I PUC – MATHEMATICS
CHAPTER - 13

Limits and Derivatives

One mark question

I. Evaluate

16. $\lim_{x\to 0} \dfrac{x^2-5x+9}{x+3x+6}$

17. $\lim_{x\to \frac{\pi}{2}} \dfrac{2\sin x+\cos x-\cos 2x}{3+\tan 2\pi}$

18. $\lim_{x\to \frac{1}{2}} \dfrac{2^x+2^{-x}}{2x-2^{-x}}$

19. $\lim_{x\to 0} \dfrac{x^4-81}{x-3}$

20. $\lim_{x\to 0} \dfrac{x^5+1}{x+1}$

21. $\lim_{x\to 0} \dfrac{\sqrt{x}-\sqrt{a}}{x-a}$

7. $\lim_{x\to 0} \left(\dfrac{3x-2}{x^2+1}\right)$

8. $\lim_{x\to \frac{\pi}{2}} \dfrac{\cos x}{x}$

9 $\lim_{x\to \frac{\pi}{2}}$ xsinx

10. $\lim_{\theta\to 0} \dfrac{3\sin\theta-4\sin^3\theta}{\theta}$

11. $\lim_{x\to 0} \dfrac{\tan x^o}{x}$

II. Differentiate the following w.r.t. 'x'

1. $\dfrac{x^4+5x-7}{x}$

2. $\dfrac{x^2+3x+1}{\sqrt{x}}$

3. (2x +1) (3x – 5)

4. x^2 cosx

5. 5 cotx – 3 cosecx

6. $\dfrac{3\cos ecx}{2}+2\sin x$

7. $\dfrac{\tan x}{2}+4x^2$

Two marks questions:
I. Evaluate :

1. $\lim_{x\to 2} \dfrac{x^2-x-2}{x^2-5x+6}$

2. $\lim_{x\to 5} \dfrac{2x^2-7x-15}{x^2-25}$

3. $\lim_{\theta\to 0} \dfrac{2\sin^2 3\theta}{\sin 5\theta\tan\theta}$

4. $\lim_{x\to 1} \dfrac{x^p-1}{x^q-1}$

5. $\lim_{\theta\to 0} \dfrac{1-\cos 2\theta}{3\theta^2}$

6. $\lim_{x\to 1} \dfrac{\sqrt{2+x+x^2}}{x-1}$

7. $\lim_{t\to 5} \dfrac{t^3-125}{t^2-6t+5}$

8. $\lim_{x\to 2} \dfrac{x^2-3x+2}{x^3-8}$

9. $\lim_{y\to 0} \dfrac{(a+y)^3-(a-y)^3}{3ay+5y^2}$

10. $\lim_{x\to 1} \dfrac{\sin \pi x}{x(x+1)}$

11. $\lim_{x\to -2} \dfrac{x^3+8}{x^5+32}$

12. $\lim_{x\to 0} \dfrac{1-\cos 5x}{1-\cos 3x}$

13. $\lim_{x\to 0} \dfrac{\sqrt{3+x}-\sqrt{3-x}}{x^2+4x}$

14. $\lim_{x\to 0} \dfrac{\sqrt{4+x}-2}{x}$

15. $\lim_{x\to 0} \dfrac{\sqrt{x^2+x+4}-2}{x}$

16. $\lim_{x\to a} \dfrac{x-a}{x\sqrt{x}-a\sqrt{a}}$

17. $\lim_{x\to 0} \dfrac{2x+3\sin x}{5x+2\sin x}$

18. $\lim_{x\to 0} \dfrac{\tan(\sin 4x)}{\sin(\tan 2x)}$

19. $\lim_{x\to 0} \dfrac{\tan 2x - x}{3x-\sin x}$

20. $\lim_{x\to 0} \dfrac{\sin^2 3x}{x\tan 2x}$

II.

1. If f(x) = x tanx then find $f^1\left(\dfrac{\pi}{4}\right)$
2. If f(x) = $x^5 + 4x^3 + \dfrac{1}{x}$ then, find $f^1(1)$
3. f(x) = 3 sinx + 4tanx – secx, find $f^1(\pi)$
4. f(x) = $x^2 - \dfrac{1}{x^2}$ find f1(-1)

III. **Differentiate the following w.r.t 'x'**

1. $\dfrac{1-x}{1+x}$

2. $\dfrac{\sin x}{x+2}$

3. $\left(x+\dfrac{1}{x}\right)^2 \cos x$

4. $\dfrac{(x-4)}{\sqrt{3}}$

5. $\left(\sqrt{x}+\dfrac{1}{x}\right)\left(x-\dfrac{1}{\sqrt{x}}\right)$

6. $\sqrt{\dfrac{1+\cos 2x}{1-\cos 2x}}$

7. $\dfrac{\tan x}{x}$

8. 4 cot x - $\dfrac{\cos x}{2}+\dfrac{3}{\cos x}-\dfrac{4}{\sin x}$

9. $\dfrac{\sin x}{1+\tan x}$

10. x^5 cosec x

11. $(3x^2 - x)$ secx

12. secx cosec x

Three marks questions

I. Evaluate :

1. $\lim_{x\to a} \dfrac{\sin x-\sin a}{x-a}$

2. $\lim_{\theta\to \frac{\pi}{2}} \dfrac{\cos 3\theta+3\cos\theta}{\left(\dfrac{\pi}{2}-\theta\right)^3}$

3. $\displaystyle\lim_{x\to\frac{\pi}{4}} \frac{(1-\tan x}{\frac{\pi}{4}-x}$

4 $\displaystyle\lim_{x\to 0} \frac{\sin(a+x)-\sin(a-x)}{x}$

5. $\displaystyle\lim_{t\to 0} \frac{(5+t)^3-125}{t}$

6. $\displaystyle\lim_{x\to 2} \frac{x^3-8}{x\sqrt{x}-2\sqrt{2}}$

7. $\displaystyle\lim_{x\to\pi} \frac{1+\cos^3 x}{\tan^2 x}$

8. $\displaystyle\lim_{x\to 1} \left[\frac{1}{1-x}-\frac{3}{1-x^3}\right]$

9. $\displaystyle\lim_{x\to 1} \left[\frac{2}{x^2-1}+\frac{1}{1-x}\right]$

10. $\displaystyle\lim_{x\to 3} \frac{x^2-9}{\sqrt{3x-4}-\sqrt{x+2}}$

11. $\displaystyle\lim_{x\to\frac{\pi}{2}}$ (secθ - tanθ)

12. $\displaystyle\lim_{x\to\frac{\pi}{2}} \frac{16x^4-1}{8x^3-1}$

13. $\displaystyle\lim_{x\to 0} \frac{\cos(a+x)-\cos(a-x)}{x}$

14. $\displaystyle\lim_{x\to\frac{\pi}{2}} \frac{1+\cos 2\theta}{(\pi-2\theta)^2}$

15. $\displaystyle\lim_{x\to 0} \frac{\sqrt{1+2x}-\sqrt{1-2x}}{x}$

16. $\displaystyle\lim_{x\to 0} \frac{\tan(2x^4)\sin^2 4x}{x^6}$

17. $\displaystyle\lim_{x\to\frac{\pi}{3}} \frac{\sin x-\sqrt{3}\cos x}{x-\frac{\pi}{3}}$

