nm^{-1} is mounted on a horizontal table as shown in the figure. A mass of 3 kg is attached to the free end of the spring. Themass is then pulled sideways to a distance of 2.0 cm and released.

   Determine (i) the frequency of oscillations,
   (ii) maximum acceleration of the mass and (iii) the maximum speed of the mass.
6. A particle executes SHM along the x-axis, its displacement varies with the time according to the equation: $x(t) = 2.5\cos(4\pi t + \pi/6)$, where $x(t)$ in metre and t is in second.
   Determine the amplitude, frequency, period and phase constantof the motion.
7. The acceleration due to gravity on the surface of moon is 1.7 ms^{-2}. What is the time period of a simple pendulum on the surface of moon if its time period on the surface of earth is 1.5s.
8. A particle describes SHM with amplitude of 5 cm and a period of 0.2 s. Find the acceleration and velocity of the particle when the displacement is (a) 5cm, (b) 3 cm and (c) 0 cm.
9. A block of mass is 1 kg is fastened to a spring. The spring has a spring constant of 50 Nm^{-1}. The block is pulled to a distance x= 10 cm from its equilibrium position at x= 0 on a frictionless surface from rest at t= 0. Calculate the kinetic, potential and total energies of the block when it is 5 cm away from the mean position.
10. The piston in the cylinder head of a locomotive has a stroke (twice the amplitude) of 1.0 m.
    If the piston moves with simple harmonic motion with an angular frequencyof 200 rad/min, what is its maximum speed and maximum acceleration?

Answers to one mark questions

1. The number of repetitions that occur per unit time (second) is called the frequency of the periodicmotion.
2. Rotation of earth about its axis, Revolution of earth around the sun, etc..
3. hertz (Hz).
4. $\upsilon = \frac{1}{T}$ or $T = \frac{1}{\upsilon}$, where υ is frequency and T is time period.
5. It is a position of the particle where the net force acting on it is zero.
6. Phase of a vibrating particle is defined as state of vibration regarding position and direction of motion at that instant of motion.
7. Zero.
8. At the extreme positions.
9. The magnitude of the maximum displacement ofthe particle in either direction of SHM is called the amplitude.
10. radianper second (rad/s).
11. $\omega = 2\pi\upsilon$, where υ is frequency and ω is angular frequency.
12. $\omega = \frac{2\pi}{T}$, where ω is angular frequency and T is time period.
13. Angular frequency: $\omega = \sqrt{\frac{k}{m}}$
14. $F(t) = -k\,x(t)$.
15. Velocity: $v(t) = -\omega A \sin(\omega t + \phi)$.
16. $\pi/2$ or 90^o.
17. πor 180^o.
18. $v = \omega A$.
19. Acceleration : $a(t) = -\omega^2 A \cos(\omega t + \phi) = -\omega^2 x(t)$
20. Kinetic energy at mean position(maximum value) = $K_{max} = \frac{1}{2}m\omega^2A^2$ or $K_{max} = \frac{1}{2}k\,A^2$
21. Zero (at extreme positions of displacement).
22. No, the total mechanical energy of a harmonic oscillator is *independent of time*.
23. The motion of a simple pendulum swinging through small angles is approximately SHM.
24. Time period = T, since the time period of simple pendulum is independent of mass of the bob.
25. The time period decreases as the value of g is more at the poles than that at equator.
26. The time period increases as the value of g is less on the moon than that on earth surface.
27. Time period of simple pendulum is directly proportional to the square root of its length.
28. The time period increases as the value of g is less on hills or in mines than that at surface.
29. Air drag, viscous force and friction at support oppose the oscillations are main causes for damping.
30. Mechanical energy of the particle decreases.
31. Amplitude decreases gradually in damped oscillations.
32. The phenomenon of increase in amplitude when the driving frequency (applied) is close to the natural frequency of the oscillatoris called resonance.
33. Frequency is zero since the gravity disappears for free fall.
34. $\omega_d = \omega$, where ω is natural angular frequency and ω_d is driven angular frequency

    OR$\upsilon_d = \upsilon$, where υ is natural frequency and υ_d is driven frequency

