12. Introduction to Three dimensional geometry (Question Bank)

I .ONE MARK QUESTIONS WITH ANSWERS:

1. Name the octant in which the point $(4,-2,-5)$ lie?

Ans:$(4,-2,-5)$ lies in 8th octant $OXY'Z'$

2. Name the octant in which the point $(-2,-4,-7)$ lie?

Ans:$(-2,-4,-7)$ lies in 7th octant$OX'Y'Z'$.

3. Name the octant in which the point $(-3,1,-2)$ lie.

Ans: $(-3,1,-2)$ lies in 6th octant$OX'YZ'$.

4. Find the distance between the pair of points $(-3,7,2)$ & $(2,4,-1)$.

Ans: Required distance= $\sqrt{(-3-2)^2+(7-4)^2+(2+1)^2}$

$$=\sqrt{25+9+9}=\sqrt{43}.$$

5. Find the distance between pair of points $(2,-1,3) and\ (-2,1,3)$.

Ans: Required distance $=\sqrt{(2+2)^2+(-1-1)^2+(3-3)^2}$

$$=\sqrt{16+4}=\sqrt{20}.$$

6. Find the coordinates of the foot of the perpendicular of the point $P(2,4,5)$ in the XZ plane.

Ans: $(2,0,5)$.

7. Write the coordinate of any point in the XY plane.

Ans:$(x,y,0)$.

8. Find the equation of YZ plane.

Ans:$x=0$.

9. Find the equation of ZX plane.

Ans: $y=0$.

10. Find the reflexion (Image) of the point $(-a,b,c) in\ the\ YZ\ plane$.

Ans: (a,b,c).

11. Find the reflexion (Image) of the point $(2,3,-4) in\ the\ XY\ plane$.

Ans: $(2,3,4)$.

12. Find the coordinate of the centroid of the triangle formed by the points $(2,-5,1),(3,2,-6)\ \&\ (1,-3,-1)$

Ans: G$\equiv \left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}, \frac{z_1+z_2+z_3}{3}\right) \equiv (2,-2,-2)$.

13. Find the coordinates of midpoint of line joining the points $A \equiv (-5,-3,7)\ \&\ B \equiv (-7,3,5)$

Ans:$P \equiv \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}, \frac{z_1+z_2}{2}\right) \equiv (-6,0,6)$.

II. TWO MARKS QUESTIONS WITH ANSWERS:

1. Show that the points $P(-2,3,5) Q(1,2,3) and\ R(7,0,-1)$ are collinear.

Ans: $Q = \sqrt{(-2-1)^2+(3-2)^2+(5-3)^2} = \sqrt{9+1+4} = \sqrt{14}$.

$QR = \sqrt{(1-7)^2+(2-0)^2+(3+1)^2} = \sqrt{36+4+16}$=$\sqrt{56} = 2\sqrt{14}$

$PR = \sqrt{(-2-7)^2+(3-0)^2+(5+1)^2} = \sqrt{81+9+36} = \sqrt{126} = 3\sqrt{14}$

$\therefore PQ + QR = PR.$

$\therefore The\ points\ P, Q\ andR$ are collinear.

2. Show that the points $(3,-2,4),(1,1,1)\ and\ (-1,4,-2)$ are collinear.

Ans: Let $P \equiv (3,-2,4)\ Q \equiv (1,1,1)\ and\ R \equiv (-1,4,-2)$

$\therefore PQ = \sqrt{(3-1)^2+(-2-1)^2+(4-1)^2} = \sqrt{4+9+9} = \sqrt{22}.$

$QR = \sqrt{(1+1)^2+(1-4)^2+(1+2)^2} = \sqrt{4+9+9} = \sqrt{22}$

$PR = \sqrt{(3+1)^2+(-2-4)^2+(4+2)^2} = \sqrt{16+36+36} = 2\sqrt{22}$

$\therefore PQ + QR = PR$

$\therefore$ The points P, Q, R are collinear.

3. Find the coordinates of the point which divides the line segment joining the points $(1,-2,3)\ and\ (3,4,-5)$ internally in the ratio $2:3$.

