Chapter 6: Work Energy and power

One mark questions

1. Define scalar product of two vectors.
2. Write the mathematical equation for dot product of two vectors.
3. What is the condition for the two vectors to be perpendicular to each other.
4. If $\vec{A} \cdot \vec{B}=0$, then what is the angle between $\vec{A}$ & $\vec{B}$
5. In the equation $\vec{A} \cdot \vec{B}=A(B\cos\theta)$, what does 'Bcos$\theta$'represent?
6. If $\vec{A} \cdot \vec{B} =AB$, then what is the angle between $\vec{A}$ &$\vec{B}$.
7. If $\vec{A} \cdot \vec{B}=-AB$, then what is the angle between $\vec{A}$ & $\vec{B}$.
8. What is the value of scalar product of a vector with itself ?
9. What is the value of dot product of unit vector with itself ?
10. Define work –energy theorem.
11. What do you mean by work done by a force?
12. What is the work done by the tension in the string of a simple pendulum.
13. Mention the dimensions of the work done .
14. What is the nature of the work done by frictional force.
15. Define the term energy.
16. What is the nature of the work done by applied force.
17. Define Kinetic energy of a body.
18. What type of energy possessed by a man standing in a moving train?
19. What dose the area under ' force-displacement' curve represent.
20. Define potential energy of a body.
21. Out of joule, calorie, kilowatt and electron volt, which one is not the unit of energy ?
22. Can potential energy of on object be negative?
23. If an object of mass 'm' is released from rest form the top of a frictionless inclined plane of height 'h' what is its speed at the bottom of the inclined plane.
24. Wather the spring force is conservative or non – conservative?

25. Mention the S.I unit of spring constant.
26. If the spring constant of a given spring is large, what it represent?
27. If the spring constant of a given spring is small, what it represent?
28. Mention the expression for the work done by a spring force.
29. What is the energy associated with 1 kg of mass.
30. What type of nuclear reaction takes place in nuclear power plant?
31. What type of nuclear reaction takes place in nuclear weapons
32. How does an arrow gains K.E, when it is shot from a bow?
33. What kind of energy transformation take place at a thermoelectric power station?
34. Which type of energy is responsible for the formation of molecules form the atoms and polymers from the molecules.
35. What is mass 'defect'?
36. State the law of conservation of energy.
37. What is power?
38. What is average power?
39. What is instantaneous power?
40. Give the practical unit of power.
41. What is the unit used to describe the out put of automobiles and motorbikes?
42. Convert 1.K.Wh in joule.
43. The energy associated with the daily food intake of a human adult is 10^7 J express it in Kilo calories.
44. What is elastic collision?
45. What is inelastic collision ?
46. Give an example for elastic collision.
47. Give an example for inelastic collision.
48. What is perfectly inelastic collision?
49. In which type of collision mechanical energy is not transformed into any other form of energy?
50. In which type of collision whole mechanical energy may be transformed into other form?
51. What is head on collision?

Two mark questions

1. Mention the two types of multiplication of vectors.
2. Explain how commutative law holds good in dot product.
3. Explain how distributive law holds good in dot product.
4. Find the value of ‘n’ so that vector $(4\hat{i}-6\hat{j}+2\hat{k})$ may be perpendicular to the vector $(6\hat{i}+8\hat{j}+n\hat{k})$
5. Find the angle between the vectors $\vec{A}=2\hat{i}-4\hat{j}-5\hat{k}$ and $\vec{B}=2\hat{i}+2\hat{j}-4\hat{k}$
6. Define work done by the force.what is value of work done by the centripetal force?
7. Under what conditions the work done by a force is maxi mum and minimum?
8. State any two conditions under which a force does no work.
9. Name the largest and smallest practical unit of enegry.
10. What is non conservative force ? Give an example.
11. What is conservative force ?Give an example.
12. Write down the expression for spring force and explain the terms.
13. If $\vec{A}=A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$ and $\vec{B}=B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$ are the two vectors in rectangular components, then find out their scalar product and find out $\vec{A} \cdot \vec{A}$.
14. what is collision ? mention its two types.
15. How do you represent graphically work done by a constant force and by a variable force ?
16. Distinguish between elastic collision & inelastic collision.

Questions carrying 4mark and 5mark

1) **Prove that for a particle in rectilinear motion under constant acceleration the change in kinetic energy of a particle is equal to the work done on it by the net force?**

2) **Obtain graphically the work done by a variable force?**
3) **Prove the work-energy theorem for a variable force?**
4) **Describe the conservation of mechanical energy of a system?**
5) **Give an illustration for the conservation of mechanical energy in case of a ball dropped from a cliff of height 'H'?**
6) **Give an illustration for the conservation of mechanical energy in case of a bob suspended by a light string completes a semi-circular trajectory in the vertical plane?**
7) **Obtain en expression for potential energy of a spring?**
8) **Define power obtain the expression for instantaneous power? Mention the units of power?**
9) **What are elastic and inelastic collisions obtain the expression for final velocities of two bodies colliding in one dimension?**
10) **Describe briefly the collisions in two-dimension?**

Answer to 1 mark questions:

