QUESTION BANK

UNIT-6 CHAPTER-8
GRAVITATION

I . One mark Questions:

1. State Kepler's law of orbits.
2. State Kepler's law of areas.
3. State Kepler's law of periods.
4. Which physical quantity is conserved in the case of law of areas?
5. State universal law of gravitation.
6. Express universal law of gravitation in mathematical form.
7. Express universal law of gravitation in vector form.
8. What is the value of gravitational constant?
9. Name the experiment that has given the value of gravitational constant.
10. Write the relation between g and G.
11. Write the expression for acceleration due to gravity at a point above the surface of the earth.
12. Write the expression for acceleration due to gravity at a point below the surface of the earth.
13. Write the expression for gravitational potential energy of a particle at a point due to the earth.
14. Write the expression for gravitational potential energy between two masses separated by a distance.
15. What is escape speed?
16. What is a satellite?
17. What is the value of period of moon?
18. What are geostationary satellites?
19. What are polar satellites?
20. Give the period of geostationary satellites?

21. Name the group of the geostationary satellites sent up by India?
22. Give an important use of geostationary satellites.
23. Write the dimensional formula for gravitational constant.
24. Define orbital speed of a satellite around the earth.
25. Name the force that provides the necessary centripetal force for the earth around the sun in an approximately circular orbit.
26. How does the escape velocity of a body varies with the mass of the earth?
27. How does speed of the earth changes when it is nearer to the sun?
28. What are central forces?
29. Give the value of escape speed for moon.
30. The Newton's law of gravitation is said to be universal law. Why?
31. Differentiate gravitation and gravity.
32. Give the relation between escape and orbital speed.
33. Write the dimensional formula of "g'.

II. <u>Two mark questions:</u>

1. State and explain Kepler's law of orbits.
2. State and explain Kepler's law of areas.
3. State and explain Kepler's law of periods.
4. State universal law of gravitation. Express it in mathematical form.
5. Moon has no atmosphere. Why?
6. Define gravitational potential energy of a body. Give an expression for it.
7. State and explain Newton's law of gravitation.
8. Derive the relation between gravitational constant and acceleration due to gravity.
9. Write the expression for escape speed on the earth. Give its value in the case of earth.
10. Give two uses of polar satellites.
11. Explain the state of weightlessness of a body.
12. Astronauts in satellite experience weightlessness. Explain why?
13. A freely falling body is acted up on by a constant acceleration. Explain why?

14. An object weighs more on the surface than at the centre of the earth. Why?
15. A body weighs more at pole than at equator of the earth. Explain why?
16. “Cavendish weighed the earth”. Why this statement is popular? Justify with the expression for the mass of the earth.
17. The radius and mass of a planet are two times that of the earth’s values. Calculate the acceleration due to gravity on the surface of the planet.
18. Who proposed ‘Geocentric theory’? Give the brief account of the theory.
19. Who proposed ‘Heliocentric theory’? Give the brief account of the theory.
20. The orbiting speed of an earth’s satellite is 10 km s^{-1}. What is its escape speed?
21. Distinguish between ‘Geostationary’ and ‘polar’ satellites.
22. Give any two applications of artificial satellites.

III. Four/Five mark questions:

1. State Kepler’s laws of planetary motion and explain law of orbits and law of areas.
2. Derive $g=(GM_E)/R_E^2$, where the symbols have their usual meaning.
3. Derive the expression for acceleration due to gravity at a point above the surface of the earth.
4. Derive the expression for acceleration due to gravity at a point below the surface of the earth.
5. Derive the expression for gravitational potential energy of a particle at a point due to the earth.
6. Obtain the expression for escape speed.
7. Derive the expression for orbital speed of a satellite/ period of a satellite around the earth.
8. Obtain the expression for energy of an orbiting satellite.
9. Show that the law of areas follows from the law of conservation of angular momentum.
10. Describe Cavendish’s experiment to determine the value of gravitational constant G.

ANSWERS

Answers to one mark questions:

1. All planets move in elliptical orbits with the sun situated at one of the foci of the ellipse.
2. The line that joins any planet to the sun sweeps equal areas in equal intervals of time.
3. The square of the time period of revolution of a planet is proportional to the cube of the semi – major axis of the ellipse traced out by the planet.
4. "Angular momentum" is conserved in the case of law of areas.
5. Every body in the universe attracts every other body with a force which is directly proportional to the product of their masses and inversely proportional to the square of the distance between them.
6. The magnitude of the force $\vec{F}$ on a point mass m_2 due to another point mass m_1 at a distance r is given by ,

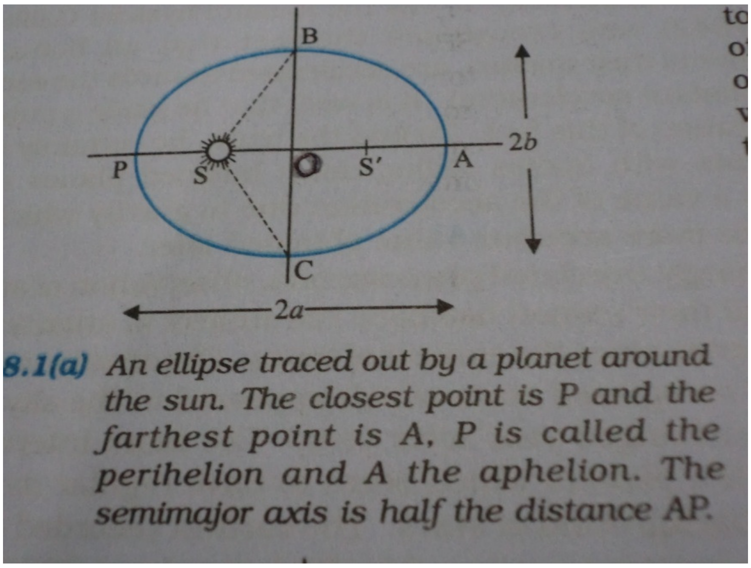
   $|\vec{F}| = F = G\,(m_1m_2)/r^2$

7. The force $\vec{F}$ on a point mass m_2 due to another point mass m_1 at a distance r is given by $|\vec{F}| = F = G\,[(m_1m_2)/r^2](-\hat{r}) = -\,G\,(m_1m_2/r^2)\,\hat{r}$, $\quad \vec{F} = -\,[(G\,m_1m_2)/|\vec{r}|^3]\,\vec{r}$
8. $G = 6.67 \times 10^{-11}\ N\,m^2/kg^2$
9. Cavendish's experiment.
10. $g = GM_E/R_E^2$ for earth,
    Where, g is acceleration due to gravity on the surface of the earth.
    G is gravitational constant,
    M_E is mass of earth,
    R_E is radius of the earth.
11. $G(h) = g\,(1 - 2h/R_E)$ where h is height of the point, R_E is radius of the earth, g is acceleration due to gravity on the surface of the earth.
12. $g\,(d) = g\,(R_E - d)/R_E = g\,(1 - d/R_E)$
    where d is depth of the point,
    R_E is the radius of the earth, g is acceleration due to gravity on the surface of the earth.
13. $W(r) = -\,GM_Em/r$
    Where G is gravitational constant,
    M_E is mass of the earth,
    m is mass of the particle,
    r is distance of the particle from the surface of the earth.
14. $V = -\,Gm_1m_2/r$
    Where G is gravitational constant,
    m_1 & m_2 are masses,
    r is distance between the masses,
15. Escape speed is the minimum speed required for an object to escape from the earth (i.e.to reach infinity (or) zero gravitational potential energy.)
16. A satellite is an object which revolves around the planet in circular or elliptical orbit.
17. The period of revolution around the earth (≈ rotation about its own axis) of moon is 27.3 days.
18. Geostationary satellites are the satellites in circular orbits around the earth in the equatorial plane with the period T = 24 hrs.

