SUBJECT: MATHEMATICS
CHAPTER – 9
SEQUENCE AND SERIES

Question Bank

One Mark Questions:

1. Write the first three terms of sequence whose nth terms is $\frac{3n-5}{6}$

2. Write the first Three terms of Sequence whose n^{th} term is 2^n

3. Write the first Three terms of sequence whose nth term is $\frac{n^2}{3^n}$

4. Write the first Three terms of Sequence whose nth term is $(-1)^{n-1}\ 2^{-n}$

5. Write the first Three terms of Sequence whose nth term is n(n-2)

6. Write the first five terms of sequence whose nth term is 2n+5

7. Write the first five terms of sequence whose nth term is $\frac{n-3}{4}$

8. Write the first five terms of the sequence whose nth term is $\frac{n}{n+1}$

9. Write the first five terms of the sequence whose nth term is $\frac{1}{n^2+1}$

10. Write the first five terms of the sequence whose nth term is 4n+1

11. If $\mathbf{a_n} = \frac{n(n-2)}{3}$, Find a_{20}

12. If $\mathbf{a_n} = \frac{4n}{n^2+1}$, Find a_6

13. If $\mathbf{a_n} = \frac{n^2}{2^n}$, Find a_5

14. If $\mathbf{a_n}$ = (n-1) (n-2), Find a_7

15. If $\mathbf{a_n} = (-1)^{n-1}\ n^3$, Find a_9

16. Find the A.M. between 7 and 13.

17. Find the A.M. between 12 and -8

18. Find the A.M. between (x-y_ & (x+y)

19. Find the 9th term of G.P. 1,4, 16, 64,……

20. Find the 12th term of G.P. $\frac{1}{a^3x^3}$, ax, $a^5\ x^5$, …….

21. Find the 5th term of G.P. $1, \frac{1}{3}, \frac{1}{9}, \ldots\ldots\ldots\ldots\ldots..$

22. Find the 8th term of G.P. 0.3, 0.06, 0.012, ……..

23. Find the nth term of G.P. $\sqrt{3}, \frac{1}{\sqrt{3}}, \frac{1}{3\sqrt{3}}, \ldots\ldots\ldots\ldots\ldots..$

24. Find the G.M. of 2 and 8

25. Find the G.M. of a^3b and ab^3

26. Find the G.M. of x^2 and y^2

27. Find the G.M. of 4 and 9

28. If ‘a’ is the G.M. of 2 and ¼. Find ‘a’

Two Mark Questions

1. If $a_1, = 3$, $a_n = 3a_{n-1} + 2 \forall n > 1$, Find the first five terms of the sequence and write the corresponding series.

2. If $a_1, = -1$, $a_n = \frac{a_{n-1}}{n}$, $n \geq 2$, Find the first five terms of the sequence and write the corresponding series.

3. If $a_1, = 2$, $a_2 = 3$, $a_n = a_{n-1} + a_{n-2}$, $n \geq 3$, Find the first five terms of the sequence and write the corresponding series.

4. Which term of the sequence 72, 70, 68, 66…….. is 40 ?

5. Which term of A.P. 84, 80, 76, …… is 0 ?

6. Which term of A.P. 13, 10, 7,……. Is – 59?

7. Which term of A.P. -3, -7, -11, …….. is -403 ?

8. Which term of A.P. 4, 9, 14, 19, …….. is 124 ?

9. How many terms are there in the sequence 3, 6, 9, 12, ……. 111 ?

10. Find the 10^{th} term of the A.P. 1, 4, 7, 10, ……..

11. Find the 18^{th} term of the A.P. $\sqrt{2}, 3\sqrt{2}, 5\sqrt{2}$ ……..

12. Find the 15^{th} term of A.P. 2, 8, 14, ……

13. Find the 17^{th} term of A.P. 9, 4, -1, …….

14. Find the 20^{th} term of A.P. 3, 7, 11, ……

15. The first term of an A.P is 5, common difference is 3 & the last term is 80. Find the numbers of terms?

16. The 6^{th} and 17^{th} terms of an A P are 19 & 41. Find the first term and common difference?

17. If the Third and Seventh terms of an A.P. are 18 & 30 respectively. Find the first term and common difference?

18. A Student purchased a pen for Rs.100. At the end of 8 years, it was valued at Rs. 20. Assuming the yearly depreciation is a constant amount. Find the annual depreciation?

19. If you save 1 paise today, 2 pase next day and 3 paise succeeding day and so on what will be your savings in 365 days?

20. Find the sum of 20 terms of the A.P. 3, 7, 11, …….

21. Find the sum of 15 terms of A.P. 2, 8, 14……..

22. Which term of G.P. $\sqrt{2}$ $\frac{1}{\sqrt{2}}, \frac{1}{2\sqrt{2}}, \frac{1}{4\sqrt{2}}, \ldots\ldots\ldots\ldots$ is $\frac{1}{512\sqrt{2}}$?

23. Which term of G.P. 18, -12, 8 …….. is $\frac{512}{729}$?

24. Which term of G.P. 2, 1, $\frac{1}{2}, \frac{1}{4},$ …… $\frac{1}{128}$?

25. Which term of G.P. 5, 10, 20, 40…….. is 5120?

26. Which term of G.P. $\sqrt{3},\ 3,\ 3\sqrt{3},$ …… is 729?

27. If the first and fourth terms of a G.P. are 9 & 72 respectively find the common ratio?

28. Find the 12^{th} term of a G.P. for which first term is 3 and second term is -6.

29. Find the sum of 7 terms of G.P. 3, 6, 12, ……

30. Find the sum of 10 terms of the G.P. 1, $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}$, ……

31. Find the sum of 8 terms of the G.P. 1, 3, 9, 27, ….……

32. Find the sum of 9 terms of the G.P. 1, –½ , $\frac{1}{4}, -\frac{1}{8}$, ……

Three Marks Question

1. If the 6^{th} term and 8^{th} terms of an A.P. are 12 & 22 find second term.
2. 9^{th} term of an A.P. is 99 & 99^{th} term is 9. Find 108^{th} term.
3. If 5 times the 5^{th} term of an A.P. be equal to 8 times the 8^{th} term. Find the 13^{th} term.
4. If Ten times the 10^{th} term of an A .P. is equal to Fifteen times the 15^{th} term S.T. 25^{th} term of an A.P. is Zero.
5. The sum of three numbers in A.P. is 24 and their product is 440. Find the numbers.
6. The sum of three numbers in A.P. is -3 and thus product is 8. Find the number.
7. Find the three Nos. of A.P. whose sum is 27 and product is 648?
8. Find the sum of 1 + 5 + 9 +…….+ 325
9. Find the sum of -29 -24 -19 -14……+91
10. Find the sum of 5 + 13 + 21 +…….+ 181
11. Find the sum of 120 + 113 + 106 +……..+ 1
12. Find the sum of 10 terms of an A.P. 50, 46, 42, ……
13. Find the sum of 12 terms of an A.P. 1, 3, 5, 7, ……

