# Chapter - 3. (Motion in a straight line)

## One mark questions

1. **When is an object said to be in motion?**
An object is said to be in motion if its position changes with time

2. **What is rectilinear motion?**
The motion of an object along a straight line is known as rectilinear motion

3. **What is kinematics?**
Kinematics deals with the study of motion of bodies without considering the causes of motion.

4. **What is required to specify the position of an object?**
To specify the position of an object, a reference point called origin is required.

5. **What is meant by path length?**
The total distance traversed by an object is called path length.

6. **What is displacement?**
The change of position in a particular direction or the distance between the initial and final position of the object is called displacement.

7. **What is meant by uniform motion?**
If an object moving along the straight line covers equal distances in equal intervals of time, then it is said to have uniform motion.

8. **What is the position - time graph?**
A graph of position (along y - axis) against time (along x - axis) is known as position time graph.

9. **Define average velocity.**
Average velocity is defined as the displacement ($\Delta x$) divided by time interval ($\Delta t$).
$$\bar{v} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{\Delta x}{\Delta t}$$

10. **Define average speed.**
Average speed is defined as the total path length traveled divided by the total time taken. Average speed = $\frac{\text{path length}}{\text{total time interval}}$

11. **Define instantaneous velocity of a body in terms of its average velocity.**
The instantaneous velocity is defined as the limit of the average velocity as the time interval $\Delta t$ tends to zero.
$$v = \lim_{\Delta t \to 0} \bar{v} = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

12. **The average velocity of a body is equal to its instantaneous velocity. What do you conclude by this?**
The body is moving with constant velocity.

13. **What does the slope of position - time graph represent?**
Velocity.

14. **Define average acceleration.**
Average acceleration is the change in velocity divided by the time interval during which the change occurs. $\bar{a} = \frac{\Delta v}{\Delta t}$.

15. **Define instantaneous acceleration of a body in terms of its average acceleration.**
The instantaneous acceleration is defined as the limit of the average acceleration as the time interval $\Delta t$ tends to zero. $a = \lim_{\Delta t \to 0} \bar{a} = \lim_{\Delta t \to 0} \frac{\Delta V}{\Delta t} = \frac{dV}{dt}$.

16. **What does the slope of velocity - time graph represent?**
Acceleration.

17. **What does area under velocity - time graph represent?**
Displacement for a given time interval.

18. **When is relative velocity of two moving objects zero?**
Relative velocity is zero when the two objects move with same velocity in same direction.

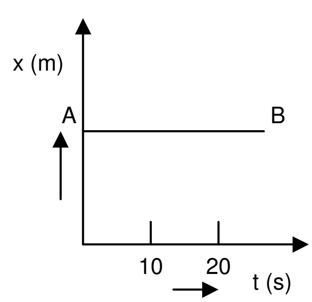
19. **What is the acceleration of a body moving with constant velocity?**
Zero.

**Two mark question**

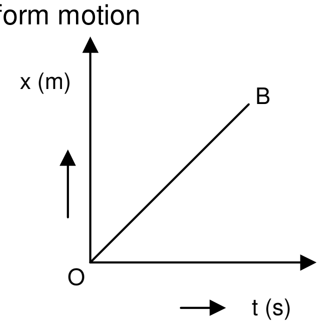
1. **Distinguish between distance and displacement.**
(i) The distance is the length of path traversed. The displacement is the change of position in a particular direction.
(ii) Distance is a scalar. But displacement is a vector.
(iii) When a body returns to initial position, then distance is not zero but displacement is zero.