18. f(x) $\begin{cases} 4x+1 & if \quad x<1 \\ 8x-3 & if \quad x>1 \end{cases}$, find $\displaystyle\lim_{x\to 1}$ f(x)

II. Differentiate the following w.r.t ‘x’ from first principles.

1. $\frac{1}{x}$ 2. x$\sqrt{x}$ 3. ax^2 + bx + c 4. tan3x

5. x^{2}sinx 6. $\frac{x-1}{x+1}$

III. Differentiate the following w.r.t. ‘x’

1. $\frac{2+3\cos x}{3-2\sin x}$ 2. $\frac{x^5-3x+2}{x^5+4x+6}$ 3. $\frac{\sqrt{x}+\sqrt{2}}{\sqrt{2}-\sqrt{x}}$ 4. x^2(3x + 2) cosec x

Five marks questions

1. Evaluate $\displaystyle\lim_{x\to 0} \frac{\cos ecx-\cot x}{x}$

2. If f(x) = $\begin{cases} 10x+1 & when & x<5 \\ 2x^2+1 & when & 5<x<6 \\ 11x+7 & when & x>6 \end{cases}$

Evaluate $\displaystyle\lim_{x\to 5}$ f(x) and $\displaystyle\lim_{x\to 6}$ f(x)

3. f(x) = $\begin{cases} \dfrac{\sin x - \cos x}{\dfrac{\pi}{4} - x} & when \quad x \neq \dfrac{\pi}{4} \\ K & when \quad x = \dfrac{\pi}{4} \end{cases}$

and if $\lim\limits_{x \to \frac{\pi}{4}}$ f(x) = f$\left(\dfrac{\pi}{4}\right)$, then find the value of K.

4. f(x) = $\begin{cases} ax^2 - 3x + 4 & when \quad x < 1 \\ 3 & when \quad x = 1 \\ bx + 5 & when \quad x > 1 \end{cases}$

find the values of 'a' and 'b' if $\lim\limits_{x \to 1}$ f(x) = f(1)

5. Evaluate $\lim\limits_{x \to 0} \dfrac{|x|}{x}$ if its exists.

6. f(x) = $\begin{cases} \dfrac{1 - \cos 4x}{2x^2} & for \quad x < 0 \\ \dfrac{4(x^2 - 1)}{(x - 1)} & for \quad x > 0 \end{cases}$ Evaluate $\lim\limits_{x \to 0}$ f(x)

7. Show that f(x) = $\dfrac{x}{1 + |x|}$ is differentiable at x = 0

8. Differentiate $\dfrac{(3x^2 + 2)\sin x}{(1 + x\cos x)}$ w.r.t. 'x'

9. If y = $\dfrac{\tan x + \sec x - 1}{\tan x - \sec x + 1}$ prove that $\dfrac{dy}{dx}$ = secx (secx + tanx)

10. Differentiate $\sqrt{\cos x}$ w.r.t. 'x' from first principles.

11. Differentiate $\dfrac{x^4 \cot x}{(x^2 + 3)}$ w.r.t. 'x'

Limits and Derivatives

Solutions to one mark questions

1. $\underset{x\to 0}{Lim} \quad \dfrac{x^2-5x+9}{x+3x+6}$

$= \dfrac{0+9}{0+6} = \dfrac{3}{2}$

2. $\underset{x\to \frac{\pi}{2}}{Lim} \quad \dfrac{2\sin x+\cos x-\cos 2x}{3+\tan 2\pi}$

$= \dfrac{2+0-\cos\pi}{3+0}$

$= \dfrac{2-(-1)}{3}$

$= 1$

3. $\underset{x\to \frac{1}{2}}{Lim} \quad \dfrac{2^x+2^{-x}}{2x-2^{-x}}$

$= \dfrac{\sqrt{2}+\dfrac{1}{\sqrt{2}}}{\sqrt{2}-\dfrac{1}{\sqrt{2}}} \quad = \quad \dfrac{\dfrac{2+1}{\cancel{\sqrt{2}}}}{\dfrac{2-1}{\cancel{\sqrt{2}}}}$

$= \dfrac{3}{1}$

$= 3$

4. $\underset{x\to 0}{Lim} \quad \dfrac{x^4-81}{x-3}$

$= \lim\limits_{x\to 3} \quad \dfrac{x^4-3^4}{x-3}$

$= 4(3)^{4-1}$

$= 4 \times 3^3$

$= 108$

5. $\underset{x\to 0}{Lim} \quad \dfrac{x^5+1}{x+1}$

$= \lim\limits_{x\to -1} \quad \dfrac{x^5-(-1)^5}{x-(-1)}$

$= 5\,(-1)^{5-1}$

$= 5 \times 1$

$= 5$

6. $\underset{x\to 0}{Lim} \quad \dfrac{\sqrt{x}-\sqrt{a}}{x-a}$

$= \lim\limits_{x\to a} \quad \dfrac{x^{1/2}-a^{1/2}}{x-a}$

$= \frac{1}{2} a^{\frac{1}{2}-1} = \frac{1}{2} a^{-1/2}$

$= \frac{1}{2\sqrt{a}}$

7. $\underset{x \to 0}{Lim} \left(\frac{3x-2}{x^2+1} \right)$

$= \left(\frac{-2}{1} \right)^3$

= -8

8. $\underset{x \to \frac{\pi}{2}}{Lim} \quad \frac{\cos x}{x}$

$= \frac{\cos \frac{\pi}{2}}{\frac{\pi}{2}} = \frac{0}{\frac{\pi}{2}}$

= 0

9. $\underset{x \to \frac{\pi}{2}}{Lim}$ xsinx

$= \frac{\pi}{2} \sin \frac{\pi}{2}$

$= \frac{\pi}{2}$ x 1

$= \frac{\pi}{2}$

10. $\underset{\theta \to 0}{Lim} \quad \frac{3\sin\theta - 4\sin^3\theta}{\theta}$

$= \lim_{\theta \to 0} \quad \frac{\sin 3\theta}{\theta}$

$= \lim_{\theta \to 0} \quad \frac{\sin 3\theta}{3\theta} \times 3$

$= 1 \times 3$

= 3

11. $\underset{x \to 0}{Lim} \quad \frac{\tan x^o}{x}$

$= \lim_{x \to 0} \quad \frac{\tan\left(\frac{\pi x}{180} \right)}{x}$

$= \lim_{x \to 0} \quad \frac{\tan\left(\frac{\pi x}{180} \right)}{\frac{\pi x}{180}} \times \frac{\pi}{180}$

$$= 1 \times \frac{\pi}{180}$$

$$= \frac{\pi}{180}$$

II.