14. Find the sum of 25 terms of an A.P. 3, $\frac{9}{2}$, 6, $\frac{15}{2}$ ……

15. Find the sum of first 30 terms of an A.P. whose n^{th} term is 3n–5?

16. How many terms of the A.P. 1, 5, 9, ……. Must be taken so that their sum is 2415?

17. How many terms of the A.P. -12, -9, -6, -3…….must be taken so that sum is 54?

18. If $\frac{1}{b+c}, \frac{1}{c+a}, \frac{1}{a+b}$, are in A.P. P.T. A^2, b^2, l^2 are also is A.P.

19. If a, b, c are in A.P. PT b + c, c + a, a + b are also is A.P.

20. If $\frac{b+c-a}{a}, \frac{c+a-b}{b}, \frac{a+b-c}{c}$ are in A.P. P.T. $\frac{1}{a}, \frac{1}{b}, \frac{1}{c},$ re also in A.P.

21. Insert 7 Arithmetic means between 2 & 42.

22. Insert 4 Arithmetic means between 4 & 19.

23. Insert 3 Arithmetic Means between -18 & 4

24. A person purchases a T.V. set for Rs.3200. Its life is estimated to be 50 years. Its price after 40 years is Rs.640 only. Assuming the yearly depreciation to be constant rate. Find the annual depreciation and its price after 30 years?

25. The sum of five consecutive odd integers is 1185. What are the numbers?

26. The fourth term of a G.P. is 27 and 7^{th} term is 729. Find the G.P.?

27. Find a G.P. for which sum of first two terms is -4 and 5^{th} term is four times the third term?

28. The seventh term of a G.P. is 8 times the fourth term and 5^{th} term is 48. Find the G.P.?

29. The fourth, seventh and last term of a G.P. are 10, 80 & 2560 respectively. Find the first term and the number of terms in the G.P.?

30. Find three numbers in G.P. whose sum is 65 and product is 3375.

31. Find three numbers in G.P. whose sum is $\frac{13}{12}$, and product is '-1'.

32. Find three numbers in G.P. whose sum is 21 and product is 216.

33. Find the sum of 1+2+4+……..+1024.

34. Find the sum of $\frac{1}{81}+\frac{1}{27}+\frac{1}{9}+.....+243$

35. Find the sum of $5-\frac{5}{2}+\frac{5}{4}-.....+\frac{5}{256}$

36. Insert 5 G.M. between 3 & 192.

37. Insert 4 G.M. between $\frac{1}{2}\&\frac{1}{486}$

38. Insert 6 G.M. between 27 & $\frac{1}{81}$

39. If a, b, c are in G.P. T a^2, b^2, c^2 are also in G.P.

40. If $a^2 + b^2$, ab + bc, $b^2 + c^2$ are in G.P. PT a, b, c, are also in G.P.

FIVE MARKS QUESTIONS:

1. Find the sum of all integers between 150 & 500 that are divisible by 7.
2. Find the sum of integers between 50 and 200 which leave remainder 5 when divided by 7.

3. Find the sum of all natural numbers between 100 & 1000 which are multiple of 5.

4. The fourth term of an A.P. is 7 and the 10^{th} term is 19. Find the sum to 'n' terms?

5. The sum of the First Ten terms of an A.P. is 185. If the 13^{th} term is 41. Find the sum to first 25 terms?

6. The sum of the third and seventh term of an A.P. is 42 and sum of the seventh and eleventh term of an A.P. is 82. Find the first term & common difference? Write the A.P.

7. The sum of first 'n' term of two A.P. are in the ratio (2n-3): (3n-2). Find the ratio of their 10^{th} terms.

8. The sum of first 'n' terms of two A.P's are in the (7n + 2) : (n + 4). Find the ratio of their 5^{th} terms.

9. The sum of n terms of two A.P's are in the ratio (5n+4) : (9n+6). Find the ratio of their 18^{th} terms?

10. If a+b+c $\neq 0$ and $\frac{b+c}{a}, \frac{c+a}{b}, \frac{a+b}{c}$ are in A.P. P.T. $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are also in A.P.

11. If a, b, c are in A.P. P.T. $\frac{1}{\sqrt{b}+\sqrt{c}}, \frac{1}{\sqrt{c}+\sqrt{a}}, \frac{1}{\sqrt{a}+\sqrt{b}}$, are also in A.P.

12. PT $[(b+c)^2-a^2]$, $[(c+a)^2 - b^2]$, $[(a+b)^2 - c^2]$ are in A.P. if a, b, c, are in A.P.

13. There are 'n' A.M's between 3 and 17. The ratio of the last mean to the first mean is 3:1. Find the value of n?

14. There are m A.M's between 1 and 31, so that the ratio of 7^{th} mean to $(m-1)^{th}$ mean is 5:9. Find m?

15. If x, y, z are in A.P. and A_1 is the A.M. of X & Y and A_2 is the A.M. of Y & Z then PT AM of A_1 & A_2 is y.

16. 25 Trees are planted in a straight line 5 meters about from each other to water them the gardener must bring water for each tree separately from a well 10mts. From the first tree is line with the trees. How far will he move in order to water all the trees beginning with first, if he starts from the well.