Ans: $P \equiv \left(\frac{2(3)+3(1)}{2+3}, \frac{2(4)+3(-2)}{2+3}, \frac{2(-5)+3(3)}{2+3}\right)$

$P \equiv \left(\frac{9}{5}, \frac{2}{5}, \frac{-1}{5}\right).$

4. Find the coordinates of the point which divides the line segment joining the points $(1,-2,3)$ and $(3,4,-5)$ externally in the ratio $2:3$.

Ans: $P \equiv \left(\frac{2(3)+3(1)}{2-3}, \frac{2(4)+3(-2)}{2-3}, \frac{2(-5)+3(3)}{2-3}\right) = \left(\frac{3}{-1}, \frac{14}{-1}, \frac{-19}{-1}\right) \equiv (-3,-14,19)$.

5. Find the coordinates of the point which divides the line segment joining the points $(-2,3,5)$ and $(1,-4,6)$ internally in the ratio $2:3$.

Ans: Let $A=(-2,3,5)$ and $B=(1,-4,6)$

Let P divides AB internally in the ratio 2:3

$P \equiv \left(\frac{2(1)+3(-2)}{2+3}, \frac{2(-4)+3(3)}{2+3}, \frac{2(6)+3(5)}{2+3}\right)$, $\therefore$ $P \equiv \left(\frac{-4}{5}, \frac{1}{5}, \frac{27}{5}\right)$.

6. Find the coordinates of the point which divides the line segment joining the points $(-2,3,5)$ and $(1,-4,6)$ externally in the ratio $2:3$.

Ans: Let $A=(-2,3,5)$ and $B=(1,-4,6)$

Let P divides AB externally in the ratio 2:3

$P \equiv \left(\frac{2(1)-3(-2)}{2-3}, \frac{2(-4)-3(3)}{2-3}, \frac{2(6)-3(5)}{2-3}\right)$, $\therefore$ $P \equiv \left(\frac{8}{-1}, \frac{-17}{-1}, \frac{-3}{-1}\right) \equiv (-8,17,3)$

7. Find the ratio in which the YZ plane divides the line segment formed by joining the points $(-2,4,7) and$ $(3,-5,8)$.

Ans: Let YZ plane divide line segment joining the points $A \equiv (-2,4,7)$ and $B \equiv (3,-5,8)$ at

$P \equiv (x,y,z) in\ the\ ratio\ k:1$.

$\therefore P \equiv \left(\frac{k(3)+1(-2)}{k+1}, \frac{k(-5)+1(4)}{k+1}, \frac{k(8)+1(7)}{k+1}\right)$

$\therefore P \equiv \left(\frac{3k-2}{k+1}, \frac{-5k+4}{k+1}, \frac{8k+7}{k+1}\right)$

Given that the point 'P' lies on YZ plane$\Longrightarrow$ $x\ coordinate\ of\ P=0 \Longrightarrow \frac{3k-2}{k+1}=0 \Longrightarrow k=\frac{2}{3}$.

Therefore ratio is 2:3.

8. The centroid of a ΔABC is the point (1, 1, 1).If the coordinates of A and B are $(3,-5,7) and$ $(-1,7,-6)$

respectively. Find the coordinates of the point 'C'.

Ans: Let $C \equiv (x,y,z)$; $A \equiv (3,-5,7)$ and $B \equiv (-1,7,-6)$

G$\equiv \left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}, \frac{z_1+z_2+z_3}{3}\right)$

$$\Rightarrow (1,1,1) \equiv \left(\frac{3-1+x}{3}, \frac{-5+7+y}{3}, \frac{7-6+z}{3}\right)$$

$$\Rightarrow (1,1,1) \equiv \left(\frac{2+x}{3}, \frac{2+y}{3}, \frac{1+z}{3}\right)$$