1. The scalar product of to vectors is defined as the product of the magnitudes of the two vectors and the cosine of the angle between them.
2. If $\vec{A}$ and $\vec{B}$ are the two vectors, and θ is the angle between them, then their dot product is given by $\vec{A} . \vec{B} = AB \cos\theta$
3. The two vectors are perpendicular to each other, if their dot product is zero i.e., if $\vec{A} . \vec{B} = O$, then $\vec{A}$ and $\vec{B}$ are perpendicular to each other
4. If $\vec{A} . \vec{B} = 0$; then the angle between them is 90^0
5. It is the projection of $\vec{B}$ along $\vec{A}$
6. The angle between $\vec{A}$ and $\vec{B}$ is zero (0) (Two vectors are parallel)
7. The angle between $\vec{A}$ and $\vec{B}$ is 180^0 (Two vectors are antiparallel)
8. Square of its magnitude ($\vec{A} . \vec{A} = A A \cos 0 = A^2$)
9. The dot product of unit vector with itself is unity $\hat{i}.\hat{i} = 1$ or $\hat{J}.\hat{J} = 1$ ($\theta = 0$)
10. It states that workdone by a force on a body is equal to the change in its kinetic energy.
11. Work is said to be done when a force applied on a body displaces it through a certain distance.
12. Zero (Tension (force) and displacement of a bob are perpeldicular to each other)
13. $[W] = [ML^2 \ T^{-2}]$
14. Negative (The motion is opposed by the frictional force)
15. The capacity to do work is called energy
16. Positive
17. The ability of a body to do work due to its motion is called kinetic energy.
18. Kinetic energy

19. Work done
20. The ability of the body to do work due to its configuration or position is called potential energy.
21. Kilowatt
22. Yes, It is negative, when forces involved are attractive
23. $\sqrt{2gh}$; g = acceleration due to gravity
24. Conservative force
25. Its unit is Newton per metre (Nm^{-1})
26. The spring is said to be stiff
27. The spring is said to be soft (or smooth)
28. $W_s = -\frac{1}{2} kx^2$: K = spring constant
    X = extension produced in the spring
29. Energy associated with 1 kg can be calculated using the relation $E = mc^2$
    $= 1 \text{ kg} \times (3 \times 10^8)^2$
    $E = 9 \times 10^{16}$ J
30. Controlled nuclear fission reaction
31. Un controlled nuclear fission reaction
32. It gains K.E. From the configuration of the bow or P.E. of the bow
33. The heat energy is converted into electrical energy
34. Chemical energy
35. The difference between the sum of the masses of the nucleons forming the nucleus and rest mass of the nucleus is called ‘ mass defect’
36. Energy can neither be created, nor destroyed i.e., the total energy of an isolated system remains constant
37. The time rate at which work is done or energy transferred is called power.
38. The ratio of the work (W) to the total time taken (t) is called average power.
    $$P_{av} = \frac{W}{t}$$
39. The limiting value of the average power when time tends to zero is called instantaneous power
    It is given by $P = \frac{dw}{dt}$
40. The practical unit of power is horse power (Hp)
41. Horse power (hp) (1 hp = 746 watt)
42. 1 Kwh = 1000 x 60 x 60 watt S
    $= 10^3 \times 3600$ $\underline{\text{Joule}}$ x S

S

1 Kwh = 3.6×10^6 J

43. Given average human consumption in a day = 10^7 J

energy consumption in K. Calorie = $\frac{10^7}{4.2}$ = 0.238×10^7cal

= 0.24×10^7 cal

= 2400×10^3 cal

(i.e. 1 calorie = 4.2 J) 10^7 J =2400K cal

44. The collision in which both the momentum and kinetic energy of the system remains conserved is called elastic collision.
45. The collision in which only momentum is conserved but kinetic energy is not conserved is called inelastic collision.
46. Examples : Collision of atoms, collision of subatomic particles like proton and electron, collision of molecules of gas (any one)
47. Examples : Collision of mud on the wall, bullet striking a block of wood collision of plastic bodies (any one)
48. If two bodies stick together after colliding, the collision is perfectly inelastic

OR

A collision in which the two particles move together after the collision is called perfectly inelastic collision
49. Elastic collision
50. Inelastic collision
51. If the initial velocities and final velocities of both the colliding bodies are along the same straight line then it is called head-on collision (one dimensional collision)

Answer to 2 mark questions

1. i) Scalar product
   ii)Vector product
2. Form defnition $\vec{A} \cdot \vec{B} = AB \cos\theta$

$\rightarrow \rightarrow$

$\vec{B} . \vec{A} = BA \cos\theta = AB\cos\theta$

Hence $\vec{A} . \vec{B} = \vec{B} . \vec{A}$

Hence commutative law holds good in dot product

3. If $\vec{A}$. $\vec{B}$ & $\vec{C}$ are the vectors then

$\vec{A}.(\vec{B}+\vec{c}) = \vec{A} . \vec{B} + \vec{A} . \vec{C}$

Hence distributive law holds good for dot product .

4. Let $\vec{A} = 4\hat{i} - 6\hat{j} + 2\hat{k}$

$\vec{B} = 6\hat{i} + 8\hat{j} + n\hat{k}$

Two vectors perpendicular to each other only If $\vec{A} . \vec{B} = O$

$(4\hat{i} - 6\hat{j} + 2\hat{k}) . (6 \hat{I} + 8 \hat{j} + n\hat{k}) = 0$

$24\ \hat{i}.\hat{i} - 48\ \hat{j}.\hat{j} + 2n\ \hat{k}.\hat{k} = 0$ $(\hat{i}.\hat{j} = \hat{j}.\hat{k} = \hat{k}.\hat{i} = 0)$

$24 - 48 + 2n = 0$ $(\hat{i}.\hat{i} = \hat{j}.\hat{j} = \hat{k}.\hat{k} = 1)$

$-24 + 2n = 0$

$2n = 24$

$n = 12$

5. WKT $\vec{A} . \vec{B} = AB \cos\theta$ and $\vec{A} . \vec{A} = A^2$ and $\vec{B} . \vec{B} = B^2$

$$\cos\theta = \frac{\vec{A}.\vec{B}}{A\,B}$$

$\vec{A}. \vec{B} = 2 \times 2 + 2\times (-4) + (-4) \times (-5)$

$= 4 - 8 + 20$

$= -4 + 20$

$\vec{A}.\vec{B}. = 16$

$A^2 = \vec{A}. \vec{A}.$

$= 2\times2+(-4)\times(-4) + (-5)\times(-5)$

$= 4 + 16 + 25 = 45$

$A = \sqrt{45}$

$$B^2 = \vec{B} \cdot \vec{B}$$

$= 2x2+(2)(2) + (-4)(-4)$

$= 4+4+16$

$= 24$

$B = \sqrt{24}$

$$\cos\theta = \frac{16}{\sqrt{45}\sqrt{24}} = \frac{16}{6.7082 \times 4.8989}$$

$$= \frac{16}{32.8628}$$

$\cos\theta = 0.4868$

$\theta = \cos(0.4868)$

$\theta = 60^{o}\, 52^{1}$

6. The work done by the force is defined as the product of component of the force along the direction of the displacement and the magnitude of the displacement,

