Chapter-5
COMPLEX NUMBERS AND QUADRATIC EQUATIONS

1. **Complex numbers and quadratic equations**

Number of teaching hours : 8
Total marks : 10

Blue - Print

1 Mark	2 Marks	3 Marks	Total
1	3	1	10
OR			
-	2	2	10
OR			
1	1	2	10

1 Mark Questions

1. Express the following in the form a + i b
   a) $\frac{5+\sqrt{2}i}{1-\sqrt{2}i}$ b) i^{-35}
2. Find the real and imaginary part of $\sqrt{3}/_2 - \frac{\sqrt{-2}}{7}$
3. Write the multiplicative inverse of
   a) 4 - 3i b) $-1 + \sqrt{3}i$
4. Write the complex conjugate of $(1 + i)^2$
5. Find the real numbers x and y if (x - iy) (1 + i) is the conjugate of -3 -2i
6. Evaluate : $i^{18} + \left(1/_i\right)^{25}$
7. Find the sqare roots of $\frac{-9}{16}$
8. Solve : $x^2 + 3 = 0$
9. Find the modulus of $\frac{2-i}{5i}$
10. Find the amplitude of 1 + i

2 Marks Questions

1. Express the following in the form a + ib

(a) $(5 - 3i)^3$ (b) $3(7 + i7) + i(7 + i7)$

(c) $(1 + i)(1+2i)$ (d) $\frac{i}{1+i}$

(e) $\frac{(3+i\sqrt{5})\ (3-i\sqrt{5})}{(\sqrt{3}+\sqrt{2}i)-(\sqrt{3}-\sqrt{2}i)}$ (f) $\left({}^{1}/_{3} + 3i\right)^3$

(g) $\frac{i}{1+i}$ (h) $\frac{3}{1+i} + \frac{2}{2-i} + \frac{2}{2-i} + \frac{2}{1-i}$

2. Find the conjugate of

a) $(1 + 2i)(2 - 3i)$ b) $\frac{1+2i}{3-i}$ c) $(2 + 5i)^2$

3. Find the multiplicative inverse of

(a) $\frac{3+4i}{3i}$ (b) $\frac{(i+1)\ (i+2)}{(i-1)\ (i-2)}$

4. If 'n' is any integer find $i^n + i^{n+1} + i^{n+2} + i^{n+3}$

5. If $Z_1 = 2 - i$, $Z_2 = -2+i$. Find imaginary part of $\frac{1}{z_1\ \overline{z_2}}$

6. Find the values of x and y if

a) $(x + 2y) + i(2x - 3y) = 5 - 4i$

b) $(1-i)x + (1+i)y = 1-3i$

c) $(x-iy)(3+5i)$ is the conjugate of $-6 -24i$.

7. If $x + iy = \frac{a+ib}{a-ib}$ Prove that $x^2 + y^2 = 1$.

8. If $1 + 4\sqrt{3}i = (a + ib)^2$ prove that $a^2 - b^2 = 1$ and $ab = 2\sqrt{3}$

9. If $Z_1 = 2 - i$, $Z_2 = 1 + i$. Find $\left|\frac{z_1+z_2+1}{z_1-z_2+i}\right|$

10. Find real and imaginary parts of : $\frac{3+\sqrt{-1}}{2-\sqrt{-1}}$ Also find its modulus.

11. Find the least +ve integer m such that $\left(\frac{1+i}{1-i}\right)^{2m} = 1$

12. If $x - iy = \sqrt{\frac{a-ib}{c-id}}$. Prove that $(x^2 + y^2)^2 = \frac{a^2+b^2}{c^2+d^2}$

13. Solve : $x^2 + {}^{x}/_{\sqrt{2}} + 1 = 0$

14. Find the modulus and amplitudes of

(a) 1+ i (b) -1 + i (c) $-\sqrt{3} - i$ (d) $\frac{1+i}{1-i}$

3 Marks Questions

1. Express complex numbers in Polar form :

(a) 1 - i (b) -1 + i

© $\sqrt{3} + i$ (d) $\frac{1-i\sqrt{3}}{2}$

(e) $\frac{-1+i\sqrt{3}}{2}$ (f) $\frac{1+3i}{1-2i}$

2. Find the conjugate of : $\frac{(3-2i)\ (2+3i)}{(1+2i)\ (2-i)}$

3. Find 'θ' so that $\frac{3+2i\ sin\ \theta}{1-2\ i\ Sin\ \theta}$ is

a) Purely real b) Purely imaginary

4. Convert complex number

(a) $\frac{-16}{1+i\sqrt{3}}$ (b) $\frac{i-1}{Cos\ {}^{\pi}/_{3}+i\sin{}^{\pi}/_{3}}$ into polar form.

