Chapter 1
Sets

One mark questions:

1. **Define set.**
   A set is defined as collection of well defined objects

2. **State whether the collection of all odd numbers between 6 and 16 is a set or not.**
   {7,9,11,13,15} is a set

3. **State whether the collection of all good cricket players in India a set or not.**
   It is not a set because good cricketer is not well defined

4. **Give an example of a collection which is not a set.**
   Collection of beautiful girls in the city is not a set

5. **Write the set of all natural numbers such that $x^2 – 4x = 0$ in the Roster form.**
   $x^2 – 4x = 0 \Rightarrow x (x -4) = 0 \Rightarrow x =0,4$, as $0 \notin N$
   Required set = {4}

6. **Write the set A = { x : x is an even prime} in the tabular form.**
   A = {2}

7. **Write set S = {1, 2, 4, 8, 16} in the set builder form.**
   $S = \{2^n \mid n = 0,1,2,3,4\}$

8. **Write the set M = {$\pm$ 1, $\pm$ 2, $\pm$ 3, ….., $\pm$ 10} in the set-builder form.**
   $M = \{n \mid -10 \le n \le 10, n \in Z, n \neq 0\}$

9. **Define an empty set.**
   The set of real roots of $x^2 + 1 = 0$ is an empty set.

10. **Give an example of a null set.**
    The set of real roots of $x^2 + 1 = 0$ is a null set.

11. **State weather the set of all prime numbers lying between 14 and 16 is a null set or not.**
    It is a null set

12. **Is the set of all natural numbers less than 2, an empty set?**
    {1} is not an empty set

13. **Define singleton set.**
    A set having only one element is called sigelton set.

14. Give an example of a singleton set.
{0} is the singleton set

15. Define a finite set.
A set containing finite number of elements is called a finite set

16. Give an example of a finite set.
A = {a,b,c} Is a finite set

17. Define an infinite set.
A set containing infinitely many elements is called an infinite set

18. Give an example of an infinite set.
Set of natural numbers is an infinite set

19. State whether the set of all points on a particular straight line is a finite or not.
It is an infinite set, because a particular straight line contains infinitely many points.

20. Define equal sets.
In two sets A and B , if every element of A is an element of B and if every element of B is also an element of A, then the sets A and B are said to be equal

21. Give an example of equal sets.
Let A = {a,b,c} and B = {c,a,b} . then A and B are equal sets.

22. Define subset.
A set A is said to be a subset of a set B if every element of A is also an element of B

23. Write a subset of A = {1,2,3}
{1} is a subset of A

24. Define power set.
The collection of all subset of a set A is called the power set of A.

25. If A = {1,2,3} then find number of elements of power set of A
n(A) = 3, therefore number of elements of power set of A = 2^3 = 8.

26. If n(P(A)) = 32, then find the number of elements of A.
n[P (A)] = 32 = 2^5, ∴ n(A) = 5

27. Write the power set of X = {1,2}
Power set of X = {Φ , x, {1} , {2}}

28. Define proper set.

Let A and B be two sets. If A **C** B and A ≠ B, then A is called a proper subset of B.

29. Write all the proper subsets of {a,b}

Proper subset of {a,b} are {a} and {b}

30. If P(A) = {Φ, A, {a} , {b} , {c} , {a,b} , {b,c} , {c,a} } then write the set A.

A = {a,b,c}

31. Define closed interval.

The interval which contains the end point also is called closed interval

i.e. [a,b] = {x / a ≤ x ≤ b} is closed interval

32. Define open interval.

Let a,b∈ R and a < b . then the set of real numbers {x : a < x < b } is called an open interval

33. Define universal set.

In the given context, a set, superset of all other sets is called the universal set.

34. Define union of two sets

Let A and B be two sets. Then the set A U B = {x/ x ∈ A or x ∈ B} is the union of A and B

35. If A = {1,2,3} and B = {2,3,4} find AUB

A U B = {1,2,3,4}

36. Find the smallest set X such that X∪ {a,b} = {a,b,c,d,e}

X = {c,d,e}

37. Define intersection of two sets

Let A and B be two sets. Then the set A∩B = {x/ x ∈ A or x ∈ B} is the intersection of A and B

38. Define difference of two sets.

Let A and B be two sets. Then the set A – B = {x/ x ∈ A or x ∉ B} is the difference of A and B

39. Define compliment of a set.

Let U be the universal set and A be the set, then A\` = U – A = {x / x ∈ U and x ∉ A} is the complement of a set.