   Work done by the centripetal force is zero,
   (force and displacement are perpendicular to each other)

7. Work done by a force is maximum, when the force and displacements are in the same direction ($\theta = O^0$) and minimum when they are perpendicular to each other ($\theta = 90^0$)

8. A force does no work when
   [i] The Displacement is Zero.
   [iii] The displacement is perpendicular to the direction of force.

9. Largest practical unit is kilowatt hour [K wh]
   Smallest practical unit is electron volt (ev)

10. If the work done by the force depends on the path fallowed by the body is called non-conservative force
    Ex: frictional force.

11. If the work done by the force depends only on the initial and final potations of the body.
Ex: Gravitational force.

12. Spring force $F_s = -\ Kx$

Where $K = -$ Spring constant

$X = -$ displacement from the equilibrium position

13. $\vec{A} \cdot \vec{B} = (A_x\hat{i} + A_y\hat{j} + A_z\hat{k}) \cdot (B_x\hat{i} + B_y\hat{j} + B_z\hat{k})$

$= A_x B_x\, \hat{i}\cdot\hat{i} + A_y B_y\, \hat{j}\cdot\hat{j} + A_z B_z\, \hat{k}\cdot\hat{k}$

$\vec{A}\,\vec{B} = A_x B_x + A_y B_y + A_z B_z$

$(\hat{i}\cdot\hat{i} = \hat{j}\cdot\hat{j} = \hat{k}\cdot\hat{k} = 1$ and $\hat{i}\cdot\hat{j} = \hat{j}\cdot\hat{k} = \hat{k}\cdot\hat{i} = 0)$

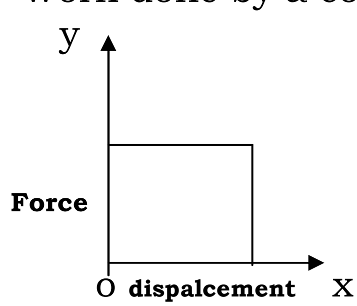
$\vec{A} \cdot \vec{A} = (A_x\hat{i} + A_y\hat{j} + A_z\hat{k}) \cdot (A_x\hat{i} + A_y\hat{j} + A_z\hat{k})$

$= A_x A_x\, \hat{i}\cdot\hat{i} + A_y A_y\, \hat{j}\cdot\hat{j} + A_z A_z\, \hat{k}\cdot\hat{k}$

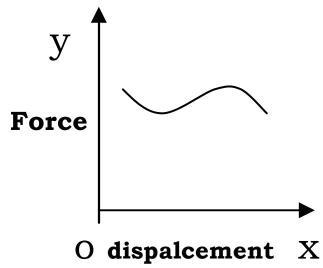
$\vec{A} \cdot \vec{A} = A^2_x + A_y^2 + A^2 z$

14. An event in which two bodies come in physical contact with other or path of one body is affected by the force due to the other body is called collisoin, [Physical contact between the two colliding bodies is not necessary, if a body can
causes change in the velocity of another body without contact, collision may occur]
There are two types of collisions, they are elastic collision and inelastic collision.

15. i. Work done by a constant force



ii. Work done by a Variable force



16.

	Elastic collision		Inelastic collision
i	The collisions in which both the momentum and K.E of the system remains conserved are called elastic collisions	i	The collisions in which only momentum is conserved, but KE is not conserved are called inelastic collisions
ii	There is no loss of KE during elastic collisions	ii	There is a loss of KE during inelastic collisions
iii	The forces involved are conservative in nature	iii	The forces involved are non-conservative in nature
iv	Mechanical energy is not transformed into any other form of energy	iv	Whole Mechanical energy may be transformed into other forms

Answers for 4 mark and 5mark

1) Prove that for a particle in rectilinear motion under constant acceleration the change in kinetic energy of a particle is equal to the work done on it by the net force?

Consider a particle in rectilinear motion with constant acceleration 'a' then equation of motion is

$$v^2 - u^2 = 2as \text{ -------- } (1)$$

Where ' u ' and ' v ' are initial and final speeds.

S the distance traveled

On multiplying equation (1) by $\frac{m}{2}$

We have,

$$\tfrac{1}{2}\,mv^2 - \tfrac{1}{2}\,mu^2 = mas \quad \text{--------} \quad (2)$$

From Newton's II law, ma = F

ie $\tfrac{1}{2}\,mv^2 - \tfrac{1}{2}\,mu^2 = FS$ **--------- (3)**

in general, for 3 – Dimensions

$$\tfrac{1}{2}\,mv^2 - \tfrac{1}{2}\,mu^2 = F.\,d \quad \text{---------} \quad (4)$$

F is the force d is the displacement

bsut $\tfrac{1}{2}\,mv^2 = Kf$ **→ Final kinetic energy**

$\tfrac{1}{2}\,mu^2 = Ki$ **→ initial kinetic energy**

F – d = W-work done

Then, $\boxed{Kf - Ki = W}$ **---------- (5)**

Hence, equation (5) is the special ease of work- Energy theorem.

ie " The change in kinetic energy of a particle is equal to the work done on it by the net forc

2) Obtain graphically the work done by a variable force.