5. If a + ib = $(x + iy)^{1/3}$ where a, b, x & y are real.

Prove that $\frac{x}{a} - {}^{y}/_{b} = -2\ (a^2 + b^2)$

6. If $Z_1 = 1 - i$, & $Z_2 = -2 + 4i$. Find imaginary part of $\left(\frac{z_1 .\ z_2}{\overline{z_1}}\right)$

7. If α and β are different complex numbers with | β |=1 then

Prove that $\left|\frac{\beta-\alpha}{1-\bar{\alpha}\beta}\right| = 1$

8. If p + iq = $\frac{(\alpha-i)^2}{2\alpha-i}$ show that $p^2 + q^2 = \frac{(\alpha^2-1)^2}{4\alpha^2+1}$

9. Solve : (a) $x^2 + 3x + 9 = 0$

(b) $3x^2 - 4x + 20/3 = 0$

© $27x^2 - 10x + 1 = 0$

(d) $ix^2 - x + 12i = 0$

SOLUTIONS

1 Mark Questions

1. a) $Z = \frac{5+\sqrt{2}i}{1-\sqrt{2}i} \times \frac{1+\sqrt{2}1}{1+\sqrt{2}i} = \frac{5+6\sqrt{2}i-2}{1+2} = \frac{3+6\sqrt{2}i}{3} = 1+\sqrt{2}i$

b) $i^{-35} = \frac{1}{i^{35}} = \frac{1}{i^{32}.i^{3}} = \frac{1}{1.i^{3}} = \frac{1}{-i} \times \frac{i}{i} = \frac{i}{1} = i = 0 + 1.i$

2. Z = ${}^{\sqrt{3}}/_{2} - \frac{\sqrt{2}i}{7}$; $Re\, z = {}^{\sqrt{3}}/_{2}$, $Img.z = -\frac{\sqrt{2}}{7}$

3. a) Z = 4 - 3i, Multiplicative inverse of Z = $\frac{\bar{z}}{|z|^2} = \frac{4+3i}{25}$

b) Z = -1 + $\sqrt{3}i$ Multiplicative inverse of Z = $\frac{-1-\sqrt{3}i}{4}$

4. Z = $(1 + i)^2$ = 1 - 1 + 2i = 2i

$\bar{Z} = -2i$

5. (x - iy) (1 + i) = -3 + 2i

(x + y) + i (x - y) = -3 + 2i

x + y = -3

x - y = 2

$\therefore$ x = ${}^{-1}/_{2}, y = {}^{-5}/_{2}$

6. $i^{18} + \left(\frac{1}{i}\right)^{25} = i^{16}.i^2 + (-i)^{25} = -1 - i^{24}.i = -1 - i$

7. $\sqrt{\frac{-9}{16}} = \pm \frac{3i}{4}$

8. $x^2 + 3 = 0$ => $x^2 = -3$

x = $\pm\sqrt{3}i$

9. Z = $\frac{2-i}{5i}$ $\therefore$ | z | = $\frac{|2-i|}{|5i|} = \frac{\sqrt{5}}{5} = {}^{1}/_{\sqrt{5}}$

10. Z = 1 + i = r (Cos θ + i sin θ)

$\left.\begin{array}{l} r = \sqrt{2}, \quad Cos\,\theta = {}^{1}/_{\sqrt{2}} \\ \quad\quad\quad Sin\ \theta = {}^{1}/_{\sqrt{2}} \end{array}\right\}$