Answers to Two marks questions

1. The motion that repeats itself at regular intervals of time is called periodic motion.
   E.g.: The motion of leaves and branches of a tree in breeze, orbital motion of planets in the solar system, oscillations of loaded spring, movement of the pendulum of a clock.
2. The smallest interval of time after which the motion is repeated is called time period.
   SI unit of period is second(s).
3. The beat frequency of heart = $75/(1 \text{ min}) = 75/(60 \text{ s}) = 1.25 \text{ s}^{-1} = 1.25$ Hz
   The time period $T = 1/(1.25 \text{ s}^{-1}) = 0.8$ s
4. $\text{Time period} = \dfrac{\text{Total time}}{\text{Number of oscillations}} = \dfrac{32 \text{ s}}{20} = 1.6 \text{ s}$

   $\text{Frequency} = \dfrac{1}{\text{Time period}} = \dfrac{1}{1.6} = 0.625 \text{ Hz}$
5. (a) $\sin 3\omega t$ is a periodic function. Period of the function is $2\pi/3\omega$

   (b) $\sin\omega t + \cos\omega t$ is a periodic function.
   $\sin\omega t + \cos\omega t = \sqrt{2} \sin(\omega t + \pi/4)$
   Now $\sqrt{2}\sin(\omega t + \pi/4) = \sqrt{2}\sin(\omega t + \pi/4 + 2\pi) = \sqrt{2}\sin[\omega(t + 2\pi/\omega) + \pi/4]$
   $\therefore$ Period of the function is $2\pi/\omega$.

   (c) $\sin\omega t + \cos 2\omega t + \sin 4\omega t$ is a periodic function.
   $\sin\omega t$ has a period of $2\pi/\omega$, $\cos 2\omega t$ has a period of $2\pi/2\omega$ and
   $\sin 4\omega t$ has a period of $2\pi/4\omega$.
   $\therefore$ Period of the function is $2\pi/\omega$.

   (d) $3\cos(\pi/4 - 2\omega t)$ is a periodic function. Period of the function is $2\pi/2\omega = \pi/\omega$

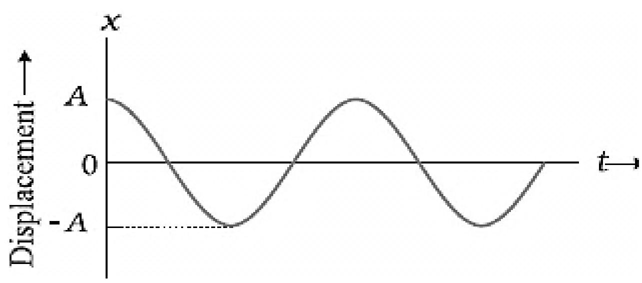
(e) $\sin^2\omega t = \frac{1}{2} - (\frac{1}{2}\cos 2\omega t)$ is a periodic function. Period of the function is $2\pi/2\omega = \pi/\omega$

   (f) $\sin^3\omega t$ is a periodic function. Period of the function is $2\pi/\omega$

   (g) $e^{-\omega t}$ is a non-periodic function.
   $e^{-\omega t}$ decreases monotonically with increasing time and tends to zero as $t \to \infty$ and never repeats its value.

   (h) $\log(\omega t)$ is a non-periodic function.
   $\log(\omega t)$ increases monotonically with time t. It never repeats its value.
   As $t \to \infty$, $\log(\omega t)$ diverges to ∞. $\therefore$ It cannot represent any kind of physical displacement.
6. $x(t) = A\cos(\omega t + \phi)$,

Where $x(t)$ is displacement as a function of time,
A is Amplitude, ω is angular frequency,
ϕ is phase constant and $(\omega t + \phi)$ is phase.



7. Displacement : $x(t) = A\cos(\omega t + \phi)$,
   The graph of x as a function of time for the SHM is shown in the adjacent figure.
8. Displacement : $x(t) = A\cos(\omega t + \phi)$,
   Velocity: $v(t) = -A\omega\sin(\omega t + \phi)$ and acceleration: $a(t) = -A\omega^2\cos(\omega t + \phi)$
9. The function $\sin\omega t - \cos\omega t$ represents SHM.
   Because $\sin\omega t - \cos\omega t = \sin\omega t - \sin(\pi/2 - \omega t) = 2\cos(\pi/4)\sin(\omega t - \pi/4) = \sqrt{2}\sin(\omega t - \pi/4)$
   This function represents a simple harmonic motion having a period $T = 2\pi/\omega$ and a phase angle $(-\pi/4)$ or $(7\pi/4)$.

