MATHEMATICS

CHAPTER: 2

RELATIONS AND FUNCTIONS

One mark questions

1. **Define ordered pair.**
   If P and Q are two sets then pair of elements of P and Q written in small brackets and grouped together in a particular order is defined as an ordered pair.

2. **Find x , if (x + 1, 3) ≡ (2, 3)**
   $(x+1, 3) \equiv (2, 3) \Rightarrow x + 1 = 2$
   $\therefore \quad x = 1$

3. **Define Cartesian product of two sets.**
   Given two non-empty sets P and Q, the cartesian product PxQ is the set of all ordered pair of elements from P and Q.
   i.e.$P \times Q = \{(p, q): p \in P, q \in Q\}$

4. **If A = {a, b} and B = {1}, find A x B.**
   $A \times B = \{ (a, 1), (b, 1) \}$

5. **If n (A) = p and n (B) = q, then find n (A x B).**
   Given $n(A) = p$ and $n(B) = q$
   $\therefore n(A \times B) = pq$

6. **If A = {1, 2, 3} and B is an empty set , then find A x B.**
   $A \times B = \{ \}$

7. **Define on ordered triplet.**
   If P, Q and R are any three sets then the elements of P, Q and R written in small brackets and grouped together in a particular order is defined as an ordered triplet.

8. **If n (A) = 3 and A x A contains (a, b) and (b, c) , write A.**
   $A = \{a, b, c\}$

9. **Define a relation R from the set A to the set B.**
   A relation R from a non-empty set A to a non – empty set B is a subset of the cartesian product A xB.

10. If R = {(2, 1) , (3, 1), (4, 2)}. Write the domain of the relation R.

Domain of R = {2, 3, 4}

11. If R = {(2,4) , (3, 6)}. Write the image of the relation R.

Image of R = {4, 6}

12. If n(A) = 3 and n(B) = 4 then find the total number of relationsfrom A to B.

n(A xB) = 12

13. If n(A x B) = 4 and n(A) = 2 find n(B).

Given n(AxB) = 4 and n(A) =2

$\therefore$ n(B) = 2

14. If n(AxA) =64 find n(A).

Given n(AxA) = 64 = 4^4

$\therefore$ n(A) = 4

15. If A={1, 2, 3}, B ={4, 5, 6} and f = { (1, 4), (2, 2), (3, 5)} is f a relation A to B.

Since (2, 2) $\notin$ AxB, f is not a relation.

16. If A={a, b, c} and B={0, 1} find the number of relations from A to B.

Given n(A) = 3, n(B) = 2.

$\therefore$ n(AxB) = 6

17. State whether R= {(1, 2), (1, 3),(2, 4)} is a relation or not on A={1, 2, 3}

Since 4 $\notin$ A , (2, 4) $\in$ R $\therefore$ *R* is not a relation

18. Write the smallest relation on N.

The smallest relation is $\emptyset$ (empty relation)

19. Write the greatest relation on {1, 2}

Greatest relation = { (1, 1) , (1, 2) , (2, 1) , (2, 2) }

20. Define a function.

A relation is said to be a function if every member of domain is related to unique element of co-domain.

21. Define a real valued function.

A function f:X $\rightarrow$Y is said to be valued function if Y is a subset of a set of reals.

22. **If f:R→R is defined by f(x) = x^2+2x-3, find the image of 1 under f.**
    f(1) = 1^2 +2 -3 =0

23. **Define range of a function.**
    The set of all images of elements of domain of a function is said to be range of the function.

24. **If f:A→A defined by {(1, a),(2, 2),(3, 3)} is identity function, find a.**
    As f is identity function ∴ a = 1.

25. **Find the domain of f(x) = x+2.**
    Domain is set of all reals.

26. **Find the domain of f(x) = $\frac{1}{x-1}$**
    Domain is the set of all reals except 1.