$$\Rightarrow (x, y, z) \equiv (1,1,2)$$

$$\Rightarrow C \equiv (1,1,2).$$

9. The vertices of a triangle are (2, 4, 6) (0,-2,-5).If origin is the centroid of the triangle find the third vertex?

Ans: Let $A \equiv (2,4,6)\ and\ B \equiv (0,-2,-5)\ C \equiv (x, y, z)$ be the vertices of ΔABC

$$G \equiv \left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}, \frac{z_1+z_2+z_3}{3}\right)$$

$$\Rightarrow (0,0,0) \equiv \left(\frac{2+0+x}{3}, \frac{4-2+y}{3}, \frac{6-5+z}{3}\right)$$

$$\Rightarrow (0,0,0) \equiv \left(\frac{2+x}{3}, \frac{2+y}{3}, \frac{1+z}{3}\right)$$

$$\Rightarrow (x, y, z) \equiv (-2,-2,-1)$$

$$\Rightarrow C \equiv (-2,-2,-1).$$

10. If the distance between the points (k,-8, 4) and (3,-5, 4) is 5. Then find k.

Ans: Let A$\equiv (k,-8,4)$ and B$\equiv (3,-5,4)$

Given AB$= 5 \Rightarrow \sqrt{(k-3)^2 + (-8+5)^2 + (4-4)^2} = 5$

$$\Rightarrow \sqrt{k^2 + 9 - 6k + 9} = 5$$

$$\Rightarrow k^2 - 6k - 7 = 0$$

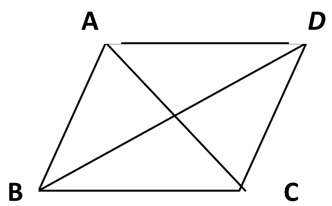
$$\Rightarrow k^2 - 7k + k - 7 = 0$$

$$\Rightarrow k(k-7) + 1(k-7) = 0$$

$$\Rightarrow k = -1\ or\ k = 7.$$

11. Three consecutive vertices of a parallelogram ABCD are $A \equiv (3,-1,2)\ B \equiv (1,2,-4)$ $and\ C \equiv (-1,1,2)$. Find the coordinates of fourth vertex D.

Ans: Let $D \equiv (x, y, z)$ be fourth vertex of parallelogram



$\Longrightarrow$ Midpoint of AC = Midpoint of BD.

$\Longrightarrow \left(\frac{3-1}{2}, \frac{-1+1}{2}, \frac{2+2}{2}\right) = \left(\frac{1+x}{2}, \frac{2+y}{2}, \frac{-4+z}{2}\right)$

$\Longrightarrow (1,0,2) = \left(\frac{1+x}{2}, \frac{2+y}{2}, \frac{-4+z}{2}\right)$

$\Longrightarrow x = 1, y = -2, and\ z = 8$

$\therefore D \equiv (1, -2, 8)$

12. If the origin is the centroid of the triangle PQR with the vertices $P \equiv (2a, 2, 6), Q \equiv (-4, 3b, -10)\ and\ R \equiv (8, 14, 2c)$ then find the values of a, b, c.

Ans: Given origin = Centroid of a triangle PQR.

$\Longrightarrow\ (0,0,0) = \left(\frac{2a-4+8}{3}, \frac{2+3b+14}{3}, \frac{6-10+2c}{3}\right)$

$\Longrightarrow\ (0,0,0) = \left(\frac{2a+4}{3}, \frac{3b+16}{3}, \frac{2c-4}{3}\right)$

$\Longrightarrow a = -2, b = -\frac{16}{3}, c = 2.$

13. Centroid of a triangle with vertices $A \equiv (a, 1, 3), B \equiv (-2, b, -5)\ and\ C \equiv (4, 7, c)$ is the origin.

Find the values of a, b, c.

Ans: Given origin = Centroid of a triangle ABC.