A graph of variable force F (X) Versus Displacement ' X ' is as shown in fig.

The area below the **Curve gives total work done,**

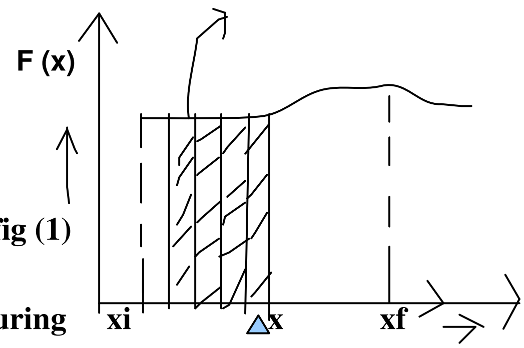
Area $\triangle A = (x)\ \triangle x$

To estimate the work done,

the area below the curve is

divided into a large no of strips as in fig (1)

If 'Δx' be the small displacement assuring



fig(1)

the force F(x) constant over a small displacement then the work done is equal to the area of

the rectangular strip

$$\text{ie} \quad W = \sum_{xi}^{xf} F(x) \Delta X$$

If the displacement is infinitesimally small

ie $\Delta X \rightarrow O$

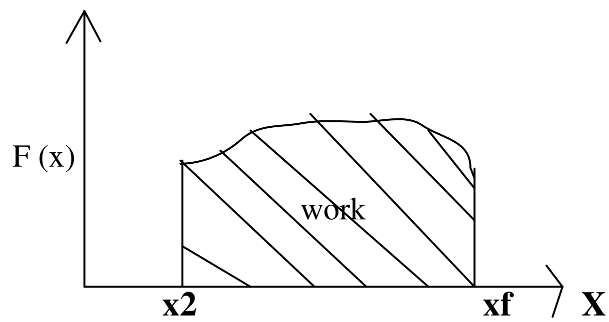
$$\text{then the work done} \quad W = \sum_{xi}^{xf} F(x) \Delta X$$

$$\text{Limit} \quad \Delta x \rightarrow O$$

taking the initial position xi and final position xf as the lower and upper limits. As in fig (2)

xf

then the work done $W = \int_{x_i} F(x)\, dx$



3)Prove work – Energy theorem for a variable force

We know that $K = \frac{1}{2} mv^2$ - (1)

The time rate of change of kinetic energy is (on differentiating k w.r.t. Time)\

$$\frac{dk}{dt} = \frac{d}{dt}\left(\frac{1}{2} mv^2\right)$$

$$\frac{dk}{dt} = m\frac{du}{dt} V \qquad - (2)$$

from Newtons II law - $m \frac{du}{dt} = F$

then $\frac{dk}{dt} = FU \quad - \quad (3)$

but $V = \frac{dx}{dt}$

ie $\frac{dk}{dt} = F\frac{dx}{dt}$

Then $dk = Fdx \quad - \quad (4)$

On integrating eqn (4) taking initial position (xi) to final position (xf), we have

$$\int_{ki}^{kf} dk = \int_{xi}^{xf} Fdx$$

Where ki and kf are the initial and final kinetic energies corresponding to xi & kf

$$\text{ie} \quad kf - ki = \int_{xi}^{xf} Fdx \quad - \quad (5)$$

We know that for a variable force

$$W = \int_{xi}^{xf} F(x)\,dx \qquad - \quad (6)$$

On comparing eqns (5) & (6) we get

$$\boxed{K_f - K_i = W}$$

Thus. The work- energy theorem is verified for a variable force.

4)Describe the Conservation of mechanical energy of a system

Consider a body in one – dimensional motion undergoes a displacement "ΔX"under the

action of a conservative force F, the from the work -Energy theorem.

We have $\Delta K = F(x)\,\Delta X \qquad ----- \qquad (1)$

If the force is conservative the potential energy function V (x) is defined as

$$-\Delta V = F(x)\ \Delta X \qquad - \ (2)$$

$$\Delta V = -F(x) \quad \Delta X \ - (2)$$

From eqns (1) & (2) $\Delta K + \Delta V = \quad F(x)\,\Delta x) - F(x)\,\Delta x$

$$\Delta K + \Delta V = O$$

$$\triangle(K + V) = O \quad - \quad (3)$$

Where (K + V) is the sum of the kinetic and potential energies of the body remains a

constant for the entire path ie from X_i to X_f

$K_i + V(x_i) = K_f + V(x_f) \quad - \quad (4)$

In general, the quantity K + V (x) is called the total mechanical energy of the system.

However the kinetic energy K and the potential energy V (x) may vary from point to

point, bnt the sum remains a constant and the force is conservative

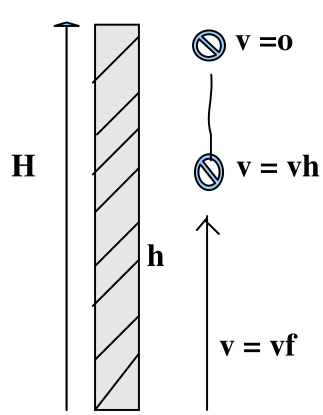
from eq (4) – it is clear that work done by the conservation force depends on initial

& final positions of body.

If $X_i = X_f$. ie for a closed path work done by the force is zero.

Thus the total mechanical energy of a system is conserved, if the force doing work on it, are conservative.

5)Give an Illustration for the conservation of mechanical energy in case of a ball dropped from a cliff of height 'H'



Consider a ball of mass m is top of a cliff height H.

The total mechanical energy on top of cliff is $Eh = mgH$ – (1)

if the ball is dropped from rest, reaches the height h from the ground then its total mechanical energy.

$$Eh = mgh + \frac{1}{2} mvh^2 \quad --- \quad (2)$$

As the ball reaches the ground then

$$Eo = \frac{1}{2} mvf^2 \quad ---- \quad (3)$$

since the work done by the gravitational force ie conservation force.