17. Find the sum of the G.P. 0.15 + 0.015 + 0.0015 + …… to 8 terms.

18. Find the sum of 5 + 55 + 555 + …… to n terms.

19. Find the sum of 0.7+0.77+0.777+ …….. to n terms.

20. Find the sum of 9 + 99 + 999 + …….. to n terms.

21. Find the sum of 0.6 + 0.66 + 0.666 + …… to n terms.

22. Find the sum of 3 + 33 + 333 + …… to n terms.

23. If a, b, c, d are in G.P. P.T a + b, b + c, c + d are also in a G.P.

24. Find the sum to n terms of the series $1^2 + 3^2 + 5^2$ + …….. to n terms.

25. Find the sum to n terms of the series 1.2 + 2.3 + 3.4 + ……. to n terms.

26. Find the sum to n terms of the series $1.2^2 + 2.3^2 + 3.4^2$ + …… to n terms.

27. Find the sum to 'n' terms of the series 3.8 + 6.11 + 9.14 + ……. to n terms.

28. Find the sum to 'n' terms of the series 3 + 15 + 35 + 63 + …… to n terms.

29. Find the sum to 'n' terms of the series. 1+5+12+22+35+…… to n terms.

30. Find the sum to 'n' terms of the series 3+7+13+21+31+……. to n terms.

31. Find the sum to 'n' terms of the series whose nth term is $2n^3+3n^2 – n+1$.

32. Find the sum to 'n' terms of the series whose nth term is $(2n-1)^2$.

33. Find the sum to 'n' terms of the series whose n^{th} term is n(n-1) (n-4).

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ANSWERS TO SEQUENCE AND SERIES

One Mark Questions:

1. an = $\frac{3n-5}{6}$ $-\frac{1}{3}, \frac{1}{6}, \frac{2}{3}$

2. $a^{n} = 2^{n}$ 2, 4, 8

3. $a_n = \frac{n^2}{3^n}$ $\frac{1}{3}, \frac{4}{9}, \frac{9}{27}$

4. $a_n = (-1)^{n-1}\ 2^{-n}$ $\frac{1}{2}, -\frac{1}{4}, \frac{1}{8}$

5. $a_n = n(n-2)$ -1, 0, 3

6. $a_n = 2n + 5$ 7, 9, 11, 13, 15

7. $a_n = \frac{n-3}{4}$ $-\frac{1}{2}, -\frac{1}{4}, 0, \frac{1}{4}, \frac{1}{2}$

8. $a_n = \frac{n}{n+1}$ $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}$

9. $a_n = \frac{1}{n^2+1}$ $\frac{1}{2}, \frac{1}{5}, \frac{1}{10}, \frac{1}{17}, \frac{1}{26}$

10. $a_n = 4n+1$ 5, 9, 13, 17, 21

11. $\mathbf{a_n} = \frac{n(n-2)}{3}$, $a_{20} = \frac{20(20-2)}{3} = 180$

12. $\mathbf{a_n} = \frac{4n}{n^2+1}$, $a_6 = \frac{24}{7}$

13. $\mathbf{a_n} = \frac{n^2}{2^n}$, $a_5 = \frac{25}{32}$

14. $\mathbf{a_n} = (n-1)\,(n-2)$, $a_7 = 30$

15. $\mathbf{a_n} = (-1)^{n-1}\ n^3$, $a_9 = 729$

16. A.M. = 10

17. A.M. = 2

18. A.M. = x

19. $a_9 = 4^8$

20. $a_{12} = a^{41}\, x^{41}$

21. $a_5 = \frac{1}{81}$

22. $a_8 = (0.3)\,(0.2)^7$

23. $a_n = \sqrt{3}\left(\frac{1}{3}\right)^{n-1}$

24. G.M.= 4

25. G.M.= $a^2 b^2$

26. G.M.= xy

27. G.M.= 6

28. $a = \frac{1}{\sqrt{2}}$

Two Mark Questions

29. $a_1 = 3$ $\quad a_n = 3a_{n-1} + 2$

First five terms are 3, 11, 35, 107, 323

Series is 3 + 11 + 35 + 107 + 323 + …….

30. $a_1 = -1$ $\quad a_n = \frac{a_{n-1}}{n}$, First five terms are -1, $\frac{-1}{2}$, $\frac{-1}{6}$, $\frac{-1}{24}$, $\frac{-1}{120}$

series is $(-1) + \left(\frac{-1}{2}\right) + \left(\frac{-1}{6}\right) + \left(\frac{-1}{24}\right) + \left(\frac{-1}{120}\right) + \ldots\ldots\ldots$

31. $a_1 = 2$ $\quad a_2 = 3$ $\quad a_n = a_{n-1} + a_{n-2}$ First five terms are 2, 3, 5, 8, 13, ……

Series is 2 + 3 + 5 + 8 + 13 + ……..

32. 72, 70, 68, 66…….. are in A.P.

$a = 72$, $\quad d = -2$

$n = 17$

$\therefore a_{17} = 40$

33. 84, 80, 76, …… are in A.P.

$a = 84$, $\quad d = 4$

$n = 20$

$\therefore a_{20} = 0$

34. 13, 10, 7,……. Are in A.P.

$a = 13$, $d = -3$

$n = 25$

$\therefore a_{25} = -59$

35. -3, -7, -11, …….. are in A.P.

$a = -3$, $\quad d = -4$

$n = 101$

$a_{101} = -403$

36.4, 9, 14, 19, …….. are in A.P.

$a = 4, \quad d = 5$

$n = 25$

$a_{25} = 124$

37.3, 6, 9, 12, ……. 111 are in A.P.

$a = 3, \quad d = 3$

$n = 37$

38.1, 4, 7, 10, …….. are in A.P.

$a = 1, \quad d = 3$

$a_{10} = 28$

39. $\sqrt{2}, 3\sqrt{2}, 5\sqrt{2}$ …….. are in A.P.

$a = \sqrt{2}, \quad d = 2\sqrt{2}$

$a_{18} = 35\sqrt{2}$

40.2, 8, 14, …… are in A.P.

$a = 2, \quad d = 6$

$a_{15} = 86$

41.9, 4, -1, ……. Are in A.P.

$a = 9, \quad d = -5$

$a_{17} = -71$

42.3, 7, 11, ……are in A.P.

