<u>Chapter-7 : PERMUTATIONS AND COMBINATIONS</u>

1. Find the number of 4 letter words, with or without meaning which can be farmed out of the letters of the word ROSE. Where the repetitions of the letters is not allowed?

Let us keep one letter in one box as shown.

This can be done in 4! Ways =4x3x2x1=24   4ways 3ways 2ways 1 way

2.Find the number of the 4 letter words with or without meaning which can be formed out of the letters of the word ROSE where the repetitions of the letters are allowed.

First box can be filled in 4ways.

Second box can be filled in 4 ways.

3rd box can be filled in 4 ways.

4th box can be filled in 4 ways.

Total number of ways=4x4x4x4 =256 ways.

3.Given four flags of different colours, how many different signals can be generated if a signal requires the use of two flags one below the other?

The number of ways of filling 2flags one below the other using 4flags of different colours , here first box can be filled by any one of the 4flags at a time(4times) and the second box can be filled in 3 ways. Therefore total number of ways = 12ways $\{4x3 = 12ways.$

4ways
3ways

4.How many 2 digit even numbers can be formed from the digits . 1,2,3,4,5. If the digits can be repeated .

Since the required even numbers contain two digits. We keep two digits in two separate boxes.

As the required number is even the unit plan can be filled by 2ways and the tenth place can be filled by 5ways. Therefore total number of ways =2x5=10.

5. Find the number of the different signals that can be generated by arranging at least 2 flags in order (one below the other) on a vertical staff, If five different flags are available

. (a) Number of possible signals with 2flags=5x4=20.

(b) Number of possible signals with 3flags =5x4x3=60

© Number of possible signals with 4flags is given by $5p_4$ ways =5x4x3x2=120ways.

(d) Number of possible signals with 4flags is given by$5p_5$ways=5x4x3x2=120ways.

∴ The required number of signals=20+60+120+120=320 ways.

6.How many 3 digit numbers can be formed from the digits 1,2,3,4and 5 assuring that

Repetition of the digits is allowed? Repetition of the digits is not allowed?

∴ Number of 3 digit numbers out of 5 digits with repetitions =5x5x5=125 ways.

∴ Number of 3 digits numbers out of 5 digits without repetitions =5x4x3=60 ways.

7 . How many 4 letter code can be formed using the first 10 letters of the English alphabet. If no letter can be repeated.

The number of 4 letter code out of 10 letters of the English alphabet =$10p_4$ =10x9x8x7=720x7=5040 ways.

8. How many 5 digits telephone numbers can be constructed using the digits 0 to 9. If each number starts with 67 and no digit appears. More than once?

Total number of 5 digit telephone numbers starting with 67 is given by 8x7x6 =56x6 = 186 ways.

9. Evaluate: 8 ! =8x7x6x5x4x3x2x1 = 40320

10. 4! - 3! = 4x3x2x1 – 3x2x1 = 6x3 = 18.

11. Is 3! + 4 ! = 7! .

No, L.H.S = 3! +4! = 3! (1+4) = 30

R.H. S = 7! = 7x6x5x4x3x2= 504. **Therefore 3!+4!≠7!**

12. How many 3 digit numbers can be formed by using the digits 1to 9 if no digit is repeated?

Required 3 digit numbers can be formed by arranging all the given 9 different digits taking 1 at a time. This can be done in 9 p_3 ways.

$\therefore$ Required 3 digit numbers = $9p_3 = \frac{9!}{(9-3)!} = \frac{9x8x7x6!}{6!}$ =9x8x7=504

13. How many 4 digits numbers are there with no digit repeated.

The thousandth place can be filled by 9 digits (except 0) and the Hundredth, tenth, units place can be filled by 9p3 ways.

$\therefore$ Required 4 digit numbers = 9x9p3 =9x504= 4536.

14. **How many3 digit even numbers can be made using the digits 1, 2, 3,4,6,7. If no digit is repeated.**

Here units place can be filled by any one number from the digit. 2,4 or 6. This can be done in 3 ways. Since the repetition of digits is not allowed therefore remaining 2 places can be filled by arranging 5 different digits. This can be done in $5p_2$ ways.

$\therefore$ Required 3 digits even numbers =3x $5p_2$ =3x5x4 = 60 ways.

15 . From a committee of 8 persons, in how many ways can we choose a chairman and a vice – chairman assuming one person cannot hold more than one position?

Since one person cannot hold more than one position. $\therefore$ we just arrange 8 persons at 2 different position this can be done in 8p2 ways.

Required number of ways = 8p2= 8x7 = 56 ways.

16. Find n if (n-1)p3 :np4 =1:9

$\frac{np4}{(n-1)p3} = \frac{9}{1} \Rightarrow \frac{\frac{n!}{(n-4)!}}{\frac{(n-1)!}{(n-1-3)!}} = \frac{n!}{(n-4)!}\frac{(n-4)!}{(n-1)\,!} = 9$ $\therefore$ on simplification we get n=9

17. How many words with or without meaning can be formed using all the letters of the word EQUATION using each letter exactly once?

