6. LINEAR INEQUALITEIS

1. Solve $-12x > 35$ when x is a natural number.

Solution: $-12x > 35 \Rightarrow -x > \frac{35}{12} \Rightarrow x < -\frac{35}{12} \Rightarrow x < -2.91.$ Thus x has no value.

2. Solve $-7x > 12$ when x is an integer.

Solution: $-7x > 12 \Rightarrow -x > \frac{12}{7} \Rightarrow x < -\frac{12}{7} \Rightarrow x < -1.71$. Thus the solution set is $\{\ldots, -4, -3, -2\}$.

3. Solve $5x < 26$ when x is a natural number.

Solution: $5x < 26 \Rightarrow x < \frac{26}{5} \Rightarrow x < 5.2$. Thus the solution set is $\{1,\ 2,\ 3,\ 4,\ 5\}$.

4. Solve $3x < 17$ when x is an integer.

Solution: $3x < 17 \Rightarrow x < \frac{17}{3} \Rightarrow x < 5.67$. thus the solution set is $\{\ldots, -1,\ 0,\ 1,\ 2,\ 3,\ 4,\ 5\}$.

5. Solve $(2x-3) > 6$ when x is an integer.

Solution: $(2x-3) > 6 \Rightarrow 2x > (6+3) \Rightarrow 2x > 3 \Rightarrow x > \frac{3}{2} \Rightarrow x > 1.5.$ Thus the solution set is $\{1,\ 2,\ 3, \ldots\}$.

6. Solve $(3x+5) > 11$ when x is a real number.

Solution: $(3x+5) > 11 \Rightarrow 3x > (11-5) \Rightarrow 3x > 6 \Rightarrow x > 2$. Thus the solution set is $(2,\ \infty)$.

7. Solve $(3x+5) \geq (5x-8)$ when x is a natural number.

Solution: $(3x+5) \geq (5x-8) \Rightarrow (5x-3x) \leq (5+8) \Rightarrow 2x \leq 13$ $\Rightarrow x \leq \frac{13}{2} \Rightarrow x \leq 6.5$. Thus the solution set is $\{1, 2,\ 3,\ 4,\ 5,\ 6\}$.

8. Solve $(3x+5) < (5x-8)$ when x is an integer

Solution: $(3x+5) < (5x-8) \Rightarrow (5x-3x) > (5+8) \Rightarrow 2x > 13$ $\Rightarrow x > \frac{13}{2} \Rightarrow x > 6.5$. Thus the solution set is $\{7,\ 8,\ 9, \ldots\}$

9. Solve $(3x+5) \geq (5x-8)$ when x is a real number.

Solution: $(3x+5) \geq (5x-8) \Rightarrow (5x-3x) \leq (5+8) \Rightarrow 2x \leq 13$ $\Rightarrow x \leq \frac{13}{2} \Rightarrow x \leq 6.5$. Thus the solution set is $(-\infty,\ 6.5]$.

10. Solve $x + \frac{x}{2} - \frac{x}{3} > 11$.

Solution: $x + \frac{x}{2} - \frac{x}{3} > 11 \;\; \Rightarrow \frac{6x + 3x - 2x}{6} > 11 \;\; \Rightarrow 7x > 66 \;\Rightarrow x > \frac{66}{7}$.

Thus the solution set is $\left(\frac{66}{7},\ \infty\right)$.

11. Solve $\frac{x}{3} \le \frac{x}{4} - 2$.

Solution: $\frac{x}{3} \le \frac{x}{4} - 2 \Rightarrow \frac{x}{3} \le \frac{x-8}{4} \;\; \Rightarrow 4x \le 3(x-8) \;\; \Rightarrow 4x \le 3x - 24$

$\Rightarrow x \le -24$. Thus the solution set is $(-\infty, -24)$.

12. Solve $\frac{2(x-3)}{4} \ge \frac{5(3-x)}{7}$.

Solution: $\frac{2(x-3)}{4} \ge \frac{5(3-x)}{7} \;\; \Rightarrow 14(x-3) \ge 20(3-x)$

$\Rightarrow (14x - 42) \ge (60 - 20x) \;\; \Rightarrow (14x + 20x) \ge (60 + 42) \;\; \Rightarrow \; 34x \ge 102$

$\Rightarrow \; x \ge 3$. Thus the solution set is $[3,\ \infty)$.