$\therefore$ Amp θ = ${}^{\pi}/_{4}$

TWO MARKS

1. a) $Z = (5 - 3i)^3 = 5^3 - 3.5^2 (-3i) + 3.5 (-3i)^2 - (3i)^3$

$= 125 - 225i - 135 + 27 i$

$= -10 - 198 i$

b) $3 (7 + i7) + i (7 + i7) = 21 + 2i\ i + 7i - 7 = 14 + 28 i$

c) $(1+i) (1+2i) = 1 + 2i + i-2 = -1 + 3i$

d) $\frac{3}{3+4i} = \frac{3(3-4i)}{9+16} = \frac{9-12i}{25} = {}^{9}/_{25} - \frac{12}{25}i$

e) $\frac{(3+i\sqrt{5})\ (3-i\sqrt{5})}{(\sqrt{3}+\sqrt{2}i)-(\sqrt{3}-\sqrt{2}i)} = \frac{9+5}{\sqrt{2}i+\sqrt{2}i} = \frac{14}{2\sqrt{2}i} = \frac{7}{\sqrt{2}i}\ \frac{i}{i} = \frac{-7i}{\sqrt{2}} = 0 - \frac{7}{\sqrt{2}}i$

f) $\left({}^{1}/_{3} + 3i\right)^3 = \frac{1}{27} + 3\frac{1}{9}.3i + 3.\frac{1}{3}\ (-9) - 27i$

$= \frac{1}{27} + i - 27i - 9$

$= \frac{-242}{27} - 26i$

g) $\frac{i}{1+i} = \frac{i(1-i)}{1+1} = \frac{i+1}{2} = {}^{1}/_{2} + i\ {}^{1}/_{2}$

h) $\frac{3}{1+i} + \frac{2}{2-i} + \frac{2}{1-i} = \frac{3(1-i)}{2} + \frac{2(2+i)}{5} + \frac{2(1+i)}{2}$

$= \frac{3}{2} - {}^{3}/_{2}\,i + {}^{4}/_{5} + \frac{2i}{5} + 1 + i$

$= \left({}^{3}/_{2} + {}^{4}/_{5} + 1\right) + \left(\frac{2}{-5}\ {-}^{3}/_{2} + 1\right) i$

$= \frac{33}{10} - \frac{1}{10}\ i$

2)a) $Z = (1 + 2i)\ (2 - 3i) = 2 - 3i + 4i + 6 = 8 + i\ \bar{z} = 8 - i$

b) $z = \frac{1+2i}{3-i} \times \frac{3+i}{3+i} = \frac{3+i+6i-2}{9+1} = \frac{1+7i}{10}$

$\bar{z} = \frac{1 - 7i}{10}$

c) $z = (2 + 5i)^2 = 4 \neq 20i - 25 = -21 + 20\ i$

$\bar{z} = -21 - 20i$

3.a) $z = \frac{3+4i}{3i} \times \frac{-3i}{-3i} = \frac{-9i+12}{+9} = \frac{12-9i}{9} = \frac{4-3i}{3}$

multiplicative inverse of $z = \frac{4+3i}{25}$

b) $z = \frac{(i+1)\ (i+2)}{(i-1)\ (i-2)} = \frac{-1+3i+3}{-1-3i+2} = \frac{2+3i}{1-3i} \times \frac{1+3i}{1+3i} = \frac{2+6i+34i-9}{1+9} = \frac{-7+9i}{10}$

