

KINETIC THEORY

ONE MARK QUESTION:

- 1.Name any one scientist who explained the behavior of gases considering it to be made up of tiny particles.
- 2.Based on which idea kinetic theory of gases explain the behavior of gases?
- 3.Mention any one measurable property of gas.
- 4.Name any one scientist who suggested that matter consists of indivisible constituents.
- 5.What is an ideal gas or a perfect gas?
- 6.Draw P-V curve or diagram for Boyle's law.
- 7.Draw P-V curve for Charle's law.
- 8.What is the value of universal gas constant 'R'?
9. On what factor does the internal energy of a gas depend on?
- 10.On what factors does internal energy of the gas doesn't depend?
- 11.Define degree's of freedom.
- 12.How many degrees of freedom a mono atomic gas molecule possess?
- 13.How many degrees of freedom a di-atomic gas molecule possess?
- 14.State the law of equipartition of energy.
- 15.What is the value of Boltzmann's constant?
- 16.What is the value of ratio of specific heats for a mono atomic gas molecule?
- 17.What is the value of ratio of specific heats of di-atomic gas molecule?
- 18.Write the general formula for ratio of specific heats for poly atomic gases.
- 19.Write the equation connecting C_p , C_v & R .
- 20.Define mean free path.
21. Write the equation for pressure of an ideal gas.
- 22.Write the order of mean free path in gases .

TWO MARK QUESTION:

23. Show that specific heat of solids $C=3R$.
24. What are the important characteristics of an ideal gas?
25. State and explain Boyle's law.
26. State and explain Charles's law.
27. Write the perfect gas equation and explain the terms.
28. State and explain Dalton's law of partial pressures.
29. Write the equation for mean free path and explain the terms.
30. Explain the term free mean free path.

FOUR MARK QUESTION:

31. Mention the salient features of kinetic theory of gases developed by Maxwell and Boltzmann.
32. Derive the equation $P = \rho RT / M_0$
33. Deduce the expression $PV = \mu RT$.
34. Determine the specific heat capacity of a mono atomic gas molecule.
35. Determine the specific heat capacity of a di-atomic gas molecule treated as a rigid rotator.
36. Deduce the equation $\gamma = \frac{(4+f)}{(3+f)}$
37. Apply law of equipartition of energy to a monoatomic gas molecule.
38. Apply the law of equipartition of energy to a di-atomic gas molecule treating it as a rigid rotator.

FIVE MARK QUESTION:

39. Mention any five assumptions of kinetic theory of gases.
40. Derive an expression for pressure of an ideal gas.
41. From kinetic theory of gases explain the kinetic interpretation of temperature.
42. Define mean free path and hence derive the expression $l = \frac{1}{(\sqrt{2}n\pi d^2)}$.

ANSWERS:-

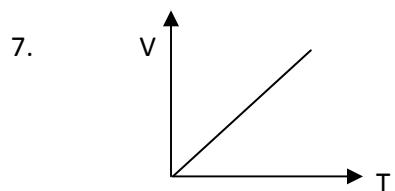
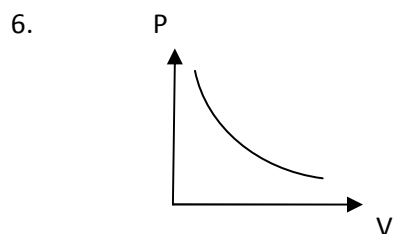
1. Newton, Boyle.

2. Gases consist of rapidly moving atoms or molecules .

3. Viscosity, conduction, diffusion, pressure.

4. Kanada, Democritus.

5. Is one which obeys Boyle's law and Charles' law at all values of temperature and pressure.



8. $8.314 \text{ J mol}^{-1} \text{ K}^{-1}$

9. Temperature.

10. Pressure, volume.

11. Number of co-ordinates required to specify the position of a molecule.

12. Three translational degrees of freedom.

13. Three translational degrees of freedom and two rotational degrees of freedom.

14. It states that for any system in thermal equilibrium, the total energy is equally distributed among its various degrees of freedom and the energy associated with each molecule per degree of freedom is $\frac{1}{2} k_B T$.