19. Polar satellites are low altitude (h ≈ 500 to 800 km) satellites that go around the poles of the earth in a north-south direction where as the earth rotates around its axis in an east-west direction.
20. The period of geostationary satellite T is 24 hrs.
21. The group of the geostationary satellites sent up by India is INSAT group of satellites.
22. Geostationary satellites are widely used for telecommunications in India.
23. The dimensional formula for G is $[M^{-1} L^{3} T^{-2}]$.
24. Orbital speed of a satellite around the earth is the speed required to put a satellite into its orbit.
25. Gravitational force between earth and the sun provides the necessary centripetal force for the earth around the sun in an approximately circular orbit.
26. The escape speed of a body is proportional to the square root of the mass of the earth.
27. The speed of the earth increases when it is nearer to the sun.
28. Central forces are always directed towards or away from a fixed point, that is along the position vector of the point of application of the force with respect to the fixed point.
29. Escape speed on the surface of the moon is 2.3 km/s.
30. Newton's law of gravitation is independent of the nature of the interacting bodies in nature. Therefore it is called as universal law.
31. The force of attraction between any two bodies in nature is gravitation where as the force of attraction between a body and the earth is gravity.
32. Escape speed, $v_e = \sqrt{2}\, v_o$. Where v_o is orbital speed.
33. The dimensional formula of g is $[M^0 L^1 T^{-2}]$.

II. Answers to two mark questions:

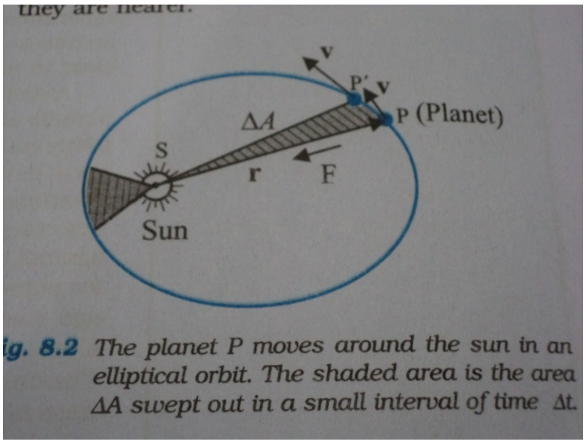
1. Kepler's law of orbits:- All planets move in elliptical orbits with the sun situated at one of the foci of the ellipse. Ellipse is a closed curve.



8.1(a) An ellipse traced out by a planet around the sun. The closest point is P and the farthest point is A, P is called the perihelion and A the aphelion. The semimajor axis is half the distance AP.

Figure shows an ellipse traced out by a planet around the sun. The closest point is P (perihelion) and the farthest point is A (aphelion).

2. Kepler's Law of areas:- The line that joins any planet to the sun sweeps equal areas in equal intervals of time.
   Planets move slower when they are farther from the sun than when they are nearer.



ig. 8.2 The planet P moves around the sun in an elliptical orbit. The shaded area is the area ΔA swept out in a small interval of time Δt.

   The planet P moves around the sun in an elliptical orbit. The shaded area is the area Δ A swept out in a small interval of time Δ t.
3. Kepler's law of periods:- The square of the time period of revolution of a planet is proportional to the cube of the semi - major axis of the ellipse traced out by the planet.

   If T is the period of revolution of the planet and a is semi - major axis of the elliptical path, then$T^2 \alpha a^3$.
4. Every body in the universe attracts every other body with a force which is directly proportional to the product of their masses and inversely proportional to the square of the distance between them. The magnitude of the force $\vec{F}$ on a point mass m_1 due to another point mass m_2 is;
   $|\vec{F}| = F = G\, m_1 m_2/r^2$
   Where r is the distance between m_1 and m_2.
5. Escape speed of a body on the surface of moon is 2.3 km/s. This value is five times smaller than the escape speed on the surface of the earth. Gas molecules if formed on the surface on the moon having velocities larger than this will escape from the gravitational pull of the moon. Because of this reason moon has no atmosphere.
6. Potential energy of a body arising out of the force of gravity is called the gravitational potential energy. The expression for gravitational potential energy of a body due to earth is
   $V = W(r) = -\,G\, M_E\, m/r$
   Where M_E is mass of the earth,
   m is mass of the body,
   r is the distance of the body from the centre of the earth.

7. Every body in the universe attracts every other body with a force which is directly proportional to the product of their masses and inversely proportional to the square of the distance between them. The force $\vec{F}$ on a point mass $m_2$due to another point mass m_1 has the magnitude
 $|\vec{F}| = F = G\, m_1 m_2/r^2$
 where G is the universal gravitational constant. The gravitational force is attractive.
 That is the force $\vec{F}_{12}$ on the body 1 due to 2 and $\vec{F}_{21}$ on the body 2 due to 1 are related as $\vec{F}_{12} = -\ \vec{F}_{21}$.

8. Let m be the mass of the body situated on the surface of the earth of radius R_E and mass M_E.
 According to Newton's law of gravitation the force between the body close to the surface of the earth and the earth is;
 $F = G\, mM_E/R_E^2$ ………. (1)
 Where G is gravitational constant.
 But according to Newton's 2nd law of motion, the gravitational force exerted on the body by the earth;
 F = mg…….(2)
 Where g is acceleration due to gravity.
 $\therefore$ From equations (1)&(2),
 $mg = GmM_E/R_E^2$
 (or) $g = GM_E/R_E^2$.
9. Expression for escape speed on the earth ;
 $v_e = \sqrt{\frac{2GM_E}{R_E^2}}$ (or) $v_e = \sqrt{2gR_E}$
 where G is gravitational constant,
 M_E is mass of the earth,
 R_E is radius of the earth,
 g is acceleration due to gravity.
 The value of escape speed on the earth is 11.2km/s.
10. Polar satellites are useful for (a). remote sensing. (b). meteorology/(c). Environmental studies of the earth.
11. Weight of a body is the force with which the earth attracts it. If there is no opposite force (support) exerted on the body it would fall down. During this free fall, the object appears to lose its weight and becomes weightless, since there is no upward force on the body. This state of a body is called weightlessness.
12. The necessary centripetal force of the satellite around the earth is provided by the gravitational force of attraction between earth and satellite. Every part in a satellite around the earth has acceleration towards the centre of the earth which is acceleration due to gravity of the earth at that position. Therefore everything inside it is in a state of free fall. This is as if we were falling towards the earth from a height. In a manned satellite astronauts inside the satellite experience no gravity. Therefore they experience a state of weightlessness.
13. The expression for acceleration due to gravity on the earth's surface is
 $g = \frac{GM_E}{R_E^2}$

where G is gravitational constant,

M_E is mass of the earth,

R_E is radius of the earth.

An object released near the surface of the earth is accelerated downward under the influence of the force of gravity. The magnitude of acceleration due to gravity is g. If air resistance is neglected, the object is said to be in free fall. If the height through which the object falls is small compared to the earth's radius, g can be taken to be constant equal to $9.8 m/s^2$. Therefore a freely falling body is acted upon by a constant acceleration.