multiplicative inverse of $z = \frac{-7-9i}{13}$

4. $i^n + i^{n+1} + i^{n+2} + i^{n+3} = i^n\ (i + i + i^2 + i^3)$

$= i^n\ (1 + i - 1 - i) = i^n - 0$

$= 0$

5. $Z_1 . \overline{Z_2} = (2 - i)\ (-2 - i) =$

$= -4 - 2i + 2i - 1 = -5$

$\therefore\ \frac{1}{z_1.\overline{z_2}} = \frac{1}{-5} = {}^{-1}/_{5}$

6.a) $x + 2y = 5 \Rightarrow 2x + 4y = 10$

$2x - 3y = -4 \qquad 2x - 3y = -4$

$7y = 14$

$y = 2, \quad x = 1$

b) $x + y = 1$

$\underline{-\ x + y = -3} \Rightarrow 2y = -2$

$\therefore\ y = -1, \quad x = 2$

c) $3x + 5xi - 3iy + 5y = -6 + 24i$

$(3x + 5y) + i(5x - 3y) = -6 + 24i$

$3x + 5y = -6$ => $9x + 15y = -18$

$5x - 3y = 24$ $25x - 15y = 120$

$34x = 102$

$x = \dfrac{102}{34} = 3$

$y = -3$

7. $x + iy = \dfrac{a + ib}{a - ib}$

$\therefore x - iy = \dfrac{a - ib}{a + ib}$

$\therefore (x + iy)(x - iy) = \dfrac{a+ib}{a-ib} \times \dfrac{a-ib}{a+ib}$

$x^2 + y^2 = 1$

8. $1 + 4\sqrt{3}i = (a + ib)^2 = (a^2 - b^2) + 2iab$ equating real and imaginary parts. $a^2 - b^2 = 1$, $ab = 2\sqrt{3}$

9. $\left|\dfrac{z_1+z_2+1}{z_1-z_2+i}\right| = \left|\dfrac{2-i+1+i+1}{2-i-1-i+i}\right| = \left|\dfrac{4}{1-i}\right| = \dfrac{4}{\sqrt{2}} = \dfrac{4\sqrt{2}}{2} = 2\sqrt{2}$

10. $z = \dfrac{3+i}{2-i} \times \dfrac{2+i}{2+i} =$

$= \dfrac{6+3i+2i-1}{5} = \dfrac{5+5i}{5} = 1 + i$ & $|z| = \sqrt{1+1} = \sqrt{2}$

Re z = 1, Img. z = 1,

11. $\left(\dfrac{1+i}{1-i}\right)^{2m} = 1 \Rightarrow \left[\dfrac{(1+i)\ (1+i)}{(1+i)\ (1+i)}\right]^{2m} = 1 \Rightarrow \left(\dfrac{1-1+2i}{2}\right)^{2m} = 1$

⇨ $i^{2m} = 1 = i^4$

⇨ $2m = 4 \Rightarrow m = 2$

12. x – iy = $\sqrt{\frac{a-ib}{c-id}}$

$\therefore$ (x – iy)^2 = $\frac{a-ib}{c-id}$

$(x^2 - y^2) - 2i\,xy = \frac{a-ib}{c-id}$

$|(x^2 - y^2) - 2i\,xy| = \left|\frac{a-ib}{c-id}\right|$

$\sqrt{(x^2 - y^2)^2 + 4x^2y^2} = \frac{|a-ib|}{|C-id|}$

$\sqrt{x^4 + y^4 - 2x^2y^2 + 4x^2y^2} = \frac{\sqrt{a^2+b^2}}{\sqrt{C^2+d^2}}$

$\sqrt{x^4 + y^4 + 2x^2y^2} = \frac{\sqrt{a^2+b^2}}{\sqrt{C^2+d^2}}$

$\sqrt{(x^2 + y^2)^2} = \frac{\sqrt{a^2+b^2}}{\sqrt{C^2+d^2}}$

$Sq. on\ both\ sides.$

$(x^2 + y^2)^2 = \frac{a^2+b^2}{c^2+d^2}$

13. $x^2 + {}^{x}/_{\sqrt{2}} + 1 = 0$

$\sqrt{2}x^2 + x + \sqrt{2} = 0$

$x = \frac{-1 \pm \sqrt{1-4\sqrt{2}.\sqrt{2}}}{2\sqrt{2}}$

$= \frac{-1 \pm \sqrt{1-8}}{2\sqrt{2}}$

$x = \frac{-1 \pm \sqrt{7}i}{2\sqrt{2}}$

14.a) Z = 1 + i = r (Cos θ + isin θ)

$r = \sqrt{1^2 + 1^2} = \sqrt{2}$

$Cos\ \theta = {}^{x}/_{r} = {}^{1}/_{\sqrt{2}}$

$Sin\ \theta = {}^{y}/_{r} = {}^{1}/_{\sqrt{2}}$

Amp (θ) = ${}^{\pi}/_{4}$

b) Z = -1 +i=r (Cos θ + isin θ)