1) $Y = \frac{x^4 + 5x - 7}{x}$

$Y = \frac{x^4}{x} + \frac{5x}{x} - \frac{7}{x}$

$Y = x^3 + 5 - \frac{7}{x}$

Diff. w.r.t 'x'

$\frac{dy}{dx} = 3x^2 + 0 - 7\left(-\frac{1}{x^2}\right)$

$= 3x^2 + \frac{7}{x^2}$

2) $Y = \frac{x^2 + 3x + 1}{\sqrt{x}}$

$Y = \frac{x^2}{\sqrt{x}} + \frac{3x}{\sqrt{x}} + \frac{1}{\sqrt{x}}$

$Y = x^{3/2} + 3\sqrt{x} + x^{-1/2}$

Diff . w.r.t 'x'

$\frac{dy}{dx} = \frac{3x^{\frac{3}{2}-1}}{2} + \frac{3}{2\sqrt{x}} - \frac{1}{2x^{\frac{1}{2}+1}}$

$= \frac{3x}{2} + \frac{3}{2\sqrt{x}} - \frac{1}{2x^{3/2}}$

3) $Y = (2x + 1)(3x - 5)$

Diff w.r.t 'x'

$\frac{dy}{dx} = (2x + 1)\frac{d}{dx}(3x-5) + (3x - 5)\frac{d}{dx}(2x+1)$

$= (2x+1)(3) + (3x - 5)(2)$

$= 6x + 3 + 6x - 10$

$= 12x - 7$

4) $Y = x^2 \cos x$

$\frac{dy}{dx} = x^2 \frac{d}{dx}(\cos x) + \cos x \frac{d}{dx}(x^2)$

$= -x^2 \sin x + 2x \cos x$

5) $Y = 5 \cot x - 3 \operatorname{cosec} x$

$\frac{dy}{dx} = -5 \operatorname{cosec}^2 x + 3 \operatorname{cosec} x \cot x$

6) $Y = \frac{3}{2}\text{cosec } x + 2\sin x$

$$\frac{dy}{dx} = -\frac{3}{2}\text{cosec } x \cot x + 2\cos x$$

7) $Y = \frac{\tan x}{2} + 4x^2$

$$\frac{dy}{dx} = \frac{\sec^2 x}{2} + 8x$$

Solutions to two marks questions

1. $= \lim_{x\to 2} \frac{\cancel{(x-2)}(x+1)}{\cancel{(x-2)}(x-3)}$

$= \frac{(2+1)}{(2-3)} = \frac{3}{-1}$

$= -3$

2. $= \lim_{x\to 5} \frac{(2x+3)\cancel{(x-5)}}{(x+5)\cancel{(x-5)}}$

$= \frac{2(5)+3}{5+5}$

$= \frac{13}{10}$

3. $\lim_{\theta\to 0} \frac{2\left(\frac{\sin 3\theta}{3\theta}\right)^2 \times 9\cancel{\theta^2}}{\left(\frac{\sin 5\theta}{5\theta}\right)\times 5\cancel{\theta}\times\left(\frac{\tan\theta}{\theta}\right)\times\cancel{\theta}}$

$= \frac{2\times 1^2\times 9}{1\times 5\times 1} = \frac{18}{5}$

4. $= \lim_{x\to 1} \frac{\frac{x^p-1}{x-1}}{\frac{x^q-1}{x-1}}$

$= \frac{p.(1)^{p-1}}{q.(1)^{q-1}} = \frac{p}{q}$

5. $= \lim_{\theta\to 0} \frac{2\sin^2\theta}{3\theta^2}$

$= \lim_{\theta\to 0} \frac{2}{3}\left(\frac{\sin\theta}{\theta}\right)^2$

$= \frac{2}{3}\times 1^2$

$$= \frac{2}{3}$$

6. $$\lim_{x\to 1} \frac{\sqrt{2+x+x^2}-2}{x-1} \times \frac{\sqrt{2+x+x^2}+2}{\sqrt{2+x+x^2}+2}$$

$$= \lim_{x\to 1} \frac{\left(\sqrt{2+x+x^2}\right)^2 - 2^2}{(x-1)[\sqrt{2+x+x^2}+2]}$$

$$= \lim_{x\to 1} \frac{2+x+x^2-4}{(x-1)[\sqrt{2+x+x^2}+2]}$$

$$= \lim_{x\to 1} \frac{x^2+x-2}{(x-1)[\sqrt{2+x+x^2}+2]}$$

$$= \lim_{x\to 1} \frac{\cancel{(x-1)}(x+2)}{\cancel{(x-1)}[\sqrt{2+x+x^2}+2]}$$

$$= \frac{1+2}{\sqrt{2+1+1}+2} = \frac{3}{2+2}$$

$$= \frac{3}{4}$$

7. $$= \lim_{t\to 5} \frac{(t-5)(t^2+5t+5^2)}{(t-5)(t-1)}$$

$$= \frac{25+25+25}{5-1}$$

$$= \frac{75}{4}$$

8. $$= \lim_{x\to 2} \frac{(x-2)(x-1)}{(x-2)(x^2+2x+4)}$$

$$= \frac{2-1}{4+4+4}$$

$$= \frac{1}{12}$$

9. $$= \lim_{y\to 0} \frac{[a+y-(a-y)][a+y^2)^2+(a+y)(a-y)+(a-y)^2]}{y[3a+5y]}$$

$$= \lim_{y\to 0} \frac{2y[(a+y)^2+(a+y)(a-y)+(a-y)^2]}{y[3a+5y]}$$

$$= \frac{2[a^2+(a)(a)+a^2]}{3a+0}$$

$$= \frac{2(3a^2)}{3a}$$

= 2a

10. $$= \lim_{x\to 1} \left(\frac{\sin \pi x}{\pi x}\right) \times \frac{\pi}{x+1}$$

$$= 1 \times \frac{\pi}{1+1} = \frac{\pi}{2}$$

11. $$= \lim_{x \to -2} \frac{\dfrac{x^3 - (-2)^3}{x(-2)}}{\dfrac{x^5 - (-2)^5}{x - (-2)}}$$

$$= \frac{3(-2)^{3-1}}{5(-2)^{5-1}} = \frac{3 \times 4}{5 \times 16}$$

$$= \frac{3}{20}$$

12. $$= \lim_{x \to 0} \frac{2\sin^2\left(\dfrac{5x}{2}\right)}{2\sin 2\left(\dfrac{3x}{2}\right)}$$

$$= [\therefore \lim_{x \to 0} \frac{\sin^2 mx}{\sin^2 nx} = \frac{m^2}{n^2}]$$

$$= \frac{\left(\dfrac{5}{2}\right)^2}{\left(\dfrac{3}{2}\right)^2} = \frac{25}{9}$$