14. The weight of a body at a place on the surface of the earth is given by W = mg. since the value of g is maximum on the surface, the object weighs more on the surface. At the centre of the earth, the value of g is zero. Therefore the weight of the body at the centre of the earth is zero.
15. The weight of a body at a place on the surface of the earth is given by W = mg. Since the value of g is maximum at poles than at the equator, the weight of the body is more at the poles than at the equator.
16. The acceleration experienced by the mass m due to the earth is called acceleration due to gravity g, which is related to the force F by Newton's 2nd law from the relation F =mg.

    Therefore $g = \frac{F}{m} = \frac{GM_E}{R_E^2}$

    Acceleration g is measurable. R_E is a known quantity. The measurement of G by Cavendish's experiment and with the knowledge of g and R_E, the mass of the earth M_E can be estimated. Because of this reason there is a popular statement regarding Cavendish, "Cavendish weighed the earth".
17. Let mass of the earth is M_E and that of planet is $M_P = 2M_E$. Also radius of the earth is R_E and that of planet is $R_P = 2R_E$.

    But we know that on the surface of the earth, $g_E = \frac{GM_E}{R_E^2}$………………………… (1)

    and on the surface of the planet, $g_P = \frac{GM_P}{R_P^2}$……………………………………(2)

    $\frac{(1)}{(2)} \Rightarrow g_E/g_P = 2$.

    Therefore $g_P = g_E/2$. But we know that $g_E = 9.8 m/s^2$.

    $g_P = 9.8/2 = 4.9 m/s^2$.
18. Geocentric theory was proposed by Ptolemy about 2000 years ago. According to this theory, all celestial objects, stars, the sun and the planets all revolved around the earth. The only motion that was thought to be possible for celestial objects was motion in a circle. Complicated schemes of motion were put forward by Ptolemy in order to describe the observed motion of the planets. The planets were described as moving in circles with the centre of the circles themselves moving in larger circles.
19. Heliocentric theory was proposed by Nicolas Copernicus. According to this theory, sun was at the centre with the planets revolving around. Planets moved in circles around a fixed central sun. This theory was discredited by the church but Galileo supported it and faced prosecution from the State. Tycho Brahe recorded observations of the planets with naked eyes. This data was analyzed by Kepler and later formulated three laws called Kepler's laws of planetary motion, that support Helio centric theory.
20. Orbiting speed of earth's satellite, v_o = 10 km/s. Escape speed, v_e = ?

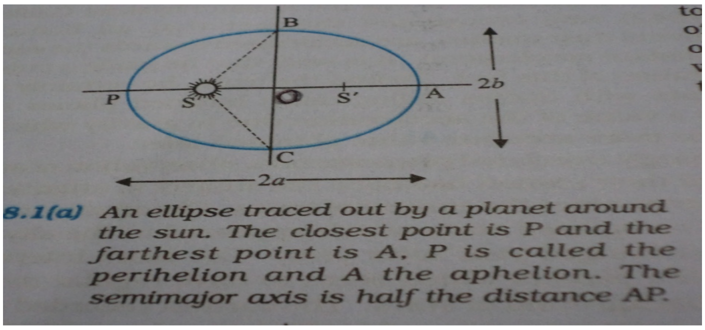
    We have $v_e = \sqrt{2}\, v_o = \sqrt{2} \times 10 = 14.14 m/s$.
21. Geostationary satellites are the satellites in circular orbits around the earth in the equatorial plane with the period T = 24hrs.

Polar satellites are low altitude (h≈500 to 800km) satellites and go around the poles of the earth in a north-south direction with a time period of around 100 minutes where as the earth rotates around its axis in an east- west direction.

22. Artificial satellites find applications in the fields like telecommunication, geophysics, meteorology.

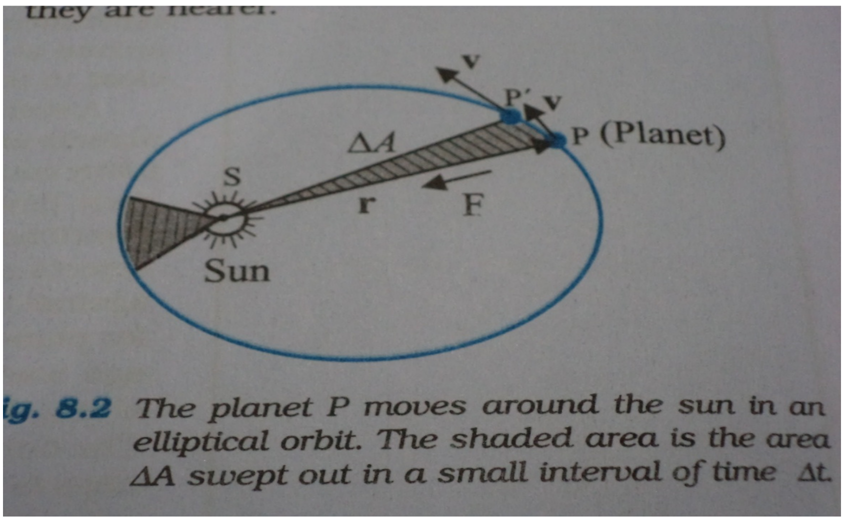
Answers to four/five mark questions:

1. Kepler's laws of planetary motion:-
   (i).Law of orbits:- All planets move in elliptical orbits with the sun situated at one of the foci of the ellipse.



8.1(a) An ellipse traced out by a planet around the sun. The closest point is P and the farthest point is A. P is called the perihelion and A the aphelion. The semimajor axis is half the distance AP.

This law was a deviation from the Copernican model which allowed only circular orbits. Circle is a closed curve and is a special case of ellipse.
The midpoint of the line PA is the centre of the ellipse O and the length PO = AO is called the semi-major axis of the ellipse. For a circle, the two foci merge into one and the semi-major axis becomes the radius of the circle.
(ii). Law of areas:- The line that joins any planet to the sun sweeps equal areas in equal intervals of time.



ig. 8.2 The planet P moves around the sun in an elliptical orbit. The shaded area is the area ΔA swept out in a small interval of time Δt.

This law comes from the observations that planets appear to move slower when they are farther from the sun than when they are nearer. Δ A is the area swept out in a small interval of time Δ t.

(iii). Law of periods:- The square of the time period of revolution of a planet is proportional to the cube of the semi-major axis of the ellipse traced out by the planet.

2. Let m be the mass of the body on the surface of the earth of radius R_E . The entire mass M_E of the earth is concentrated at the centre of the earth.

 The magnitude of the force acting on the mass m is

 $F=\frac{GmM_E}{R_E^2}$ ……………………(1)

 If the entire earth is assumed to be of uniform density ρ, its mass is

 $M_E = \frac{4}{3}\pi R_E^3 \rho$………………….(2) (since volume of the earth $V_E = \frac{4}{3}\pi R_E^3$)

 Therefore from (1)and (2) $F = \frac{Gm(\frac{4}{3})\pi\rho R_E^3}{R_E^2}$ (But $(4/3)\pi\rho = \frac{M_E}{R_E^3}$)

 Therefore $F = Gm(\frac{M_E}{R_E^3})\frac{R_E^3}{R_E^2}$

 That is $F = \frac{GmM_E}{R_E^2}$…………………….(3)

 By Newton's 2nd law of motion, F = mg ………………….(4) is the force on the mass m and g is the acceleration experienced by the mass m.

 Therefore from (3) and (4)

 $g = \frac{F}{m} = \frac{GmM_E}{R_E^2 m}$

 that is, $g = \frac{GM_E}{R_E^2}$.