40. If U = {0,1,2,3,…..9} and A={2,4,6,8} then find A\`

A' = {0,1,3,5,7,9}

41. Let U be the set of all even natural numbers and A is of all even multiples of 3. Write (A')'

(A')' = {6,12,18,…..}

Two marks question:

1. **Which are the two methods of representation of sets.**
   The two methods of representation of sets are
   i) Roster or tabular form (method)
   ii) Set builder or rule form (method)

2. **Examine whether the set A = {x / x^3-x = 0 , x > 1} is an empty set or not.**
   $x^3 - x = 0 \Rightarrow x(x^2 - 1) = 0$
   ⇨ $x(x-1)(x+1) = 0$
   ⇨ $x = 0$ or $x = 1$ or $x = -1$

3. **Is the set B = {x / x = 2n-1, n < 3, x ∈ N} an empty set?**
   If n < 3, then x = 3, 1, -1, -3, …
   Since x ∈ N, B = {3, 1}
   ∴ B is nonempty.

4. **Is the set C= {x / x^2-1 = 0, x ∈ N } a singleton set?**
   $X^2 - 1 = 0 \Rightarrow x^2 = 1 \Rightarrow x = \pm 1$
   Since x ∈ N, x = 1. ∴ C = {1}
   ∴ C is a singleton set

5. **State whether the set D = {x / 10^2 < x < 11^2 , x ∈ Z} is singleton or not.**
   10^2 = 100 and $11^2 = 121$
   ∴ 100 < x < 121
   Since x ∈ Z, x = 101, 102, …. 120
   ∴ D = {101, 102, 103, …..120}
   ∴ D is not singleton

6. **State whether the set of positive factors of 25 which are greater than 5 is finite or not.**
   Positive factors of 25 are 1, 5, 25
   ∴ set of positive factors of 25 which are greater then 5 is {25}. This is finite.

7. **State whether the set of all multiples of 5 which are greater then 25 is finite or not.**
   Set of all multiples of 5 which are greater than 25 is {30, 35 , 40, …}
   This is infinite

8. **Are the following pair of sets equal. Justify your answer.**
   A = {x / x is a letter of the word STAGE}
   B = {x / x is a letter of the word GATES}
   A = {S , T, A, G, E} and B = {G, A , T, E, S}.
   Since A and B contains same elements the sets A and B are equal

9. State whether the set A = {x / x is a multiple of 3 lying between 5 and 20} and the set B = {6, 9, 12, 15, 16, 18} are equal.

A = {6, 9, 12, 15, 18}

Since 16 $\notin$ A, the set A and B are not equal

10. Write all the subsets of the set A = {4}.

The set of all subsets of A is { Φ, A }

11. If A = {0,1,2,3,4} and B = {x / x $\in Z, -1 < x < 4$}. State whether A is a subset of B or not.

B = {0, 1, 2, 3}

Since 4 $\notin$ B , A is not a subset of B.

12. Write all the subset of the set A = {x / 4x < 5, x $\in$ N}.

A = {1}

$\therefore$ The set of all subsets of A is { Φ, A }

13. If A = {2}, B = {{2}, 3} and C = {{2} , 3, 4} is A $\subset$ C . Justify your answer.

2 $\in$ A . But 2 $\notin$ C

$\therefore$ A is not a subset of C.