Since the repetition of letters is not allowed therefore given problem is just equivalent to arranging all the 8 letters of word EQUATION taken all at a time. This can be done in $8p_8$ or 8! ways

Required number of words $=8p_8$ = 8x7x6x5x4x3x2x1 = 20320.

18. How many words with or without meaning can be made from the letters of the word MONDAY, assuming that no letter is repeated . If,

(1) 4 letters are used at a time.

(2) all letters are used at a time

(3) all letters are used but first letter is vowel?

(1) Since the repetition of digits is not allowed, therefore number of words formed by using 4 letters at a time is just equivalent to arranging the 6 different letters taken 4 at a time this can be done in $6p_4$ ways required number of words = $6p_4$ = 6! = 6x5x4x3x2 = 360.

(2) When all letters are used at a time: required number of words = $6p_6$ = 6! = 6x5x4x3x2x =720

(3) Given that first letter is vowel. It means first place in each word is fixed with A or O

∴ First letter of each word can be selected in 2ways (either A or O).

The remaining 5places can be filled up by the remaining 5 letters (M.N.D.Y and one vowel which is not used) this can be done in $5p_5$ ways.

∴ required number of words = $2x5p_5$ =2x5! =2x5x4x3x2 =240.

19. In many of the distinct permutations of the letters in MISSISSIPPI do the Four Is not came together?

The word MISSISSIPPI Contains 11 letters, out of which I Occurs 4 times, S occurs 4 times. And P occurs 2 times.

If no restriction is given then number of words formed by taking all the letters = $\frac{11!}{4!4!2!}$

$4\frac{11x10x9x8x7x6x5x4!}{4x3x2x1x2x1 \quad 4!}$ =34650.

Now we will subtract those words in which 4 I' s occur together.

Let us consider 4I's as a Single letter say X Now we have 8 letters (M,S,S, S,S,P, P,X) number of words formed by taking those.

8letters $=\frac{8!}{4!2!} = \frac{8x7x6x5x4!}{4!x2x1}$ = 8x7x5x3 = 840

∴ Required number of words = 34650 -840 = 33810.

20. In how many ways can the letters of the word PERMUTATIONS be arranged if the

(i)Words start with P and end with S

(ii)Vowels are together.

(iii)There are always 4 letters between P and S the word PERMUTATIONS contains 12 letters out of which T occurs 2times.

(i)Since each word start with P and with S therefore first and last place of each word is reserved for letters P and S respectively.

The remaining 10 places can be filled up by remaining 10 letters. This can be done in $10p_{10}$ or 10! ways.

But the letter T occurs twice

Required number of words formed $=\frac{10!}{2!}$

(ii)Vowels are together.

$$\text{PERMUTATIONS}\begin{cases} vowels(A,E,I,O,U) \\ consonants(P,R,M,T,T,N,S) \end{cases}$$

Let us consider all the vowels as a single letters say X, now we have 8 letters (P,R,M,T,T,S,X).These 8 letters can be shuffled in $\frac{8!}{2!}$ways.

But 5 vowels can interchange their positions in 5! Ways

Required number of words formed $=\frac{8!}{2!}$ x5!=8x7x6x5x4x3 x120=2419200.

(iii) exactly 4 letters between P& S can be placed

Position of P	1	2	3	4	5	6	7	

Position of S	6	7	8	9	10	11	12	

∴ there are 7 ways in which P and S can be placed. But P and S can interchange their position in 2ways .

Number of ways in P and S can be placed such that there are exactly 4letters between them =7x2=14.

Now the remaining 10 letters in $\frac{10!}{2!}$ways(∵the letter T is repeating twice)

∴total number of ways =14x$\frac{10!}{2!}$=25401600.

21.If nc_8=nc_2 find nc_2

We know that if nc_a=nc_b then either a=b or a+b=n, Here nc_8=nc_2⇒8+2=n ∴n=10

22.Determine n if (i) $2nc_3$:nc_2=12:1

$\Rightarrow\frac{\frac{2n(2n-1)(2n-2)}{3x2x1}}{\frac{n(n-1)}{2x1}}=12 \qquad \Rightarrow\frac{(2n)2(n-1)(2n-1)}{3(n)(n-1)}=12 \qquad \Rightarrow 4(2n-1)=36$

∴2n-1=9∴n=5

23. (ii)$2nc_3$:nc_3 =11:1

$\frac{\frac{(2n)(2n-1)(2n-2)}{1x2x3}}{\frac{(n)(n-1)(n-2)}{1x2x3}}=\frac{11}{1}$ ⇒ (2n)(2n-1)(2n-2)=11(n)(n-1)(n-2)[by crossmultiplication]