$a = 3, \quad d = 4$

$a_{20} = 79$

43.$a = 5, \quad d = 3, \quad a_n = 80 \quad n = 26$

44.$a_6 = 19 \Rightarrow a + 5d = 19$

$a_{17} = 41 \Rightarrow a + 16d = 41$

$d = 2, \quad a = 9$

45. $a_3 = 18 \quad \Rightarrow \quad a + 2d = 18$

$a_7 = 30 \quad \Rightarrow \quad a + 6d = 30$

$d = 3, \quad a = 12$

46. Original cost of pen = a = Rs.100 Let 'd' be the annual depreciation price after eight years = $a_8 = 20$

$\therefore d = Rs.10$

47. $a = 1, \quad d = 1, \quad n = 365, \quad S =$ Total savings.

$\therefore S = Rs.\ 667.95$

48. $a = 3, \quad d = 4, \quad n = 20$

$S_{20} = 820$

49. $a = 2, \quad d = 6, \quad n = 15$

$S_{15} = 660$

50. $a = \sqrt{2}$, $r = ½ \qquad n = 11$

$$a_{11} = \frac{1}{512\sqrt{2}}$$

51. $a = 18, \quad r = \frac{-2}{3}$

$n = 9$

$$a_9 = \frac{512}{729}$$

52. $a = 2, \quad r = ½ \qquad a = 9$

$$\therefore a_9 = \frac{1}{128}$$

53. $a = 5, \quad r = 2 \qquad n = {}_{11} \qquad a_{11} = 5120$

54. $a = \sqrt{3}, \quad r = \sqrt{3} \qquad n = 12 \qquad a_{12} = 729$

55. $a = 9 \qquad a_4 = 72 \quad \Rightarrow \quad ar^3 = 72 \quad \therefore r = 2$

56.a = 3

$a_2 = -6 \Rightarrow ar = -6$

$\Rightarrow r = -2 \qquad a_{12} = -6144$

57.a = 3, r = 2

$s_7 = 381$

58.a = 1, r = ½ $\qquad s_{10} = \frac{1023}{512}$

59.a = 1, r = 3 $\qquad s_8 = 3280$

60.a = 1, r = -½ $\qquad s_9 = \frac{513}{256}$

Three Marks Question

61.$a_6 = 12 \Rightarrow a + 5d = 12$

$a_8 = 22 \Rightarrow a + 7d = 22$

$d = 5, a = -13$

$a_2 = -8$

62.$a_9 = 99 \Rightarrow a + 8d = 99$

$a_{99} = 9 \Rightarrow a + 98d = 9$

$\therefore d = -1, \quad a = 107 \qquad a_{108} = 0$

63.$5a_5 = 89_8 \qquad a_{13} = 0$

64.$10\ a_{10} = 15a_{15}$

$2\ a_{10} = 3a_{15} \qquad \therefore\ a_{25} = 0$

65.a-d, a, a + d are in A.P.

$(a - d) + a + (a + d) = 24$

$(a - d)\ a\ (a + d) = 440$

$a = 8, \quad d = \pm 3$

The Nos. are 5, 8, 11

66.(a – d), a, (a + d) are in A.P.

$a = -1, \quad d = \pm 3$

$\therefore$ The Nos. are -4, -1, 2.

67.(a – d), a, (a + d) are in A.P.

$a = 9, \quad d = \pm 3$

$\therefore$ The Nos. are 6, 9, 12

68.1, 5, 9, …… 325 are in A.P.

$n = 82$

$S_{82} = 13366$

69.-29, -24, -19, -14, ……91 are in A.P.

$n = 25$ $\qquad$ $S_{25} = 775$

70. 5, 13, 21, ……. 181 are in A.P.

$n = 23$ $\qquad$ $S_{23} = 2139$

71. 120, 113, 106, ……. 1 are in A.P.

$n = 18$ $\qquad$ $S_{18} = 1089$

72. $a = 50, \quad d = -4, \quad n = 10$ $\qquad$ $S_{10} = 320$

73. $a = 1, \quad d = 2, \quad n = 12$ $\qquad$ $S_{12} = 144$

74. $a = 3, \quad d = \frac{3}{2}, \quad n = 25$ $\qquad$ $S_{25} = 525$

75.$a_n = 3n - 5$

$a_1 = 3 - 5 = -2$

$a_{30} = 3 * 30 - 5 = 90 - 5 = 85$

$\therefore S_{30} = 1245$

76. $a = 1, \quad d = 4, \quad n = ?$ $\qquad$ $S_n = 2415$

$S_n = \frac{n}{2}[2a+(n-1)d]$ $\qquad$ $2n^2 - n - 2415 = 0$

$\Rightarrow (n-35)(2n+69) = 0 \qquad \Rightarrow n = 35 \;\&\; n = \frac{-69}{2}$ not admissible

$\therefore\ n = 35$

77. $a = -12, \quad d = 3, \quad n = ? \quad S_n = 54$

$$S_n = \frac{n}{2^2}[2a + (n-1)d]$$

$$3n^2 - 27n - 108 = 0$$

$$\Rightarrow (n - 12)(3n + 9) = 0$$

$\Rightarrow n = 12 \;\&\; n = \frac{-9}{3} = -3$ not admissible

$\therefore\ n = 12$

78. $\frac{1}{b+c}, \frac{1}{c+a}, \frac{1}{a+b}$, are in A.P.

$\therefore\ \frac{1}{c+a} - \frac{1}{b+c} = \frac{1}{a+b} - \frac{1}{c+a}$

Simplify & we get $b^2 - a^2 = c^2 - b^2$

$\Rightarrow a^2, b^2, c^2$ are in A.P.

79. $b + c, c + a, a + b$ will be in A.P.

if $(c + a) - (b + c) = (a + b) - (c + a)$

$\Rightarrow a - b = b - c \qquad \Rightarrow b - a = c - b$

$\Rightarrow$ a, b, c, are in A.P. which is true

$\therefore\ b + c, c + a, a + b$ are also in A.P.

80. $\frac{b+c-a}{a}, \frac{c+a-b}{b}, \frac{a+b-c}{c}$ are in A.P.

$\Rightarrow \left\{\frac{b+c-a}{a}+2\right\}, \left\{\frac{c+a-b}{b}+2\right\}, \left\{\frac{a+b-c}{c}+2\right\}$ are in A.P.

{Adding 2 to each term}

$\Rightarrow \frac{a+b+c}{a}, \frac{a+b+c}{b}, \frac{a+b+c}{c}$ are in A.P. Dividing by $a + b + c$

$\frac{1}{a}, \frac{1}{b}, \frac{1}{c},$ are in A.P.