Two marks questions:

1. **If (3x + y, 5) = (4, x+1) find x and y.**
   3x +y = 4 ---(1)   x + 1 = 5---(2)
   x =4
   Put x=4 in (1),
   y = -8

2. **If A= {2, 3, 4} , B= { -1, 2, 5} and C={3, 5} find Ax(B –C).**
   B –C ={-1, 2}
   AxB-C = {(2, -1), (2,2), (3, -1), (3, 2), (4, -1), (4, 2)}

3. **If AxB={(1, a), (2, b), (1, b), (2, a), (3, a), (3, b)}. Write A and B.**
   A={1, 2, 3}   B={a, b}

4. **If A={x:x=3n^2+1, n=0, 1, 2} and B={-1, 0, 1} find BxA.**
   A={1, 4, 13}
   B={-1, 0, 1}
   BxA={(-1, 1), (-1, 4), (-1, 13), (0,1), (0,4), (0, 13), (1, 1), (1, 4), (1, 13)}

5. **Let X={a, b, c}, Y={b, c, d} and Z={b, d, e} find (X ∩ Y) xZ.**
   X∩Y ={b, c}
   Z={b, d, e}
   (X∩Y) xZ={(b, b), (b, d), (b, e), (c, b), (c, d), (c, e)}

6. **If A={x, y},B={a, z} show that AxB and BxA are disjoint sets.**
   AxB={(x, a), (x, z), (y, a), (y, z)}
   BxA={(a, x), (a, y), (z ,x), (z, y)}
   ∴ (AxB) ∩ (BxA) = 4 ∴ AxB and BxA are disjoint sets.

7. **If n(AxA)=9 and if (-2, 1) and (2, 1) are two elements of AxA, find A.**
   (AxA) =9 ∴ n(A) =3
   ∴ A={-2, 2, 1}

8. **Write the domain and range of the relation R={ (2x, 3x-1): x∈ (1, 2, 3) }**
   R= {(2, 2), (4, 5), (6, 8)}
   Domain = {2, 4, 6}
   Range = {2, 5, 8}

9. **Let A={1, 2, 3, 4, 5}. Let R be a relation on A defined by R={(a-1,a+1): a, b ∈ A}. Find the range of R.**
   R={(1, 3), (2, 4), (3, 5)}
   Range = {3, 4 , 5}

10. **Let A={4, 5, 6, 7, 8, 9}. Let R be a relation onA defined by R={(x, x+2): x∈ A}. find the domain of R.**
    R={(4, 6), (5, 7), (6, 8), (7, 9)}
    Domain = {4, 5, 6, 7}

11. **Write the domain and range of the relation R= {(2x-1, 3-x)/ x≤ 3, x ∈ N}**
    R={(1, 2), (3, 1)}
    Domain ={1, 3}
    Range= {2, 1}

12. **If AxB ={(1, a) (2, a) (3, a) (1, b) (2, b) (3, b)}, then write the sets A and B.**
    A={1, 2, 3}
    B={a, b}

13. **State whether the relation R={(1, 2), (2, 3), (1, 3)} is a function or not on A={1, 2, 3}. Give reason.**
    R is not a function because the element has two images.

14. **Let f:R→R be defined as f(x)=$\begin{cases} 3-2x, & x \le 1 \\ 5-3x, & x \ge 1 \end{cases}$. verify whether f is a function or not.**

f={(1, 1), (1, 2),, (2, -1), (3, -4)……} f is not a function. since element has two Images

15. If f is a real valued function defined by f(x) = $\frac{x+1}{x-1}$, show that f(f(x)) =x.

f(x) $=\frac{x+1}{x-1}$

f(f(x))= $\frac{\frac{x+1}{x-1}+1}{\frac{x+1}{x-1}-1} = \frac{x+1+x-1}{x+1-x+1} = \frac{2x}{2} =$ X

16. If f: A → R is defined by f(x) = 2x^2 -3 and if A={0, 1, 2 } find the range of f.

f={(0, -3), (1, -1), (2, 5)}
Range of f={-3, -1, 5}

17. If f(x) =x^2 , evaluate $\frac{f(x+2)-f(2)}{2}$

$\frac{f(x+2)-f(2)}{2} = \frac{x^2+2x+4-4}{2} = \frac{x(x+2)}{2}$

18. Draw the graph of identity function.

19. Draw the graph of the constant function y=2.

20. Draw the graph of the modulus function f(x) = | x |

21. Draw the graph of f(x) = $\begin{cases} \mathbf{1}, & \boldsymbol{if\ x} > 0 \\ \mathbf{0}, & \boldsymbol{if\ x = 0} \\ \mathbf{-1}, & \boldsymbol{if\ x} < 0 \end{cases}$