2. **Draw the position - time graph for an object (i) at rest (ii) with uniform motion.**

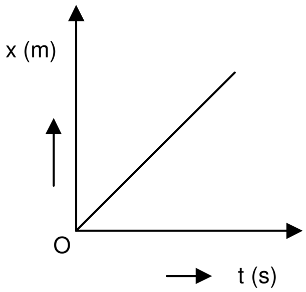
(i) rest

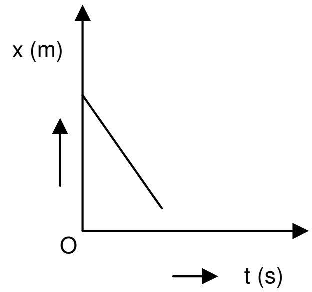



(ii) uniform motion



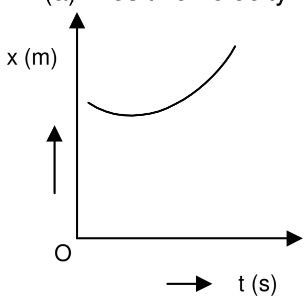

3. **Draw the position - time graph for an object (a) moving with positive velocity and (b) moving with negative velocity.**
(a) Positive velocity (b) Negative velocity

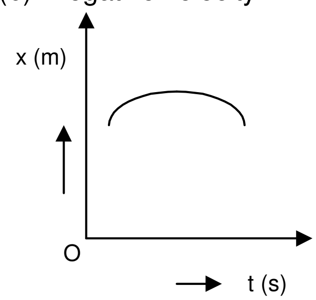


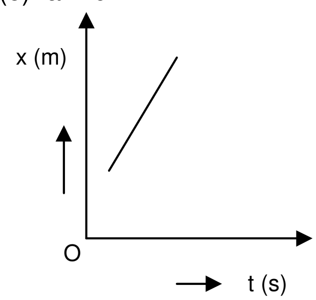




**4. Draw position - time graph for motion with (a) positive acceleration (b) negative acceleration (c) zero acceleration.**

(a) Positive velocity (b) Negative velocity (c) a = 0

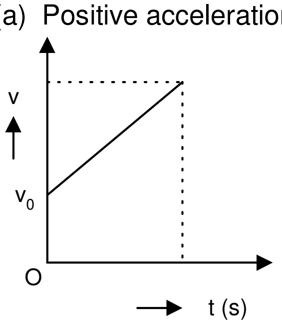


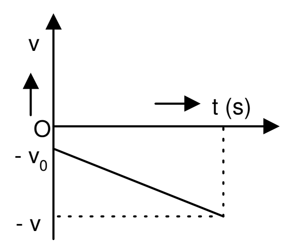



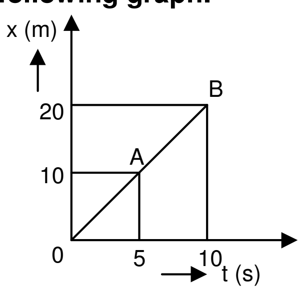

**5. Draw velocity - time graphs for motion in (a) positive direction with positive acceleration (b) negative direction with negative acceleration.**

(a) Positive acceleration (b) Negative acceleration




**6. Find the velocity of the particle for the time interval t = 5 to t = 10 s from the following graph.**




The velocity for the time interval $t_1 = 5s$ to $t_2 = 10s$ is given by

$$v = \frac{x_2 - x_1}{t_2 - t_1} = \frac{20 - 10}{10 - 5} = \frac{10}{5} = 2ms^{-1}$$

**7. The displacement (in metre) of a particle moving along x - axis given by $x = 20t + 10t^2$. Calculate the instantaneous velocity at t = 2s.**

We have $x = 20t + 10t^2$.

Velocity $v = \frac{dx}{dt} = 20 + 20t$

Instantaneous velocity at t = 2 s is v = 20 + 20 x 2 = 60 $ms^{-1}$.

**8. A ball is thrown vertically upward and it reaches a height of 90 m. Find the velocity with which it was thrown.**

v = 0, x = 90 m, g = 9.8 $ms^{-2}$, $v_0$ = ?

Using the equation $v^2 = v_0^2 + 2gx$ we get

$$0 = v_0^2 - 2 \times 9.8 \times 90$$

$$\therefore v_0^2 = 2 \times 9.8 \times 90$$

$$v_0 = \sqrt{2 \times 9.8 \times 90} = 42\ ms^{-1}$$

**9. Define relative velocity with an example.**

Relative velocity means velocity of one object w.r.t the other object.