3. Consider a point mass m at a height h above the surface of the earth as shown in figure.

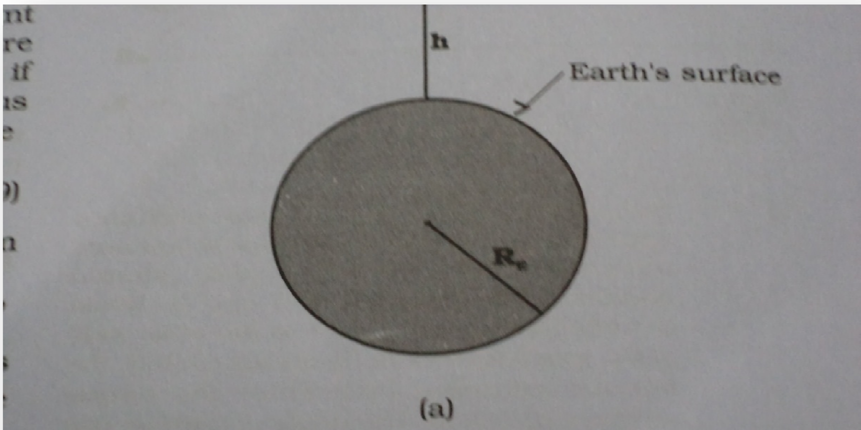


Fig. 8.8 (a) g at a height h above the surface of the earth.

The radius of the earth is R_E . Since this point is outside the earth, its distance from the centre of the earth is $(R_E + h)$. If F (h) is the magnitude of the force on the point mass m,
Then $F(h) = GM_Em/(R_E+h)^2$ ……………………………………….(1)
The acceleration experienced by the point mass is F(h)/m = g(h) and we get
$g(h) = F(h)/m = GM_E/(R_E+h)^2$……………………....................(2)
This is clearly less than the value of g on the surface of earth, $g = GM_E /R_E^2$
For $h \ll R_E$ we can expand the RHS of equation (2), $g(h) = GM_E/[R_E^2(1+h/R_E)^2]$
$= g (1+h/R_E)^{-2}$
For $h/R_E \ll 1$, using binomial expression,
$g(h) = g(1-2h/R_E)$…………………….(3)
Equation (3) tells that for small heights h above the surface of the earth, the value of g decreases by a factor $(1-2h/R_E)$.

4. Consider a point mass m at a depth d below the surface of the earth as shown in figure.

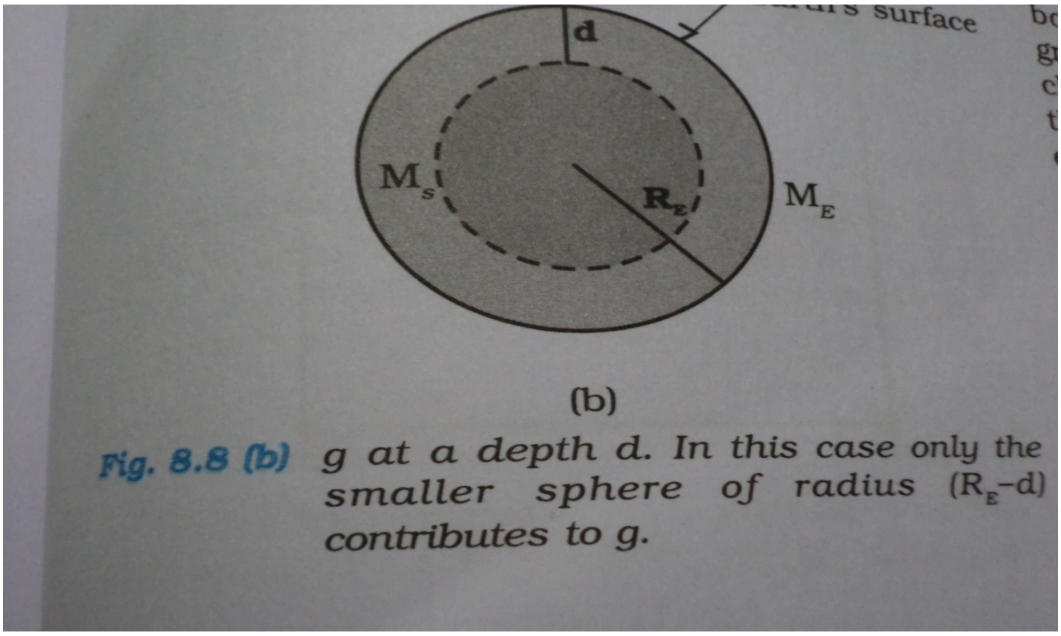


Fig. 8.8 (b) g at a depth d. In this case only the smaller sphere of radius (R_E-d) contributes to g.

Its distance from the centre of the earth is (R_E-d). The earth can be thought of as being composed of a smaller sphere of radius $(R_E – d)$ and a spherical shell of thickness d. The force on m due to the outer shell of thickness d is zero. The point mass m is outside the smaller sphere of radius $(R_E – d)$. The force due to this smaller sphere is just as if the entire mass of the smaller sphere is concentrated at the centre. If M_S is the mass of the smaller sphere, then $M_S/M_E = (R_E – d)^3/R_E^3$……………………………..(1)
(Since mass of a sphere is proportional to the cube of its radius).
Therefore the force on the point mass is $F(d) = GM_Sm/(R_E – d)^2$………………………(2)
Substituting for M_S we get
$F(d) = GM_Em(R_E – d)/R_E^3$..................................(3)
Therefore the acceleration due to gravity at a depth d is,
$g(d) = F(d)/m$
that is $g(d) = [GM_E/R_E^3](R_E – d)$

$= g\,(R_E - d)/R_E$

$g(d) = g(1 - d/R_E)$……………………………(4)

Therefore as we go down below earth's surface, the acceleration due to gravity decreases by a factor $(1 - d/R_E)$.

5. Gravitational potential energy of a particle at a point due to the earth is defined as the amount of work done in displacing the particle from infinity to that point in the gravitational field. It is denoted by V. Consider a particle of mass m placed at a distance r from the centre of the earth of mass M . The gravitational force of attraction between the earth and the particle is,

   $F = GMm/r^2$.

   If the particle of mass m is displaced through a small distance dr towards the earth then, work done is given by ,

   $dW = F\,dr = [GMm/r^2]\,dr$……………………………….(1)

   Amount of work done in displacing the particle of mass m from infinity to a distance r with respect to the earth is given by $\int_\infty^r dW = \int_\infty^r [GMm/r^2]dr$

   $= GMm\ \left[\frac{r^{-1}}{-1}\right]^{r}_{\text{infinity}}$

   $= -\,GMm\,[1/r]^{r}_{\text{infinity}}$

   Therefore $W = -GMm[(1/r)-(1/\infty)] = -\,GMm/r$

   The amount of work done is equal to the gravitational potential energy V.

   Therefore $V = -\,GMm/r$.

6. The minimum speed required to project an object vertically upwards from the surface of the earth, so that it escapes from the gravitational influence of the earth and never returns to the earth, is called escape speed.

   Consider an object of mass m at a distance of r from the centre of the earth.

   The gravitational force acting on the mass m is given by

   $F = GMm/r^2$ …………………………(1)

   This force acts towards the centre of the earth.

   Let dr be the small distance covered by the object away from the centre of the earth.

   Therefore the work done on the object against the gravitational force of attraction of the earth is

   $dW = \vec{F}.\overrightarrow{dr} = F\ dr\ \cos 180^0 = -\,F\,dr$

   From equation (1) we get, $dW = -\,[GMm/r^2]\,dr$……………………(2)

   Therefore total work done to displace the object from the surface of the earth [i.e, r=R] to [$r = \infty$] is calculated by integrating equation(2) between the limits R and ∞ .