22. If [.] indicates greatest integer function, evaluate f(x) =[x^2 + 1] + [x-1],when x= -2.

When x= -2

$[x^2 + 1] = [5] = 5$

$[x-1] = [-3] = -3$

$\therefore f(x) = 5-3 = 2$

23. If f(x) = x^2 -1 and g(x) = 2x +3, find (f+g) (x) and (f-g) (x)

$(f+g)(x) = f(x) + g(x)$

$= x^2 - 1 + 2x + 3$

$= x^2 + 2x + 2$

$(f-g)(x) = f(x) - g(x)$

$= x^2 - 1 - (2x + 3)$

$= x^2 - 1 - 2x - 3$

$= x^2 - 2x - 4$

24. If f (x) = x^2 + 2 and g(x) = x, find (fg) (x) and (f/g) (x).

$(fg)(x) = f(x)\,g(x)$

$= (x^2 + 2)(x)$

$= x^3 + 2$

$\left(\frac{f}{g}\right) x = \frac{f\,(x)}{g\,(x)} = \frac{x^2+2}{x}$

25. If f(x) = 2-5x and g(x) = x+1, find (f+g) (x) and (f-g)(x).

$(f+g)(x) = f(x) + g(x)$

$= 2 - 5x + x + 1$

$= 3 - 4x$

$(f - g)(x) = f(x) - g(x)$

$= 2 - 5x - (x+1)$

$= 2 - 5x - x - 1$

$= 1 - 6x$

26. If f(x) = $\sqrt{x^2 - 3}$ prove that f(-x) = f(x).

$f(-x) = \sqrt{(-x)^2 - 3} = \sqrt{x^2 - 3} = f(x)$

27. Find the domain of the function f(x) = $\sqrt{3 - x}$

$3 - x \geq 0 \Rightarrow 3 \geq x$ or $x \leq 3$

$\therefore$ domain = $\{x : x \in R, x \leq 3\}$

28. Find the range of f(x)= $\frac{1}{x-3}$, x $\neq$ 3

Let $y = \frac{1}{x-3}$. Then $\frac{1}{y} = x - 3$

$\therefore x = \frac{1}{y} + 3$

This is defined $\forall\, y \in R, y \neq 0$

$\therefore$ range = $\{y : y \in R, y \neq 0\}$

29. If f(x) = | x - 3 | + | 4 - 2x |, find f(4).

$f(4) = |4 - 3| + |4 - 2(4)|$

$= |1| + |4 - 8|$

$= 1 + |-4|$

$= 1 + 4$

$= 5$

30. If f(x)= $\frac{1}{\sqrt{|x| - x}}$, find f(2) and f(-2).

$f(2) = \frac{1}{\sqrt{|2| - 2}} = \frac{1}{\sqrt{2-2}} = \frac{1}{\sqrt{0}}$. This is not defined

$f(-2) = \frac{1}{\sqrt{|-2| - (-2)}} = \frac{1}{\sqrt{2-(-2)}} = \frac{1}{\sqrt{2+2}} = \frac{1}{\sqrt{4}} = \frac{1}{2}$

Three marks questions:

1. **If A={a, b, c}, B= { b, c, d} and C={ a, c, e}, verify that Ax(B∩ C) = (A x B) ∩ (A x C)**

   B ∩ C = {c}

   A x (B ∩ C) = { a, b, c} x{c}

   = {(a, c), (b, c), (c, c)} ………..(1)

   A x B = {(a, b), (a, c), (a, d), (b, c), (b, d), (c, b), (b, b), (c, c), (c, d)}

   A x C = {(a, a), (a, c), (a, e), (b, a), (b, c), (b, e), (c, a), (c, c), (c, e)}

   ∴ (A x B) ∩ (A x C) = {(a, c), (b, c), (c, c)}……….(2)

   From equations (1) and (2) A x (B ∩ C) = (A x B) ∩ (A x C)

2. **If A= {-1, 0, 1}, B={2, 3} and C= { -2, 3}, verify that Ax(B-C) = (A x B) – (Ax C)**

   B –C = {2} A={-1, 0 1}

   ∴A x (B - C) = {(-1, 2). (0, 2), (1, 2)}………(1)

   A x B = {(-1, 2), (-1, 3), (0, 2), (0, 3), (1, 2), (1, 3)}

   A x C = { (-1, -2), (-1, 3), (0, -2), (0, 3), (1, -2), (1, 3)}

   (A x B) – (Ax C) = { (-1, 2), (0, 2), (1, 2)}…….(2)