13. Solve $\frac{1}{3}\left(\frac{2x}{5} - 3\right) < \frac{1}{4}(3x - 5)$.

Solution: $\frac{1}{3}\left(\frac{2x}{5} - 3\right) < \frac{1}{4}(3x - 5) \;\; \Rightarrow \frac{1}{3}\left(\frac{2x - 15}{5}\right) < \frac{1}{4}(3x - 5)$

$\Rightarrow 4(2x - 15) < 15(3x - 5) \;\;\; \Rightarrow (8x - 60) < (45x - 75)$

$\Rightarrow (45x - 8x) > (75 - 60) \;\;\; \Rightarrow 37x > 15 \;\;\; \Rightarrow x > \frac{15}{37}$.

Thus the solution set is $\left(\frac{15}{37},\ \infty\right)$.

14. Solve $\{3(2x - 5) - 7\} \ge 9(x - 5)$.

Solution: $\{3(2x - 5) - 7\} \ge 9(x - 5) \;\; \Rightarrow (6x - 15 - 7) \ge (9x - 45)$

$\Rightarrow (6x - 9x) \ge (22 - 45) \;\;\; \Rightarrow -3x \ge -23 \;\; \Rightarrow 3x \le 23 \;\; \Rightarrow x \le \frac{23}{3}$.

Thus the solution set is $\left(-\infty,\ \frac{23}{3}\right)$.

15. Solve $\frac{2x-5}{3} - \frac{7x-3}{5} \le \frac{3x}{4}$

Solution: $\frac{2x-5}{3} - \frac{7x-3}{5} \le \frac{3x}{4} \Rightarrow \frac{5(2x-5) - 3(7x-3)}{15} \le \frac{3x}{4}$

$\Rightarrow \frac{10x - 25 - 21x + 9}{15} \le \frac{3x}{4} \;\; \Rightarrow \frac{-11x - 16}{15} \le \frac{3x}{4} \;\;\; \Rightarrow 4(-11x - 16) \le 45x$

$\Rightarrow -44x - 64 \le 45x \;\;\; \Rightarrow -44x - 45x \le -64 \;\;\; \Rightarrow -99x \le -64$

$\Rightarrow 99x \ge 64 \;\;\; \Rightarrow x \ge \frac{64}{99}$. Thus the solution set is $\left(\frac{64}{99},\ \infty\right)$.

16. Solve $\frac{2x-1}{3} > \left\{\frac{3x-2}{4} - \frac{2-x}{5}\right\}$

Solution: $\frac{2x-1}{3} > \left\{\frac{3x-2}{4} - \frac{2-x}{5}\right\} \Rightarrow \frac{2x-1}{3} > \left\{\frac{5(3x-2)-4(2-x)}{20}\right\}$

$\Rightarrow \frac{2x-1}{3} > \left\{\frac{15x-10-8+4x}{20}\right\} \quad \Rightarrow \frac{2x-1}{3} > \left\{\frac{19x-18}{20}\right\}$

$\Rightarrow 20(2x-1) > 3(19x-18) \Rightarrow (40x-20) > (57x-54)$

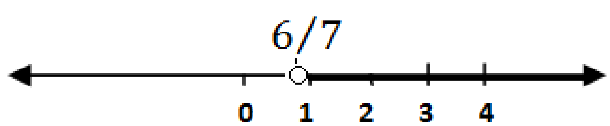
$\Rightarrow (40x-57x) > (20-54) \quad \Rightarrow -17x > -34 \quad \Rightarrow -x > -2$

$\Rightarrow x < 2$. Thus the solution set $(-\infty, 2)$.

17. Solve the inequality $(2x-5) > (1-5x)$ and represent the solution graphically on the number line.

Solution: $(2x-5) > (1-5x) \Rightarrow (2x+5x) > (1+5) \Rightarrow 7x > 6 \Rightarrow x > \frac{6}{7}$.

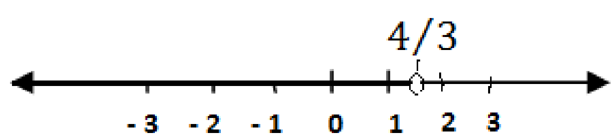
The solution set is $\left(\frac{6}{7}, \infty\right)$. The graphical representation of the solution is



18. Solve the inequality $(7x+1) \leq (4x+5)$ and represent the solution graphically on the number line.

Solution: $(7x+1) \leq (4x+5) \quad \Rightarrow (7x-4x) \leq (5-1) \quad \Rightarrow 3x \leq 4$

$\Rightarrow x \leq \frac{4}{3}$. The solution set is $\left(-\infty, \frac{4}{3}\right]$.