14. If the number of proper subsets of a set is 127. Find the number of element in the set.

If the given set contains n elements then the number of proper subsets is $2^n - 1$

Given $2^n - 1 = 127$

$\therefore 2^n = 127 + 1 = 128 = 2^7$

$\therefore$ n = 7

15. If A = {x / x $\in$ N 1 < x < 6 } and B = {x / x $\in$ Z -2 < x < 3 } then find A U B.

A = {2, 3, 4, 5} and B = {-1, 0, 1, 2}

$\therefore$ A U B = {-1, 0, 1, 2, 3, 4, 5}

16. If A = {x / x = 3n + 1, n = 0, 1, 2 } and B = {x / x = 2n + 1, n = 0, 1, 2, 3 } then find A $\cap$ B.

A = {1, 4, 7} and B = {1, 3, 5, 7}

$\therefore$ A $\cap$ B = {1, 7}

17. If A = {2, 3, 5} and B = {3, 5, 6}. Find A - B and B – A.

A – B = { 2 }

B – A = { 6 }

18. If A = {x / x = 4n - 3, n = 1, 2, 3 } and B = {1, 5, 14} . Find B – A.

A = {1, 5, 9}

$\therefore$ B – A = { 14 }

19. If A = {3, 5, 7, 9} , B = {4, 5, 6, 8} and C = {7, 8, 9, 10} . Find B - C and C – A.

B – C = {4, 6, 8}

C – A = {8, 10}

20. If A = {a, b, c, d} , B ={b, d, e, f} and C = {a, c, f, g} . find A – (C - B).

C – B = {a, c, g}

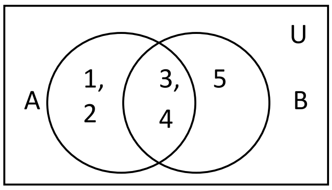
∴ A – (C - B) = {b, d}

21. If A = {2, 4, 6}, B = {4, 6, 8} and C = {6, 8} . find (A - B) ∩ C.

A – B = { 2 }

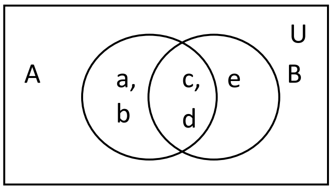
∴ (A – B) ∩ C = Φ

22. If A = {1, 2, 3, 4} , B = {3, 4, 5}. Draw the Venn diagram of A ∩ B.



A ∩ B = {3, 4}

23. If A = {a, b, c, d} , B ={c, d, e}. Draw the Venn diagram of A – B.



A-B = {a, b}

24. If n (A ∪ B) = 30 and n (A ∩ B) = 10. Find n (A) + n (B) .

n (A ∪ B) = n (A) + n (B) – n (A ∩ B)

∴ n (A) + n (B) = n (A ∪ B) + n (A ∩ B)

= 30 + 10 = 40

25. If n (A – B) = 5 , and n (B – A) = 2 and n (A ∩ B) = 4. Find n (A ∪ B)

n (A ∪ B) = n (A - B) + n (A ∩ B) + n (B - A)

∴ n (A ∪ B) = 5 + 4 + 2

= 11

26. If A = Φ , B = {1, 3, 5}. Find A ∩ B and A ∪ B.

A ∩ B = Φ

A ∪ B = {1, 3, 5}

27. Let U = {1, 2, 3, 4, 5, 6}, A = {2, 4, 6} and B = {1, 3, 5}. Find A ∩ B and A' ∪ B' .

A ∩ B = Φ

A' ∪ B' = {1, 3, 5 } ∪ {2, 4, 6 } = {1, 2, 3, 4, 5, 6 }

28. If A = {2, 4, 6, 8, 10, 12}, B = {3, 6, 9, 12, 15}, C = {4, 8, 12, 16} and D = {5, 10, 15, 40}. Find A – B and C – D.

A – B = {2, 4, 8, 10}
C – D = {4, 8, 12, 16}

29. If A = {-1, 0, 1} and B = {x / x^2 + 1 = 0, x ∈ Z}. Find A – B and A ∩ B.

A = {-1, 0, 1} and B = Φ
A – B = {-1, 0, 1}
A ∩ B = Φ

30. Prove that the difference on sets is not commutative.

Let us prove this by giving the counter example.
Let A = {1, 2, 3} and B = {2, 3, 4}
A – B = { 1 } and B – A = { 4 }
∴ A – B ≠ B – A