10. **Kinetic energy:** $K = \frac{1}{2}m\omega^2A^2\sin^2(\omega t+\phi)$ or $K = \frac{1}{2}kA^2\sin^2(\omega t+\phi)$

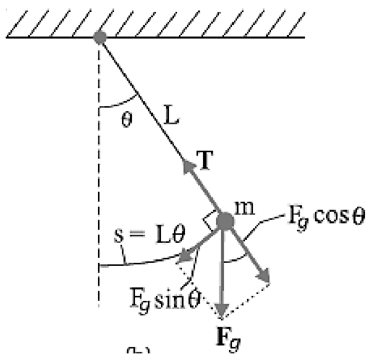
    Where m is mass of the particle executing SHM, $\omega$is angular frequency,Ais Amplitude, $\omega$is angular frequency,ϕ is phase constant, ($\omega t+\phi$) is phase and k is force constant.

11. **(i) Minimum at extreme positions (ii) maximum at mean position.**

12. **The time period of a simple harmonic oscillator:** $T = 2\pi\sqrt{\frac{m}{k}}$

    Where m is the mass of the load attached andk is the force constant.

13. **Free-body diagram of the simple pendulum is shown in the adjacent diagram.**
    Where m is the mass of the bob of the pendulum,
    L is the length of the pendulum,
    T is the tension in the string,
    F_g(= mg) is the gravitational force
    $F_g\cos\theta$ radial component of gravitational force and
    $F_g\sin\theta$ is the tangential component of gravitational force.



14. **Potential energy** $U = \frac{1}{2}kx^2 = \frac{1}{2}m\omega^2A^2\cos^2(\omega t+\phi)$ or $U = \frac{1}{2}kA^2\cos^2(\omega t+\phi)$

    Where m is mass of the particle executing SHM, $\omega$is angular frequency,A is Amplitude, $\omega$is angular frequency,ϕ is phase constant, ($\omega t+\phi$) is phase and k is force constant.

15. **(i) Minimum at mean position (ii) maximum at extreme positions.**

16. **Total mechanical energy:** $E = K+U = \frac{1}{2}m\omega^2A^2\sin^2(\omega t+\phi)+\frac{1}{2}m\omega^2A^2\cos^2(\omega t+\phi) = \frac{1}{2}kA^2$

    Where A is Amplitude and k is force constant.

17. **Time period of oscillation of the pendulum:** $T = 2\pi\sqrt{\frac{L}{g}}$

    Where L is the length of the pendulum and g is acceleration due to gravity.

18. **Time period of oscillation of the given pendulum:T = 2 s**

    Time period: $T = 2\pi\sqrt{\frac{L}{g}}$

    OR $L = \left(\frac{gT^2}{4\pi^2}\right) = \left(\frac{9.8\times2^2}{4\times3.142^2}\right) = 0.9927\text{m} \approx 1\text{m}$

19. **Time period of oscillation of loaded spring:** $T = 2\pi\sqrt{\frac{m}{k}}$,

    Where m is the mass of the load attached and k is the spring constant.

20. **The oscillations made by a body (particle) when it is left free itself, it oscillates with a frequency of its natural frequency are called free oscillations.**
    E.g.: The oscillations of a pendulum, the oscillations of loaded spring, oscillations of the prongs of a tuning fork, etc..

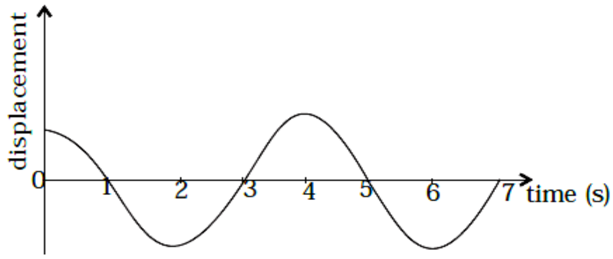
21. **The oscillations of a simple pendulum (or any other oscillating particle) are opposed by air drag and friction at its support. As a resultpendulum makes oscillations with decreasing amplitude. Such oscillations are called damped oscillations.**
    E.g.: The oscillations of a pendulum, the oscillations of loaded spring, oscillations of the prongs of a tuning fork, etc..