13. $$= \lim_{x \to 0} \frac{\sqrt{3+x} - \sqrt{3-x}}{x^2 + 4x} \times \frac{\sqrt{3+x} + \sqrt{3-x}}{\sqrt{3+x} + \sqrt{3-x}}$$

$$= \lim_{x \to 0} \frac{(3+x) - (3-x)}{(x^2 + 4x)[\sqrt{3+x} + \sqrt{3-x}]}$$

$$= \lim_{x \to 0} \frac{2x}{x(x+4)[\sqrt{3+x} + \sqrt{3-x}]}$$

$$= \frac{2}{4[\sqrt{3} + \sqrt{3}]} = \frac{2}{4(2\sqrt{3})}$$

$$= \frac{1}{4\sqrt{3}}$$

14. $$= \lim_{x \to 0} \frac{(\sqrt{4+x} - 2)(\sqrt{4+x} + 2)}{x(\sqrt{4+x} + 2)}$$

$$= \lim_{x \to 0} \frac{(\sqrt{4+x})^2 - 2^2}{x(\sqrt{4+x} + 2)}$$

$$= \lim_{x \to 0} \frac{x}{x(\sqrt{4+x} + 2)}$$

$$= \frac{1}{\sqrt{4+0} + 2}$$

$$= \frac{1}{4}$$

15. $$= \lim_{x\to 0} \frac{(\sqrt{x^2+x+4})^2 - 2^2}{x[\sqrt{x^2+x+4}+2]}$$

$$= \lim_{x\to 0} \frac{x^2+x}{x[\sqrt{x^2+x+4}+2]}$$

$$= \lim_{x\to 0} \frac{x(x+1)}{x[\sqrt{x^2+x+4}+2}$$

$$= \frac{1}{\sqrt{0+4}+2} = \frac{1}{4}$$

16. $$= \lim_{x\to a} \frac{x-a}{x^{3/2}-a^{3/2}}$$

$$= \frac{1}{\frac{3}{2}[a]^{3/2-1}} = \frac{2}{3a^{1/2}}$$

$$= \frac{2}{3\sqrt{a}}$$

17. $$= \lim_{x\to 0} \frac{x[2+3\frac{\sin x}{x}]}{x[5+2\frac{\sin x}{x}]}$$

$$= \frac{2+3\times 1}{5+2\times 1}$$

$$= \frac{5}{7}$$

18. $$\lim_{x\to 0} \frac{\tan(\sin 4x)}{\sin(\tan 2x)}$$

$$= \lim_{x\to 0} \frac{\frac{\tan(\sin 4x)}{\sin 4x}\times\frac{\sin 4x}{4x}\times 4x}{\frac{\sin(\tan 2x)}{\tan 2x}\times\frac{\tan 2x}{2x}\times 2x}$$

$$= \frac{1\times 1\times 4}{1\times 1\times 2} = 2$$

19. $$= \lim_{x\to 0} \frac{x\left[\frac{\tan 2x}{x}-1\right]}{x\left[3-\frac{\sin x}{x}\right]}$$

$$= \lim_{x\to 0} \left[\frac{\frac{\tan 2x}{2x} \times 2 - 1}{3 - \frac{\sin x}{x}} \right]$$

$$= \frac{1\times 2 - 1}{3-1} \quad = \frac{1}{2}$$

20. $= \lim_{x\to 0} \quad \dfrac{\sin^2 3x}{x \tan 2x}$

$$= \lim_{x\to 0} \frac{\left(\frac{\sin 3x}{3x}\right)^2 \times 9x^2}{x\left(\frac{\tan 2x}{2x}\right) \times 2x}$$

$$= \lim_{x\to 0} \quad \frac{1\times 9}{1\times 2}$$

$$= \frac{9}{2}$$

II.

1. f(x) = x tan x

   $f^1(x) = x \dfrac{d}{dx}(\tan x) + \tan x \dfrac{d}{dx}(x)$

   = x sec2 x + tanx

   $f^1\left(\dfrac{\pi}{4}\right) = \dfrac{\pi}{4}(\sqrt{2)}^2 + 1 = \dfrac{\pi}{2} + 1$

2. $f(x) = x^5 + 4x^3 + \dfrac{1}{x}$

   $f^1(x) = 5x^4 + 12x^2 - \dfrac{1}{x^2}$

   $f^1(1) = 5 + 12 - 1$

   = 16

3. f(x) = 3sin x + 4 tan x – sec x

   $f^1(x) = 3 \cos x + 4 \sec^2 x - \sec x \tan x$

   $f^1(\pi) = 3\cos \pi + 4\sec^2 \pi - \sec\pi \tan\pi$

   $= 3 (-1) + 4(-1)^2 - 0$

   = - 3 + 4

   = -1

4. $f(x) = x^2 - \dfrac{1}{x^2}$

   $f^1(x) = 2x - \left(\dfrac{-2}{x^3}\right)$

   $= 2x + \dfrac{2}{x^3}$

   $f^1(-1) = 2(-1) + \dfrac{2}{(-1)^3}$

$= -2 - 2$
$= -4$

III.

1. y = $\frac{1-x}{1+x}$

Diff. w.r.t. 'x'

$$\frac{dy}{dx} = \frac{(1+x)\frac{d}{dx}(1-x) - (1-x)\frac{d}{dx}(1+x)}{(1+x)^2}$$

$$= \frac{(1+x)(-1) - (1-x)(1)}{(1+x)^2}$$

$$= \frac{-1-x-1+x}{(1+x)^2}$$

$$\therefore \frac{dy}{dx} = \frac{-2}{(1+x)^2}$$

2. y = $\frac{\sin x}{x+2}$

diff w.r.t. 'x'

$$\frac{dy}{dx} = \frac{(x+2)\frac{d}{dx}(\sin x) - \sin x\frac{d}{dx}(x+2)}{(x+2)^2}$$

$$\frac{dy}{dx} = \frac{(x+2)\cos x - \sin x}{(x+2)^2}$$

3. y = $\left(x+\frac{1}{x}\right)^2 \cos x$

y = $\left(x^2+\frac{1}{x^2}+2\right)\cos x$

diff w.r.t. 'x'

$$\frac{dy}{dx} = \left(x^2+\frac{1}{x^2}+2\right)(-\sin x) + \cos x\left(2x-\frac{2}{x^3}\right)$$

$$= -\sin x\left(x^2+\frac{1}{x^2}+2\right) + 2\cos x\left(x-\frac{1}{x^3}\right)$$

4. y = $\frac{(x-4)\cot x}{\sqrt{3}}$

diff w.r.t 's'

$$\frac{dy}{dx} = \frac{1}{\sqrt{3}}\left[(x-4)\frac{d}{dx}(\cot x) + \cot x\frac{d}{dx}(x-4)\right]$$

$$= \frac{1}{\sqrt{3}}\left[-(x-4)\operatorname{cosec}^2 x + \cot x(1)\right]$$

$$= \frac{(4-x)\operatorname{cosec}^2 x + \cot x}{\sqrt{3}}$$

5. $y = \left(\sqrt{x} + \frac{1}{x}\right)\left(x - \frac{1}{\sqrt{x}}\right)$

$y = x\sqrt{x} - \sqrt{x} \cdot \frac{1}{\sqrt{x}} + \frac{1}{x}.x - \frac{1}{x\sqrt{x}}$

