

QUESTION BANK FOR FIRST YEAR P U C

CHAPTER: 14

MATHEMATICAL REASONING

One mark questions

1. What is a mathematically acceptable statement?
2. What kind of sentences are not statements?

NOTE : While dealing with statements ,small letters p,q,r,..... are used.

Eg : p : Ice is cold

3. Check whether the following sentences are statements.Give reasons
 - a. 6 is less than 2
 - b. The moon is a natural satellite of the earth
 - c. Mathematics is interesting
 - d. How far is Delhi from here?
 - e. There are 32 days in a month
 - f. The sum of 3 & 8 is greater than 11
 - g. Square of a number is an even number
 - h. Today is a sunny day
 - i. All natural numbers are real numbers
 - j. Sum of -1 and 9 is 6
 - k. How beautiful the rose is !

4. What is negation of a statement?

NOTE : If p : Ice cream is hot,its negation is written as $\sim p$ and it is Ice cream is not hot

5. Write negation of the following statements
 - a. Jaipur is a city
 - b. Opposite sides of a rectangle have same length
 - c. $\forall a,b \in I, a-b \in I$
 - d. 6 is irrational
6. When is a compound statement with connective 'and' true?
7. When is a compound statement with connective 'and' false?

8. When is a compound statement with connective 'or' true?
9. When is a compound statement with connective 'or' false?
10. What are quantifiers?
11. How is p implies q denoted by?

Two marks questions

1. Write the negation and check whether resulting statements are true or false
 - a. Asia is a continent
 - b. There does not exist a triangle which has all its sides equal
 - c. Product of 2 and 3 is equal to 5
 - d. 2 is the only even prime number.
 - e. Earth is stationary.
2. Define a compound statement with an example.
3. Find the components of the following compound statements and also the connecting word and check whether the compound statement is true or false
 - a. 2 is a positive number or a negative number
 - b. All natural numbers are either even or odd.
 - c. A person who has taken mathematics or electronics can go for masters degree.
 - d. 0 is less than every positive integer and every negative integer
 - e. Bangalore is the capital of Karnataka and Andhra Pradesh.
 - f. 3 is a prime number or it is odd.
 - g. 100 is divisible by 3 and 5
 - h. All animals have 2 legs and 2 eyes
4. State whether 'or' used in the following statements is exclusive or inclusive. Give reasons.
 - a. An ice cream or juice is available with a meal in a hotel.
 - b. A
 student who has studied mathematics or computer science can apply for M.B.A
 - c. To obtain a driving license you need a ration card or a passport.
 - d. The sum of any 2 natural numbers is even or odd.

- e. Banks are closed if it is a holiday or a Sunday.
- f. Students can take science or arts as their combination.
- g. Sun rises or moon sets
- h. $\sqrt{7}$ (square root of 7) is rational or irrational.

5. Identify the quantifier and write negation of the following

- a. There exists a number which is equal to its square.
- b. For every real number x , $x > x+2$
- c. There exists a kingdom for every king in India.

6. Write each of the following in the form "If then"

- a. A rectangle is a square if all the 4 sides are equal
- b. All natural numbers are odd if square of a natural number is even
- c. 24 is divisible by 6 if 25 is a multiple of 5
- d. 5 divides 20 if 5 divides -20

7. Write the converse and contra positive of the following implications

- a. If a parallelogram is a square then it is a rhombus
- b. If 2 sides of a triangle are not equal the the triangle is isosceles
- c. If x is a prime number then x is even
- d. If 2 lines are intersecting then they are not parallel
- e. If 2 is the only even prime number then 6 is divisible by 3

8. Write the following statements with "If then" in 5 different ways conveying the same meaning

- a. If a triangle is equi angular then it is an obtuse angled triangle
- b. If a natural number is even then its square is also even

9. Show that the statement if n is an odd integer then n is prime is false by giving counter example.

10. Combine these 2 statements using 'If and only If'

- a. p : If a rectangle is a square then all its 4 sides are equal
 q : If all the 4 sides of a rectangle are equal ,then the rectangle is a square.
- b. p : If the sum of digits of a number is divisible by 3,then the number is divisible by 3
 q : If a number is divisible by 3,then the sum of its digits is divisible by 3.

11. Given p : 25 is multiple of 5

q : 25 is multiple of 8

Write the compound statements connecting these 2 statements with “and” and “or”. In both cases check the validity of the compound statement

12. Show that the statement “For any real numbers a and b $a^2 = b^2$ implies that $a = b$ ” is not true by giving a counter example.