22. The oscillations in which the amplitude of the oscillation is maintained/sustainedwith the help of external agency (force) are called forcedor driven oscillations.
E.g.: When the stem of vibrating tuning fork pressed on a table, table executes forced oscillations with a frequency equal to frequency of tuning fork.

23. The two basic characteristics of a simple harmonicmotion:
(i) Restoring force is proportional to the displacement of the particle from mean position,
(ii) Restoring force is directed towards the meanposition.

24. Maximum acceleration: $a_m = A\,\omega^2$.............(1)
Maximum velocity: $v_m = A\,\omega$..............(2)
$\Rightarrow$ Required ratio, (1)÷(2) gives $a_m/v_m = \omega$, isthe angular frequency.

25. Ratio between the distance travelled by the oscillatorin one time period and amplitude:
Distance travelled in by the oscillatorin one time period is 4A, where A is Amplitude.
Required ratio = 4A/A = 4

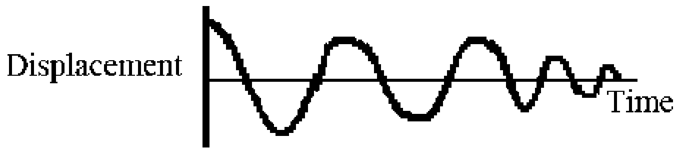


26. (i) Time period of S.H.M is 4s
(ii)The phase of the particle at t = 2s is 'π'

27. Damping force : $F_d = -\,b\,v$
Where b is damping constant (always positive) and v is velocity.

Note:The negative sign indicates that the force is opposite to the velocity at every moment.

28. Displacement-time graph for damped oscillations is as shown in the adjacent figure.

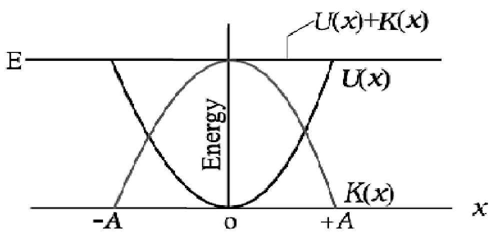


29. Angular frequency of damped oscillator: $\omega' = \sqrt{\dfrac{k}{m} - \dfrac{b^2}{4m^2}}$
Where b is damping constant, m is the mass and k is the force constant.

30. The total mechanical energy of the damped oscillator: $E = \dfrac{1}{2} k\, A^2\, e^{-bt/m}$
Where A is Amplitude, k is force constant, m is the mass and b is damping constant.

31. Yes,the clock gives correct timeduring the free fall.
The motion in the wristwatch depends on spring action and has nothing to do with acceleration due to gravity.
i.e., The motion of spring in the watch is not affected by acceleration due to gravity.

32. The graphical representation ofthe variation of potential energy U(x), kinetic energy K(x) and the total energy E as functions of position x for a linear harmonic oscillator with amplitude A is as shown in the adjacent diagram.



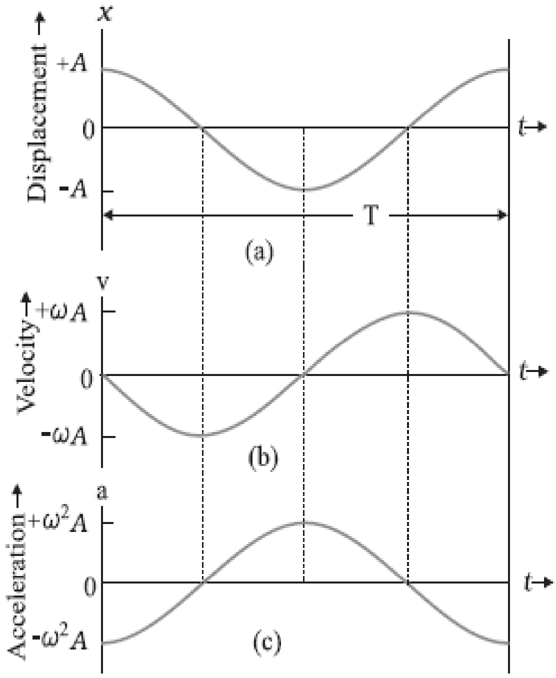
Answers to Four/Five marks questions

1. Simple harmonic motion is the motion executed by a particle subject to a force, which is proportional to the displacement of the particle and is directed towards the mean position.
Consider a particle of mass m executing SHM with an angular frequency ω.
From Newton's second law: $F(t) = ma = -m\omega^2 x(t)$........(1) since acceleration $a = -\,\omega^2 x(t)$