$y = x^{3/2} - 1 + 1 - \frac{1}{x^{3/2}}$

diff w.r.t. 'x'

$$\frac{dy}{dx} = \frac{3}{2}x^{\frac{3}{2}-1} - \frac{1}{\left(\frac{3}{2}\right)x^{3/2+1}}$$

$$= \frac{3\sqrt{x}}{2} - \frac{2}{3x^{5/2}}$$

6. $y = \sqrt{\frac{1+\cos 2x}{1-\cos 2x}}$

$y = \sqrt{\frac{2\cos^2 x}{2\sin^2 x}}$

y = cotx

$\therefore \frac{dy}{dx}$ = - cosec^{2}x

7. $y = \frac{\tan x}{x}$

$$\frac{dy}{dx} = \frac{x.(\sec^2 x) - \tan x(1)}{x^2}$$

$$= \frac{x\sec^2 x - \tan x}{x^2}$$

8. y = 4 cotx - $\frac{\cos x}{2} + \frac{3}{\cos x} - \frac{4}{\sin x}$

y = 4 cot x - $\frac{\cos x}{2} + 3\sec x - 4\operatorname{cosec} x$

$\therefore \frac{dy}{dx}$ = - 4cosec^{2}x + $\frac{\sin x}{2}$ + secx tanx + 4 cosecx cotx

9. y = (3x^2 – x) secx

$\frac{dy}{dx}$ = (3x^2 – x) secx tanx + secx(6x – 1)

= secx [x(3x-1) tanx + 6x – 1]

10. $y = \frac{\sin x}{1+\tan x}$

$$\frac{dy}{dx} = \frac{(1+\tan x)\cos x - \sin x(\sec^2 x)}{(1+\tan x)^2}$$

$$= \frac{\cos x + \tan x \cos x - \sin x\left(\frac{1}{\cos^2 x}\right)}{(1+\tan x)^2}$$

$$\frac{dy}{dx} = \frac{\cos x + \sin x - \sec x \tan x}{(1+\tan x)^2}$$

11. y = secx cosecx

$$\frac{dy}{dx} = \text{secx (-cosecx cotx) + cosecx(secx tanx)}$$

$$= - \frac{1}{\cos x} \times \frac{1}{\sin x} \times \frac{\cos x}{\sin x} + \frac{1}{\sin x} \times \frac{1}{\cos x} \times \frac{\sin x}{\cos x}$$

$= - \text{cosec}^2 \text{ x} + \sec^2 \text{ x}$

Solutions to three marks questions

I.

1. $\lim\limits_{x \to a} \dfrac{\sin x - \sin a}{x - a}$

$$= \lim_{x \to a} \frac{2\cos\left(\frac{x+a}{2}\right)\sin\left(\frac{x-a}{2}\right)}{(x-a)}$$

$$= \lim_{x \to a} 2 \cos \left(\frac{x+a}{2}\right) \times \frac{\sin\left(\frac{x-a}{2}\right)}{\frac{(x-a)}{2}} \times \frac{1}{2}$$

$$= \cos \left(\frac{a+a}{2}\right) \times 1$$

= cos a

2. $\lim\limits_{\theta \to \frac{\pi}{2}} \dfrac{\cos 3\theta + 3\cos\theta}{\left(\frac{\pi}{2} - \theta\right)^3}$

$$= \lim_{\theta \to \frac{\pi}{2}} \frac{4\cos 3\theta - 3\cos\theta + 3\cos\theta}{\left(\frac{\pi}{2} - \theta\right)^3}$$

$$= \lim_{\theta \to \frac{\pi}{2}} \frac{4\cos^3\theta}{\left(\frac{\pi}{2} - \theta\right)^3}$$

put $\frac{\pi}{2} - \theta = \text{t}$ As $\theta \to \frac{\pi}{2} = \text{t} \to 0$

$$= \lim_{\theta \to \frac{\pi}{2}} \frac{4\left[\cos(\frac{\pi}{2} - t)\right]^3}{t^3}$$

$$= \lim_{t \to 0} \frac{4\sin^3 t}{t^3}$$

= 4 x 1

$= 4$

3. $\lim_{x \to \frac{\pi}{4}} \dfrac{1-\tan x}{\left(\frac{\pi}{4}-x\right)}$

$= \lim_{x \to \frac{\pi}{4}} \dfrac{1-\tan x}{1+\tan x} \times \dfrac{1+\tan x}{\left(\frac{\pi}{4}-x\right)}$

$= \lim_{x \to \frac{\pi}{4}} \dfrac{\tan\left(\frac{\pi}{4}-x\right)(1+\tan x)}{\left(\frac{\pi}{4}-x\right)}$

$= 1 \times \left(1+\tan\frac{\pi}{4}\right)$ $\quad\quad\quad$ $\therefore$ As $x \to \frac{\pi}{4} = \frac{\pi}{4} - x \to 0$

$= 1 + 1$

$= 2$

4. $\lim_{x \to 0} \dfrac{\sin(a+x)-\sin(a-x)}{x}$

$= \lim_{x \to 0} \dfrac{2\cos(a)\sin(x)}{x}$

$= 2 \cos a \times 1$

$= 2\cos a$

5. $\lim_{x \to 0} \dfrac{\cos(a+x)-\cos(a-x)}{x}$

$= \lim_{x \to 0}\ - \dfrac{2\sin\left(\frac{a+x+a-x}{2}\right)\sin\left(\frac{a+x-a+x}{2}\right)}{x}$

$= \lim_{x \to 0} \dfrac{-2\sin a \cdot \sin x}{x}$

$= -2\sin a \ .\ \lim_{x \to 0} \dfrac{\sin x}{x}$

$= -2 \sin a \times 1 = -2 \sin a$

6. $\lim_{x \to \frac{\pi}{2}} \dfrac{1+\cos 2\theta}{(\pi-2\theta)^2}$

$= \lim_{x \to \frac{\pi}{2}} \dfrac{2\cos^2\theta}{(\pi-2\theta)^2}$

$= \lim_{x \to \frac{\pi}{2}} \dfrac{2 \cdot \sin^2\left(\frac{\pi}{2}-\theta\right)}{2^2\left(\frac{\pi}{2}-\theta\right)^2}$

$$= \frac{1}{2} \lim_{x\to\frac{\pi}{2}} \frac{\sin^2\left(\frac{\pi}{2}-\theta\right)}{2\left(\frac{\pi}{2}-\theta\right)^2}$$

$$= \frac{1}{2} \times 1 = \frac{1}{2}$$

7. $\lim\limits_{x\to 0} \dfrac{\sqrt{1+2x}-\sqrt{1-2x}}{x}$

$$= \lim_{x\to 0} \frac{(1+2x)-(1-2x)}{x[\sqrt{1+2x}+\sqrt{1-2x}]}$$

$$= \lim_{x\to 0} \frac{4}{\sqrt{1+2x}+\sqrt{1-2x}}$$

$$= \frac{4}{2} = 2$$

8. $\lim\limits_{x\to 0} \dfrac{\tan(2x^4)\sin^2(4x)}{x^6}$

$$= \lim_{x\to 0} 2. \frac{\tan 2x^4}{2x^4} \times \left(\frac{\sin 4x}{4x}\times 4\right)^2$$

$$= 2\times 16 = 32$$

9. $\lim\limits_{x\to\frac{\pi}{3}} \dfrac{\sin x-\sqrt{3}\cos x}{x-\frac{\pi}{3}}$

$$= \lim_{x\to\frac{\pi}{2}} \frac{2.\left(\frac{1}{2}\sin x-\frac{\sqrt{3}}{2}\cos x\right)}{x-\frac{\pi}{3}}$$

$$= \lim_{x\to\frac{\pi}{2}} 2. \frac{\sin\left(x-\frac{\pi}{3}\right)}{(x-\frac{\pi}{3})}$$

$$= 2\times 1 = 2$$

10. f(x) = 4x + 1 if x< 1

= 8x – 3 if x > 1

LHL = $\lim\limits_{x\to 1-}$ f(x) = $\lim\limits_{x\to 1-}$ (4x + 1) = 5

RHL = $\lim\limits_{x\to 1+}$ f(x) = $\lim\limits_{x\to 1+}$ (8x – 3) = 5

As LHL = RHL . Limit of the function at x = 1 exists and $\lim\limits_{x\to 1}$ f(x) = 5