81. Let $A_1, A_2, A_3, A_4, A_5, A_6, A_7$ are 7 A.M's between 2 & 42

Then 2, A_1, ……A_7, 42 are in A.P.

$\therefore a = 2, \quad n = 9, \quad a_n = 42$

$\therefore d = 5$

$\therefore A_1 = 7_1, A_2 = 12, A_3 = 17, A_4 = 22, A_5 = 27, A_6 = 32, A_7 = 37$

82. Let A_1, A_2, A_3, A_4 are 4 A.M's between 4 & 19

Then 4, A_1, A_2, A_3, A_4, 19 are in A.P.

$a = 4, \quad n = 6, \quad a_n = 19$

$\therefore d = 3$

four A.M's are 7, 10, 13, 16

83. Let A_1, A_2, A_3 are 3 A.M's between -18 & 4

Then -18, A_1, A_2, A_3, 4 are in A.P.

$a = -18, n = 5, a_n = 4$

$\therefore d = \frac{22}{4}$

Three A.M's are $-\frac{50}{4}, -\frac{28}{4}, -\frac{6}{4}$

84. Cost of T.V. = a = Rs.3200

'd' be the annual depreciation

$a_{40} = 640$

$\therefore d = Rs.64$

$a_{30} = Rs.1280$

85. $S_5 = 1185, \quad n = 5, \quad d = 2$

$S_n = \frac{n}{2}[2a + (n-1)d]$

$\Rightarrow 5a = 1165$

$\therefore a = 233$

Consecutive add integers are 233, 235, 237, 239, 241.

86. $a_4 = 27 \quad \Rightarrow ar^3 = 27$

$a_7 = 729 \quad \Rightarrow ar^6 = 729$

$\therefore r = 3, \quad a = 1$

G.P. is 1, 3, 9, 27, ……

87. 'a' is the first term and 'r' is the common ratio

$a + ar = -4 \quad \Rightarrow a(1 + r) = -4$

$a_5 = 4a_3 \quad \Rightarrow ar^4 = 4ar^2$

$\Rightarrow r = \pm 2$

$\therefore a = \frac{-4}{3}$ & $a = 4$

When $a = \frac{-4}{3}$ & r = 2 G.P. is $\frac{-4}{3}, \frac{-8}{3}, \frac{-16}{3}$, …….

When a = 4 & r = -2 G.P. is 4, -8, 16, -32 ……..

88. Let 'a' is the first term & 'r' be the common ratio

$a_7 = 8\, a_4 \quad \Rightarrow \quad ar^6 = 8\, ar^3 \quad \Rightarrow \quad r = 2$

$a_5 = 48 \quad \Rightarrow \quad ar^4 = 48 \quad \Rightarrow \quad a = 3$

$\therefore$ G.P. is 3, 6, 12, ……..

89. $a_4 = 10 \quad \Rightarrow \quad ar^3 = 10$

$a_7 = 80 \quad \Rightarrow \quad ar^6 = 80$

$\Rightarrow \quad r = 2 \quad \therefore a = \frac{10}{8}$

$a_n = 2560 \quad \Rightarrow ar^{n-1} = 2560 \quad \therefore n = 12$

90. Let $\frac{a}{r}$, a, ar are three numbers in G.P.

$\frac{a}{r} \times a \times ar = 3375$

$\Rightarrow a = 15 \quad \frac{a}{r} + a + ar = 65 \quad \Rightarrow r = 3$ & $r = \frac{1}{3}$

when a = 15, r = 3, G.P. is 5, 15, 45

when a = 15, $r = \frac{1}{3}$ G.P. is 45, 15, 5

91. Let $\frac{a}{r}$, a, ar are three numbers in G.P.

$\frac{a}{r}$ x a x ar = -1

=> a = -1

$\frac{a}{r}$ + a + ar = $\frac{13}{12}$

=> r = $\frac{-4}{3}$ & r = $\frac{-3}{4}$

when a = -1, r = $\frac{-4}{3}$ G.P. is $\frac{3}{4}$, -1, $\frac{4}{3}$

when a = -1, r = $\frac{-3}{4}$ G.P. is $\frac{4}{3}$, -1, $\frac{3}{4}$

92. Let $\frac{a}{r}$, a, ar are three numbers in G.P.

$\frac{a}{r}$ x a x ar = 216

=> a = 6

$\frac{a}{r}$ + a + ar = 21

=> r = 2 & r = ½

when a = 6, r = 2 G.P. is 3, 6, 12

when a = 6, r = ½ G.P. is 12, 6, 3

93. 1, 24, …….. 1024 are in G.P.

a = 1, r = 2

$a^n = ar^{n-1}$ => n = 11

$S_n = \frac{a(r^n - 1)}{r - 1}$

$\therefore S_{11} = 2048$

94. $\frac{1}{81}, \frac{1}{27}, \frac{1}{9}, \ldots\ldots\ldots\ldots\ldots$ 243 are in G.P.

$a = \frac{1}{81}, \quad r = 3 \qquad a_n = ar^{n-1} \qquad \Rightarrow \quad n = 10$

$S_n = \frac{a(r^n - 1)}{r - 1} \qquad \therefore S_{10} = \frac{9841}{81}$

95. $5, \frac{-5}{2}, \frac{5}{4} \ldots\ldots\ldots\ldots\ldots\ldots \frac{5}{256}$ are in G.P.

$a = 5, \quad r = -½$

$a_n = ar^{n-1} \Rightarrow \quad n = 9$

$S_n = \frac{a(1 - r^n)}{1 - r} \qquad \therefore S_9 = \frac{855}{256}$

96. Let G_1, G_2, G_3, G_4, G_5, are 5 G.M's between 3 & 192

$\therefore$ 3, G_1, G_2, G_3, G_4, G_5, 192 are in G.P.

$a = 3, \quad n = 7, \quad a_n = 192$

$a_n = ar^{n-1} \Rightarrow \quad r = 2$

$\therefore$ 6, 12, 24, 48, 64 are 5 G.M's

97. Let G_1, G_2, G_3, G_4 are 4 G.M's between $\frac{1}{2} \& \frac{1}{486}$

$\therefore \frac{1}{2}, G_1, G_2, G_3, G_4, \frac{1}{486}$ are in G.P.