The restoring force Facting on the particle $F(t) = -\,k\,x(t)$........(2) , k is force constant.

From (1) and (2), $k = m\omega^2 \therefore \omega = \sqrt{\dfrac{k}{m}}$

2. **The diagram(a) Represents Displacement–time graph, The diagram(b) Represents velocity–time graph and The diagram(c) Represents acceleration–time graph.**



Displacement: $x(t) = A\cos(\omega t)$

Velocity: $v(t) = -A\omega\sin(\omega t)$

Acceleration: $a(t) = -A\omega^2\cos(\omega t)$.

Where A is Amplitude, ω is angular frequency, and (ωt) is phase.

3. **<u>Derivation of expression for time period of oscillation of a mass attached to a spring:</u>**

Consider the small oscillations of a block of mass mfixed to a spring, which in turnis fixed to a support.

From Newton's second law,

$F(t) = ma$, but acceleration $a = -\omega^2 x(t)$

i.e., $F(t) = -m\omega^2 x(t)$……………(1)

The magnitude of the restoring forceacting on the massis proportional to the deformationor the displacement and acts in opposite direction

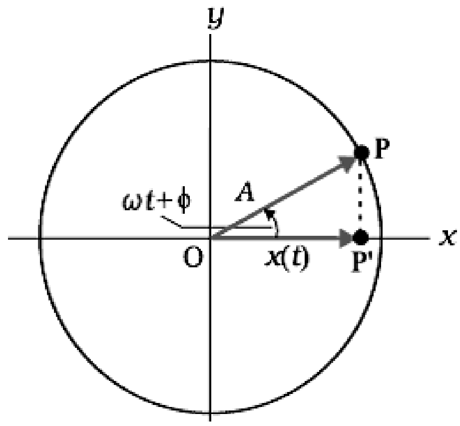
i.e., $F(t) = -k\,x(t)$……………..(2)

From (1) and (2), $k = m\omega^2$ or $\omega = \sqrt{\dfrac{k}{m}}$

Time period of oscillation: $T = \dfrac{2\pi}{\omega} = 2\pi\sqrt{\dfrac{m}{k}}$

4. **The motion of a reference particle P executing a uniform circular motion with constant angular speed ω in a reference circle is shown in the adjacent figure.**

The radius Aof the circle is the magnitude of the particle's position vector.



At any time t, the angular position of the particle is $(\omega t + \phi)$, where ϕ is its angular position att = 0.

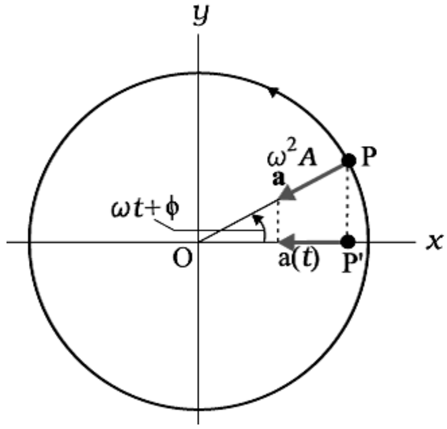
The projection of particle P on the x-axis is a point P′, which we can take as a secondparticle. The projection of the position vector ofparticle P on the x-axis gives the location $x(t)$ ofP′.

Thus we have,$x(t) = A\cos(\omega t + \phi)$.

This showsthat if the reference particle P moves in auniform circular motion, its projection particleP′ executes a simple harmonic motion along adiameter of the circle.

ThusSimple harmonic motionis the projection of uniform circular motionon a diameter of the circle in which thelatter motion takes place.