11. $\lim_{t \to 0} \dfrac{(5+t)^3 - 125}{t}$

$= \lim_{x \to 5} \dfrac{x^3 - 5^3}{x - 5}$ put 5 + t = x

As t $\to$ 0 = x $\to$ 5

$= 3(5)^{3-1}$

$= 3 \times 25$

$= 75$

12. $\lim_{x \to 2} \dfrac{x^3 - 8}{x\sqrt{x} - 2\sqrt{2}}$

$= \lim_{x \to 2} \dfrac{\dfrac{x^3 - 2^3}{x - 2}}{\dfrac{x^{3/2} - 2^{3/2}}{x - 2}}$ $= \dfrac{3(2)^{3-1}}{\frac{3}{2}(2)^{3/2-1}}$

$= \dfrac{2 \cdot 4}{2^{1/2}}$ $= 4\sqrt{2}$

13. $\lim_{x \to \pi} \dfrac{1 + \cos^3 x}{\tan^2 x}$

$= \lim_{x \to \pi} \dfrac{(1 + \cos x)[1 + \cos x + \cos^2 x]}{\dfrac{\sin^2 x}{\cos^2 x}}$

$= \lim_{x \to \pi} \dfrac{(1 + \cos x)(1 + \cos x + \cos^2 x)}{(1 + \cos x)(1 - \cos x)} \times \cos^2 x$

$= \dfrac{(1 + \cos\pi + \cos^2 \pi)}{1 - \cos\pi}$

$= \dfrac{(1 - 1 + 1)(-1)^2}{1 - (-1)}$

$= \dfrac{1}{2}$

14. $\lim_{x \to 1} \dfrac{1}{1-x} - \dfrac{3}{1-x^3}$

$= \lim_{x \to 1} \dfrac{1}{1-x}\left[1 - \dfrac{3}{1 + x + x^2}\right]$

$= \lim_{x \to 1} \dfrac{1}{1-x}\left[\dfrac{(x+2)(x-1)}{1 + x + x^2}\right]$

$= \lim_{x \to 1} -\dfrac{(x+2)}{1 + x + x^2}$ $= \dfrac{-3}{3}$ = -1

15. $\lim_{x \to 1} \left[\dfrac{2}{x^2 - 1} + \dfrac{1}{1-x}\right]$

$$= \lim_{x\to 1} \left[\frac{2}{(x-1)(x+1)} + \frac{1}{1-x}\right]$$

$$= \lim_{x\to 1} \left(\frac{1}{(x-1}\right) \left[\frac{2}{x+1} - 1\right]$$

$$= \lim_{x\to 1} \frac{1}{(x-1)} \times \frac{1-x}{x+1}$$

$$= \lim_{x\to 1} \frac{-1}{x+1} = -1/2$$

16. $\lim_{x\to 3} \dfrac{x^2-9}{\sqrt{3x-4}-\sqrt{x+2}}$

$$= \lim_{x\to 3} \frac{(x+3)(x-3)[\sqrt{3x-4}+\sqrt{x+2}]}{(3x-4)-(x+2)}$$

$$= \lim_{x\to 3} \frac{(x+3)(x-3)(\sqrt{3x-4}+\sqrt{x+2})}{2(x-3)}$$

$$= \frac{6\times 2\sqrt{5}}{2}$$

$$= 6\sqrt{5}$$

17. $\lim_{\theta\to\frac{\pi}{2}}$ (secθ - tanθ)

$$= \lim_{\theta\to\frac{\pi}{2}} \left(\frac{1}{\cos\theta} - \frac{\sin\theta}{\cos\theta}\right)$$

$$= \lim_{\theta\to\frac{\pi}{2}} \frac{1-\sin\theta}{\cos\theta} \times \frac{1+\sin\theta}{1+\sin\theta}$$

$$= \lim_{\theta\to\frac{\pi}{2}} \frac{1-\sin^2\theta}{\cos\theta(1+\sin\theta)}$$

$$= \lim_{\theta\to\frac{\pi}{2}} \frac{\cos\theta}{1+\sin\theta}$$

$$= \frac{0}{1} = 0$$

18. $\lim_{x\to 1/2} \dfrac{16x^4-1}{8x^3-1}$

$$= \lim_{x\to 1/2} \frac{(4x^2-1)(4x^2+1)}{(2x-1)(4x^2+2x+1)}$$

$$= \lim_{x\to 1/2} \frac{(2x-1)(2x+1)(4x^2+1)}{(2x-1)(4x^2+2x+1)}$$

$$= \frac{2\times 2}{1+1+1} = 4/3$$

II.

1. f(x) = $\frac{1}{x}$

   $f^1(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$

   $= \lim_{\Delta x \to 0} \frac{\frac{1}{x+\Delta x} - \frac{1}{x}}{\Delta x}$

   $= \lim_{\Delta x \to 0} \frac{x-(x+\Delta x)}{x(x+\Delta x)\Delta x}$

   $= \lim_{\Delta x \to 0} \frac{-\Delta x}{x(x+\Delta x)\Delta x}$

   $= - \frac{1}{x(x+0)}$

   $= - \frac{1}{x^2}$

2. f(x) = x$\sqrt{x}$

   f(x) = $x^{3/2}$

   $f^1(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$

   $f^1(x) = \lim_{\Delta x \to 0} \frac{(x+\Delta x)^{3/2} - x^{3/2}}{(x+\Delta x) - x}$

   $= \frac{3}{2}(x)^{3/2-1}$

   $f^1(x) = \frac{3}{2}x^{\frac{1}{2}}$

   $f^1(x) = \frac{3\sqrt{x}}{2}$

3. f(x) = ax^2 + bx + c

   $f(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$

   $= \lim_{\Delta x \to 0} \frac{a(x+\Delta x)^2 + b(x+\Delta x) + c - \{ax^2 + bx + c\}}{\Delta x}$

   $= \lim_{\Delta x \to 0} \frac{a(x+\Delta x)^2 + b(x+\Delta x) - ax^2 - bx}{\Delta x}$

   $= \lim_{\Delta x \to 0} \frac{ax^2 + 2a(\Delta x) + 2ax\Delta x + bx + b\Delta x - ax^2 - bx}{\Delta x}$