19. Mr. A obtained 15 and 18 in first two tests. Find the minimum mark he should get in the third test to have an average of 16.5.

Solution: Let x be the mark he should get in the third test.

By the given condition $\frac{15+18+x}{3} \geq 16.5$.

$\therefore \frac{33+x}{3} \geq 16.5 \Rightarrow 33+x \geq 49.5 \Rightarrow x \geq 49.5-33 \Rightarrow x \geq 16.5$.

Thus the minimum mark he must get is 16.5.

20. Find all pairs of consecutive odd positive integers both of which are lesser than 18 and such that their sum is more than 15.

Solution: Let x and $x+2$ are two consecutive odd positive integers. Now $x \leq 17$, $(x+2) \leq 17$. Hence $x \leq 15$. Also $(x+x+2) > 15$

$\Rightarrow\ (2x+2)\ > 15\ \ \Rightarrow\ 2x\ > 13\ \ \Rightarrow\ x\ > \frac{13}{2} = 6.5.$

Thus the possible pairs of consecutive odd positive numbers are

$(7,\ 9),\ (9,\ 11),\ (11,\ 13),\ (13,\ 15),\ (15,\ 17).$

21. The longest side of a triangle is 4 times the shortest side and the third side is 3 cm shorter than the longest side. If the perimeter of the triangle is atleast 89 cm, find the minimum length of the shortest side.

Solution: Let x be the length of the shortest side. Then the longest side is $4x$. Also the third side is $4x-3$. By the given condition, the perimeter

$\frac{x\ +\ 4x\ +\ (4x\ -\ 3)}{3} \geq\ 89\ \ \Rightarrow (9x-3) \geq 267\ \ \ \Rightarrow 9x \geq 270\ \ \ \Rightarrow x \geq 30.$

Thus minimum length of the shortest side is 30 cm.

22. A man wants to cut three lengths from a single piece of board of length 115 cm. The second length is to be 5 cm longer than the shortest and the third length is to be thrice as long as the shortest. Find the possible lengths of the shortest board if the third piece is to be at least 7 cm longer than the second.

Solution: Let x be the length of the shortest piece. Then the other two lengths are $x+5$ and $3x$.

By the given condition, $\{3x-(x+5)\} \geq 7$ and $(x+x+5+3x) \leq 113.$

Now $\{3x-(x+5)\} \geq 7\ \ \Rightarrow 2x\ \geq 12$ or $x\ \geq 6.$

Also $(x+x+5+3x) \leq 113\ \Rightarrow (5x+5) \leq 115\ \ \Rightarrow 5x \leq 110$

$\Rightarrow x \leq 21$. Thus the length of the shortest piece is ≥ 6 cm and ≤ 21 cm.

23. Solve the inequality $2x+y\ < 4$ and represent the solution region graphically.

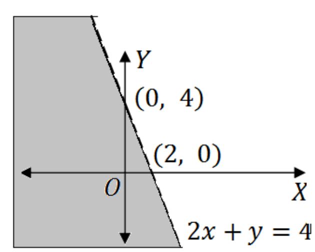
Solution: Consider the straight line $2x+y=4.$

When $x=0$, we get, $y=4.$

When $y=\ 0$, we get, $x=2.$

Thus the line pass through the points $(0,4)$ and $(2,0).$

When $x=0$ and $y=0$ we get, $2x+y=0\ < 4.$



Thus origin lies in the solution region. Thus the solution region is the half plane lying to the left of the line $2x+y=4$. Also line is not included in the solution region.

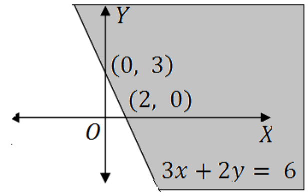
24. Solve the inequality $3x + 2y \geq 6$ and represent the solution graphically.

Solution: Consider the straight line $3x + 2y = 6$.

When $x = 0$, we get, $y = 3$.

When $y = 0$, we get, $x = 2$.