$a = ½, \quad n = 6, \quad a_n = \frac{1}{486} \qquad a_n = ar^{n-1} \Rightarrow \quad r = \frac{1}{3}$

$\therefore \frac{1}{6}, \frac{1}{18}, \frac{1}{54}, \frac{1}{162}$ are 4 G.M's.

98. Let $G_1, G_2, G_3, G_4, G_5, G_6$ are 6 G.M's between 27 & $\frac{1}{81}$

$\therefore$ 27, G, $G_2, \ldots\ldots G_6, \frac{1}{81}$ are in G.P.

a= 27 n = 8 $a_n = \frac{1}{81}$ $a_n = ar^{n-1}$ => $r = \frac{1}{3}$

$\therefore$ 9, 3, 1, $\frac{1}{3}$, $\frac{1}{9}$, $\frac{1}{27}$ are 6 G.M's.

99. a, b, c, are in G.P.

=> $\frac{b}{a} = \frac{c}{b} = r$ => $b^2 = ac$

=> b = ar, c = br = ar^2

Now a^2, b^2, c^2 are in G.P.

If $(b^2)^2 = a^2c^2$

ie. If $b^4 = a^2c^2$

ie. If $(ar)^4 = a^2 (ar^2)^2$

ie. If $a^4r^4 = a^4r^4$ which is true

$\therefore$ a^2, b^2, c^2 are in G.P.

100. Given $a^2 + b^2$, ab + bc, $b^2 + c^2$ are in G.P.

=> $(ab + bc)^2 = (a^2 + b^2)(b^2 + c^2)$

Simplify & get $b^2 = ac$

=> a, b, c are in G.P.

FIVE MARKS QUESTIONS:

101. The required integers are 154, 161, 168, …….497, which are in A.P.

a = 154, d = 7, $a_n = 497$

$a_n = a + (n-1)d$ => n = 60

$\therefore S_{60} = 19,530$

102. The required integers are 54, 61, 68, ……194, which are in A.P.

a = 54, d = 7, an = 194

$a_n = a + (n–1)$ d => n = 21

$\therefore S_{21} = 2604$

103. Required natural Nos. are 105, 110, 115, …...…….995, which are in A.P.

$a_n = a+(n-1)\,d \quad => \quad n = 179$

$\therefore\ S_{179} = 98{,}450$

104. $a_4 = 7 \quad => \quad a + 3d = 7$

$a_{10} = 19 \quad => \quad a + 9d = 19$

$\therefore\ d = 2$ and $a = 1$

$S_n = n^2$

105. $S_{10} = 185 \quad => \quad 2a + 9d = 37 \quad \rightarrow \underline{1}$

$a_{13} = 41 \quad => \quad a + 12d = 41 \quad \rightarrow \underline{2}$

From $\underline{1}$ & $\underline{2}$ we get $d = 3$

$\therefore\ a = 5$

$\therefore\ S_{25} = 1025$

106. $a_3 = a + 2d$, $a_7 = a + 6d$, $a_{11} = a + 10d$

$a_3 + a_7 = 42 \qquad a_7 + a_{11} = 82$

$=> a_{11} - a_3 = 40 \qquad => d = 5$

$\therefore\ a = 1 \qquad \therefore$ A.P. are 1, 6, 11, 16, 21, ……..

107. Let a, d, s_n and A, D, S_n be respectively first term, c.d. and sum to 'n' terms of two A.P's

Given $\frac{s_n}{S_n} = \frac{2n-3}{3n-2}$

$$\therefore\ \frac{\frac{n}{2}[2a+(n-1)d]}{\frac{n}{2}[2A+(n-1)D)]} = \frac{2n-3}{3n-2} \quad => \quad \frac{a+\frac{(n-1)}{2}d}{A+\frac{(n-1)}{2}D} = \frac{2n-3}{3n-2} \mapsto \boxed{1}$$

To get 10^{th} term, $\frac{n-1}{2} = 9 \quad => \quad n = 19$

$\therefore \boxed{1}$ becomes $\frac{a+9d}{A+9D} = \frac{2x19-3}{3x19-2} = \frac{7}{11}$

Hence ratio of 10^{th} terms is 7:11

108. Let a, d, s_n and A, D, S_n are respectively 1st term,

C.D. & Sum to n terms of two A.P's.

$$\frac{s_n}{S_n} = \frac{7n + 2}{n + 4}$$

$$\therefore \frac{\frac{n}{2}[2a+(n-1)d]}{\frac{n}{2}[2A+(n-1)D]} = \frac{7n+2}{n+4} \qquad \Rightarrow \frac{a+\frac{(n-1)}{2}d}{A+\frac{(n-1)}{2}D} = \frac{7n+2}{n+4} \mapsto \boxed{1}$$

To get 5th term, $\frac{n-1}{2} = 4$ => $n = 9$

$\therefore$ (1) becomes $\frac{a+4d}{A+4D} = \frac{7x9+2}{9+2} = \frac{65}{13} = \frac{5}{1}$

Hence ratios of 5th terms is 5:1

109. Let a, d, s_n & A, D, S_n are respectively 1st term, C.D. & Sum to 'n' terms of two A.P's.

$$\frac{s_n}{S_n} = \frac{5n+4}{9n+6} \qquad \frac{\frac{n}{2}[2a+(n-1)d]}{\frac{n}{2}[2A+(n-1)D]} = \frac{5n+4}{9n+6}$$

$$\Rightarrow \frac{a+\frac{(n-1)}{2}d}{A+\frac{(n-1)}{2}D} = \frac{5n+4}{9n+6} \mapsto \boxed{1}$$ To get 18th term, $\frac{n-1}{2} = 17$ => $n = 35$

$\therefore$ (1) becomes $\frac{a+17d}{A+17D} = \frac{5x35+4}{9x35+6} = \frac{179}{321}$

The 18th terms of Two A.P's are in ratio 179:321

110. Let $\frac{b+c}{a}, \frac{c+a}{b}, \frac{a+b}{c}$ are in A.P.

$$\frac{c+a}{b} - \frac{b+c}{a} = \frac{a+b}{c} - \frac{c+a}{b}$$

Simplify & get $\frac{1}{b} - \frac{1}{a} = \frac{1}{c} - \frac{1}{b}$

$\Rightarrow \frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in A.P.