   $= \lim_{\Delta x \to 0} \frac{(\Delta x)\{a\Delta x + 2ax + b\}}{\Delta x}$

   = a (0) + 2ax + b

   = 2ax+b

4. f(x) = tan3x

$f^1(x) = \lim\limits_{h\to 0} \dfrac{f(x+h)-f(x)}{h}$

$= \lim\limits_{h\to 0} \dfrac{\tan(3x+3h)-\tan 3x}{h}$

$= \lim\limits_{h\to 0} \dfrac{\tan(3x+3h-3x)\{1+\tan(3x+3h)\tan 3x\}}{h}$

$= \lim\limits_{h\to 0} \dfrac{\tan 3h\{1+\tan(3x+3h)\tan 3\}}{3h}\times 3$

$= 1 \times \{1 + \tan 3x \tan 3x\} \times 3$

$= 3\,(1 + \tan^2 3x)$

$f^1(x) = 3\sec^2 3x$

5. $f(x) = x^2 \sin x$

$f^1(x) = \lim\limits_{h\to 0} \dfrac{f(x+h)-f(x)}{h}$

$= \lim\limits_{h\to 0} \dfrac{(x+h)^2\sin(x+h)-x^2\sin x}{h}$

$= \lim\limits_{h\to 0} \dfrac{x^2\sin(x+h)+h^2\sin(x+h)+2xh\sin(x+h)-x^2\sin x}{h}$

$= \lim\limits_{h\to 0} \dfrac{x^2[\sin(x+h)-\sin x]+h[h\sin(x+h)+2x\sin(x+h)]}{h}$

$= \lim\limits_{h\to 0}$

$$\left\{ \frac{x^2\times 2\cos\left(\frac{x+h+x)}{2}\right)\sin\left(\frac{x+h-x}{2}\right)}{h} + \frac{h[h\sin(x+h)+2x\sin(x+h)}{h} \right\}$$

$$= \lim_{h\to 0} \frac{2x^2\cos\left(\frac{2x+h}{2}\right)\sin\left(\frac{h}{2}\right)}{\frac{h}{2}\times 2} + 2x\sin x$$

$= x^2 \cos x \times 1 + 2x \sin x$

$= x^2 \cos x + 2\,x \sin x$

6. $f(x) = \dfrac{x-1}{x+1} = 1 - \dfrac{2}{x+1}$

$f1(x) = \lim\limits_{h\to 0} \dfrac{f(x+h)-f(x)}{h}$

$$= \lim_{h\to 0} \frac{\left(1-\frac{2}{x+h+1}\right)-\left(1-\frac{2}{x+1}\right)}{h}$$

$$= \lim_{h\to 0} \frac{\frac{2}{x+1}-\frac{2}{x+h+1}}{h}$$

$= \lim\limits_{h\to 0} \dfrac{2[x+h+1-(x+1)]}{h(x+1)(x+h+1)}$

$$= \lim_{h\to 0} \frac{2h}{h(x+1)(x+h+1)}$$

$$= \frac{2}{(x+1)(x+1)} = \frac{2}{(x+1)^2}$$

III.

1. y = $\dfrac{2+3\cos x}{3-2\sin x}$

Diff w.r.t. 'x'

$$\frac{dy}{dx} = \frac{(3-2\sin x)\frac{d}{dx}(2+3\cos x)-(2+3\cos x)\frac{d}{dx}(3-2\sin x)}{(3-2\sin x)^2}$$

$$= \frac{(3-2\sin x)(-3\sin x)-(2+3\cos x)(-2s\cos x)}{(3-2\sin x)^2}$$

$$= \frac{-9\sin x+6\sin^2 x+4\cos x+6\cos^2 x}{(3-2\sin x)^2}$$

$$= \frac{dy}{dx} = \frac{6+4\cos x-9\sin x}{(3-2\sin x)^2}$$

2. y = $\dfrac{x^5-3x+2}{x^5+4x+6}$

Diff w.r.t. 'x'

$$\frac{dy}{dx} = \frac{(x^5+4x+6)(5x^4-3)-(x^5-3x+2)(5x^4+4)}{(x^5+4x+6)^2}$$

$$\frac{5x^9-3x^5+20x^5-12x+30x^4-18-5x^9-4x^5+15x^5+12x-10x^4-8}{(x^5+4x+6)^2}$$

$$= \frac{28x^5+20x^4-26}{(x^5+4x+6)^2}$$

$$= \frac{2[14x^5+10x^4-13]}{(x^5+4x+6)^2}$$

3. y = $\dfrac{\sqrt{x}+\sqrt{2}}{\sqrt{2}-\sqrt{x}}$

$$\frac{dy}{dx} = \frac{(\sqrt{2}-\sqrt{x})\left(\frac{1}{2\sqrt{x}}\right)-(\sqrt{x}+\sqrt{2})\left(\frac{-1}{2\sqrt{x}}\right)}{(\sqrt{2}-\sqrt{x})^2}$$

$$= \frac{\frac{1}{2\sqrt{x}}[\sqrt{2}-\sqrt{x}+\sqrt{x}+\sqrt{2}]}{(\sqrt{2}-\sqrt{x})^2}$$

$$= \frac{2\sqrt{2}}{2\sqrt{x}\left(\sqrt{2}-\sqrt{2}\right)^2}$$

$$= \frac{\sqrt{2}}{\sqrt{2}(\sqrt{2}-\sqrt{x})^2}$$

4. $y = x^2(3x+2)\ \text{cosec}x$

$$\frac{dy}{dx} = x^2\ (3x+2)\ (-\text{cosec}x) + x^2\ \text{cosec}\ x(3) + (3x+2)\ \text{cosec}x\ (2x)$$

$$= -\ x^2\ (3x+2)\ \text{cosec}x + 3x^2\ \text{cosec}\ x + 2x(3x+2)\ \text{cosec}x$$

Solution to five marks questions

1. $$\lim_{x\to 0} \frac{\cos ecx - \cot x}{x}$$

$$= \lim_{x\to 0} \frac{\frac{1}{\sin x} - \frac{\cos x}{\sin x}}{x}$$

$$= \lim_{x\to 0} \frac{1-\cos x}{x\sin x}$$

$$= \lim_{x\to 0} \frac{1-\cos x}{x\sin x} \times \frac{1+\cos x}{1+\cos x}$$

$$= \lim_{x\to 0} \frac{1-\cos^2 x}{x\sin x(1+\cos x)}$$

$$= \lim_{x\to 0} \frac{\sin^2 x}{x\sin x(1+\cos x)}$$

$$= \lim_{x\to 0} \frac{\sin x}{x(1+\cos x)}$$

$$= \frac{1}{1+\cos o} \qquad (\therefore \lim_{x\to 0} \frac{\sin x}{x} = 1)$$

$$= \frac{1}{2}$$

2. LHL = $\lim_{x\to 5^-}$ f(x) = $\lim_{x\to 5^-}$ 10x+1 = 10 (5) + 1 = 51

RHL = $\lim_{x\to 5^+}$ f(x) = $\lim_{x\to 5^+}$ $(2x^2+1) = 2(5)^2 + 1 = 51$

LHL = RHL $\therefore$ $\lim_{X\to 5}$ f(x) = 51

LHL = $\lim_{X\to 6^-}$ f(x) = $\lim_{X\to 6^-}$ $2x^2 + 1 = 2(6)^2 + 1 = 72 + 1 = 73$