5. Consider the a particle executing SHM,



Let the displacement: $x(t) = A\cos(\omega t+\phi)$

Differentiating displacement w.r.t. 't',

Velocity: $v(t) = \frac{d}{dt}[x(t)] = -A\omega\sin(\omega t+\phi)$

Differentiating velocityw.r.t. 't',

Acceleration: $a(t) = \frac{d}{dt}[v(t)]$

$= -A\omega^2\cos(\omega t+\phi) = -\omega^2 x(t)$

$\Rightarrow$ Acceleration: $a(t) \propto x(t)$

$\therefore$ In SHM, the acceleration is proportional to thedisplacement at any instant of time.

6. <u>Derivation of expression for the kinetic energy of a harmonic oscillator:</u>

Kinetic energy: $K = \frac{1}{2}mv^2$, where v = velocity= $-A\omega\sin(\omega t+\phi)$, m is mass of the particle

i.e., $K = \frac{1}{2}m\omega^2 A^2\sin^2(\omega t+\phi) = \frac{1}{2}k A^2\sin^2(\omega t+\phi)$, where $k = m\omega^2$

<u>Derivation of expression for potential energy of a harmonic oscillator:</u>

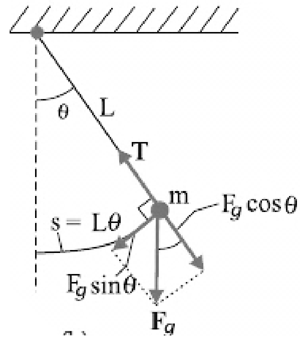
Potential energy: $U = \frac{1}{2}k x^2$ where $x = A\cos(\omega t + \phi)$and k is force constant

i.e., $U = \frac{1}{2}k A^2\cos^2(\omega t+\phi) = \frac{1}{2}m\omega^2 A^2\cos^2(\omega t+\phi)$

Where$\omega$is angular frequency,Ais Amplitude,

$\omega$is angular frequency,ϕ is phase constant and $(\omega t+\phi)$ is phase.

7. <u>Derivation of the expression for the time period of simple pendulum:</u>



Let mbe the mass of the bob of the simple pendulum executing oscillations about the mean position as shown in the diagram.

L be the length of the simple pendulum,

Tbe the tension in the string,

$F_g(= mg)$ be the gravitational force acting vertically down,

$F_g\cos\theta$be the radial component of gravitational force and

$F_g\sin\theta$be the tangential component of gravitational force.

Torque $\tau = -L\,(mg\sin\theta)$ and from rotational motion, $\tau = I\,\alpha$

$\Rightarrow I\,\alpha = -mgL\sin\theta$, where$\alpha$is the angular acceleration and

Iis thependulum's rotational inertia of the system about the pivot point.

$\Rightarrow \alpha = -\left(\frac{mgL}{I}\right)\sin\theta$ and for small value of θ, $\sin\theta \approx \theta$

$\therefore \alpha = -\left(\frac{mgL}{I}\right)\theta$ comparing this equation with $a = -\omega^2 x$as they are similar.

We have, $\omega^2 = \frac{mgL}{I} \Rightarrow \omega = \sqrt{\frac{mgL}{I}}$

Time period: $T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{I}{mgL}}$

The string of simple pendulum is massless. So moment of inertia of the bob is $I = mL^2$

$\therefore$ Time periodof the simple pendulum is $T = 2\pi\sqrt{\frac{L}{g}}$

8. Let the damping force: $F_d = -b\,v$ ……………(1)

Where b is a positive constant called damping constant and v is velocity of the particle. The negative sign indicates that the force is opposite to the velocity at every moment.

When the mass mis attached to the spring and released, the spring will elongate a little and the mass will settle at some height. This position is the equilibrium position of the mass. If the mass is pulled down or pushed up a little, the restoring force on the block due to the spring is $F_S = -kx$, where $x$is the displacement of the mass from its equilibrium position. Thus the total force actingon the mass at any time tis

$F = -k\,x - b\,v$……………………(2)

If a(t)is the acceleration of the mass at time t, then by

Newton's second law of motion F = ma(t)

(2) $\Rightarrow ma(t) = -kx(t) - bv(t)$ ……………......(3)

Using $a(t) = \frac{d^2x}{dt^2}$ and $v(t) = \frac{dx}{dt}$ in (3)

We get, $m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + k\,x = 0$ ……………… (4)

The solution of equn.(4) describes the motion of the block under the influence of a damping force which is proportional to velocity.