111. $\frac{1}{\sqrt{b}+\sqrt{c}}, \frac{1}{\sqrt{c}+\sqrt{a}}, \frac{1}{\sqrt{a}+\sqrt{b}}$, will be in A.P.

if $\frac{1}{\sqrt{c}+\sqrt{a}} - \frac{1}{\sqrt{b}+\sqrt{c}} = \frac{1}{\sqrt{a}+\sqrt{b}} - \frac{1}{\sqrt{c}+\sqrt{a}}$

simplify & get $b - a = c - b$

=> a, b, c are in A.P.

=> $\frac{1}{\sqrt{b}+\sqrt{c}}, \frac{1}{\sqrt{c}+\sqrt{a}}, \frac{1}{\sqrt{a}+\sqrt{b}}$ are in A.P.

112. $[(b+c)^2-a^2]$, $[(c+a)^2 - b^2]$, $[(a+b)^2 - c^2]$ will be in A.P.

If $(b + c + a)(b + c - a)$, $(c+ a + b)(c + a - b)$, $(a + b+c)(a+b-c)$ in A.P.

i.e. If $b + c - a$, $c + a - b$, $a + b - c$ in A.P. (Divided by $a + b + c$)

i.e. If $(c + a - b) - (b + c - a) = (a + b - c) - (c + a - b)$

i.e. If $2(a - b) = 2(b - c)$

i.e. If $b - a = c - a$

i.e. If a, b, c are in A.P.

Thus $[(b + c)^2 - a^2]$, $[(c + a) - b^2]$, $[(a + b)^2 - c^2]$ are in A.P.

113. Let $A_1, A_2, \ldots\ldots\ldots\ldots\ldots$An are 'n' A.M's between 3 & 17

Then $3, A_1, A_4\ldots\ldots\ldots\ldots$An, 17 are in A.P.

$a = 3, \quad a_{n+2} = 17$

$$d = \frac{b-a}{n+1} = \frac{17-3}{n+1}$$

$$\Rightarrow d = \frac{14}{n+1}$$

$$An = a + n\,d \qquad = 3 + n\frac{14}{n+1} = \frac{17n+3}{n+1}$$

$$A_1 = a + d = 3 + \frac{14}{n+1} = \frac{3n+17}{n+1}$$

$$\frac{A_n}{A_1} = \frac{3}{1}$$

$$\frac{17n+3}{3n+17} = 3 \quad \Rightarrow \quad n = 6$$

114. A_1, A_2 ……..Am are 'm' A.M. b/n 1 & 31

Then 1, A_1, A_2, …….A_m, 31 are in A.P.

$$d = \frac{b - a}{m + 1} = \frac{31 - 1}{m + 1} = \frac{30}{m+1}$$

Now $A_7 = a + 7d = 1+7 \frac{30}{m+1} = \frac{m+211}{m+1}$

$$A_{m-1} = a + (m-1)\, d = 1+(m-1) \frac{30}{m+1} = \frac{31m-29}{m+1}$$

Given $\frac{A_7}{A_{m-1}} = \frac{5}{9}$

Put A_7, A_{m-1} values & get m = 14

115. x, y, z are in A.P.

$\Rightarrow y - x = z - y$

$\Rightarrow 2y = x + z \quad \rightarrow (1)$

A_1 is AM of x & y

$\therefore A_1 = \frac{x+y}{2} \qquad \rightarrow (2)$

A_2 is AM of y & z

$\therefore A_2 = \frac{y+z}{2} \qquad \rightarrow (3)$

A is AM of A_1 & A_2

$$A = \frac{A_1 + A_2}{2} \qquad A = y \quad [\text{using (2) \& (3)}]$$

116.

Distance covered by gardener to water 1st tree = OA_1 = 10m

Distance covered by gardener to water

2nd tree = $A_1O + OA_2 = 10 + 15 = 25$m

Distance covered by gardener to water

3rd tree = $A_2O + OA_3 = 15 + 20 = 35$m

Distance covered by gardener to water

4^{th} tree = $A_3O + OA_4 = 20 + 25 = 45m$

$\therefore$ Total distance covered by the gardener to water all trees = D

D = 10 + 25 + 35 + 45 + …… to 25 terms.

= 10 + [25 + 35 + 45 + …….. to 24 terms]

25, 35, 45, ……. Are in A.P.

a = 25 d = 10, n = 24

$\therefore S_{24} = 3360$

$\therefore$ D = 10 + 3360 = 3370mts.

117. 0.15, 0.015, 0.0015, …… to 8 terms.

$\frac{15}{100}, \frac{15}{1000}$, ……are in G.P.

$a = \frac{15}{100}, r = \frac{1}{10} < 1, n = 8$

$S_n = \frac{9(1-r^n)}{1-r}$

$S_8 = \frac{\frac{15}{100}\left[1-\left(\frac{1}{10}\right)^8\right]}{1-\frac{1}{10}} = \frac{15}{100}\frac{\left[1-\frac{1}{10^8}\right]}{\frac{9}{10}} = \frac{5}{6}\left[1-\frac{1}{10^8}\right]$

118. 5 + 55 + 555 + ……. to n terms.

Take 5 as C.F. and Multiply & divide by 9, we get

$= \frac{5}{9}$ [9 + 99 + 999 + ……. + to n terms]

$= \frac{5}{9}$ [(10 - 1) + (10^2 – 1) + (10^3 – 1) + ……. + to n terms]

$= \frac{5}{9}$ [(10 + 10^2 + 10^3 +…….+ 10^n) – (1 + 1 + 1 +…….+1)]

$= \frac{5}{9}\left[10\left\{\frac{10^n-1}{10-1}\right\}-n\right] = \frac{5}{9}\left[\frac{10}{9}(10^n-1)-n\right]$

$= \frac{5}{81}\left[(10^{n+1}-10-9n\right] = \frac{5}{81}\left[(10^{n+1}-9n-10\right]$

119. 0.7 + 0.77 + 0.777 + …… to n terms.

= 7 x 0.1 + 7 x 0.11 + 7 x 0.111 + …….. to n terms.