Hence, the mechanical energy is conserved

ie $EH = EO$

$$mgh = \frac{1}{2} mvf^2$$

$$Vf^2 = 2gh \qquad Vf = \sqrt{2gh} \quad --- (4)$$

this is the eqn for a freely falling body

Further $EA = Eh$

$$mgH = mgh + \frac{1}{2} mvh^2$$

$$g H = g h + \frac{1}{2} v h^2$$

$$\frac{1}{2} Vh^2 = g\,h - g\,h$$

$$Vh = \sqrt{2g\,(H - h)}$$

Which is an equation for motion in 1 – dimensional motion Hence, at the height H . the energy is purely

potential, it is partly potential & partly kinetic at height h and is finally kinetic at ground level

This illustrates the conservation of mechanical energ

6) Give an Illustration for law of conservation of mechanical energy

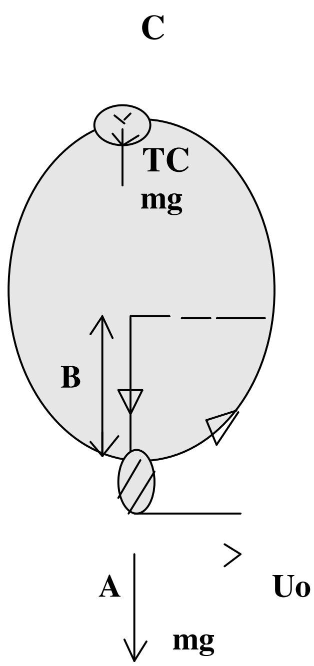
Consider a bob of mass ' m ' is Suspended by a hight string of length L , Let it be given a

horizontal velocity vo at the lowest point A such that if

completes a semi – Circular trajectory in vertical plane

With the string becoming slack only on reaching the

top most point C : as in fig



At A, the bob is under the action of two forces

(1) W = mg weight the to gravity (11) tension 'T' in the (spring force) String & if there is no work,

since Displacement of the bob is normal to the force (spring force)

The potential energy of the bob is zero at A.

The total mechanical energy of the system is

$$E = \tfrac{1}{2}\, m v_o^2 \qquad - (1)$$

for equilibrium at .A. $\quad T_A - mg = \dfrac{m v_o^2}{L} \quad - (2)$

(SUF = SDF)

($\dfrac{m v_o^2}{L}$ **is centripetal force)**

' L ' radius

At the highest point C, the string slackens as the tension in the string become zero thus at C.

Total mechanical energy $E = \tfrac{1}{2} m v_c^2 + 2\, mgL \quad - (3) \quad (mgL + mgL)$

we know,

that $mg = \dfrac{m v_c^2}{L} \quad (W = CF) \quad - (4)$

eq (4) in (3) $\qquad m v_c^2 = mgL$

$$\therefore\; E = \tfrac{1}{2}(mgL) + 2\, mgL$$

$$E = \frac{mgL}{2} + 2mgL$$

$$E = \frac{5}{2}\, mgL \qquad - (5)$$

equating (5) with (1)

$$\frac{5}{2}\, mgL = \tfrac{1}{2}\, m v_o^2$$

$$\boxed{VO = \sqrt{5gL}}$$

from eq (4) $\boxed{VC = \sqrt{gL}}$

A & B The energy is

$E = \frac{1}{2} mv_B^2 + mgL$ --- (6)

equating (6) with (4)