RHL = $\lim_{X\to 6^+}$ f(x) = $\lim_{X\to 6^+}$ (11x + 7) = [11(6) +7] = 73

LHL = RHL

$\therefore$ $\lim_{X\to 6^-}$ f(x) = 73

3. Given $\lim_{X\to \frac{\pi}{4}}$ f(x) = $f\left(\frac{\pi}{4}\right)$

$$\rightarrow \lim_{X\to\frac{\pi}{4}} \left(\frac{\sin x - \cos x}{\frac{\pi}{4} - x}\right) = K$$

$$\rightarrow \lim_{X\to\frac{\pi}{4}} \frac{\sqrt{2}\left(\frac{1}{\sqrt{2}}\sin x - \frac{1}{\sqrt{2}}\cos x\right)}{\left(\frac{\pi}{4} - x\right)} = K$$

$$\rightarrow \lim_{X\to\frac{\pi}{4}} \frac{\sqrt{2}\sin\left(\frac{\pi}{4} - x\right)}{\left(\frac{\pi}{4} - x\right)} = k$$

$\rightarrow \sqrt{2} \times 1 = K$

$\therefore$ K = $\sqrt{2}$

4. Given $\lim_{X\to 1^-}$ f(x) = $\lim_{X\to 1^+}$ f(x) = f(1)

   Consider $\lim_{X\to 1^-}$ f(x) = f(1)

   a(1) – 3(1) + 4 = 3

   ➔ a = 2

   also $\lim_{X\to 1^+}$ f(x) = f(1)

   b(1) + 5 = 3

   b = -2

5. $$|x| = \begin{cases} x & if \quad x > 0 \\ 0 & if \quad x = 0 \\ -x & if \quad x < 0 \end{cases}$$

   RHL = $\lim_{X\to 0^+} \frac{|x|}{x} = \lim_{X\to 0^+} \frac{\cancel{x}}{\cancel{x}} = 1$

   LHL = $\lim_{X\to 0^-} \frac{|x|}{x} = \lim_{X\to 0^-} \frac{= x}{x} = 1$

   LHL $\neq$ RHL

   $\therefore \lim_{X\to 0} \frac{|x|}{x}$ doesn't exist

6. LHL = $\lim_{X\to 0^-}$ f(x) $= \lim_{X\to 0^-} \frac{1-\cos 4x}{2x^2} = \lim_{X\to 0} \frac{2\sin^2 x}{2x^2}$

   $= \lim_{X\to 0} \left(\frac{\sin 2x}{2x}\right)^2 \times 4$

   $= 1 \times 4 = 4$

   RHL = $\lim_{X\to 0^+}$ f(x) $= \lim_{X\to 0^+} \frac{4(x^2-1)}{(X-1)}$

   $= \lim_{X\to 0^+} \frac{4(x+1)\cancel{(x-1)}}{\cancel{(x-1)}}$ = 4 (0 +1)

$\therefore \underset{X\to 0}{Lim}$ f(x) = 4

7. $f(x) = \dfrac{x}{1+|x|} = \begin{cases} \dfrac{x}{1+x} & if \quad x>0 \\ 0 & if \quad x=0 \\ \dfrac{x}{1-x} & if \quad x<0 \end{cases}$

$Rf^1(0) = \underset{X\to 0^+}{Lt} \dfrac{f(x)-f(0)}{x-0}$

$= \underset{X\to 0^+}{Lt} \dfrac{\dfrac{x}{1+x}-0}{x} = 1$

$Lf^1(0) = \underset{X\to 0^-}{Lt} \dfrac{f(x)-f(0)}{x-0}$

$= \underset{X\to 0^-}{Lt} \dfrac{\dfrac{x}{1-x}}{x}$

$= \underset{X\to 0^-}{Lt} \dfrac{1}{1-x}$

= 1

$Rf^1(0) = Lf^1(0) \quad \therefore f^1(0) = 1$

8. $f(x) = \dfrac{(3x^2+2)\sin x}{1+x\cos x}$

$f1(x) = \dfrac{(3x^2+2)\sin x.\dfrac{d}{dx}(1+x\cos x)-(1+x\cos x)\dfrac{d}{dx}\left((3x^2+2)\sin x\right)}{(1+x\cos x)^2}$

$= \dfrac{(3x^2+2)\sin x[-x\sin x+\cos x]-(1+x\cos x)\left((3x^2+2)\cos x+6x\sin x\right)}{(1+x\cos x)^2}$

9. $y = \dfrac{\tan x+\sec x-1}{\tan x-\sec x+1}$

$y = \dfrac{(\tan x+\sec x)-(\sec x+\tan x)(\sec x-\tan x)}{(\tan x-\sec x+1)}$

$= \dfrac{(\sec x+\tan x)[\cancel{1-\sec x+\tan x}]}{(\cancel{\tan x-\sec x+1})}$

Y = secs + tanx

Diff w.r.t. ‘x’

$\dfrac{dy}{dx}$ secx tanx + $\sec^2$ x

= secx [tanx + secx]

10. $y = \sqrt{\cos x}$

$\dfrac{dy}{dx} = \underset{h\to 0}{Lim} \dfrac{f(x+h)-f(x)}{h}$

$= \underset{h\to 0}{Lim} \dfrac{\sqrt{\cos(x+h)}-\sqrt{\cos x}}{h}$

$$= \lim_{h\to 0} \frac{\left(\sqrt{\cos(x+h)} - \sqrt{\cos x}\right)\left[\sqrt{\cos(x+h)} + \sqrt{\cos x}\right]}{h\left[\sqrt{\cos(x+h)} + \sqrt{\cos x}\right]}$$

$$= \lim_{h\to 0} \frac{\cos(x+h) - \cos x}{h\left[\sqrt{\cos(x+h)} + \sqrt{\cos x}\right]}$$

$$= \lim_{h\to 0} \frac{-2\sin\left(\frac{2x+h}{2}\right)\sin\left(\frac{h}{2}\right)}{h\left[\sqrt{\cos(x+h)} + \sqrt{\cos x}\right]}$$

$$= \lim_{h\to 0} \frac{-\cancel{2}\sin\left(\frac{2x+h}{2}\right)\sin\left(\frac{h}{2}\right)}{\left[\sqrt{\cos(x+h)} + \sqrt{\cos x}\right]\left(\frac{h}{2}\right)\times\cancel{2}}$$

$$= \frac{\sin x \times 1}{\left[\sqrt{\cos x} + \sqrt{\cos x}\right]}$$

$$= \frac{\sin x}{2\sqrt{\cos x}}$$

11. y = $\dfrac{x^4 \cot x}{(x^2+3}$

diff w.r.t ‘x’

$$\frac{dy}{dx} = \frac{(x^2+3)\frac{d}{dx}(x^4\cot x) - x^4\cot x\frac{d}{dx}(x^2+3)}{(x^2+3)^2}$$

$$= \frac{(x^2+3)[-x^4\cos ec^2 x + 4x^3\cot x] - x^4\cot x(2x)}{(x^2+3)^2}$$

$$\frac{dy}{dx} = \frac{x^3(x^2+3)[-x\cos ec^2 x + 4\cot x] - 2x^5\cot x}{(x^2+3)^2}$$

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