15. $1.38 \times 10^{-23} \text{ J K}^{-1}$

16. $\frac{5}{3}$

17. $\frac{7}{5}$

$$18. \gamma = \frac{(4+f)}{(3+f)}$$

$$19. (C_p - C_v) = R$$

20. The average distance travelled by a gas molecule between two successive collisions .

$$21. P = \frac{1}{3} n m \bar{v}^2$$

$$22. 1000 A^0$$

$$23. \text{For a mole of solid } U = 3K_B T N_A = 3RT$$

At constant pressure $\Delta Q = \Delta U$

$$\therefore C = \frac{\Delta Q}{\Delta T} = \frac{\Delta U}{\Delta T} = 3R$$

24. The molecules of a gas are point masses, inter molecular force among molecules is zero.

25. At constant temperature T, the volume V of a given mass of gas is inversely proportional to its pressure P i.e. $V \propto \frac{1}{P}$ or $PV = K$.

26. At constant pressure P, the volume V of a given mass of gas is directly proportional to its absolute temperature T i.e. $\frac{V}{T} = \text{constant}$ or $V = KT$.

27. $PV = \mu RT$ where P is the pressure, V volume, μ number of moles, R gas constant, T temperature.

28. The total pressure of a mixture of ideal gases is the sum of the partial pressures

Consider a mixture of two ideal gases μ_1 number of moles of gas 1, μ_2 number of moles of gas 2

At a temperature T, pressure P $PV = (\mu_1 + \mu_2)RT$

$$P = \mu_1 \frac{RT}{V} + \mu_2 \frac{RT}{V}$$

$$P = P_1 + P_2 \quad \text{where } P_1 \text{ is pressure of gas 1 and } P_2 \text{ is pressure of gas 2}$$

$$29. l = \frac{1}{\sqrt{2} \pi n d^2} \quad \text{where } n \text{ is number of moles, } d \text{ diameter of the molecule.}$$

30. During the motion of molecules of a gas, the molecule suffers number of collisions with other gas molecules. During two successive collisions, a molecule of a gas moves in a straight line with constant velocity and the average distance travelled by a molecule during all collisions is known as mean free path.

31. A gas consists of a large number of tiny particles called molecules which are perfectly rigid and elastic

- The molecules are in a state of continuous random motion moving in all directions with all possible velocities
- At ordinary temperature and pressure, the size of the molecules is negligible compared with the average distance between the molecules.
- The molecules have velocities ranging from 0 to ∞ , so that average velocity of random motion is zero.
- The collision between the two molecules is perfectly elastic that is the kinetic energy remains conserved in the collision .
- There is no inter molecular force of attraction between the molecules of the gas .
- Between two successive collisions a molecule traverses straight line path with constant speed called mean free path of the molecule.

32. Perfect gas equation is $PV = \mu RT$ where $R = N_A k_B$, N_A number of moles in the sample, k_B Boltzmann constant

Choosing Kelvin's we have $\mu = \frac{M}{M_0} = \frac{N}{N_A}$, M is mass of the gas containing N molecules, M_0 is molar mass

$$PV = \mu RT = \frac{M}{M_0} RT \text{ or } P = \frac{M}{V} \cdot \frac{RT}{M_0}$$

$$\text{Or } p = \frac{\rho}{M_0} RT \text{ where } \frac{M}{V} = \rho$$

33. From Boyle's law $V \propto \frac{1}{P}$ OR $V = \frac{K}{P}$ (1) at constant temperature

From Charle's law $V \propto T$ at constant pressure

$$\text{OR } V = KT \text{(2)}$$

From equation (1) and (2) $V \propto \frac{T}{P}$ OR $\frac{PV}{T} = \text{constant}$

For one mole of gas, the constant has same value for all gases and is called universal gas constant R

$$\therefore \frac{PV}{T} = R \text{ ie } PV = RT$$

For μ number of moles $PV = \mu RT$.

34. A mono atomic gas molecule has three translational degrees of freedom. The average energy of a molecule at temperature T is $\frac{3}{2} k_B T$

$$\text{Total internal energy } U = \frac{3}{2} k_B T N_A = \frac{3}{2} RT$$

The molar specific heat at constant volume $C_v = \frac{dU}{dT} = \frac{3}{2} R$

We have $C_p = C_v + R = \frac{3}{2}R + R = \frac{5}{2}R$

\therefore ratio of specific heats $\gamma = \frac{C_p}{C_v} = \frac{5/2}{3/2} = \frac{5}{3}$