$\Longrightarrow\ (0,0,0) = \left(\frac{a-2+4}{3}, \frac{1+b+7}{3}, \frac{3-5+c}{3}\right)$

$\Longrightarrow a = -2, b = -8, c = 2.$

III. FIVE MARKS QUESTIONS WITH ANSWERS:

1. Show that the points $\boldsymbol{A} \equiv (\mathbf{1}, \mathbf{2}, \mathbf{3})\ \boldsymbol{B} \equiv (-\mathbf{1}, -\mathbf{2}, -\mathbf{1})\ \boldsymbol{C} \equiv (\mathbf{2}, \mathbf{3}, \mathbf{2})\ \boldsymbol{and\ D} \equiv (\mathbf{4}, \mathbf{7}, \mathbf{6})$ are the vertices of parallelogram ABCD but it is not a rectangle.

Ans: To show ABCD is a parallelogram we need to show that two opposite sides are equal.

$$\boldsymbol{AB} = \sqrt{(\mathbf{1}+\mathbf{1})^2 + (\mathbf{2}+\mathbf{2})^2 + (\mathbf{3}+\mathbf{1})^2} = \sqrt{\mathbf{4}+\mathbf{16}+\mathbf{16}} = \sqrt{\mathbf{36}} = \mathbf{6}.$$

$$\boldsymbol{BC} = \sqrt{(-\mathbf{1}-\mathbf{2})^2 + (-\mathbf{2}-\mathbf{3})^2 + (-\mathbf{1}-\mathbf{2})^2} = \sqrt{\mathbf{9}+\mathbf{25}+\mathbf{9}} = \sqrt{\mathbf{43}}$$

$$\boldsymbol{CD} = \sqrt{(\mathbf{2}-\mathbf{4})^2 + (\mathbf{3}-\mathbf{7})^2 + (\mathbf{2}-\mathbf{6})^2} = \sqrt{\mathbf{4}+\mathbf{16}+\mathbf{16}} = \sqrt{\mathbf{36}} = \mathbf{6}$$

$$\boldsymbol{DA} = \sqrt{(\mathbf{1}-\mathbf{4})^2 + (\mathbf{2}-\mathbf{7})^2 + (\mathbf{3}-\mathbf{6})^2} = \sqrt{\mathbf{9}+\mathbf{25}+\mathbf{9}} = \sqrt{\mathbf{43}}$$

$$AC = \sqrt{(1-2)^2+(2-3)^2+(3-2)^2} = \sqrt{3}$$

$$BD = \sqrt{(-1-4)^2+(-2-7)^2+(-1-6)^2} = \sqrt{25+81+49} = \sqrt{155}.$$

AC≠BD.Diagonals are not equal.∴ ABCD is not a rectangle.

2. Show that the points (-1,2,1) (1,-2,5) (4,-7,8) and (2,-3,4) are the vertices of the parallelogram.

Ans: Let $A \equiv (-1,2,1) \quad B \equiv (1,-2,5) \quad C \equiv (4,-7,8) \quad and\ D \equiv (2,-3,4)$

$$\mathbf{AB} = \sqrt{(-1-1)^2+(2+2)^2+(1-5)^2} = \sqrt{4+16+16} = \sqrt{36} = 6.$$

$$BC = \sqrt{(1-4)^2+(-2+7)^2+(5+8)^2} = \sqrt{9+25+9} = \sqrt{43}$$

$$CD = \sqrt{(4-2)^2+(-7+3)^2+(8-4)^2} = \sqrt{4+16+16} = \sqrt{36} = 6$$

$$DA = \sqrt{(-1-2)^2+(2+3)^2+(1-4)^2} = \sqrt{9+25+9} = \sqrt{43}$$

$$\therefore AB = CD\ and\ BC = DC$$

$$AC = \sqrt{(-1-4)^2+(2+7)^2+(1-8)^2} = \sqrt{25+81+49} = \sqrt{155}$$
$$BD = \sqrt{(1-2)^2+(-2+3)^2+(5-4)^2} = \sqrt{1+1+1} = \sqrt{3}.$$

AC≠BD.Diagonals are not equal.∴ ABCD is a Parallelogram.