31. If Z is the set of integers and N is the set of natural numbers, then find Z – N and N – Z.

Clearly N C Z
Z – N = {…., -3, -2, -1, 0 }
N – Z = Φ

32. If A = {x / x = 2^n, n ≤ 5, n ∈ N } and B = {x / x = 4^n, n ≤ 3, n ∈ N }. Find A – B and A ∩ B.

A = {2, 4, 8, 16, 32}
B = {4, 16, 64 }
A ∩ B = {4, 16 }
A – B = {2, 8, 32}

33. If A = {x ∈ N / x^2 – 9 = 0 } and B = {-3, 3}, then verify either A = B or not

$x^2 - 9 = 0 \Rightarrow x = \pm 3$
∴ A = { 3 } and B = { -3, 3 }
∴ A ≠ B

34. If A ∩ B = A ∪ B, then prove that A = B

Given A ∩ B = A ∪ B
Let x ∈ A => x ∈ A ∪ B
∴ x ∈ A ∩ B => x ∈ B
Similarly x ∈ B implies x ∈ A
∴ A = B

35. If U = {1, 2, 3, ….. 15}. Find A' when

i) A = {1, 3, 5, 7}
ii) A = {2, 4, 6, 8, 10, 12, 14}

i) A' = {2, 4, 6, 8, 9, 10, 11, 12, 13, 14, 15}
ii) A' = {1, 3, 5, 7, 9, 11, 13, 15 }

Three mark questions:

1. **Write all the subsets of A = {a, b, c}**

The set of all subsets of A is

P (A) = { Φ, A, {a} , {b}, {c}, {a, b}, {b, c}, {a, c} }

2. **If A = {a, b, c} B = {b, c, d, e} and C = {a, c, d, e}. verify that (A ∩ B) ∩ C = A ∩ (B ∩ C).**

A ∩ B = {b , c}

∴ (A ∩ B) ∩ C = { c }

B ∩ C = {c, d, e}

∴ A ∩ (B ∩ C) = { c }

∴ (A ∩ B) ∩ C = A ∩ (B ∩ C).

3. **If A = {1, 2, 3} , B = {2, 4, 6} and C = {1, 3, 5}. Verify that (A ∪ B) ∪ C = A ∪ (B ∪ C).**

A ∪ B = {1, 2, 3, 4, 6}

∴ (A ∪ B) ∪ C = {1, 2, 3, 4, 5, 6}

B ∪ C = {1, 2, 3, 4, 5, 6}

∴ A ∪ (B ∪ C) = {1, 2, 3, 4, 5, 6}

∴ (A ∪ B) ∪ C = A ∪ (B ∪ C).

4. **If A = {0, 1, -1}, B = {0, 1, 2} and C = {1, 2, 3}. Verify that A ∩ (B ∪ C) = (A ∩ B) ∪ (A ∩ C).**

B ∪ C = {0, 1 , 2 , 3}

A ∩ (B ∪ C) = {0 , 1}

A ∩ B = {0 , 1} and A ∩ C = {1}

∴ (A ∩ B) ∪ (A ∩ C) ={0 ,1} ∴ A ∩ (B ∪ C) = (A ∩ B) ∪ (A ∩ C).

5. **If A = {a, b} , B = {b, c} and C = {c, d }. Verify that A ∪ (B ∩ C) = (A ∪ B) ∩ (A ∪ C).**

B ∩ C = {C}

∴ A ∪ (B ∩ C) = { a,b,c}

A ∪ B= { a, b, c} and A ∪ C ={ a, b, c, d}

∴ (A ∪ B) ∩ (A ∪ C) ={ a, b, c}

∴ A ∪ (B ∩ C) = (A ∪ B) ∩ (A ∪ C)

6. **If A = {x / x = 2n where n = 0, 1, 2 }, B = {2, 4, 6}. Find A – B and B – A . Are they equal?**

A = { 0 , 2, 4}

A – B ={0} and B – A ={6}

A – B and B – A are not equal.