$\frac{1}{2} mvo^2 = \frac{1}{2} m\, v_B^2 + mgL$

$\frac{1}{2} mv_B^2 = \frac{1}{2} mvo^2 - mgL$

but $vo^2 = 5gL$

$\frac{1}{2} mv_B^2 = \frac{1}{2} m\,(5gL) - mgL$

$\frac{1}{2} mv_B^2 = mgL\left(\frac{5}{2} - 1\right)$

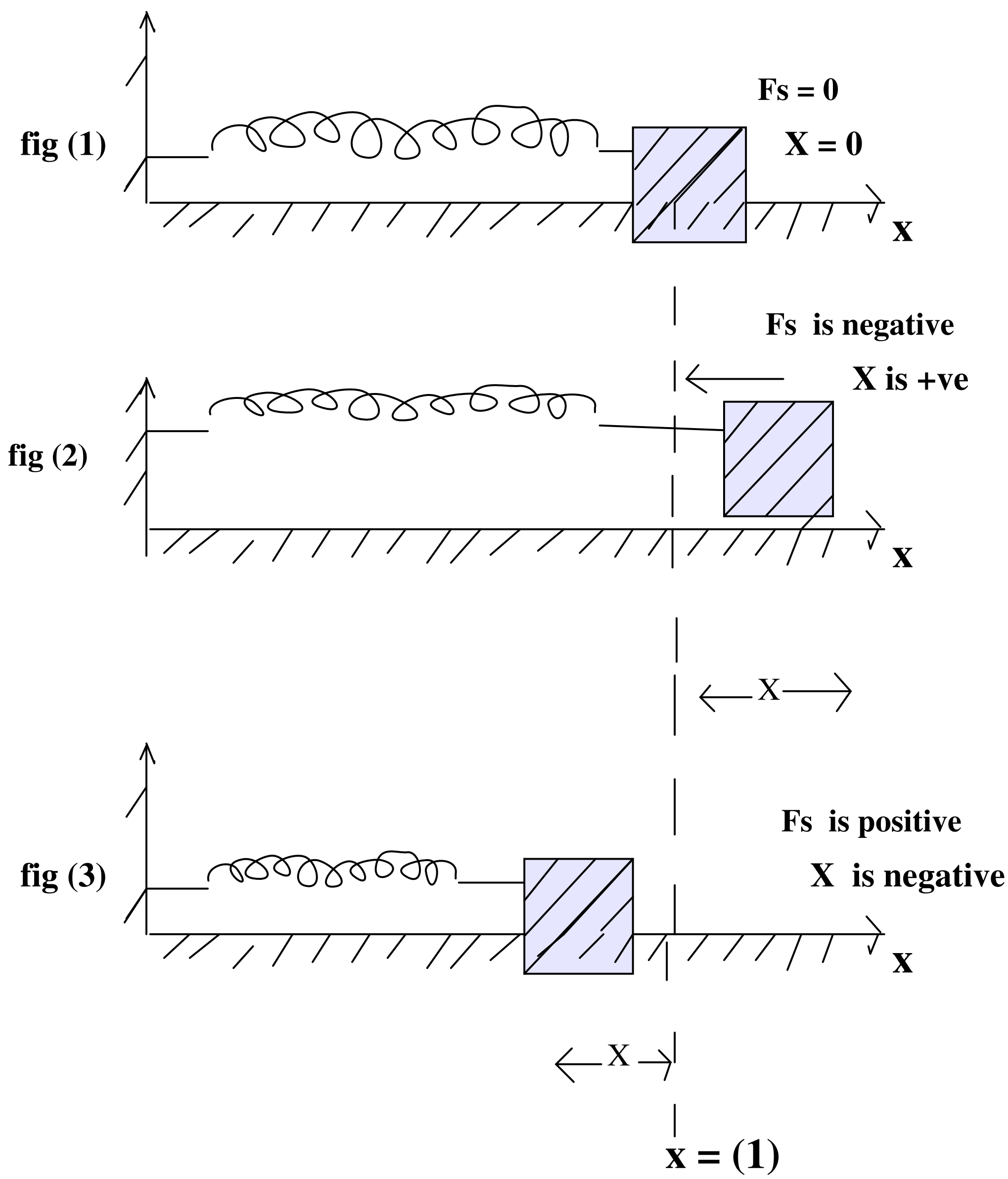
$\frac{1}{2} mv_B^2 = \frac{3}{2} mgL$

$v_B^2 = 3gL$

$$\boxed{V_B = \sqrt{3gL}}$$

7) Obtain the expression for potential energy of spring :

Consider on ideal spring , resting on a horizontal surface its one end is fixed to a rigid wall & its other end is attached to a block as in fig – (1)



Let 'Fs' be the spring force and ' X' is the Displacement of the block from the equilibrium

Position. The displacement could be either positive in fig (2) or negative as in fir (3).

From, Hooke's law $F_s = -KX$

The constant 'K' is called the spring constant, if 'K' is large the spring is said to be stiff

other wise it is soft.

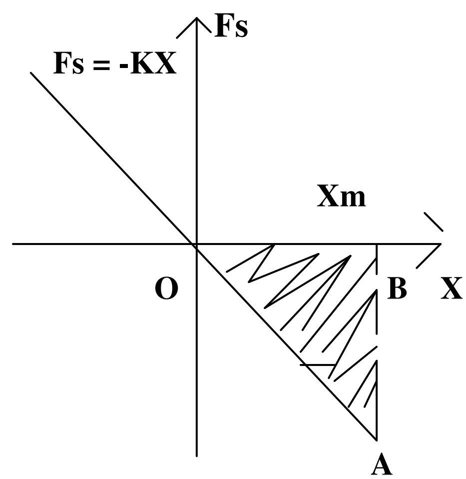
When the spring is pulled outward with the as in fig (2) extension xm

The work done by the spring force is

$$W_s = \int_O^{x_m} F_s\,dx = -\int_O^{x_m} kx\,dx$$

$$\boxed{W_s = -\frac{KX_m^2}{2}}$$

This expression may also be obtained by considering the area of the triangle as in fig (4)



work done = area of △ AOB

$W_s = \frac{1}{2} \times OB \times AB$

$W_s = \frac{1}{2} X_m \times F_s$

$$Ws = \frac{1}{2} Xm(-kXm)$$

$$\boxed{Ws = -\frac{1}{2} KXm^2}$$

8) Define power obtain an expression for instantaneous power mention the units of power.

Power is defined as the time rate at which work is done or energy is transferred.

The instantaneous power is defined as the limiting value of the average power as time interval

approaches zero.

ie $$P = \frac{dw}{dt} \quad - (1)$$

The work done dw by a force F for a diplacement dr is dw= F.dr.

The instantaneous power can also expressed as

$$P = F \frac{dr}{dt}$$

$$P = F . U . \text{ where } U = \frac{dr}{dt} \quad \text{instantaneous velocity}$$

SI unit of power is watt(W)

1 watt = 1JS^{-1}

power is also expressed in horse power (hp)

1hp = 746 W

9) What are elastic and inelastic collision. Obtain the expression for final velocities of this bodies

Colliding each other while in motion along a straight line ?(ie one Dimension)

The collision in which there is a conservation of both momentum and energy is called elastic

Collision.

Ex:- Collision between billiard balls marbles, ivory balls etc

The collision energy in which there is no conservation of kinetic energy and only momentum

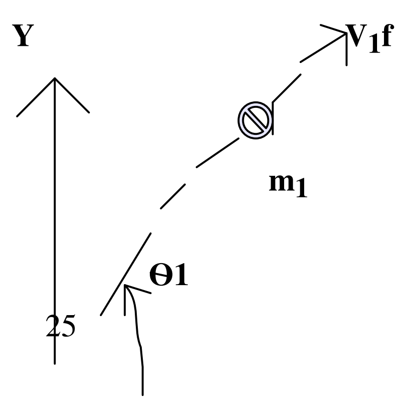
is conserved is called inelastic collision.