35. Di-atomic gas molecule has three translational degrees of freedom and two rotational degrees of freedom, totally five. Average energy $U = \frac{5}{2} k_B T N_A = \frac{5}{2} RT$

$$C_v = \frac{dU}{dT} = \frac{5}{2} R$$

$$C_p = C_v + R = \frac{5}{2}R + R = \frac{7}{2}R$$

$$\gamma = \frac{C_p}{C_v} = \frac{7/2 R}{5/2 R} = \frac{7}{5}$$

Note: If the di-atomic molecule is not a rigid rotator, it has a additional vibration mode

$$U = \left[\frac{5}{2} k_B T + k_B T \right] N_A = \frac{7}{2} RT$$

$$C_v = \frac{dU}{dT} = \frac{7}{2} R$$

$$C_p = C_v + R$$

$$\therefore \gamma = \frac{C_p}{C_v} = \frac{9/2 R}{7/2 R} = \frac{9}{7}$$

36. For poly atomic gas molecule, there are three translational three rotational and 'f' vibrational degrees of freedom .

The average internal energy $U = \left[\frac{3}{2} k_B T + \frac{3}{2} k_B T + f \times k_B T \right] N_A$

$$= (3+f) k_B T N_A = (3+f) RT$$

$$C_v = \frac{dU}{dT} = (3+f) R$$

$$C_p = C_v + R$$

$$= (3+f) R + R = (4+f) R$$

$$\gamma = \frac{C_p}{C_v} = \frac{(4+f)}{(3+f)}$$

37. A monoatomic gas molecule is free to move in space and has 3 translational degrees of freedom .

Each translational degree of freedom contributes for the total energy and $= \frac{1}{2} k_B T$.

The kinetic energy of a monoatomic molecule is

$$E_T = \frac{1}{2}mV_x^2 + \frac{1}{2}mv_y^2 + \frac{1}{2}mV_z^2$$

For a gas at temperature T, average value of energy is denoted by $\langle E_T \rangle$

$$\langle E_T \rangle = \frac{1}{2}mV_x^2 + \frac{1}{2}mV_y^2 + \frac{1}{2}mV_z^2 = \frac{3}{2}k_B T$$

Since there is no preferred direction $\langle \frac{1}{2}mV_x^2 \rangle = \langle \frac{1}{2}mV_y^2 \rangle = \langle \frac{1}{2}mV_z^2 \rangle = \frac{1}{2}k_B T$.

38. Diatomic molecule has three translational degrees of freedom in addition to that, it can also rotate about its center of mass i.e., two independent rotational degrees of freedom. Since the diatomic molecule is treated as a rigid rotator, the molecule does not vibrate. Each degree of freedom contributes to the total energy consisting of translational energy E_t and rotational energy E_r .

$$E_t + E_r = \frac{1}{2}mV_x^2 + \frac{1}{2}mV_y^2 + \frac{1}{2}mV_z^2 + \frac{1}{2}I_1\omega_1^2 + \frac{1}{2}I_2\omega_2^2$$

where ω_1 and ω_2 are angular speeds about the axes 1 and 2, I_1 and I_2 are corresponding moments of inertia.

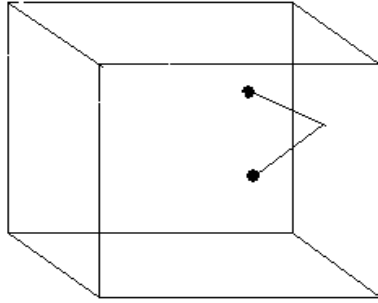
39. The following are the assumptions

- a) A gas consists of a large number of tiny particles called molecules which are perfectly rigid and elastic.
- b) The molecules are in a state of continuous motion moving in all directions with all possible velocities.
- c) At ordinary temperature and pressure, the size of the molecule is negligible compared with the average distance between the molecules.
- d) The molecules have velocities ranging from 0 to ∞ , so that average velocity of random motion is zero.
- e) The collision between two molecules is perfectly elastic that is kinetic energy remains conserved in the collision.
- f) Between two successive collisions, molecules traverse a straight line path with constant speed called free path of the molecule.
- g) There is no intermolecular force of attraction between the molecules of the gas.
- h) The collisions are almost instantaneous and the molecules obey Newton's laws of motion.

40. Consider a gas enclosed in a cube of side l . Let the axes be parallel to the sides of the cube. A molecule with velocity (V_x, V_y, V_z) hits the planar wall parallel to yz plane of area l^2 . Since the collision is elastic, the molecule rebounds with same velocity, its y and z components do not change in the collision, but the x component reverses its sign. Velocity after collision is given by $(-V_x, V_y, V_z)$.

Change in momentum of the molecule is given by $-mV_x - (+mV_x) = -2mV_x$.