3. Show that the points $(0,7,-10), (1,6,-6)\ and (4,9,-6)$ are the vertices of an isosceles right angled triangle.

Ans: Let $A \equiv (0,7,-10), B \equiv (1,6,-6)\ and\ C \equiv (4,9,-6)$

Now $AB = \sqrt{(0-1)^2+(7-6)^2+(-10+6)^2} = \sqrt{1+1+16} = \sqrt{18}$

$$BC = \sqrt{(1-4)^2+(6-9)^2+(-6+6)^2} = \sqrt{9+9+0} = \sqrt{18}$$

$$AC = \sqrt{(0-4)^2+(7-9)^2+(-10+6)^2} = \sqrt{16+4+16} = \sqrt{36}$$

Clearly $AB = BC\ and\ AB^2 + BC^2 = AC^2$

∴ The given points are the vertices of an isosceles right angled triangle.

4. Show that the points $(0,7,10), (-1,6,6)\ and (-4,9,6)$ are the vertices of an isosceles right angled triangle.

Ans: Let $A \equiv (0,7,10), B \equiv (-1,6,6)\ and\ C \equiv (-4,9,6)$

Now $AB = \sqrt{(0+1)^2+(7-6)^2+(10-6)^2} = \sqrt{1+1+16} = \sqrt{18}$

$$BC = \sqrt{(-1+4)^2 + (6-9)^2 + (6-6)^2} = \sqrt{9+9+0} = \sqrt{18}$$

$$AC = \sqrt{(0+4)^2 + (7-9)^2 + (10-6)^2} = \sqrt{16+4+16} = \sqrt{36}$$

Clearly $AB = BC \text{ and } AB^2 + BC^2 = AC^2$

∴ The given points are the vertices of an isosceles right angled triangle.

5. Find the lengths of the medians of the triangle with the vertices A(0,0,6) B(0,4,0) and C (6,0,0).

Ans: Let D =Midpoint of BC

$$\therefore D = \left(\frac{0+6}{2}, \frac{4+0}{2}, \frac{0+0}{2}\right) = (3, 2, 0)$$

∴Length of median $AD = \sqrt{(0-3)^2 + (0-2)^2 + (6-0)^2} = \sqrt{9+4+36} = \sqrt{49} = 7$

Let E =Midpoint of CA

$$\therefore E = \left(\frac{6+0}{2}, \frac{0+0}{2}, \frac{0+6}{2}\right) = (3, 0, 3)$$

∴Length of median $BE = \sqrt{(0-3)^2 + (4-0)^2 + (0-3)^2} = \sqrt{9+16+9} = \sqrt{34}$

Let F =Midpoint of AB

$$\therefore F = \left(\frac{0+0}{2}, \frac{0+4}{2}, \frac{6+0}{2}\right) = (0, 2, 3)$$

∴Length of median$CF = \sqrt{(6-0)^2 + (0-2)^2 + (0-3)^2} = \sqrt{36+4+9} = \sqrt{49} = 7.$

6. Show that the points $(0, 4, 1), (2, 3, -1), (4, 5, 0) \text{ and } (2, 6, 2)$ are the vertices of the square.

Ans. Let $A \equiv (0, 4, 1), B \equiv (2, 3, -1), C \equiv (4, 5, 0) \text{ and } D \equiv (2, 6, 2)$

Now $AB = \sqrt{(0-2)^2 + (4-3)^2 + (1+1)^2} = \sqrt{4+1+4} = \sqrt{9} = 3$

$$BC = \sqrt{(2-4)^2 + (3-5)^2 + (-1-0)^2} = \sqrt{4+4+1} = \sqrt{9} = 3$$

$$CD = \sqrt{(4-2)^2 + (5-6)^2 + (0-2)^2} = \sqrt{4+1+4} = \sqrt{9} = 3$$

$$DA = \sqrt{(2-0)^2 + (6-4)^2 + (2-1)^2} = \sqrt{4+4+1} = \sqrt{9} = 3$$

$$\therefore AB = BC = CD = DA = 3$$

∴ All the sides are equal.

$$Also\ \ AC = \sqrt{(0-4)^2 + (4-5)^2 + (1-0)^2} = \sqrt{16+1+1} = \sqrt{18}$$

$$BD = \sqrt{(2-2)^2 + (3-6)^2 + (-1-2)^2} = \sqrt{0+9+9} = \sqrt{18}$$

$\therefore \boldsymbol{AC = BD}$ $\therefore$ The diagonals are equal. $\therefore$ $\boldsymbol{ABCD}$ is a square.