The solution of the equn.(4) is of the form,

$x(t) = A\,e^{-b\,t/2m}\cos(\omega'\,t + \phi)$……………(5)

where $Ae^{-b\,t/2m}$is the amplitude and ω' is the angular frequency of the damped oscillator.

Solutions to Four/Five marks Problems

1. Given, maximum velocity: $v_m = A\,\omega =$ 30 cm/s = 0.3m/s ………….(1)

maximum acceleration: $a_m = A\,\omega^2 = 60\ cm/s^2 = 0.6 m/s^2$ ………….(2)

(2)÷(1) $\Rightarrow \omega = 2$ or $2\pi/T = 2 \Rightarrow$ T = π second.

2. A body oscillates with SHM according to the equation:$x = 5\cos(2\pi t + \pi/4)$ metre.

At t = 2.5 s,

(a) Displacement = $5\cos[(2\pi)\times 2.5\ s + \pi/4]$

$= 5\cos[(5\pi + \pi/4)] = -5\cos(\pi/4) = -5\times 0.7071 m = -3.536$ m

(b) Differentiating $x = 5\cos(2\pi t + \pi/4)$ w.r.t. 't',

Velocity: $v = \frac{dx}{dt} = -5[\sin(2\pi t + \pi/4)]\times(2\pi) = -10\pi\sin(2\pi t + \pi/4)$

At t = 2.5 s,

Velocity $= -10\pi\sin[2\pi\times(2.5) + \pi/4)] = -10\pi\sin(5\pi + \pi/4) = -10\pi\sin(\pi + \pi/4) = 10\pi\sin(\pi/4)$

$= 10(3.142)(0.7071) = 22.22$ m/s

(c) Differentiating velocity $v = -10\pi\sin(2\pi t + \pi/4)$ w.r.t. 't',

Acceleration: $a = \frac{dv}{dt} = -10\pi\,[\cos(2\pi t + \pi/4)] \times (2\pi) = -20\pi^2\sin(2\pi t + \pi/4)$

At t = 2.5 s,

Acceleration: $a = -20\pi^2\cos[2\pi\times(2.5) + \pi/4] = -20\pi^2\cos(5\pi + \pi/4) = 20(3.142)^2\cos(\pi/4)$
$= 139.6\ m/s^2$

3. Given mass m = 5 kg, Spring constant k= 500 Nm^{-1} and amplitude A= 10.0 cm= 0.01m

(a) The period of oscillation as given by $T = 2\pi\sqrt{\frac{m}{k}} = 2(3.142)\sqrt{\frac{5}{500}} = \frac{6.284}{10} = 0.6284\ s$

(b) The maximum speed: $v_m = A\omega = A\sqrt{\frac{k}{m}} = 0.1\times\sqrt{\frac{500}{5}} = 1\ m/s$

(c) The maximum acceleration: $a_{max} = -\omega^2 A = \frac{k}{m}\times A = \frac{500}{5}\times 0.1 = 10\ m/s^2$

4. The spring constant: $k = \frac{F}{l} = \frac{mg}{l} = \frac{50\times 9.8}{0.2} = 2450\ N/m$

Time period: $T = 2\pi\sqrt{\frac{m}{k}} \Rightarrow 0.6 = 2\pi\sqrt{\frac{m}{2450}} \Rightarrow m = 22.34\,kg$

Weight of the body is W= m g = 22.34 × 9.8 = 218.9 N

5. Given spring constant k =1200 Nm^{-1}, mass m= 3 kg and amplitude A =2.0 cm = 0.02m

(i)The frequency of oscillations: $\upsilon = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$

$= \frac{1}{6.284}\sqrt{\frac{1200}{3}} = \frac{1}{6.284}\sqrt{400} = \frac{20}{6.284} = 3.183\,Hz$

(ii) The maximum acceleration of the mass: $a_m = \omega^2 A = \frac{k}{m}A = \frac{1200}{3}\times 0.02 = 8.0\ m/s^2$