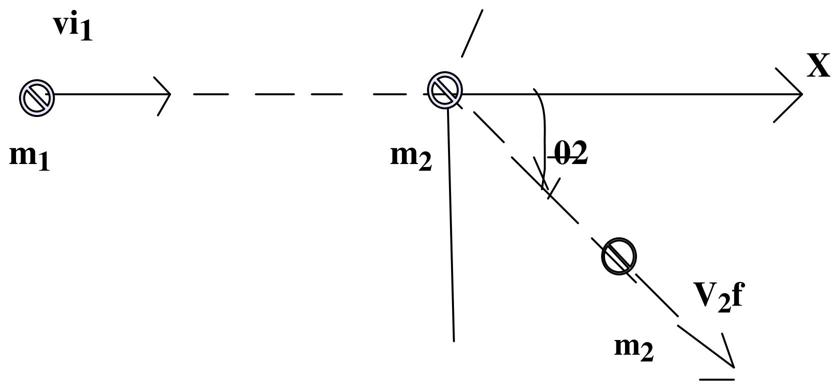
Ex :- (1) A bullet hitting a hard target get embedded into it.

(2) Collision between a person and electron.

Consider two masses m_1 and m_2 , the particle m_1 is moving with speed V_1. And m_2 is at

rest the m_1 collides with the stationary mass m_2 as in fig





The masses m_1 & m_2 fly off at directions $\Theta 1$ & $\Theta 2$

W.r.t.X – axis. as in fig.

Consider first a completely inelastic collision in one Dimension ie $\Theta 1 = \Theta 2 = \Theta$

then law of conservation of momentum.

$$V_f = \frac{m_1}{m_1 + m_2} V_i \quad - (1)$$

The loss in kinetic energy on collission is

$$\triangle K = \tfrac{1}{2} m_1 V^2_{1i} - \tfrac{1}{2} (m_1 + m_2) V^2_f - (2)$$

eq (1) in (2)

$$\triangle K = \tfrac{1}{2} m_1 V^2_{1i} - \tfrac{1}{2} \frac{m_1^2}{m_1 + m_2} V^2_{1i}$$

$$\triangle K = \tfrac{1}{2} m_1 V^2_{1i} \left(1 - \frac{m_1}{m_1 + m_2} \right)$$

$$\left(m_1 m_2 \right)$$

$$\triangle K = \frac{1}{2}\,\frac{\qquad}{m_1+m_2}\,V^2_{1i}$$

Consider next an elastic collision with $\Theta 1=\Theta 2=\Theta$.

The momentum and kinetic energy conservation equations are

$$m_1V_{1i} = m_1V_{1f} + m_2\,V_{2f} \qquad \text{--- (3)}$$

$$m_1V^2_{1i} = m_1V^2_{1f} + m_2V^2_{2f} \qquad - \ (4)$$

from eqns (3) & (4) it follows that

$$m_1V_{1i}\ (V_{2f} - V_{1i}\ ) = m_1V_{1f}\,(V_{2f} - V_{1f}\ )$$

$$V_{2f}\,(V_{1i} - V_{1f}) = V^2_{1i} - V^2_{1f}$$

$$V_{2f}\,\cancel{(V_{1i} - V_{1f})} = \cancel{(V_{1i} - V_{1f})}\,(V_{1i} + V_{1f})$$

$$V_{2f} = V_{1i} + V_{1f} \quad - (5)$$

Hence,

eq (3) in eq (5) , we obtain.

$$V_{1f} = \frac{(m_1 - m_2)}{(m_1+m_2)}\,V_{1i}$$

and

$$V_{1f} = \frac{2m_1\,V_{1i}}{m_1 + m_2}$$

Case – (i) if the two masses are equal $V_{1f} = O$

$V_{2f} = V_{1i}$

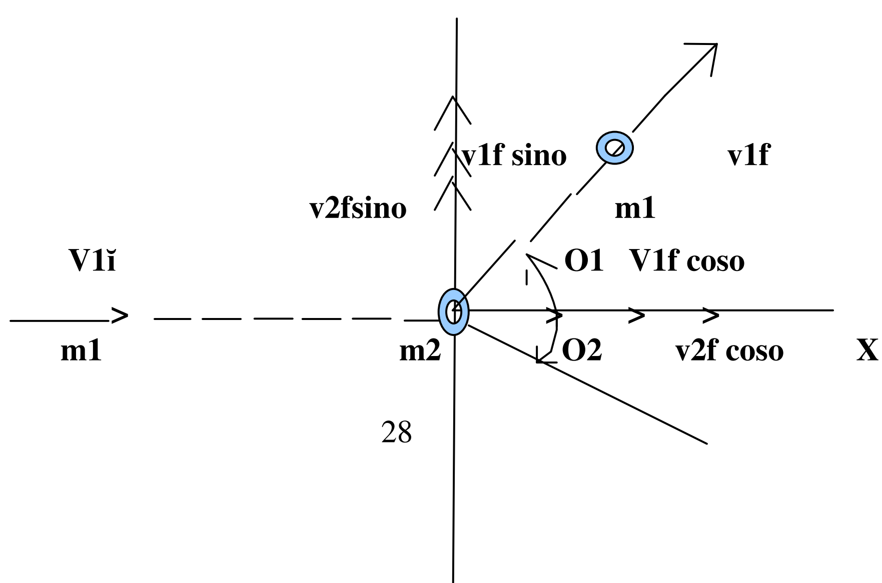
The first mass comes to rest and push off the second mass with its initial speed collision.

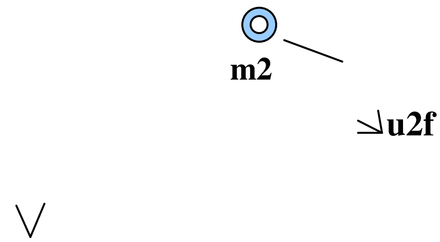
Case – (ii) if one mass dominates ex : m2 > > m1.

$V_{1f} \approx - V_{1i}$ $V_{2f} \approx O$

The heavier mass is undisturbed while the highter masses reverses its velocity.