   From equations (1) and (2)

   A x (B - C) = (A x B) – (Ax C)

3. **If P= {2, 3}, Q={3, 4} and R={2, 4}, verify that P x (Q∪ R) = (P x Q) ∪ (P x R)**

   Q ∪ R = {2, 3, 4} P = {2, 3}

   P x (Q ∪ R) = {(2, 2), (2, 3), (2, 4), (3, 2), (3, 3), (3, 4)}………..(1)

   P x Q = {(2, 3), (2, 4), (3, 3), (3, 4)}

   P x R = {(2, 2), (2, 4), (3, 2), (3, 4)}

   ∴ (P x Q) ∪ (P x R) = {(2, 3), (2, 4),(3, 3), (3, 4), (2, 2), (3, 2)}………..(2)

   From (1) and (2) P x (Q ∪ R) = (P x Q) ∪ (P x R)

4. **If A= {x: x = $\frac{n+1}{2}$, n=0, $\pm$ 1} and B= {-1, 2}, find A x B and B x A.**

   A ={ ½, 1, 0} Given B= {(-1, 2)}

   ∴ A x B = {(1/2, -1). (1/2, 2), (1, -1), (1, 2), (0, -1), (0, 2)}

   B x A = {(-1, ½), (-1, 1), (-1, 0), (2, ½), (2, 1), (2, 0)}

5. **Let A= {1, 2, 3, 4, 5 } and B= {1, 2, 3, 4,………, 36}. If R is a relation from B to A defined by R= {(x, y): x = y^2} write R, domain of R and range of R.**

   R = {(1, 1), (2, 4)}

   Domain of R = {1, 2}

   Range of R = {1, 4}

6. **Determine the domain and range of the relation R defined by R= {(x, x^2 -1)/ x∈ N and x≤ 10}.**
R = {(2, 3), (3, 8)}
Domain of R = {2, 3}
Range of R = {3, 8}

7. **Let A= {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}. Let R be a relation defined on A by R= {(x, y) : y=2x}. Write R in the Roster form. Also write the domain and range of R.**
R = {(1, 2), (2, 4), (3, 6), (4, 8), (5, 10)}
Domain of R = { 1, 2, 3, 4, 5}
Range of R = {2, 4, 6, 8, 10}

8. **Let A={1, 2, 3, 4, 5, 6, 7, 8, 9, 10}. If R={(x, y): x+2y=12} is a relation on A. Find the domain and range of R.**
R = {(10, 1), (8, 2), (6, 3), (4, 4), (2, 5)}
Domain of R = {10, 8, 6, 4,2}
Range of R = { 1, 2, 3, 4, 5}

9. **Let R be a relation defined on A = { 1, 2, 3, 4, 5} by xRy iff x-y is a natural number. Find the domain and range.**
R ={(2, 1), (3, 1), (3, 2), (4, 1), (4, 2), (4, 3), (5, 1), (5, 2), (5, 3), (5, 4)}
Domain of R = {2, 3, 4, 5}
Range of R = { 1, 2, 3, 4}

10. **If R is a relation defined on A ={1, 2, 3, 4} by xRy iff x ≤ y find the domain and range of R.**
R = {(1,1) , (2, 2), (3, 3), (4, 4), (1, 2), (1, 3), (1, 4) (2, 3), (2, 4), (3, 4)}
Domain of R = {1, 2, 3}
Range of R = {1, 2, 3, 4}

11. **Let f={(1, 5),(-1, 1),(0, 3)} be a function from A={0, 1, -1} to Z, defined by y=ax +b. Find a and b.**

$y = ax + b$
If $x = 0$ $y = b \Rightarrow b = 3$
If $x = 1$ $y = a + b \Rightarrow a + b = 5$
$\therefore a = 5 - b$ $\quad\therefore a = 2$
If $x = -1$ $y = -a + b \Rightarrow -a + b = 1$
$\therefore b = 1 + a$ $\quad\therefore b = 1 + 2 = 3$

12. If f(x)=$\begin{cases} \mathbf{3x+1}, & \mathbf{0 \le x \le 2} \\ \mathbf{1+9x}, & \mathbf{2 < x < 3} \\ \mathbf{30+2x}, & \mathbf{x \ge 3} \end{cases}$ Find f(1), f(5/2) and f(4)