   Therefore $\int_R^\infty dW = \int_R^\infty -[GMm/r^2]$ dr

   $W = -GMm\left[\frac{r^{-1}}{-1}\right]^{\infty}_{R} = GMm\left[\frac{1}{r}\right]^{\infty}_{R} = GMm\left[\frac{1}{\infty} - \frac{1}{R}\right]$

   Or $W = -\,GMm/R$ [since $\frac{1}{\infty} = 0$] this work done is equal to the potential energy (V) of the object of mass m. That is $V = -\,GMm/R$. Let v_e is the escape speed of the object of mass m then its kinetic energy is $K.E = \frac{1}{2}mv_e^2$. If kinetic energy of the object = magnitude of potential energy of the object then,

   $\frac{1}{2}mv_e^2 = GMm/R$

   Or $v_e^2 = \frac{2GM}{R}$

$v_e = \sqrt{\frac{2GM}{R}}$

but $GM/R^2 = g$

or $GM = g R^2$

Therefore $v_e = \sqrt{2gR}$.

7. The speed with which a satellite moves in its orbit around the earth is called orbital speed. The time taken by a satellite to complete one revolution in its orbit around the earth is called period of a satellite.

   Let a satellite of mass m revolves around the earth in an orbit at a height of h from the surface of the earth. If R is the radius of the earth, then the radius of the orbit of the satellite is R + h. Let v_o be the orbital speed of the satellite.

   The gravitational force of attraction between the earth and the satellite provides the necessary centripetal force to the satellite to move in a circular orbit around the earth.

   i.e, Gravitational force = centripetal force

   $$\frac{GMm}{(R+h)^2} = \frac{mv_o^2}{(R+h)}$$

   That is $v_o^2 = \frac{GM}{(R+h)}$

   Therefore $v_o = \sqrt{\frac{GM}{(R+h)}}$

   But $\frac{GM}{R^2} = g$ or $GM = gR^2$

   Therefore $v_o = \sqrt{\frac{gR^2}{(R+h)}}$

   If the satellite is very close to the earth i.e, h<<

   Then (R+h) ≈ R

   Therefore $v_o = \sqrt{gR}$.

   Period of a Satellite $T = \frac{Circumference\ of\ the\ orbit}{orbital\ speed}$

   i.e, $T = \frac{2\pi(R+h)}{v_o}$

   we know that $v_o = \sqrt{\frac{gR^2}{(R+h)}}$

   Therefore $T = \frac{2\pi(R+h)}{\sqrt{\frac{gR^2}{(R+h)}}}$

   i.e, $T = \frac{2\pi}{R}\sqrt{\frac{(R+h)^3}{g}}$.

   If the satellite is very close to the earth, h<<

   Then (R+h) ≈ R

   Therefore $T = 2\pi\sqrt{\frac{R}{g}}$

8. The total energy of a satellite in Its orbit is equal to the sum of the potential energy and kinetic energy of the satellite. Potential energy of the satellite comes due to the gravitational force of attraction between the earth and the satellite. Kinetic energy of the satellite comes due to the orbital motion of the satellite around the earth.

   Potential energy of the satellite is given by

   $P.E = \frac{-GmM_E}{R_E+h}$....................................(1)

Where G is gravitational constant,

m is the mass of the satellite,

M_E is mass of the earth,

R_E is radius of the earth,

h is height of the satellite above earth's surface.

Kinetic energy of the satellite is given by

K.E = $\frac{1}{2}$m v_o^2……………………………….(2)

Where m is mass of the satellite,

v_o is orbital speed of the satellite.

The centripetal force on the satellite is equal to the gravitational force of attraction between earth and satellite.

That is, $\frac{mv_o^2}{R_E+h} = \frac{GmM_E}{(R_E+h)^2}$

Or $mv_o^2 = \frac{GmM_E}{(R_E+h)}$…………………………..(3)

Substituting equation (3) in (2),

K.E = $\frac{GmM_E}{2(R_E+h)}$……………………………….(4)

From the equations (1) and (4), the total energy of the satellite is,

E = P.E + K.E = $\frac{-GmM_E}{R_E+h} + \frac{GmM_E}{2(R_E+h)}$

Or E = $\frac{-GmM_E}{2(R_E+h)}$

Negative sign in the above expression implies that the satellite is bound to the earth.

9. Law of areas:- The line that joins any planet to the sun sweeps equal areas in equal intervals of time.

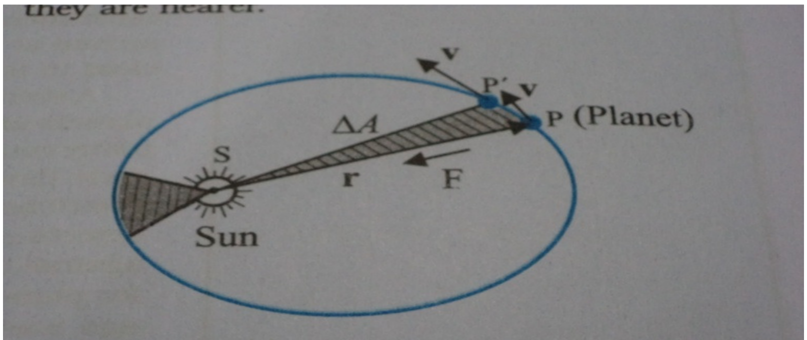


Fig. 8.2 The planet P moves around the sun in an elliptical orbit. The shaded area is the area ΔA swept out in a small interval of time Δt.

The law of areas can be understood as a consequence of conservation of angular momentum which is valid for any central force. A central force is such that the force on the planet is along the vector joining the sun and the planet. Let the sun be at the origin and let the position and momentum of the planet be denoted by $\vec{r}$ and $\vec{p}$ respectively. Then the area swept out by the planet of mass m in time interval Δt is Δ $\vec{A}$ given by

Δ $\vec{A}$ = $^1/_2$ ($\vec{r}$ x $\vec{v}$ Δt)……………………………………..(1)

$\frac{\Delta\vec{A}}{\Delta t}$ = $^1/_2$ ($\vec{r}$ x $\vec{p}$)/m (since $\vec{v} = \frac{\vec{p}}{m}$)

= $\vec{L}/(2m)$…………………………………………………(2)

Where $\vec{v}$ is the velocity, $\vec{L} = \vec{r} \times \vec{p}$,is the angular momentum.
For a central force, which is directed along $\vec{r}$, $\vec{L}$ is a constant as the planet goes around.
$\frac{\Delta \vec{A}}{\Delta t}$ is a constant according to the last equation. This is the law of areas. Gravitation is a central force and hence the law of areas follows.

10. The value of the gravitational constant G entering the Universal law of gravitation can be determined experimentally and this was first done by English scientist Henry Cavendish in 1798, the apparatus used by him is as shown in figure.