4(2n-1)=11(n-2)⇒8n-4=11n-22⇒ 3n=18 ∴ n=6

24. How many chords can be drawn through 21 points on a circle?

Required number of chords is equal to the number of straight lines obtained from 21 points by taking 2 points at a time. This can be done in $21c_2$ ways

Required number of chords =$21c_2$=$\frac{21x20}{1x2}$ =210

25. In how many ways can a team of 3boys and 3girls be selected from 5 boys and 4girls ?

A team of 3boys and 3 girls, number of ways of selecting 3boys from 5boys =$5c_3$

number of ways of selecting 3boys from 4girls = $4c_3$

Total number of ways of selecting the team=$5c_3 x 4c_3 = 5c_2 x 4c_1$ [$\because 5c_3=5c_2$ &$4c_3=4c_1$] $\Rightarrow \frac{5x4}{1x2}$ x 4 =40ways.

26.Find the number of ways of selecting 9balls from 6 red balls,5white balls,and 5 blue balls if each selection consists of 3balls of each colour .

Number of ways of selecting 3 red balls from 6 red balls=$6c_3$

Number of ways of selecting 3 white balls from 5 white balls=$5c_3$

Number of ways of selecting 3 blue balls from 5blue balls=$5c_3$

Required number of selections=$6c_3$ x $5c_3$ x $5c_3$ =$\frac{6x5x4}{3x2x1}$ x $\frac{5x4x3}{3x2x1}$ x$\frac{5x4x3}{3x2x1}$ =20x10x10=2000

27. Determine the number of 5card combinations out of a deck of 52 cards if there is exactly one ace in each combination.

Given that each 5 card combination should have exactly 1 ace card and 4 non ace cards , we know that a pack of 52 cards contains 4 ace cards and 48 non ace cards.

One ace card from 4ace cards can be selected in $4c_1$ ways , also 4 non ace cards from48 non ace cards can be selected in 48c4ways .

Required number of 5 card combination=$4c_1$ x$48c_4$ = 4x $\frac{48x47x46x45}{1x2x3x4}$ =778320

28. In how many ways can one select a cricket team of eleven from 17 players in which only 5 players can bowl if each cricket team of 11 must include exactly 4 bowlers?

Solution: 17 players $\begin{cases} 5 bowlers \\ 12 non\ bowlers(others) \end{cases}$

We need a team of 11 players containing 4 bowlers and 7 others .

4 bowlers can be selected in $5c_4$ ways and 7 others can be selected in $12c_7$ ways , Total number of ways to select cricket team=$5c_4$ x$12c_7$

=5c1 x$12c_5$[$nc_r=nc_{n-r}$]

= 5x$\frac{12x11x10x9x8}{5x4x3x2x1}$ =3960

29. A bag contains 5 black balls and 6 red balls, determine the of ways in which 2 black and 3red balls can be selected ?

Number of ways of selecting 2 black balls from 5 black balls =$5c_2$

Number of ways of selecting 3 red balls from 6 red balls $=6c_3$

Total number of ways $=5c_2 \times 6c_3 = \frac{5x4}{2x1} \frac{x \quad 6x5x4}{3x2x1}$ =200

30.In how many ways can a student choose a program of 5 courses are available and 2 specific courses are compulsory for every student?

since 2 specific courses are compulsory, therefore a student will select 3 more courses from the remaining 7 courses . This can be done in $7c_3$ ways,Total number of ways = $7c_3 = \frac{7x6x5}{3x2x1}$ =35.

31. How many words with or without meaning ,each of 2vowels and 3 consonants can be formed from the letters of the word DAUGHTER?

DAUGHTER$\begin{cases} A, U, E \\ D, G, H, T, R \end{cases}$

It is the mixed problem of permutation and combination

2 vowels out of 3vowels can be selected in $3c_2$ ways

3 consonants out of 5 consonants can be selected in $5c_3$ ways

Total number of ways to select 5 letters=$3c_2$ x$5c_3$ =$3c_1$ x$5c_3$ =3x $\frac{5x4x3}{3x2x1}$ =30ways

Now these selected 5 letters can be arranged in 5! Ways ,

Therefore total number of words formed = $3c_1 x 5c_3 x 5!$=30 x5x4x3x2x1=3600ways

32. How many words with or without meaning can be formed using all the letters of the word EQUATION at a time so that vowels and consonants occur together?

EQUATION$\begin{cases} A, E, I, O, U \\ N, Q, T, \end{cases}$

Let us consider all the vowels as single letter say X, and all the consonants as other single letter Y , now these two letters Xand Y can shuffle in 2! Ways. but 5 vowels can interchange their positions in 5! Ways and 3 consonants can interchange their positions in 3! Ways, ∴total number of words formed =2!x3!x5! =2x6x120=1440

33.A committee of seven has to be formed from 9 boys and 4 girls , In how many ways can this be done when the committee consists of

i) exactly 3 girls ii) at least 3 girls iii) at most 3 girls

i) 3 girls from 4 girls can be selected in $4c_3$ ways.