By the principle of conservation of momentum, the momentum imparted to the wall $= 2mV_x$. In a small time interval Δt a molecule with a velocity V_x will hit the wall if it is in a distance $V_x \Delta t$ from the wall. That is all molecules within the volume $AV_x \Delta t$ only can hit the wall in time Δt . On an average half the molecules move towards the wall and the other half away from the wall.



No of molecules with velocity (V_x, V_y, V_z) hitting the wall in a time Δt is $\frac{1}{2}AV_x \Delta t n$

where n - no of molecules per unit volume. The total momentum transferred to the wall by these molecules in a time Δt is $Q = (2mV_x) \left(\frac{1}{2}AV_x \Delta t n \right)$.

The force on the wall is the rate of momentum transfer $\frac{Q}{\Delta t}$ and pressure is the force per unit area.

$$P = \frac{Q}{A \Delta t} = (2mV_x) \left(\frac{1}{2}AV_x \Delta t n \right) / A \Delta t$$

$$\text{Or } P = nmV_x^2$$

Actually all the molecules in a gas do not have same velocity. The above equation gives pressure due to a group of molecules with speed V_x in the x direction and n stands for number density of that group of molecules.

The total pressure is obtained by summing over the contribution due to all the groups.

$P = nm\overline{V_x^2}$ where $\overline{V_x^2}$ is the average of V_x^2 . By symmetry

$\overline{V_x^2} = \overline{v_y^2} = \overline{V_z^2}$. Therefore

$$P = nm[\overline{V_x^2} + \overline{v_y^2} + \overline{V_z^2}]/3$$

$$P = \frac{1}{3}nm\overline{V^2} \text{ where}$$

$$\overline{V^2} = \overline{V_x^2} + \overline{v_y^2} + \overline{V_z^2}.$$

41. The pressure exerted by a gas is given by $P = \frac{1}{3}nm\overline{v^2}$ ---(1)

Multiplying by V on both sides,

$$PV = \frac{1}{3}nmV\overline{v^2}$$

$$PV = \frac{2}{3}N \frac{1}{2}m\overline{v^2} \text{---(2)}$$

Where $N = nV$, no of molecules in the sample.

The average translational kinetic energy of the molecules is the internal energy E of the ideal gas.

$$E = N \frac{1}{2}m\overline{v^2} \text{----(3)}$$

$$(2) \rightarrow PV = \frac{2}{3}E \text{----(4)}$$

Combining (4) and perfect gas equation $PV = \mu RT$

$$E = \frac{3}{2} \mu RT = \frac{3}{2} N k_B T \text{---(5)}$$

$$\text{From (3) and (5), } \frac{E}{N} = \frac{1}{2}m\overline{v^2} = \frac{3}{2} k_B T$$

Therefore average kinetic energy of a molecule is proportional to absolute temperature of the gas but is independent of pressure, volume or nature of the ideal gas.

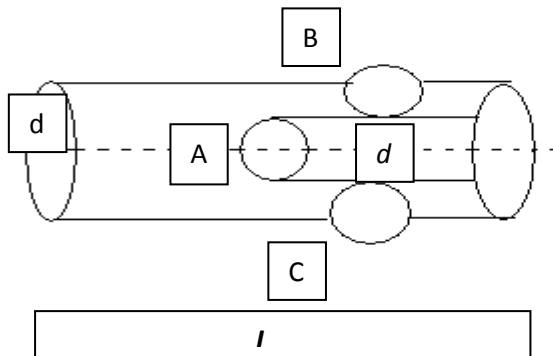
42. The Consider a gas in a container having n molecules per unit volume. Let d be the diameter of a molecule A which is assumed to be motion, while other molecules are at rest. The molecule A collides with other molecules like B and C whose centres are at a distance d from the centre of the molecule A as shown in the figure. If the molecule moves a distance L , then this molecule makes collisions with all molecules lying inside the cylinder of volume $\pi d^2 L$.

If n be the number of molecules per unit volume , then total number of molecules $= n \times \pi d^2 L$
 .Therefore number of collisions suffered by the molecule A $=$ total number of molecules $= \pi n d^2 L$ Now
 mean free path of a molecule is given by $l = \frac{\text{Total distance travelled}}{\text{Number of collisions suffered}}$

$$= \frac{L}{\pi n d^2} = \frac{1}{\pi n d^2}$$

In the above derivation ,it is assumed outer molecules to be at rest . Taking in to consideration the motion of all gas molecules , the mean free path is given by

$$l = \frac{1}{\sqrt{2} \pi n d^2} .$$



$\pi \pi$