Three marks questions

1. Write the rules to check whether a statement is true or not

2. Check whether the following is true or false

a. If $x, y \in \mathbb{Z}$ or such that x, y are even then xy is even

3. Check whether the following is true or false by proving its contra positive is $x, y \in \mathbb{Z}$ such that xy is even then both x and y are even.

4. Verify the following by method of contradiction

a. $\sqrt{2}$ is irrational

b. $\sqrt{5}$ is irrational

5. By the method of contra positive, check the validity of the statement “If $a, b \in \mathbb{Z}$ such that ab is odd, then both ‘ a ’ and ‘ b ’ are odd.

6. By giving a counter example, Show that the following statements are not true.

(i) p : If all the angles of a triangle are equal then the triangle is an obtuse angled triangle.

(ii) q : The equation $x^2 - 1 = 0$ does not have a root lying between 0 and 2.

7. Show that the statement “If x is a real number such that $x^3 + 4x = 0$, then x is zero” is true by

(i) Direct method

(ii) Method of contradiction

(iii) Method of contrapositive

ANSWERS

One mark questions

1. A sentence is called mathematically acceptable statement if it is either true or false but not both.
2. The sentences having exclamation, order, question containing variables like today, tomorrow, yesterday, pronouns like he, she etc.
3.
 - a. It is a statement because this sentence is false as $6 > 2$
 - b. This is a statement because it is a true sentence.
 - c. It is not a statement because it differs from person to person
 - d. It is not a statement because it is a question.
 - e. It is a statement because this sentence is always false irrespective of the month as maximum number of days in a month is 31
 - f. It is a statement because the sentence is false
 - g. It is not a statement because in some cases it is true and in some cases it is false .
Eg : square of 7 is odd number but square of 4 is even.
 - h. It is not a statement because it is not clear which day is referred to.
 - i. It is a statement because the sentence is true.
 - j. It is a statement because it is false sentence.
 - k. It is not a statement because it has exclamation
4. It is denial of a statement
5.
 - a. Jaipur is not a city
 - b. Opposite sides of a rectangle do not have same length
 - c. $\forall a, b \in \mathbb{I}, a-b$ does not belong to \mathbb{I}
 - d. 6 is not irrational
6. Compound statement with connective 'and' is true if all its component statements are true.
7. It is false if any one of its component statement is false
8. It is true when one atleast one component statement is true or both the component statements are true

9. It is false when both the component statements are false
10. Quantifiers are phrases like “there exists” and “for all”.
11. $p \Rightarrow q$

Two marks questions

1.

- a. Asia is not a continent. The statement is false
- b. There exists a triangle which has all its sides equal. This statement is true because we know that equilateral triangle has all its sides equal.
- c. Product of 2 and 3 is not equal to 5. This statement is true.
- d. 2 is not the only even prime number. This statement is false.
- e. Earth is not stationary. This is a true statement.

2. It is a statement which is made up of 2 or more statements. Each statement is called a component statement. Eg : The sky is black and milk is white.

3.

- a. p : 2 is a positive number
 q : 2 is a negative number
 connecting word is “or”. The compound statement is true .
- b. p : all natural numbers are even
 q : all natural numbers are odd
 connecting word is “or”. The compound statement is false.
- c. p : a person who has taken mathematics can go for masters degree
 q : a person who has taken electronics can go for masters degree
 connecting word is “or”. The compound statement is true
- d. p : 0 is less than every positive integer
 q : 0 is less than every negative integer
 connecting word is “and”. The compound statement is false.
- e. p : Bangalore is the capital of Karnataka

q: Bangalore is the capital of Andhra Pradesh
connecting word is "and". The compound statement is false.

- f. p : 3 is a prime number
q : 3 is odd number
connecting word is "or". The compound statement is true.
- g. p : 100 is divisible by 3
q : 100 is divisible by 5
connecting word is "and". The compound statement is false.
- h. p : All animals have 2 legs
q : All animals have 2 eyes
connecting word is " and". The compound statement is false.

4.