7. **If A = {4, 5, 6}, B = {5, 7, 8}. Find A – B and A ∩ B. Write the intersection of A – B and A ∩ B.**

A – B ={ 4, 6}

A ∩ B = {5}

∴ (A – B) ∩ (A ∩ B) = Φ

8. If A = {x / x = n^2 – 3, where n = 0, 1, 2} and B = {-2, 1, 2} . Find A – (A ∩ B).

A = {-3 , -2, 1 }

∴ A ∩ B = {- 2, 1 }

∴ A - (A ∩ B) = { -3 }

9. If U = { ± 1, ± 2, ± 3, ± 4} and A = {± 1, ± 3} , B = {± 3, ± 4}. Show that (A ∪ B)` = A` ∩ B`.

A ∪ B = { ±1, ±3, ±4 }

∴ (A ∪ B)' = { ± 2 }

A' = { ±2, ±4 } and B'= { ±1, ±2}

∴ A' ∩ B' = { ±2} ∴ (A ∪ B)' = A' ∩ B'

10. If U = {a, b, c, d} and A = {a, c}, B = {b, c}. Show that (A ∩ B)` = A` ∪ B`

A ∩ B = { c }

∴ (A ∩ B)' = { a, b, d }

A'= { b, d} and B' ={a , d }

∴ A' ∪ B'= { a , b , d } ∴ (A ∩ B)` = A` ∪ B`

11. If U = {1, 2, 3, ……., 15} A = {2, 4} and B = {3, 4, 6, 10, 12, 15}. Verify that (A ∪ B)` = A` ∩ B`.

A ∪ B = { 2, 3, 4, 6, 10, 12, 15}

∴ (A ∪ B)' = {1, 5, 7, 8, 9, 11, 13, 14 }

A' = { 1, 3, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15}

B' = { 1, 2, 5, 7, 8, 9, 11, 13, 14}

∴ A` ∩ B`= { 1, 5, 7, 8, 9, 11, 13, 14}

∴ (A ∪ B)` = A` ∩ B`

12. If U = {1, 2, 3, ………, 11} , A = {2, 5, 9, 10} and B = {1, 4, 7, 9} , then verify that (A ∩ B)` = A` ∪ B`

A ∩ B = {9}

∴ (A ∩ B)` = {1, 2, 3, 4, 5, 6, 7, 8, 10, 11}

A` = {1, 3, 4, 6, 7,8, 11}

B` = {2, 3, 5, 6, 8, 10, 11}

∴ A` ∪ B` = {1, 2, 3, 4, 5, 6, 7, 8, 10 , 11}

∴ (A ∩ B)` = A` ∪ B`

13. If A = { x / x = 2n +1, n ≤ 5 n ∈ N } B = { x / x = 3n – 2, n ≤ 5 n ∈ N }. Find i) A ∪ B ii) A ∩ B

A = {3, 5, 7, 9, 11}

B = {1, 4, 7, 10, 13}

∴ A ∪ B = {1, 3, 4, 5, 7, 9, 10, 11, 13 }

A ∩ B = { 7 }

14. If A = {2, 3, 4, 6 } B = { 3, 5, 8} and C = {1, 3, 5, 8, 10}. Then find (B - C) – (C - A)

B – C = { } = Φ

C – A = { 1, 5, 8, 10}

∴ (B - C) – (C - A) = Φ – { 1, 5, 8, 10} = Φ

15. If A = {1, 2, 3, 4, 5, 6} , B = {1, 2}. Then verify that A – (A - B) = A ∩ B.

A –B ={ 3, 4, 5, 6}

∴ A – (A –B) = {1, 2}

A ∩ B = { 1, 2 }

∴ A – (A - B) = A ∩ B

16. If X and Y are two sets such that X ∪ Y has 60 elements , X has 25 elements and Y has 48 elements. How many elements does X ∪ Y have ?

n (X ∪ Y) = 60 , n (X) =25 , n (Y) =48

n (X ∪ Y) = n (X) + n (Y) – n (X ∩ Y)

∴ n (X ∩ Y) = n (X) + n (Y) – n (X ∪ Y)

= 25 + 48 -60

= 13

17. A class has 80 students. 60 students speak Kannada language, 40 students speak English language. Find how many students speak both languages.