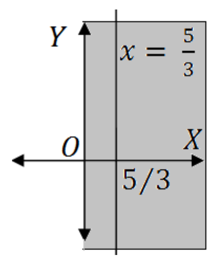
Thus the line pass through the points $(0, 3)$ and $(2, 0)$.



When $x = 0$ and $y = 0$ we get, $3x + 2y = 0 \ngeq 6$.

Thus origin does not lie in the solution region. Thus the solution region is the half plane lying to the right of the line $3x + 2y = 6$. Also line is included in the solution region.

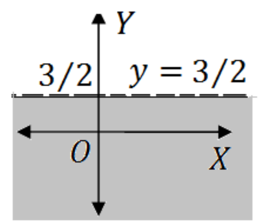
25. Solve the inequality $3x \geq 5$ and represent the solution region graphically.



Solution: $3x \geq 5 \Rightarrow x \geq \frac{5}{3}$. Thus solution set is $\left[\frac{5}{3}, \infty\right)$.

The solution region is the half plane to the right of the line $x = \frac{5}{3}$. Also the line, $x = \frac{5}{3}$ is included in the solution region.

26. Solve the inequality $2y < 3$ and represent solution set graphically.



Solution: $2y < 3 \Rightarrow y < \frac{3}{2}$. Thus the solution set is $\left(-\infty, \frac{3}{2}\right)$. The solution region is the half plane lying below the line $y = \frac{3}{2}$.

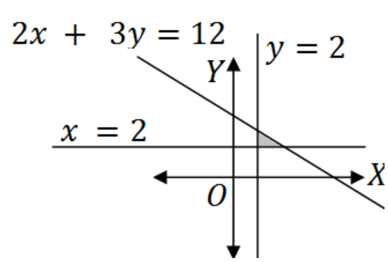
Also the line $y = \frac{3}{2}$ is not included in the solution region.

27. Solve the inequalities $2x + 3y < 12,\ x \geq 2,\ y \geq 2$ graphically.

Solution: Consider the straight line $2x + 3y = 12$.

When $x = 0,\ y = 4$ and

when $y = 0,\ x = 6$.



When $x = 0,\ y = 0$, we get,

$2x + 3y = 0 < 12$.

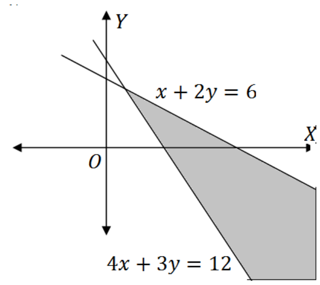
Thus solution region is the part of half plane separated by the line $2x + 3y = 12$ containing the origin.

The solution region of the set of inequalities is the region bounded by the lines $2x + 3y = 12,\ x = 2$ and $y = 2$.

28. Solve the inequalities $x + 2y \leq 6,\ 4x + 3y \geq 12$ graphically.

Solution: Consider the line $x + 2y = 6$. This passes through the points $(0, 3)$ and $(6,\ 0)$. Also when $x = 0,\ y = 0$, the number $x + 2y = 0 < 6$.

Solution region is the part of the half plane separated by $x + 2y = 6$ containing the origin.



Consider the line $4x + 3y = 12$.

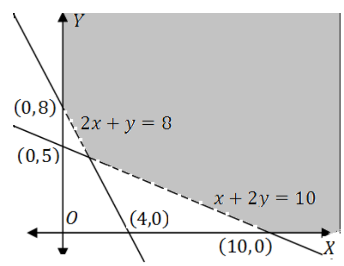
This passes through the points $(0,\ 4)$ and $(3,\ 0)$.

Also when $x = 0,\ y = 0$, the number $4x + 3y = 0 < 12$. Solution region is the part of the half plane separated by $4x + 3y = 12$ not containing the origin. The common enclosed region is the solution region. Also the two lines are included in the solution region.

29. Solve the inequalities $2x + y > 8,\ x + 2y > 10$ graphically.

Solution: Consider the line $2x + y = 8$. This passes through the points $(0, 8)$ and $(4,\ 0)$. Also when $x = 0,\ y = 0$, the number $2x + y = 0 \ngtr 8$.

Solution region is the part of the half plane separated by $2x + y = 8$ not containing the origin.