- a. This is called exclusive 'or' since a person cannot have both ice cream and juice
- b. This is inclusive 'or' since students who have taken both maths and computer science can apply for MBA as well as the students who have taken only one of these statements.
- c. This is inclusive 'or' because a person can have both a passport and a ration card.
- d. This is called exclusive 'or' because it is not possible for sum of 2 natural numbers to be both even and odd.
- e. This is inclusive 'or' since banks are closed on holidays as well as on Sundays.
- f. This is called exclusive 'or' because a student cannot take both arts and science combination at a time.
- g. This is called exclusive 'or' because only one action takes place at time.
- h. This is inclusive 'or' since $\sqrt{7}$ is irrational.

5.

- a. "There exists " is the quantifier
There does not exists a number which is equal to its square
- b. "For every" is the quantifier
There exists real number x such that x is not greater than x+2
- c. "There exists" is the quantifier
There does not exist a kingdom for every king in India.

6.

- a. If all the 4 sides in a rectangle are equal then it is a square.
- b. If square of a natural number is even then all natural numbers are odd.
- c. If 25 is a multiple of 5 then 24 is divisible by 6.
- d. If 5 divides -20 then 5 divides 20.

7.

- a. p : a parallelogram is a square
 q : it is a rhombus
converse : $q \Rightarrow p$ If it is a rhombus then parallelogram is a square
contrapositive : $\sim q \Rightarrow \sim p$ If it is not a rhombus then parallelogram is not a square.
- b. p : 2 sides of a triangle are not equal
 q : triangle is isosceles
converse : $q \Rightarrow p$ If the triangle is isosceles then 2 sides of a triangle are not equal
contrapositive : $\sim q \Rightarrow \sim p$ If the triangle is not isosceles then 2 sides of a triangle are equal
- c. p : x is a prime number
 q : x is even
converse : $q \Rightarrow p$ If x is even then x is a prime number
contrapositive : $\sim q \Rightarrow \sim p$ If x is not even then x is not a prime number
- d. p : 2 lines are intersecting
 q : They are not parallel
converse : $q \Rightarrow p$ If they are not parallel then 2 lines are intersecting
contrapositive : $\sim q \Rightarrow \sim p$ If they are parallel then 2 lines are not intersecting
- e. p : 2 is only even prime number
 q : 6 is divisible by 3
converse : $q \Rightarrow p$ If 6 is divisible by 3 then 2 is the only even prime number
contrapositive : $\sim q \Rightarrow \sim p$ If 6 is not divisible by 3 then 2 is not the only even prime number

8.

- a.
 - 1) A triangle is equiangular implies that it is an obtuse angled triangle
 - 2) If a triangle is not obtuse angled then it is not equiangular
 - 3) 3) A triangle is equiangular only if it is an obtuse angled triangle
 - 4) 4) For an obtuse angled triangle it is sufficient that it is equiangular.

5) 5) For an equiangular triangle it is necessary that it is an obtuse angled triangle

b.

- 1) A natural number is even implies that its square is also even
- 2) A natural number is even only if its square is even
- 3) For a natural number to be even it is necessary that its square is even
- 4) For the square of a natural number to be even it is sufficient that the number is even
- 5) If the square of a natural number is not even then the natural number is not even

9. This statement is if p then q.

$n=9$ is the counter example because 9 is an odd number which is not prime .

10.

- a. $p \Rightarrow q$: A rectangle is a square if and only if all its 4 sides are equal
- b. $p \Rightarrow q$: A number is divisible by 3 if and only if the sum of its digits is divisible by 3

11. The compound statement with "and " is

25 is a multiple of 5 and 8 \rightarrow This is a false statement

The compound statement with "or" is

25 is a multiple of 5 or 8 \rightarrow This is a true statement

12. When $a = -4$, $a = +4$ then $a^2 = b^2$,

but a is not equal to b. Therefore statement is not true.

Three marks questions

1. Rule 1 : If p, q are mathematical statements, then in order to show that statement p and q is true, following steps are followed
Step 1 : Show that statement p is true
Step 2 : Show that statement q is true
Rule 2 : If p, q are mathematical statements, then in order to show that statement p or q is true, we need to show that any one of the following is true
Case i : By assuming that p is false, show that q must be true
Case ii : By assuming that q is false, show that p must be true
Rule 3 : If p, q are mathematical statements, then in order to prove the statement If p then q we need to show that any one of the following is true
Case i : By assuming that p is true, prove that q must be true (Direct method)
Case ii : By assuming that q is false, prove that p must be false (Contra positive method)
Rule 4 : : If p, q are mathematical statements, then in order to prove the statement If and only If we need to show that
Case i : If p is true then q is true and
Case ii : If q is true then p is true

2.