When x = 1, f (x) = 3x + 1

$\therefore$ f (1) = 4

When x = $\frac{5}{2}$, f (x) = 1 + 9x

$\therefore f\left(\frac{5}{2}\right) = 1 + 9\left(\frac{5}{2}\right) = 1 + \frac{45}{2} = \frac{47}{2}$

When x = 4 , f (x) = 30 + 2 x

$\therefore$ f (1) = 30 + 8 = 38

13. Draw the graph of the greatest integer function f:R→R , f(x) =[x].

14. If f(x) = $\sqrt{x}$ and g(x) = $\frac{1}{x}$ be two functions defined over the set of position real numbers , find (f g) (3) , $\left(\frac{f}{g}\right)$ (2) , (f-g) (1).

(f g) (3) = f (3) . g (3)

$= \sqrt{3} \cdot \frac{1}{3} = \frac{1}{\sqrt{3}}$

$\left(\frac{f}{g}\right)(2) = \frac{f(2)}{g(2)} = \frac{\sqrt{2}}{\frac{1}{2}} = 2\sqrt{2}$

(f - g) (1) = f (1) – g (1) = 1 - $\frac{1}{1}$ = 0

15. Find the domain and range of f(x) = $\frac{x-1}{x+1}$

f (x) is not defined for x = -1

$\therefore$ Domain = {x / x ∈ R and x ≠ -1}

Given y = $\frac{x-1}{x+1}$

⇨ y (x + 1) = x – 1

$\therefore$ xy + y = x – 1

$\therefore$ xy – x = -y -1

x (y - 1) = - (y +1)

$\therefore x = \frac{y+1}{1-y}$ $\therefore y \neq 1$

∴ Range = {y / y ∈ R and y ≠ 1}

16. Find the domain and range of f(x) = $\sqrt{2x-1}$

f (x) cannot be defined for x < ½

∴ Domains = {x / x ∈ R and x ≥ 1/2}

Given y = $\sqrt{2x-1}$

$y^2 = 2x - 1$

x = $\frac{y^2+1}{2}$ exist for all values of y

∴ Range = {y / y ∈ R}

17. Find the domain and range of f(x) = $\frac{x}{x-2}$

F (x) is not defined when x = -2

∴ Domain = {x / x ∈ R, and x ≠ -2}

Given f (x) = $\frac{x}{x-2}$

y = $\frac{x}{x-2}$ => yx + 2y = x

∴ x (1 - y) = 2y

x = $\frac{2y}{1-y}$ ∴ y ≠ -1

∴ Range = {y / y ∈ R and y ≠ -1}

18. Find the domain and range of f(x) = $\sqrt{4-\ x^2}$

f (x) is not defined for x > 4

∴ Domain = {x / x ∈R, and x ≤ 4}

Given , y = $\sqrt{4-\ x^2}$

$y^2 = 4 - x^2$

$x^2 = 4 - y^2$

Range = {y / y ∈ R }

19. Find the domain and range of f(x) = $\frac{x^2-2}{x^2+2}$

Here f(x) is defined for all values of x ∈ R

∴ Domain = R

Given, y = $\frac{x^2-2}{x^2+2}$

$y\ (x^2 + 2) = x^2 - 2$

$x^2y + 2y = x^2 - 2$

$x^2\ (1 - y) = 2y + 2$

$x^2 = \frac{2\ (y+1)}{(1-y)}$

$$x = \pm\sqrt{\frac{2\,(y+1)}{(1-y)}},\quad y \neq 1 \text{ and } y < 1$$

Range = { y/ y ∈ R, and y < 1}

20. If f(x) = [x] + [2x] + [x-3], where[.] indicates greatest integer function, find f (1.6)

f (x) = [x] + [2x] + [x - 3]

f (1.6) = [1.6] + [3.2] + [-1.4]

= 1 + 3 – 2 = 2

21. If f(x) = |x + 2| + | 2x – 1|, -1 < x < 0, simplify f(x). Also find f($-\frac{1}{2}$).

f (x) = | x + 2| + |2x - 1|, -1 < x < 0

f (x) = x +2 – 2x +1

f (x) = 3 – x, -1 < x < 0

∴ f (- 1/2) = 3 + ½ = $^7/_2$

****