(iii) The maximum speed of the mass: $v_m = A\omega = A\sqrt{\frac{k}{m}} = 0.02\times\sqrt{\frac{1200}{3}} = 0.04\ m/s$

6. Displacement: $x(t) = 2.5\cos(4\pi t + \pi/6)$.

Comparing it with, $x(t) = A\cos(\omega t + \phi)$

Amplitude A =2.5m,

Angular frequency: $\omega = \frac{2\pi}{T} = 4\pi \Rightarrow$ Time period: T = 0.5s

Frequency $= \frac{1}{T} = \frac{1}{0.5} = 2\,Hz$ and phase constant $\phi = \pi/6$.

7. The time period of oscillation of simple pendulum: $T = 2\pi\sqrt{\frac{L}{g}}$

On earth surface: $T_1 = 1.5 = 2\pi\sqrt{\frac{L}{9.8}}$ ....(1)

On moon surface: $T_2 = 2\pi\sqrt{\frac{L}{1.7}}$ ....(2)

(2)÷(1) $\Rightarrow \frac{T_2}{1.5} = \sqrt{\frac{L}{1.7}}\times\sqrt{\frac{9.8}{L}} = \sqrt{\frac{9.8}{1.7}} \Rightarrow T_2 = 1.5\times 2.401 = 3.601\,s$

8. Amplitude A =5 cm = 0.05 m and time period T = 0.2 s.

Angular frequency: $\omega = \frac{2\pi}{T} = \frac{2(3.142)}{0.2} = 31.42$ rad/s

(a) Displacement x =5 cm = 0.05 m

Here displacement=amplitude, so to find acceleration and velocity at extreme positions

At extreme positions the acceleration is maximum and the velocity is minimum.

$\therefore$ Maximum acceleration = a_{max}= A ω^2=0.05 × 31.42^2 = 49.36 m/s^2

$\therefore$ Minimum velocity = v_{min} = 0

(b) Displacement x= 3 cm= 0.03 m,

Displacement: $x(t)$ = 0.05cos(ωt)

But given $x(t)$= 0.03m $\Rightarrow$ 0.03 =0.05cos(ωt)

$\therefore$ cos(ωt)= 3/5 and hence sin(ωt)= 4/5 = 0.8

The velocity of the at x= 3 cm is v=Aωsin(ωt) = 0.05 × (31.42) × 0.8 = 1.257 m/s

(c) Displacement x=0 cm, which is the mean position.

At mean position acceleration is minimum and velocity is maximum.

$\therefore$ Minimum acceleration = a_{min} = 0

$\therefore$ Maximum velocity = v_{max} = A ω=0.05 ×31.42 = 1.571 m/s

9. Mass m = 1 kg, spring constant k=50 Nm^{-1}, Amplitude A =10cm=0.1m and x= 5 cm=0.05 m.

Angular frequency: $\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{50}{1}}$ = 7.071 rad/s

Displacement: $x(t)$ = 0.1 cos (7.071t)

But given $x(t)$ = 5 cm = 0.05m $\Rightarrow$ 0.05 = = 0.1 cos (7.071t)

$\therefore$cos (7.071t) = 0.5 = ½ and hence sin(7.071t) = $\frac{\sqrt{3}}{2}$ = 0.866

The velocity of the at $x(t)$ = 5 cm is v=Aωsin (ωt) = 0.1 × (7.071) × 0.866 = 0.6123 m/s

Kinetic energy: $K = \frac{1}{2}mv^2 = \frac{1}{2}(1)(0.6123)^2 = 0.1874$ J

Potential energy: $U = \frac{1}{2}k\,x^2 = \frac{1}{2}(50)(0.05)^2 = 0.0625$ J

Total energy of the block: E= K+U = 0.1874+0.0625 = 0.2499 J

10. Amplitude: A = $\frac{1.0\text{ m}}{2}$ = 0.5 m, angular frequency:ω = 200 rad/min = 200rad/60s=10/3 rad/s,

The maximum speed: v_m=Aω= 0.5× $\frac{10}{3}$ = 1.667 m/s

The maximum acceleration: a_m= A ω^2=0.5 × $\left(\frac{10}{3}\right)^2$ = 5.556 m/s^2