Modulus $= \sqrt{1+1} = \sqrt{2}$

$Cos\ \theta = {}^{x}/_{r} = {}^{-1}/_{\sqrt{2}}$

$Sin\ \theta = {}^{y}/_{r} = {}^{1}/_{\sqrt{2}}$

Amp (θ) = $\pi - {}^{\pi}/_{4} = {}^{3\pi}/_{4}$

c) Z = -$\sqrt{3} - i = r\ (cos\theta\ + i \sin\theta)$

Modulus $= \sqrt{3+1} = 2$

$Cos\ \theta = -{}^{\sqrt{3}}/_{2}$

$Sin\ \theta = {}^{-1}/_{2}$

Amp (θ) = ${}^{-5\pi}/_{6}$

d) $Z = \frac{1+i}{1-i} = \frac{(1+i)^2}{2} = \frac{1-1+2i}{2} = i$

$r = \sqrt{0+1} = 1$

$Cos\ \theta = 0$

$Sin\ \theta = 1$

$Amp\ (\theta) = {}^{\pi}/_{2}$

3 Marks Questions

1.

a) Let Z = 1 – i = r (Cos θ + I sin θ)

$r = \sqrt{1+1} = \sqrt{2}$

$$\left.\begin{aligned} Cos\,\theta &= {}^{1}/_{\sqrt{2}} \\ Sin\;\theta &= {}^{-1}/_{\sqrt{2}} \end{aligned}\right\}\theta = -{}^{\pi}/_{4}$$

$1 - i = \sqrt{2}\,\left[Cos\left({}^{-\pi}/_{4}\right) + i\sin\left({}^{-\pi}/_{4}\right)\right]$

b) Let Z = -1 + i = r (Cos θ + I sin θ)

$r = \sqrt{1+1} = \sqrt{2}$

$$\left.\begin{aligned} Cos\,\theta &= {}^{-1}/_{\sqrt{2}} \\ Sin\;\theta &= {}^{1}/_{\sqrt{2}} \end{aligned}\right\}\theta = \pi - {}^{\pi}/_{4} = {}^{3\pi}/_{4}$$

$-1 + i = \sqrt{2}\,\left(Cos\;{}^{3\pi}/_{4} + Sin\;{}^{3\pi}/_{4}\right)$

c) Let Z = $\sqrt{3} + i$

$r = \sqrt{3+1} = 2$

$$\left.\begin{aligned} Cos\,\theta &= {}^{\sqrt{3}}/_{2} \\ Sin\;\theta &= {}^{1}/_{2} \end{aligned}\right\}\theta = {}^{\pi}/_{6}$$

$\sqrt{3} + i = 2\,\left(Cos\;{}^{\pi}/_{6} + i\,Sin\;{}^{\pi}/_{6}\right)$

d) Let Z = $\frac{1-i\sqrt{3}}{2}$

$r = \sqrt{{}^{1}/_{4} + {}^{3}/_{4}} = 1$

$$\left.\begin{aligned} Cos\,\theta &= {}^{1}/_{2} \\ Sin\;\theta &= -{}^{\sqrt{3}}/_{2} \end{aligned}\right\}\theta = -{}^{\pi}/_{3}$$

$\frac{1-i\sqrt{3}}{2} = Cos\left({}^{-\pi}/_{3}\right) + i\sin\left({}^{-\pi}/_{3}\right)$

e) Let Z = $\frac{1-i\sqrt{3}}{2}$

$$r = \sqrt{1/4 + 3/4} = 1$$

$$\left.\begin{aligned} Cos\ \theta &= -1/2 \\ Sin\ \theta &= +\sqrt{3}/2 \end{aligned}\right\} \theta = \pi - \pi/3 = 2\,\pi/3$$

$$\frac{-1+i\sqrt{3}}{2} = Cos\ 2\pi/3 + i \sin 2\pi/3$$

f) Let Z = $\frac{1+3i}{1-2i} \times \frac{1+2i}{1+2i}$

$$= \frac{1+2i+3i-6}{1+4} = \frac{-5+5i}{5}$$

= -1 + i

$$r = \sqrt{1+1} = \sqrt{2}$$

$$\left.\begin{aligned} Cos\ \theta &= -1/\sqrt{2} \\ Sin\ \theta &= 1/\sqrt{2} \end{aligned}\right\} \theta = \pi - \pi/4 = 3\pi/4$$