= 7 [0.1 + 0.11 + 0.11 1 + …… to n terms]

$= \frac{7}{9}$ [0.9 + 0.99 + 0.999 + …….. to n terms]

$= \frac{7}{9}\left[\left(1-\frac{1}{10}\right)+\left(1-\frac{1}{100}\right)+.....\right]$…….to n terms

Simplify & get = $\frac{7}{81}\left[9n-1-\frac{1}{10^n}\right]$

120. 9 + 99 + 999 + …… + to n terms.

= (10 - 1) + (100 - 1) + (1000 - 1) +…… + to n terms.

= $(10 + 10^2\ 10^3 + \ldots\ldots\ldots\ldots +10^n) - (1 + 1 + 1 + \ldots\ldots + 1)$

$= 10\frac{\{10n-1\}}{10-1} - n$ 10, 10^2, 10^3…….. are in G.P.

$= \frac{10^{n+1}-10}{9} - n$ $= \frac{1}{9}\left[10^{n+1}-9n-10\right]$

121. 0.6 + 0.66 + 0.666 + …….. to n terms.

= 6 x 0.1 + 6 x 0.11 + 6 x 0.111 + …….. to n terms.

$= \frac{6}{9}$[0.9 + 0.99 + 0.999 + ……. to n terms]

$= \frac{6}{9}\left[\left(1-\frac{1}{10}\right)+\left(1-\frac{1}{10}\right)+\left(1-\frac{1}{1000}\right)+.....\right]$……. to n terms.

$= \frac{6}{9}\left[(1+1+.....+1)-\left(\frac{1}{10}+\frac{1}{100}+\frac{1}{1000}\right)+.....\right]$…..to n terms.

$= \frac{6}{9}\left[n-\frac{\frac{1}{10}\left(1-\frac{1}{10^n}\right)}{1-\frac{1}{10}}\right]$ $= \frac{6}{9}\left[n-\frac{1}{9}\left(1-\frac{1}{10^n}\right)\right]$

$= \frac{6}{81}\left[9n-1+\frac{1}{10^n}\right]$, $= \frac{6}{81}\left[\frac{1}{10^n}+9n-1\right]$,

122. Solve similar to 18 and get the answer

$$\frac{3}{81}\left[10^{n+1}-9n-10\right]$$

123. a, b, c, d are in G.P.

$$\Rightarrow \frac{a}{b}=\frac{c}{b}=\frac{d}{c}=r \qquad \Rightarrow b=ar,\ c=br=ar^2\ d=cr=ar^3$$

$a + b = a(1 + r)$, $b + c = ar(1 + r)$ $c + d = ar^2(1 + r)$

$a + b, b + c, c + d$ are in G.P.

if $\frac{b+c}{a+b}=\frac{c+d}{b+c}$ i.e. if $(b+c)^2=(a+b)(c+d)$

Consider $(b + c)^2 = [ar(1 + r)]^2$

$= a^2r^2(1 + r)^2$

$= [a(1 + r)][ar^2(1 + r)] = (a + b)(c + d)$

Hence $a + b, b + c, c + d$ are in G.P.

124. $a_n = 4n^2 - 4n + 1$

$$\therefore S_n=\sum_{k=1}^{n} a_k = \sum_{k=1}^{n} 4k^2-4k+1$$ Simplify & get $S_n = \frac{n}{3}(4n^2 - 1)$

125. $a_n = n^2 + n$

$$\therefore S_n=\sum_{k=1}^{n} a_k = \sum_{k=1}^{n} k^2+k$$ Simplify & get $S_n = \frac{n}{3}[n^2 + 3n + 2)$

126. Get $a_n = n^3 + 2n^2 + n$

$$S_n=\sum_{k=1}^{n} a_k \qquad Sn=\sum_{k=1}^{n} k^3+2k^2+k$$

Simplify & get $S_n = \frac{n(n+1)(n+2)(3n+5)}{12}$

127. $a_n = 9n^2 + 15n$ $$S_n=\sum_{k=1}^{n} a_k = \sum_{k=1}^{n} 9k^2+15k$$

Simplify & get $S_n = 3n(n + 1)(n + 3)$

128. Using method of difference

$a_n = 4n^2 - 1$

$S_n = \sum_{k=1}^{n} a_k = \sum_{k=1}^{n} 4k^2 - 1$

Simplify and get $S_n = \frac{n}{3}[4n^2 + 6n-1]$

129. Using method of difference

$a_n = ½ (3n^2 - n)$

$S_n = \sum_{k=1}^{n} a_k \qquad S_n = \sum_{k=1}^{n} ½(3k^2 - k)$

$S_n = ½ n^2(n + 1)$

130. Using method of difference

$a_n = n^2 + n + 1$

$S_n = \sum_{k=1}^{n} a_k = \sum_{k=1}^{n} k^2 + k + 1$

$S_n = \frac{n}{3}(n^2 + 3n + 5)$

131. Using method of difference

$a_n = n^2 + 3n + 1$

$S_n = \sum_{k=1}^{n} a_k = \sum_{k=1}^{n} k^2 + 3k + 1$

$S_n = \frac{n}{3}(n^2 + 12n + 16)$

132. $a_n = 2n^2 - 3n + 5$

$S_n = \sum_{k=1}^{n} a_k = \sum_{k=1}^{n} 2k^2 - 3k + 5$

$S_n = \frac{n}{6}(4n^2 - 3n + 23)$

133. $a_n = 2n^3 + 3n^2 - n + 1$

$$S_n = \sum_{k=1}^{n} a_k = \sum_{k=1}^{n} 2k^3 + 3k^2 - k + 1$$

$$S_n = \frac{n}{2}(n^3 + 4n^2 + 3n + 2)$$

134. $a_n = (2n-1)^2 \quad = \quad 4n2 - 4n + 1$

$$S_n = \sum_{k=1}^{n} a_k = \sum_{k=1}^{n} 4k^2 - 4k + 1$$

$$S_n = \frac{n}{3}(4n^2 - 1)$$

135. $a_n = n(n - 1)(n - 4) \quad = n^3 - 5n^2 + 4n$

$$S_n = \sum_{k=1}^{n} a_k = \sum_{k=1}^{n} k^3 - 5k^2 + 4k$$

$$S_n = \frac{n(n+1)}{12}(3n^2 - 17n + 14)$$

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