7. Show that the points $\boldsymbol{A \equiv (1,3,4), B \equiv (-1,6,10), C \equiv (-7,4,7)}$ $\boldsymbol{and\ D \equiv (-5,1,1)}$ are the vertices of rhombus.

Ans: $\boldsymbol{AB = \sqrt{(1+1)^2 + (3-6)^2 + (4-10)^2} = \sqrt{4+9+36} = \sqrt{49} = 7}$

$$\boldsymbol{BC = \sqrt{(-1+7)^2 + (6-4)^2 + (10-7)^2} = \sqrt{36+4+9} = \sqrt{49} = 7}$$

$$\boldsymbol{CD = \sqrt{(-7+5)^2 + (4-1)^2 + (7-1)^2} = \sqrt{4+9+36} = \sqrt{49} = 7}$$

$$\boldsymbol{DA = \sqrt{(-5-1)^2 + (1-3)^2 + (1-4)^2} = \sqrt{36+4+9} = \sqrt{49} = 7}$$

$\therefore \boldsymbol{AB = BC = CD = DA = 7}$

$\therefore$ All the sides are equal.

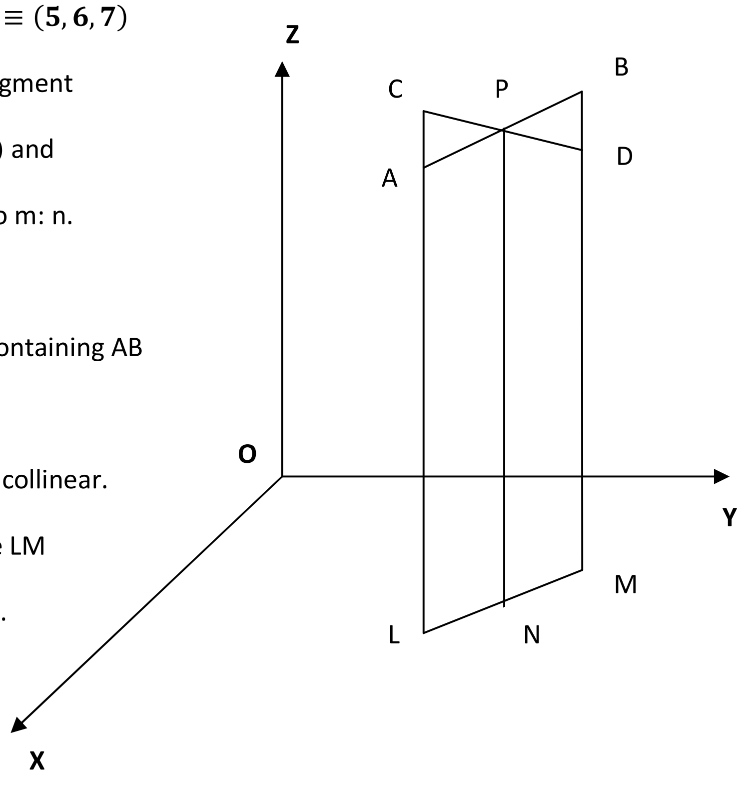
$$\boldsymbol{Also\ \ AC = \sqrt{(1+7)^2 + (3-4)^2 + (4-7)^2} = \sqrt{64+1+9} = \sqrt{74}}$$

$$\boldsymbol{BD = \sqrt{(-1+5)^2 + (6-1)^2 + (10-1)^2} = \sqrt{16+25+81} = \sqrt{122}}$$

$\therefore \boldsymbol{AC \neq BD}$ $\therefore$ The diagonals are not equal. $\therefore$ The given points form a rhombus.