$$Z = \sqrt{2}\ \left(Cos\ 3\pi/4 + i \sin 3\pi/4\right)$$

2. $Z = \frac{(3-2i)\,(2+3i)}{(1+2i\,)\,(2-i)} = \frac{6+9i-4i+6}{2-i+4i+2} = \frac{12+5i}{4+3i}$

$$= \frac{(12+5i)\,(4-3i)}{16+9} = \frac{48-36i+20i+15}{25}$$

$$= \frac{63-16\,i}{25}$$

$$\bar{Z} = \frac{63+16\,i}{25}$$

3. $Z = \frac{3+2i\, Sin\,\theta}{1-2i\, Sin\,\theta} \times \frac{1+2i\, Sin\,\theta}{1+2i\, Sin\,\theta}$

$= \frac{3+6\, i\, Sin\,\theta+2i\, Sin\,\theta-4\, Sin^2\theta}{1+4\, Sin^2\theta}$

$= \frac{(3-4\, Sin^2\,\theta)+8\, i\, Sin\,\theta}{1+4\, Sin^2\,\theta}$

a) 'Z' is purely real

=> $\frac{8\, Sin\,\theta}{1+4Sin^2\theta} = 0$

Sin θ = 0

$\therefore$ θ = n π : n ∈ I.

b) Z is purely imaginary

=> $\frac{3-4\, Sin^2\theta}{1+4Sin^2\theta} = 0$

$3 - 4\ Sin^2\,\theta = 0$

$Sin^2\,\theta = \frac{3}{4}.$

$Sin\,\theta = \pm\frac{\sqrt{3}}{2} = Sin\left(\pm\, {}^{\pi}/_{3}\right)$

$\therefore\ \theta = n\,\pi \pm (-1)^n \,.\, {}^{\pi}/_{3}$

4. a) $Z = \frac{-16}{1+i\sqrt{3}} \times \frac{1-i\sqrt{3}}{1-i\sqrt{3}} = \frac{-16\,(1-i\sqrt{3})}{1+3} = \frac{16\,(1-i\sqrt{3})}{4}$

$= 4\,(-1 + i\,\sqrt{3}) = 4 + i4\,\sqrt{3} = r\,(Cos\,\theta + I\,sin\,0)$

$r = \sqrt{46+48} = \sqrt{64} = 8$

$$\left.\begin{array}{l} Cos\,\theta = \ {}^{-4}/_{8} = {}^{-1}/_{2} \\ Sin\ \theta = \ +\frac{4\sqrt{3}}{8} + {}^{\sqrt{3}}/_{2} \end{array}\right\}\theta = \pi - {}^{\pi}/_{3} = {}^{2\pi}/_{3}$$

$Z = 8\left(Cos\ {}^{2\pi}/_{3} + i\ \sin {}^{2\pi}/_{3}\right)$

b) $$Z = \frac{i-1}{Cos\,{}^{\pi}/_{3}+i\sin{}^{\pi}/_{3}} = \frac{i-1}{\frac{1}{2}+i\,{}^{\sqrt{3}}/_{2}} = \frac{2\,(i-1)}{1+i\sqrt{3}} \times \frac{1-i\sqrt{3}}{1-i\sqrt{3}}$$

$$= \frac{2\left(i+\sqrt{3}-1+i\sqrt{3}\right)}{1+3} = \frac{2\,\left(\sqrt{3}-1\right)+i\,\left(\sqrt{3}+1\right)}{4}$$

$$= \left(\frac{\sqrt{3}-1}{2}\right) + i\,\left(\frac{\sqrt{3}+1}{2}\right) = r\,(Cos\,\theta + i\sin\theta)$$

$$r = \sqrt{\left(\frac{\sqrt{3}-1}{2}\right)^2 + \left(\frac{\sqrt{3}+1}{2}\right)^2 = 2}$$

$$\left.\begin{aligned} Cos\,\theta &= \frac{\sqrt{3}-1}{2\sqrt{2}} \\ Sin\,\,\theta &= \frac{\sqrt{3}+1}{2\sqrt{2}} \end{aligned}\right\}\theta = {}^{5\pi}/_{12}$$