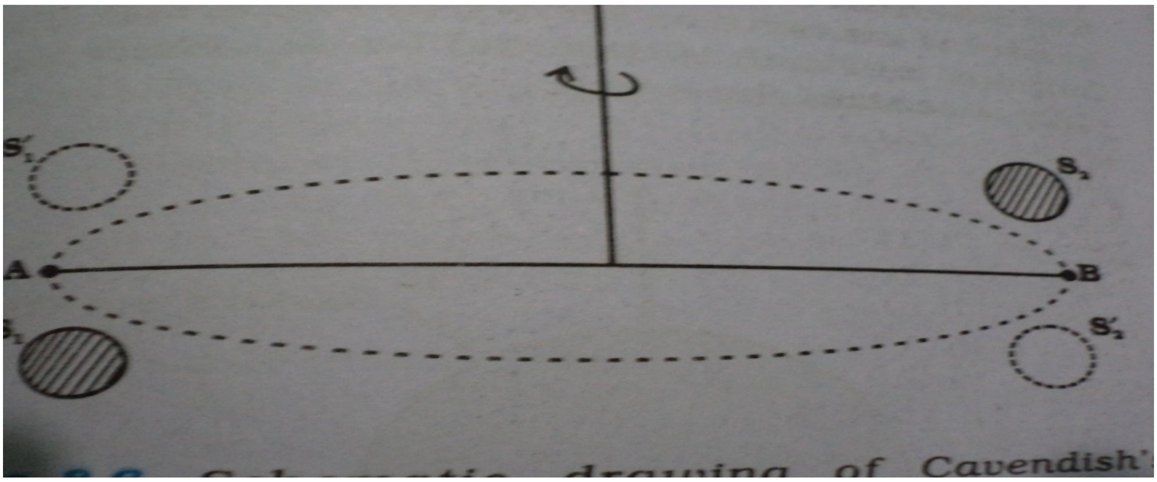


Figure.8.6:- P.T.O

Fig 8.6: Schematic drawing of Cavendish's experiment. S_1 and S_2 are large spheres which are kept on either side (shown shades) of the masses at A or B. When the big spheres are taken to the other side of the masses (shown by dotted circles), the bar AB rotates a little since the torque reverses direction. The angle of rotation can be measured experimentally.

The bar AB has two small lead spheres attached at its ends. The bar is suspended from a rigid support by a fine wire. Two large lead spheres are brought close to the small ones but on opposite sides as shown. The big spheres attract the nearby small ones by equal and opposite force as shown. There is no net force on the bar but only a torque which is clearly equal to F times the length of the bar, where F is the force of attraction between a big sphere and its neighbouring small sphere. Due to this torque, the suspended wire gets twisted till such time as the restoring torque of the wire equals the gravitational torque . If is the angle of twist of the suspended wire, the restoring torque is proportional to equal to τ , where τ is the restoring couple per unit angle of twist. τ can be measured independently e.g. by applying a known torque and measuring the angle of twist. The gravitational force between the spherical balls is the same as if their masses are concentrated at their centres. Thus if d is the separation between the centres of the big and its neighbouring small ball, M and m their masses, the gravitational force between the big sphere and the neighbouring small ball is

$F = GMm/d^2$……………………………………(1)

If L is the length of the bar AB, then the torque arising out of F is, F multiplied by L . At equilibrium, this is equal to the restoring torque and hence

FL = τ ?.

$G (Mm/d^2) L =$ τ ? ………………………………(2)

Observation of ? thus enables one to calculate G from this equation.

From the above equation (2), G can be calculated and its value is

$G = 6.67\text{x}10^{-11}\ Nm^2/kg^2$.

Chapter 8

Gravitation

Problems for Question bank

1) The orbital radius of the Neptune planet around the Sun is ***n*** times that of earth. The Neptune planet takes 164.3 years to complete one revolution. Find the value of ***n.*** (The planetary orbits are assumed to be circular)

(Ans $\boldsymbol{n = 30}$)

2) The Planet Mars take 1.88 years to complete on revolution around the sun. The mean distance of the earth from the Sun is 1.5×10^8 km. Calculate that of planet Mars?

(Ans $\boldsymbol{r_M = 2.286 \text{ x } 10^8 \ km}$)

3) A Satellite orbiting at a height of 2.5R above the earth's surface takes 6√2 hours to complete one revolution. Show that another satellite orbiting at a height of 6R from the earth's surface is a geostationary satellite. 'R' is the radius if the earth.

(Ans $\boldsymbol{T_2 = 24 \ hours}$, **Therefore , it is a geostationary satellite** .)

4) The mass of the Planet Jupiter is 2×10^{27} kg and the mass of the Sun is 1000 times the mass of Jupiter. The mean distance between the Sun and Jupiter is 7.8×10^8 km. Calculate the value of gravitational constant **G**, given the force of gravitation between the Sun and Jupiter is 4.276×10^{23}N.

(Ans $\boldsymbol{G = 6.5037 \ X \ 10^{-11} \ Nm^2kg^{-2}}$)

5) The gravitation force of attraction between Earth and Sun is 35.47×10^{21} N, calculate the mass of the Sun, given the mass of the earth is 5.98×10^{24} kg and the mean distance between Earth and Sun is 1.496×10^{11}m.Given $G = \ 6.67 \ X \ 10^{-11} Nm^2kg^{-2}$.

(Ans $\boldsymbol{M_S = 1.99 \ X \ 10^{30} \ kg}$)

6)The mass and diameter of a planet are three times that of the Earth. What is the acceleration due to gravity on the surface of that planet? Given *g* on earth's surface is 9.8 ms^{-2}.

$\boldsymbol{(Ans \ \ g_P = \ 3.2666 \ ms^{-2}})$

7) Calculate the mean radius of the earth, given its mass is 5.98×10^{24} kg and acceleration due to the gravity on its surface is g=9.8ms^{-2}. If the radius of the Earth were to shrink by 1% with its

mass remaining same, what would be the value of acceleration due to gravity? Given $G = 6.67\,X\,10^{-11}Nm^2kg^{-2}$.

(Ans $\boldsymbol{R = 6.3796\,X\,10^6 m\,, g' = 6.315\, ms^{-2}}$)

8) Estimate the value of acceleration due to gravity at the peak of Mount Everest, which is 8848m above sea level. The value of ***g*** at sea level is 9.8ms^{-2} and the mean radius of the earth is 6.37x10^6m.

(Ans $\boldsymbol{g(h) = 9.773\, ms^{-2}}$)

9) At what height above the surface of the Earth will acceleration due to gravity becomes half its value at Earth's surface. Given the mean radius of Earth is 6.37x10^6m.

(Ans $\boldsymbol{h = 2.6385\,X\,10^6\, m = 2638.5\, km}$)

10) What is the value of acceleration due to the gravity at distance of 3000 km from the centre of Earth. Given its value on the surface is 9.8 ms^{-2} and the mean radius of the Earth is 6.37x10^6m.

(Ans $\boldsymbol{g(d) = 4.6158\, ms^{-2}}$)

11) Four particles each of mass 1kg are packed at four vertices of a square of side 1m. Find the gravitational potential energy of this system . Given $G = 6.67\,X\,10^{-11}Nm^2kg^{-2}$.

(Ans $\boldsymbol{V_T = -36.11\,X\,10^{-11}\mathrm{J}}$)

12) Given the mass of the moon M_m=7.35x10^{22} kg and the radius of the moon R_m =1.7x10^6 m, estimate the escape speed for moon . Given $G = 6.67\,X\,10^{-11}Nm^2kg^{-2}$.

(Ans $\boldsymbol{v_e = 2401 ms^{-1} = 2.401\, kms^{-1}}$)

13) Calculate the period of Earth's revolution around the sun. Given the mass of the Sun is M_s = 2x10^{30} kg, mean radius of the Earth's orbit, R=1.5x10^{11}m. G= 6.67x10^{-11} Nm^2kg^{-2}.