8. Derive an expression for the coordinates of a point that divides the line joining the points.

$\boldsymbol{A \equiv (x_1, y_1, z_1)\ and\ B \equiv (x_2, y_2, z_2)}$ Internally in the ratio m: n Hence find the coordinates of the midpoint of AB.where $\boldsymbol{A \equiv (1,2,3)\ and\ B \equiv (5,6,7)}$



Ans: Let the $\boldsymbol{P \equiv (x,y,z)}$ devide the line segment

Joining the two points $\boldsymbol{A \equiv (x_1, y_1, z_1)}$ and

$\boldsymbol{B \equiv (x_2, y_2, z_2)}$ Internally in the ratio m: n.

Draw AL, BM and PN ⊥ to XY plane.

Clearly AL, BM and PN lie on the plane containing AB

But ⊥ to XY plane.

The feet of perpendiculars L, M and N are collinear.

Through P draw a line parallel to the line LM

Meets AL produced and BM at C and D.

Clearly Δ PAC and ΔPDB are equiangular

And hence similar.

$\therefore \frac{PA}{PB} = \frac{AC}{BD}$ ——————— (1)

Now $AC = CL - AL = PN - AL = z - z_1$

$BD = BM - DM = BM - PN = z_2 - z$

Now (1) becomes

$\frac{m}{n} = \frac{z - z_1}{z_2 - z}$

$\Longrightarrow n(z - z_1) = m(z_2 - z)$

$\Longrightarrow nz - nz_1 = mz_2 - mz$

$\Longrightarrow (m + n)z = mz_2 + nz_1$

$\Longrightarrow z = \frac{mz_2 + nz_1}{m+n}.$

Similarly we can prove that

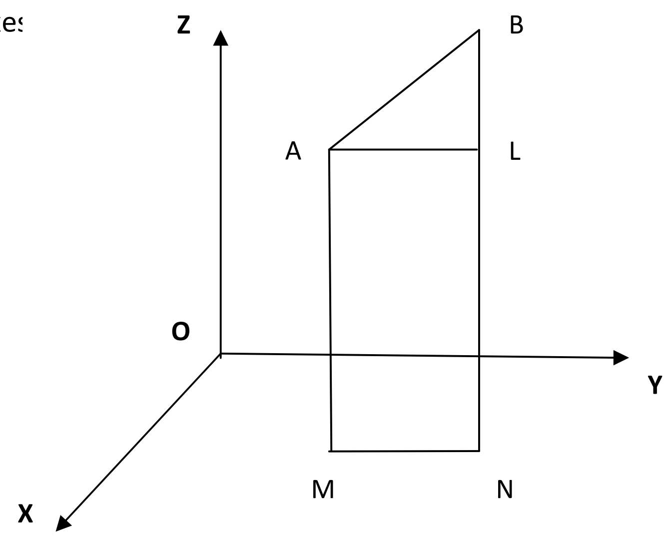
$x = \frac{mx_2 + nx_1}{m+n}, \; y = \frac{my_2 + ny_1}{m+n}$

$\therefore P \equiv \left(\frac{mx_2+nx_1}{m+n}, \frac{my_2+ny_1}{m+n}, \frac{mz_2+nz_1}{m+n}\right).$

Midpoint of $AB \equiv \left(\frac{1+5}{2}, \frac{2+6}{2}, \frac{3+7}{2}\right)$

9. Find the distance between two points in a three dimensional plane and hence find the distance between the points$P \equiv (-2,3,5)$ and $Q \equiv (1,2,3)$.

Ans: Let'O'be the origin. OX, OY, OZ be the axes



Let $A \equiv (x_1, y_1, z_1)$ and $B \equiv (x_2, y_2, z_2)$

Be any two points in the 3D-plane.

Draw AM and BN ⊥ to XY-plane and

AL⊥ to BN.

Clearly $MN = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Now $AL = MN = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Also$BL = BN - LN = BN - AM = z_2 - z_1$.

From the right angled ΔALB we have,

$$AB^2 = AL^2 + BL^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2$$

$$\therefore AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

Also,

Distance $PQ = \sqrt{(-2-1)^2 + (3-2)^2 + (5-3)^2}$

$= \sqrt{9+1+4} = \sqrt{14}$.