5. $(a + ib) = (x+iy)^{1/3}$

$(a+ib)^3 = x + iy$

$a^3 + 3a^2.\ ib - 3ab^2 - ib^3 = x+iy$

$(a^3 - 3ab^2) + i\,(3a^2b - b^3) = x + iy$

$\therefore\ x = a^3 - 3ab^2$ $\qquad$ $y = 3a^2b - b^3$

$\frac{x}{a} = a^2 - 3b^2$ $\qquad$ $\frac{y}{b} = 3a^2 - b^2$

$\frac{x}{a} - \frac{y}{b} = a^2 - 3b^2 - 3a^2 + b^2 = -2\,(a^2 + b^2)$

6. $$\frac{z_{1}.\ z_2}{\bar{z}_1} = \frac{(1-i)\ (-2+4i)}{1+i} = \frac{-2+4i+2i+4}{1+i}$$

$$= \frac{2+6i}{1+i} \times \frac{1-i}{1-i} = \frac{2+6i-2i+6}{1+1}$$

$$= \frac{8+4i}{2} = 4 + 2i$$

Imaginary part of $\left(\frac{z_1.z_2}{\bar{z}_1}\right) = 2$ $\qquad$ $|z|^2 = z.\bar{z}$

7. $\left|\frac{\beta-\alpha}{1-\bar{\alpha}\beta}\right|^2 = \frac{|\beta-\alpha|^2}{|1-\bar{\alpha}\beta|^2} = \frac{(\beta-\alpha)\left(\overline{\beta-\alpha}\right)}{(1-\bar{\alpha}\beta)\left(\overline{1-\bar{\alpha}\beta}\right)}$

$= \frac{(\beta-\alpha)\ \left(\bar{\beta}-\bar{\alpha}\right)}{(1-\bar{\alpha}\beta)\ \left(1-\alpha\bar{\beta}\right)} = \frac{|\beta|^2-\bar{\alpha}\beta-\alpha\bar{\beta}+|\alpha|^2}{1-\alpha\bar{\beta}-\alpha\bar{\beta}+\ |\alpha l^2\ |\beta|^2}$

$= \frac{1-\bar{\alpha}\beta-\ \alpha\bar{\beta}+\ |\alpha|^2}{1-\bar{\alpha}\beta-\ \alpha\bar{\beta}+|\alpha|^2}$

= 1

$Hence\quad \left|\frac{\beta-\alpha}{1-\bar{\alpha}\beta}\right| = 1$

8. $P\ \rightarrow pq = \frac{(\alpha-i)^2}{2\alpha-i}$

$\therefore\ \ p - iq\ = \frac{(\alpha+i)^2}{2\alpha+i}$

(p+iq) (p-iq) $= \frac{(\alpha-i)^2}{2\alpha-i} . \frac{(\alpha+i)^2}{2\alpha+i}$

$p^2 + q^2 = \frac{\left(\alpha^2+1\right)^2}{4\alpha^2+1}$

9. a) $x^2 + 3x + 9 = 0$

$x = \frac{-3\pm\sqrt{9-36}}{2} = \frac{-3\pm\sqrt{-27}}{2}$

$= \frac{-3\pm i3\sqrt{3}}{2}$

b) $3x^2 - 4x + \frac{20}{3} = 0$

$9x^2 - 12x + 20 = 0$

$$x = \frac{+12 \pm \sqrt{144-720}}{18} = \frac{12 \pm \sqrt{-576}}{18} = \frac{12 \pm 24i}{18}$$

$$= \frac{2 \pm 4i}{3}$$

c) $27x^2 - 10x + 1 = 0$

$$x = \frac{10 \pm \sqrt{100-108}}{2 \times 27} = \frac{10 \pm 2\sqrt{2}i}{2 \times 27}$$

$$= \frac{5 \pm \sqrt{2}i}{27}$$

d) $ix^2 - x + 12i = 0$

$$x = \frac{1 \pm \sqrt{1+48}}{2i} = \frac{1 \pm 7}{2i}$$

$$x = \frac{8}{2i} \quad x = \frac{-6}{2i}$$

$x = -4i$ $x = 3i$.