Consider the line $x + 2y = 10$. This passes through the points $(0,\ 5)$ and $(10,\ 0)$. Also when $x = 0,\ y = 0$, the number $x + 2y = 0 \ngtr 10$.

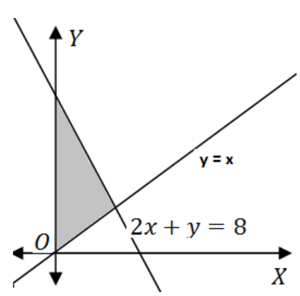
Solution region is the part of the half plane separated by $x + 2y = 10$ not containing the origin.

The common region of the above two is the solution region. Also the two lines are not included in the solution region.

30. Solve the inequalities $2x + y \leq 8,\ y > x,\ x >$ graphically.

Solution: Consider the line, $2x + y = 8$. This passes through the points $(0, 8)$ and $(4,\ 0)$. Also when $x = 0,\ y = 0$, the number $2x + y = 0 < 8$.

Solution region is the part of the half plane separated by $2x + y = 8$ containing the origin.

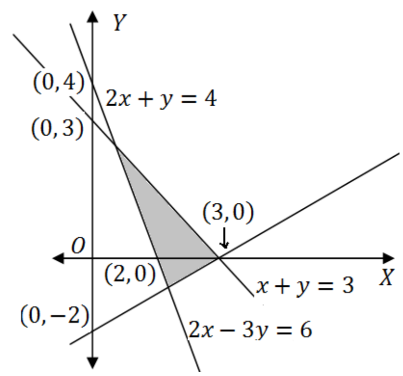


$y = x$ is a line passing through the origin. $y > x$ is the region lying to the left of the line $y = x$. Since, $x > 0$ the region lies only in the first quadrant. The shaded region gives the solution region.

31. Solve the inequalities $(2x+y) \geq 4,\ (x+y) \leq 3,\ (2x-3y) \leq 6$ graphically.

Solution: Consider the line, $2x+y=4$. This passes through the points $(0,4)$ and $(2,\ 0)$. Also when $x=0,\ y=0$, the number $2x+y=0 \ngeq 4$.

Solution region is the part of the half plane separated by $2x+y=4$ not containing the origin.



Consider the line, $x+y=3$. This passes through the points $(0,3)$ and $(3,0)$. Also when $x=0,\ y=0$, the number $x+y=0 \leq 3$.

Solution region is the part of the half plane separated by $x+y=3$ containing the origin.

Consider the line, $2x-3y=6$.

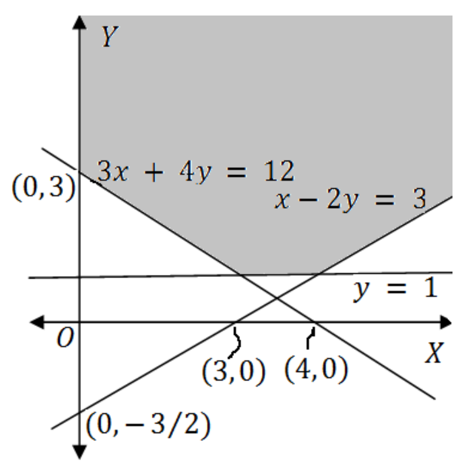
This passes through the points $(0,-2)$ and $(3,0)$. Also when $x=0,\ y=0$, the number $2x-3y=0 \leq 6$.

Solution region is the part of the half plane separated by $2x-3y=6$ containing the origin.

The shaded region gives the solution region.

32. Solve the inequalities $(x-2y) \leq 3,\ (3x+4y) \geq 12,\ x \geq 0,\ y \geq 1$ graphically.

Solution: Consider the line, $x-2y=3$. This passes through the points $(0,-3/2)$ and $(3,\ 0)$. Also when $x=0,\ y=0$, the number $x-2y=0 \leq 3$. Solution region is the part of the half plane separated by $x-2y=3$ containing the origin.



Consider the line, $3x+4y=12$. This passes through the points $(0,3)$ and $(4,\ 0)$.

Also when $x=0,\ y=0$, the number $3x+4y=0 \ngeq 12$.

Solution region is the part of the half plane separated by $3x+4y=12$ not containing the origin. $x \geq 0$ represents the region lying to the right of the y-axis. $y=1$ is a line parallel to x-axis. $y \geq 1$ is the region lying above the line $y=1$.