10) Describe briefly the Collision in TWO – Dimension?





Consider the collision of a moving mass m1 with the stationary mass m2 in the (x -y) plane

Linear momentum is conserved and its components in three directions (x.y.z) are taken.

$$M_1 V_{1i} = m_1 V_{1f} \cos\theta_1 + m_2 v_{2f} \cos\theta_2 \quad -(1)$$

$$0 = m_1 V_{1f} \sin\theta_1 - m_2 V_{2f} \sin\theta_2 \quad -(2)$$

If $\theta_1=\theta_2=\theta$. We get eqn for 1- Dimensional collision

eq (1) → $m_1 V_{1i} = m_i V_{1f} + m_2 V_{2f}$

If the Collision is elastic

$$\tfrac{1}{2} m_1 V_{1i}^2 = \tfrac{1}{2} m_1 V_{1f}^2 + \tfrac{1}{2} m_2 V_{2f}^2$$

Problems: Work Energy and power

1) **A variable force given by F=x+8 acts on a particle. Calculate the work done by the force**

during the displacement of the particle from X = 1m to X= 3m

Solution : given F = X+8

The work done during a small Displacement dx is

dw = F. dx

$$\therefore \text{Total work done} \quad W = \int_{X=1}^{X=3} dW = \int_{1}^{3} F dx$$

$$W = \int_{1}^{3} (X + 8)\, dx$$

$$W = \left.\frac{X^2}{2}\right]_1^3 + \left. 8X \right]_1^3$$

$$W = \left(\frac{9}{2} - \frac{1}{2}\right) + \left(8 \times 3 - 8 \times 1\right)$$

$$W = 4 + 16 = 20\ J$$

2) **A bullet of mass 50g strikes a wooden plank with a velocity of 200ms $^{-1}$ and energy out**

with a velocity of 50m51. Calculate the work done by the bullet against the resistive

force offered by the plank.

Given : m = 50g = 50 x 10 – 3 kg U = $200mS^{-1}$ V = $50mS^{-1}$

Work done = loss of kinetic energy

$$W = 1/2Mu^2 - ½ Mv^2$$

$$W = 1/2M (u^2 - v^2)$$

W = 1/2x50x10-3 (4x104 – O.25x104)

W = 25 x 10-3 (3.75) x 104

W = 93.75 x 10

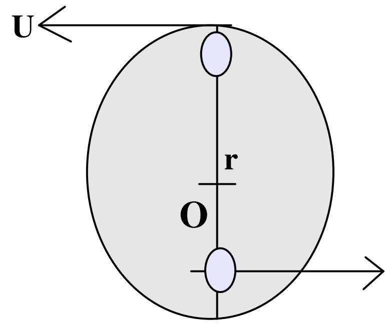
W = 937.5 J

3)

A metal bob is tied to one of an inextensible string of negligible mass and is rotated in a vertical circle of radius 8m. If the speed of the sphere at the highest point of the circle is

$80mS^{-1}$. Calculate its speed at the lowest point of the circle ($g= 10mS^{-2}$)

Solution :



Let m be the mass of the metal sphere,Radius of the circle r =8m

U

Speed at the highest point U = 80 mS^{-1}

Ep = O

Let - V ' be the speed at the lowest point of the circle, According to the law of conservation of energy.

(EK + EP) lowest point = (E1K + E1P) highest point.

EP = O at lowest point

(½ MV2 + O) = ½ MU2 + mg (2r) (: . h = 2r)

. : V2 = U2 + 4gr

V2 = 802 + 4 x 10 x 8

V2 = 6720

V = $\sqrt{6720}$

$$V = 82\ mS^{-1}$$

4) **Two bodies of masses 0.2kg and 0.1 kg moving in the same direction on a straight line with**

the velocities 0.6m51 & 0.4m51. Respectively suffer head – 0n collision, calculate their

velocities after collision.

Given : $M_1 = 0.2kg.$ $M_2 = 0.1kg.$ $U_1 = 0.6m51.$ $U_2 = 0.4m51.$

$V_1 = ?$ $U_2 = ?$

We know that $U_1 = \dfrac{m_1 - m_2}{m_1 + m_2} U_1 + \dfrac{2m_2}{m_1 + m_2} U_2$

and $U_2 = \dfrac{m_1 - m_2}{m_1 + m_2} U_2 + \dfrac{2m_1}{m_1 + m_2} U_1$

$$V_1 = \left(\frac{0.2 - 0.1}{0.1 + 0.2}\right) 0.6 + \left(\frac{2 \times 0.1}{0.1 + 0.2}\right) 0.4$$

$$V_1 = 0.47mS^{-1}$$

$$\left(0.1 - 0.2\right) \qquad \left(2 \times 0.2\right)$$

and $V_2 =$ $\frac{\quad}{0.1 + 0.2}$ 0.4 + $\frac{\quad}{0.1 + 0.2}$ 0.6

$V_2 = 0.67mS^{-1}$

5) A Pump on the ground floor of a building can pump mp water to fill a tank of volume $30m^{-3}$ in

in 15min if the tank is 40m above the ground, and the efficiency of the pump is 30% how much

electric power is consumed by the pump ?

Solution : given volume $V = 30m^{-3}$. T =15min

L = 40m efficiency 30%= 0.03

work done by the pump = Ep (potential energy)

Ep = mgh

Ep = V x S x g x h

Ep = 30 x 103 x 10 x 40

Ep = 3 x 40 x 105

Ep = 12 x 106J = power out put

$$\text{efficiency} = \frac{\text{Pout put}}{\text{Pin put}}$$

Power consumed by the pump = P input

$$\text{P input} = \frac{\text{P out put}}{\text{efficiency}} = \frac{12 \text{ x } 106}{0.03} = 4\text{x}108 \text{ W}$$