Let A: Set of students who speak Kannada

B: Set of students who speak English

Here n (A)= 60 , n (B) = 40 and n (A ∪ B) = 80

Required is n (A ∩ B)

n (A ∩ B) = n (A) + n (B) – n (A ∪ B)

= 60 + 40 – 80

= 20

18. If X = {a, b, c, d, e} and Y = {a, e, i ,o}. find (i) X – Y (ii) Y – X (iii) X ∩ Y

X – Y = { b, c, d}

Y – X = { i, o }

X ∩ Y = { a, e }

19. Given A = {x / 3 ≤ x ≤ 6, x ∈ Z } B = {x / 6 < x ≤ 9, x ∈ Z}, then find A ∪ B, A ∩ B and A - B

A = { 3, 4, 5, 6} and B = { 7, 8, 9}

A ∪ B = { 3, 4, 5, 6, 7, 8, 9}

A ∩ B = { } = Φ

A – B = { 3, 4, 5, 6}

20. If U = {1, 2, 3, 4, 5, 6, 7, 8, 9} A = {1, 3, 5, 7, 9} B = {1, 2, 3, 4, 5}.
Find i) A' (ii) (A ∪ B)' (iii) (A - B)'

A' = {2, 4, 6, 8}

A ∪ B = {1, 2, 3, 4, 5, 7, 9 }

$\therefore (A \cup B)' = \{6, 8\}$

$A - B = \{5, 7, 9\}$

$\therefore (A - B)' = \{1, 2, 3, 4, 6, 8\}$

21. If A and B are two sets such that A has 60 elements, A ∪ B has 80 elements and A ∩ B has 15 elements, how many elements does B have?

$n(A) = 60$, $n(A \cup B) = 80$, $n(A \cap B) = 15$

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$\therefore n(B) = n(A \cup B) + n(A \cap B) - n(A)$

$= 80 + 15 - 60$

$= 35$

22. In a class of 40 students, 30 play cricket and 18 play Hockey. If each student plays either cricket or hockey, find the number of students who play hockey only.

Let A be the set of students who play cricket and B be the set of students who play hockey.

Here $n(A) = 30$ and $n(B) = 18$ and $n(A \cup B) = 40$

$n(A \cap B) = n(A) + n(B) - n(A \cup B)$

$= 30 + 18 - 40$

$= 8$

$\therefore$ The number of students who play hockey only

$= n(B) - n(A \cap B) = 18 - 8 = 10$

23. A market research group conducted a survey of 1000 consumers. 850 consumers liked product A and 420 liked product B. What is the least number that must have liked both products?

Let P be the set of consumers who liked product A and let Q be the set of consumers who liked product B.

Here $n(P) = 850$, $n(Q) = 420$ and $n(P \cap Q) \leq 1000$.

$n(P \cap Q) = n(P) + n(Q) - n(P \cup Q)$

$\geq 850 + 420 - 1000$

$\geq 1270 - 1000$

≥ 270

$\therefore$ atleast 270 liked both the products

24. In a survey of 300 students in a school, 75 students are found to be drinking tea and 125 drinking coffee, 50 were drinking both tea and coffee. Find how many students drink neither tea nor coffee.

Let A be the set of students who drink tea and B be the set of students who drink coffee.

Here $n(A) = 75$, $n(B) = 125$ and $n(A \cap B) = 50$.

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$= 75 + 125 - 50$

$= 130$

$\therefore$ number of students who drink neither tea nor coffee $= 300 - 130 = 170$

25. Prove that for any two sets A and B

$A - B = A - (A \cap B)$

Let $x \in A - B$

Then $x \in A$ and $x \notin B$

$\therefore x \in A$ and $x \notin (A \cap B)$

$\therefore x \in A - (A \cap B)$

$\therefore (A - B)$ **C** $(A - (A \cap B))$ ……….(1)

Let $x \in A - (A \cap B)$

Then $x \in A$ and $x \notin A \cap B$

$\therefore x \in A$ and $x \notin B$ (since $x \in A$)

$\therefore x \in A - B$

$\therefore (A - (A \cap B))$ **C** $(A - B)$………….(2)

$\therefore$ from (1) and (2) we get $A - B = A - (A \cap B)$