Since each committee contains 7 persons ∴ Remaining 4 boys from 9boys can be selected in $9c_4$ ways. By F.P.C total number of ways $=4c_3 \times 9c_4=4c_1 \times 9c_4$ [∵ $nc_r = nc_{n-r}$] $=4\times\frac{9x8x7x6}{1x2x3x4} = 504$

(ii)Here two cases arise ,CaseI:When each committee consists of 3 girls

Total number of ways $=4c_3 \times 9c_4=4c_1 \times 9c_4=4\times\frac{9x8x7x6}{1x2x3x4} = 504$

Case II When each committee consists of 4 girls :

All 4girls from 4girls can be selected in $4c_4$ ways and 3boys from 9boys can be selected in $9c_3$ ways . Total number of ways = $4c_4 \times 9c_3= 1\times\frac{9x8x7}{3x2x1} =84$

Therefore total number of ways =504+84=588

(iii)Here 4 cases arise :

cases	9 boys	4girls
I	4	3
II	5	2
III	6	1
IV	7	0

Case I: When each committee consists of 3 girls:

3 girls from 4 girls can be selected in $4c_3$ ways

4 boys from 9 boys can be selected in $9c_4$ ways

Case II: When each committee consists of 2 girls

2 girls from 4 girls can be selected in $4c_2$ ways

5 boys from 9 boys can be selected in $9c_5$ ways

Case III: When each committee consists of 1 girl

1 girl from girls can be selected in $4c_1$ ways

6 boys from 9 boys can be selected in $9c_6$ ways

Case IV: When each committee consists of no girl

In this case all the seven members are boys .Also, 7 boys from 9 boys can be selected

In $9c_7$ ∴ Total number of ways $=4c_3X9c_4+4c_2X9c_5+4c_1X9c_6+4c_0X9c_7$

$=4c_1X9c_4+4c_2X9c_4+4X9c_3+1X9c_2$ [$\because nc_r=nc_{n-r}$]

$= 4X\frac{9x8x7x6}{4x3x2x1} \quad +\frac{4x3}{1x2}X\frac{9x8x7x6}{1x2x3x4}$

34.**If the different permutations of all the letters of the word EXAMINATION are listed as the dictionary, how many words are there in this list before the first word starting with E?**

The alphabetical order of all the letters of the word EXAMINATION are as follows ; A,E,I,M,N,O,X since the letter A comes before the letter E, therefore fix A at the first place. The remaining '10' letters{E,X,A,M,I,N,T,I,O, N} can be arranged in $\frac{10!}{2.!2!}$ ways. (it may be noted that the letters I and N are repeating twice).

$\therefore$ The number of words formed with first letter A $=\frac{10!}{2!2!}$=907200

Hence the number of words formed before the first word starting with E=907200

35. How many six digit numbers can be formed from the digits 0,1,3,5,7,9 which are divisible by 10 and no digit is repeated?

Since each number is divisible by 10, therefore a number must have 0 at the units place. Also repetition is not allowed $\therefore$ remaining 5 places can be filled by digits 1,3,5,7,9 in $5p_5$ or 5! Ways $\therefore$ total number of 6 digit numbers formed $=5p_5=5!=120$.

36. Compute: $\frac{8!}{6!2!}=\frac{8x7x6x5x4x3x2x1}{6x5x4x3x2x1}$ **=28**

37. If $\frac{1}{6!}+\frac{1}{7!}=\frac{x}{8!}$ **Find x**

$\frac{7!+6!}{6!7!}=\frac{x}{8!} \rightarrow \frac{6!(7+1)}{6!7!}=\frac{x}{8!}$ $\Rightarrow$x=64(on simplification)

38. Evaluate : $\frac{n!}{(n-r)!}$ **When n=6 and r=2**

$\frac{6!}{(6-2)!}=\frac{6x5x4!}{4!}$ **=30**

39. Evaluate : $\frac{n!}{(n-r)!}$ **When n=9 and r=5**

$\frac{9!}{(9-5)!}=\frac{9x8x7x6x5x4!}{4!}$ **=72x42x5=72x210=15120.**

40. It is required to seat 5men and 4 women in a row so that the women occupy the even places how many such arrangements are possible?

Women can occupy even places (that is 2^{nd},4^{th},$6^{th}$and 8^{th}). Man can occupy odd places (1^{st}, 3rd, 5th, and 9th). Now 4women can be seated at even places in 4! Ways. 5 men can be seated at odd places in 5! Ways By fundamental principle of counting, total number of arrangements=4!x5!=24x120=2880.