- a. Let $p : x, y \in \mathbb{Z}$ such that x, y are even
 $q : xy$ is even
we apply case 1 of Rule 3 i.e assume that p is true, then q is true
 p is true means x, y are even integers then
 $x=2n, n \in \mathbb{I} \quad y=2m, m \in \mathbb{I} \quad$ Thus
 $xy = 4nm = 2(2nm)$ which is even. Therefore the given statement is true.
Proof : By contra positive method
Assume q is not true
 $\sim q =$ product of xy is odd
This is possible only if either x or y is odd,
This shows that p is not true, therefore $\sim q \Rightarrow \sim p$

3. Let $p : xy$ is even,

q : Both x and y are even

we have to check whether $p \Rightarrow q$ is true or not true i.e

by checking contra positive statement i.e $\sim q \Rightarrow \sim p$

$\sim q$: It is false that both x and y are even

i.e. x (or y) is odd then $x=2n+1$, $n \in \mathbb{I}$

Therefore $xy = (2n+1)y$, $n \in \mathbb{I}$

Therefore xy is odd i.e. $\sim p$ is true

$\sim q \Rightarrow \sim p$ Therefore the given statement is true by method of contradiction

4.

- a. Let given statement be false i.e. we assume $\sqrt{2}$ to be rational. Therefore there exists positive integers a and b such that $\sqrt{2}=a/b$ where a & b have no common factors. squaring we get $2=a^2/b^2$

$$\Rightarrow a^2=2b^2 \Rightarrow 2 \text{ divides } a$$

Therefore there exists c such that $a=2c$, then $a^2=4c^2$ and $a^2=2b^2 \Rightarrow 2b^2=4c^2$, therefore $b^2=2c^2 \Rightarrow 2 \text{ divides } b$.

But we have already shown that 2 divides a. This implies that 2 is a common factor of both a & b which contradicts our assumption that a & b have no common factors. Therefore our assumption that $\sqrt{2}$ is rational is wrong.

Therefore the statement $\sqrt{2}$ is irrational is true.

- b. Let given statement be false i.e. we assume $\sqrt{5}$ to be rational. Therefore there exists positive integers a and b such that $\sqrt{5}=a/b$ where a & b have no common factors. squaring we get $5=a^2/b^2$

$$\Rightarrow a^2=5b^2 \Rightarrow 5 \text{ divides } a$$

Therefore there exists c such that $a=5c$, then $a^2=25c^2$ and $a^2=5b^2 \Rightarrow 5b^2=25c^2$, therefore $b^2=5c^2 \Rightarrow 5 \text{ divides } b$.

But we have already shown that 5 divides a. This implies that 5 is a common factor of both a & b which contradicts our assumption that a & b have no common factors. Therefore our assumption that $\sqrt{5}$ is rational is wrong.

Therefore the statement $\sqrt{5}$ is irrational is true.

5. Taking p : ab is odd

q : both a and b are odd

To check $\sim q \Rightarrow \sim p$

$\sim q$: It is false that both 'a' and 'b' are odd

\Rightarrow One of a, b is even, let a be even

$$a = 2m \text{ for } m \in \mathbb{Z}$$

$$ab = 2mb = 2(mb) = \text{even number} = \sim p$$

It means ab is even

Therefore $\sim p$ is true

Therefore $\sim q \Rightarrow \sim p$ is true

6. (i) If each angle of a triangle is 60° , it will form an acute angled triangle. Therefore the statement is not true.

(ii) The roots of the equation $x^2 - 1 = 0$ are +1, -1 and the root +1 lies between 0 and 2. Therefore the statement is not true.

7.

(i) When $x=0$, $x^3 + 4x = 0 + 4(0) = 0$

Therefore $x=0$ is a solution

(ii) Given x is a real number and $x^3 + 4x = 0$, assume that x is not equal to 0

Then $x(x^2 + 4) = 0 \Rightarrow x = +2i, -2i$ which is not a real number, but $x \in \mathbb{R}$ which is a contradiction, therefore our assumption that x not equal to zero is wrong

(iii) when x is not equal to zero, then $x^3 + 4x = x(x^2 + 4)$ is not equal to zero.

Because x is not equal to zero and since x is a real number, $x^2 + 4$ is not equal to zero.