10. The midpoints of the sides of a triangle are $(5,7,11), (0,8,5)\ and (2,3,-1)$ Find its vertices.

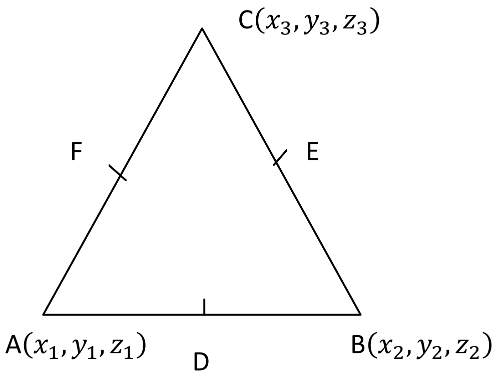
Ans: Let $D = Midpoint\ of\ AB = (5,7,11)$

$E = Midpoint\ of\ BC = (0,8,5)$

$F = Midpoint\ of\ AC = (2,3,-1)$

Where $A \equiv (x_1, y_1, z_1), B \equiv (x_2, y_2, z_2)\ and\ C \equiv (x_3, y_3, z_3)$

Are the vertices of ΔABC.



Now $(5,7,11) = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}, \frac{z_1+z_2}{2}\right)$

$\Rightarrow x_1 + x_2 = 10$ ———— (1)

$y_1 + y_2 = 14$ ———— (2)

$z_1 + z_2 = 22$ ———— (3)

$(2,3,-1) = \left(\frac{x_2+x_3}{2}, \frac{y_2+y_3}{2}, \frac{z_2+z_3}{2}\right)$

$\Rightarrow x_2 + x_3 = 4$ ———— (4)

$y_2 + y_3 = 6$ ———— (5)

$z_2 + z_3 = -2$ ———— (6)

Also,$(0,8,5) = \left(\frac{x_1+x_3}{2}, \frac{y_1+y_3}{2}, \frac{z_1+z_3}{2}\right)$

$\Rightarrow x_1 + x_3 = 0$ ———— (7)

$y_1 + y_3 = 16$ ———————— (8)

$z_1 + z_3 = 10$ ———————— (9)

Adding $(1) + (4) + (7)$

$\Longrightarrow\ 2x_1 + 2x_2 + 2x_3 = 14$

$\Longrightarrow x_1 + x_2 + x_3 = 7$

$\Longrightarrow 10 + x_3 = 7 \Longrightarrow \quad x_3 = -3$

From (7)

$x_1 + x_3 = 0 \Longrightarrow x_1 = 3$

From (1) $x_1 + x_2 = 10 \Longrightarrow x_2 = 7$

$(2) + (5) + (8)$ gives,

$\Longrightarrow\ 2y_1 + 2y_2 + 2y_3 = 36$

$\Longrightarrow y_1 + y_2 + y_3 = 18$

$\Longrightarrow 14 + y_3 = 18 \Longrightarrow \quad y_3 = 4$

$From\ (8),\ y_1 + y_3 = 16 \Longrightarrow y_1 = 12$

From (2), $y_1 + y_2 = 14 \Longrightarrow y_2 = 2$

$(3) + (6) + (9)$ gives,

$\Longrightarrow\ 2z_1 + 2z_2 + 2z_3 = 30$

$\Longrightarrow z_1 + z_2 + z_3 = 15$

$\Longrightarrow 10 + z_3 = 15 \Longrightarrow \quad z_3 = 5$

$From\ (9), \quad z_1 + z_3 = 10 \Longrightarrow z_1 = 5$

From (3), $z_1 + z_2 = 22 \Longrightarrow z_2 = 17$

$A \equiv (x_1, y_1, z_1) \equiv (3,12,5)$

$B \equiv (x_2, y_2, z_2) \equiv (7,2,17)$

$C \equiv (x_3, y_3, z_3) \equiv (-3,4,5)$ are the required points.