(Ans $T = \boldsymbol{3.1587X10^7\, s\ = 365.5\, days}$)

14) A satellite of mass 500kg orbits the Earth at a height of 400km above the surface. How much energy is required to shift it to height orbit at a height 600km? Given the mean radius of the Earth, R_e=6.4x10^6m, mass of the Earth, M_E=6x10^{24}kg, and G=6.7x$10^{-11}$$Nm^2kg^{-2}$.

(Ans $\boldsymbol{\Delta E = 42.04X10^7 J}$)

Chapter 8 GRAVITATION – SOLUTIONS TO PROBLEMS

1) Given,

$r_N = n\, r_E \quad \Rightarrow \quad \frac{r_N}{r_E} = n$

$n = ?$

$T_N = 164.3 year, \; T_E = 1 \; year \quad \Rightarrow \quad \frac{T_N}{T_E} = 164.3$

According to Kepler's laws, $T^2 \;\alpha\; r^3$

$$\left(\frac{T_N}{T_E}\right)^2 = \left(\frac{r_N}{r_E}\right)^3$$

$$\left(\frac{164.3}{1}\right)^2 = (n)^3$$

$$\Rightarrow n = (164.3)^{2/3}$$

Using logs and simplifying, $\boldsymbol{n = 30}$

2) Given,

$T_M = 1.88 year, \; T_E = 1 \; year \quad \Rightarrow \quad \frac{T_M}{T_E} = 1.88$

$r_E = 1.5 \; X \; 10^8 \; km \,, \quad r_M = \; ?$

According to Kepler's laws, $T^2 \;\alpha\; r^3$

$$\left(\frac{T_M}{T_E}\right)^2 = \left(\frac{r_M}{r_E}\right)^3$$

$$(1.88)^2 = \left(\frac{r_M}{r_E}\right)^3$$

$$\frac{r_M}{r_E} = (1.88)^{2/3}$$

$$r_M = (1.88)^{2/3} \; X \; r_E$$

$$r_M = (1.88)^{2/3} \text{ x } 1.5 \text{ x } 10^8 \; km$$

$$r_M = 1.524 \text{ x } 1.5 \text{ x } 10^8 \;\; km$$

$$\boldsymbol{r_M = 2.286 \text{ x } 10^8 \;\; km}$$

3) Given, that the Satellite's orbit is at a height of 2.5R above the earth's surface. Therefore its radius is

$r_1 = R + 2.5\,R = 3.5\,R$

Similarly, $r_2 = R + 6\,R = 7\,R$

$T_1 = 6\sqrt{2}\; hour$, T_2 =?

Using to Kepler's laws, $T^2 \;\alpha\; r^3$

$$\left(\frac{T_2}{T_1}\right)^2 = \left(\frac{r_2}{r_1}\right)^3$$

$$\frac{T_2}{T_1} = \left(\frac{7R}{3.5R}\right)^{3/2} = 2^{3/2} = 2\sqrt{2}$$

$$T_2 = 2\sqrt{2}\;\; T_1 = 2\sqrt{2}\,\mathrm{X}6\,\sqrt{2} = 24\; hours.$$

$$\boldsymbol{T_2 = 24\; hours.}$$

Therefore, it is a geostationary satellite.

4) Given,

$M_J = 2\,X\,10^{27}\; kg$,

$M_S = 1000\;M_J = 1000\,X\,2\,X\,10^{27}\;kg = 2\,X\,10^{30}\;kg$,

$r = 7.8\,X\,10^8\;km = 7.8\,X\,10^{11}\;m$

$F = 4.276\,X\,10^{23} N$

$G = ?$

From Newton's laws of universal Gravitation ,

$$F = \frac{G\,M_J\;M_S}{r^2} \;\Rightarrow G = \frac{Fr^2}{M_J\;M_S}$$

$$G = \frac{(4.276\,X\,10^{23})X\,(7.8\,X\,10^{11})^2}{(2\,X\,10^{27})\,X\,(2\,X\,10^{30})}$$

$$G = \frac{(4.276\ X\ 10^{23}\ )X\ (60.84\ X\ 10^{22}\ )}{4\ X\ 10^{57}} = 65.037\ X\ 10^{-12}$$

$$\boldsymbol{G = 6.5037\ X\ 10^{-11}\ Nm^2 kg^{-2}}$$

5) Given,

$F = 35.47\ X\ 10^{21} N$

$M_E = 5.98\ X\ 10^{24}\ kg\ ,\ \ M_S = ?$

$r = 1.496\ X\ 10^{11}\ m$

$G = \ \ 6.67\ X\ 10^{-11} Nm^2 kg^{-2}$

Using the Newton's laws of universal Gravitation,

$$F = \frac{G\ M_S\ \ M_E}{r^2} \Rightarrow\ M_S = \frac{Fr^2}{G\ M_E}$$

$$M_S = \frac{(35.47\ X\ 10^{21}) X\ (1.496\ X\ 10^{11}\ )^2}{(6.67\ X\ 10^{-11})\ X\ (5.98\ X\ 10^{24})}$$

$$\boldsymbol{M_S = 1.99\ X\ 10^{30}\ kg}$$

6) Given,

$M_P = 3\ M_E\ \ ,\ \ \ D_P = 3\ D_E\ \ \Rightarrow\ R_P = 3\ R_E$

$$g_E = \frac{G\ M_E}{{R_E}^2}\ \ and\ g_P = \frac{G\ M_P}{{R_P}^2}\ \ = \ \frac{G\ 3M_E}{9{R_E}^2}$$

$$\frac{g_P}{g_E} = \frac{1}{3}\ \ \Rightarrow\ g_P = \ \frac{g_E}{3} = \frac{9.8}{3} = \ \ 3.2666\ ms^{-2}$$

$$\Rightarrow\ \boldsymbol{g_P = \ \ 3.2666\ ms^{-2}}$$

7) Given,

$M_E = 5.98\ X\ 10^{24}\ kg$

$g = 9.8\ ms^{-2}$

$G = \ \ 6.67\ X\ 10^{-11} Nm^2 kg^{-2}$

$R = ?$

We have

$$g = \frac{G\,M}{R^2} \;\Rightarrow\; R = \sqrt{\frac{GM}{g}}$$

$$R = \sqrt{\frac{(6.67\ X\ 10^{-11})(5.98\ X\ 10^{24})}{9.8}}$$

$$\boldsymbol{R = 6.3796\ X\ 10^6 m = 6379.6\ km}$$

If radius is shrinks by 1 %

$$R' = R - 1\%\ of\ R = R - \left({}^{1}/_{100}\right)\ R = 0.99\ R$$

$$R' = 0.99\ R = 0.99\ X\ 6.3796\ X\ 10^6 = 6.3158\ X\ 10^6 m$$

$$g' = \frac{G\,M}{R'^2}$$

$$g' = \frac{(6.67\ X\ 10^{-11})\ (5.98\ X\ 10^{24}\ )}{(6.3158\ X\ 10^6)^2} = 6.315\ X\ 10^1 = 6.315\ ms^{-2}$$

$$\boldsymbol{g' = 6.315\ ms^{-2}}$$

8) Given,

$h = 8848\ m$

$g = 9.8\ ms^{-2}$

$g(h) =?$

$R_E = 6.37\ X\ 10^6 m$

We have

$$g(h) = g\ \left(1 - \frac{2h}{R_E}\right)$$

$$g(h) = 9.8\ \left(1 - \frac{2\ X\ 8848}{6.37\ X\ 10^6}\right)$$

$$g(h) = 9.8\,(1 - \ 0.002778)$$

$$g(h) = 9.8\,(0.997222)$$

$$\boldsymbol{g(h) = 9.773\ ms^{-2}}$$

9) Given,

$g(h) = {}^{g}/_{2}$

$h = ?$

$R_E = \ 6.37\ X\ 10^6 m$

We have

$$g(h) = g\,\left(\frac{R_E}{R_E + h}\right)^2$$

$$\frac{g}{2} = g\,\left(\frac{R_E}{R_E + h}\right)^2 \ \Rightarrow \quad \frac{1}{2} \ = \ \left(\frac{R_E}{R_E + h}\right)^2$$

Therefore,

$$\left(\frac{R_E}{R_E + h}\right) = \frac{1}{\sqrt{2}}$$

$$R_E + h = \sqrt{2}\,R_E \quad \Rightarrow \quad h = \ \sqrt{2}\,R_E - R_E \ = \left(\sqrt{2} - 1\right)\,R_E$$

$$h = \ (1.4142 - 1)\ \ 6.37\ X\ 10^6 \ = (0.4142)\ X\ 6.37\ X\ 10^6 \ = 2.6385\ X\ 10^6\ m$$

$$\boldsymbol{h = 2.6385\ X\ 10^6\ m = 2638.5\ km}$$

10) The point under consideration lies inside the Earth's surface, because its distance from the centre of the earth $x = \ 3000\ km = 3\ X\ 10^6 m$ is less then $R_E = \ 6.37\ X\ 10^6 m$.

The depth of this point is, $d = R_E - \ x = \ 6.37\ X\ 10^6 - \ 3\ X\ 10^6 m \ = 3.37\ X\ 10^6 m$.

Given,

$R_E = \ 6.37\ X\ 10^6 m$

$g = 9.8\, ms^{-2}$

$g(d) = ?$

We have,

$$g(d) = g\left(1 - \frac{d}{R_E}\right)$$

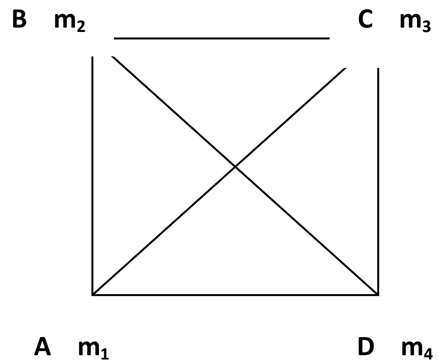
$$g(d) = 9.8\left(1 - \frac{3.37\ X\ 10^6}{6.37\ X\ 10^6}\right)$$

$$g(d) = 9.8\ (1 - \ 0.529)$$

$$g(d) = 9.8\ (0.471)$$

$$\boldsymbol{g(d) = 4.6158\ ms^{-2}}$$

11) Given



$m_1 = m_2 = m_3 = m_4 = m = 1\ kg$

$AB = BC = CD = DA = x = 1m$

$Diagonal\ AC = BD = \sqrt{2}\, x\, m$

$G = \ 6.67\ X\ 10^{-11} Nm^2 kg^{-2}$

$V_T = ?$

The gravitational potential energy of a two particles is

$$V = \frac{-G\ m_1 m_2}{r}$$

The total gravitational potential energy is

$$V_T = [V_{AB} + V_{BC} + V_{CD} + V_{DA}] + [V_{AC} + V_{BD}]$$

$$But\ V_{AB} = V_{BC} = V_{CD} = V_{DA} = \frac{-G\ m\ m}{x} = \frac{-G\ X\ 1\ X\ 1}{1} = -G\ \mathbf{J}$$

$$V_{AC} = V_{BD} = \frac{-Gmm}{\sqrt{2}\,x} = \frac{-G\ X\ 1\ X\ 1}{\sqrt{2}} = \frac{-G}{\sqrt{2}}\ \mathbf{J.}$$

$$V_T = 4(-G) + 2\left(\frac{-G}{\sqrt{2}}\right) = -4G - \frac{2G}{\sqrt{2}}$$

$$V_T = G\left(-4 - \frac{2}{\sqrt{2}}\right) = G\left(-4 - \sqrt{2}\right) = -5.4142\ G$$

$$V_T = -5.4142\ X\ 6.67\ X\ 10^{-11}$$

$$\boldsymbol{V_T = -36.11\ X\ 10^{-11}\mathrm{J}}$$

12) Given,

$M_m = 7.35\ X\ 10^{22}\ kg$ $\quad$ $R_m = 1.7\ X\ 10^{6}\ m$

$G = 6.67\ X\ 10^{-11} Nm^2kg^{-2}$

$v_e = ?$

We have

$$v_e = \sqrt{2\ g\ R}$$

But,

$$g = \frac{G\ M}{R^2} \quad \Rightarrow \quad v_e = \sqrt{\frac{2GM}{R}}$$

For moon,

$$v_e = \sqrt{\frac{2GM_m}{R_m}}$$

$$v_e = \sqrt{\frac{2X(6.67\ X\ 10^{-11})X(7.35\ X\ 10^{22})}{1.7\ X\ 10^{6}}}$$

$$\boldsymbol{v_e = 2401 ms^{-1} = 2.401\ kms^{-1}}$$

13) Given,

$M_s = 2X\ 10^{30}\ kg$ $\quad$ $R = 1.5\ X\ 10^{11}\ m$

$G = \ 6.67\, X\, 10^{-11} Nm^2 kg^{-2}$

$T = \ ?$

From the derivation of Kepler's 3rd law,

$$T^2 = \frac{4\pi^2 R^3}{GM_S} = \frac{4X3.14^2X(1.5X10^{11})^3}{(6.67X10^{-11})(2X10^{30})}$$

$$T^2 = 9.977X10^{14}$$

$$T = 3.1587X10^7\ s$$

$$\Rightarrow \quad T = \frac{3.1587X10^7}{60X60X24} days$$

$$\boldsymbol{T = 365.5\ days}$$

14) Given,

$m = 500\ kg$

$R_E = 6.4X\ 10^6\ m$

$h_1 = 400\ km = 0.4\ X\ 10^6\ m \ \Rightarrow \ R_E + h_1 = 6.8\ X\ 10^6\ m$

$h_2 = 600\ km = 0.6\ X\ 10^6\ m \ \Rightarrow \ R_E + h_2 = 7\ X\ 10^6\ m$

$M_E = 6\ X\ 10^{24}\ kg$

$G = \ 6.67\, X\, 10^{-11} Nm^2 kg^{-2}$

$$Total\ energy \quad E = \frac{-GmM}{2(R_E + h)}$$

Initially,

$$E_i = \frac{-GM_E m}{2(R_E + h_1)} = \frac{-GM_E m}{2\ X\ 6.8\ X\ 10^6}$$

Finally,

$$E_f = \frac{-GM_E m}{2(R_E + h_2)} = \frac{-GM_E m}{2\ X\ 7\ X\ 10^6}$$

Change in Energy, $\Delta E = E_f - E_i$

$$\Delta E = \frac{-GM_E m}{2 \, X \, 7 \, X \, 10^6} + \frac{GM_E m}{2 \, X \, 6.8 \, X \, 10^6}$$

$$\Delta E = \frac{GM_E m}{2 \, X \, 10^6} \left[\frac{1}{6.8} - \frac{1}{7}\right]$$

$$\Delta E = \frac{6.67 \, X \, 10^{-11} \, X \, 6 \, X \, 10^{24} \, X \, 500}{2 \, X \, 10^6} \left[\frac{0.2}{(6.8)\,(7)}\right]$$

$$\boldsymbol{\Delta E = 42.04 X 10^{7} J.}$$