Example: Consider two trains on parallel tracks with same velocity in same direction. Although both the trains are in motion w.r.t the ground, for an observer in one train, the other train does not appear to move. In this case the relative velocity becomes zero.

**10. A car travels with a uniform velocity of 20 $ms^{-1}$. The driver applies the brakes and the car comes to rest in 10 second. Calculate the retardation.**

$v_0 = 20\ ms^{-1}$, v = 0, t = 10 s.

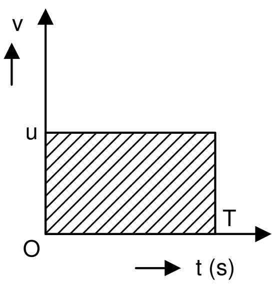
$$a = \frac{v - v_0}{t} = \frac{0 - 20}{10} = -2\ ms^{-2}$$

$\therefore$ Retardation = 2 $ms^{-2}$

**Four Mark questions**

1. a) What is the velocity - time graph?
   b) Show that area under velocity - time graph is equal to displacement.

When instantaneous velocities of a particle in motion are plotted against time, the resultant graph is called velocity - time graph.

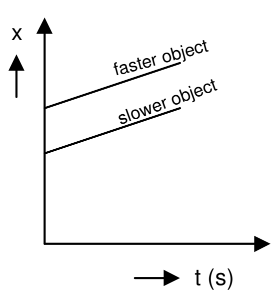



Area under the v - t graph is the area of the rectangle of height u and base T.
Therefore Area = u x T …………………… (1)
By definition, displacement during this time interval = u x T…………… (2)
Equation equations (1) and (2),
Area under velocity - time graph is equal to displacement

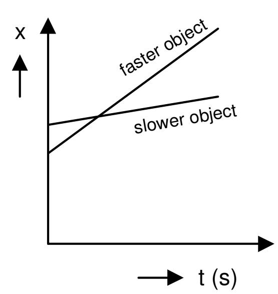
2. a) Define relative velocity of an object w.r.t another.
   b) Draw position - time graphs of two objects moving along a straight line when their relative velocity is (i) zero and (ii) non - zero

Relative velocity is the velocity of one object w.r.t another.
For example if A and B are two objects moving uniformly with average velocities $v_A$ and $v_B$ in one dimension, then the velocity of object B relative to object A is $v_B - v_A$. i.e. $v_{BA} = v_B - v_A$.
Similarly, velocity of object A relative to object B is $v_{AB} = v_A - v_B$

(i)




(ii)



3. What is the significance of velocity - time graph?

Significance of velocity - time graph

(i) It represents the nature of motion of the particle.

(ii) Instantaneous velocity and instantaneous acceleration can be obtained from the curve.

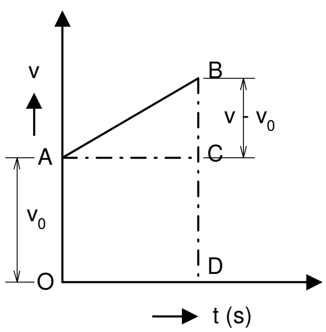
(iii) Equations of motion along a straight line can be derived.

(iv) Area under velocity - time graph in a given time interval represents the distance traveled by the particle in that time interval.

**Five marks theory questions**

1. Derive the equation of motion $x = v_0 t + \frac{1}{2}at^2$ from v - t graph.

Consider an object moving with an initial velocity $v_0$ under constant acceleration 'a'. After 't' second, let v be its velocity and x the displacement. Let AB represent the velocity - time graph of the object. Here OA represents $v_0$, DB represents v and OD represents 't'.




The area under v - t graph represents the displacement.

$\therefore$ x = area under AB

= area of the rectangle OACD + area of $\Delta$ ABC.

= (OA x OD) + ½ (AC x BC)

$= v_0 t + \frac{1}{2} t(v - v_0)$ ……… (1)

But $v = v_0 + at$

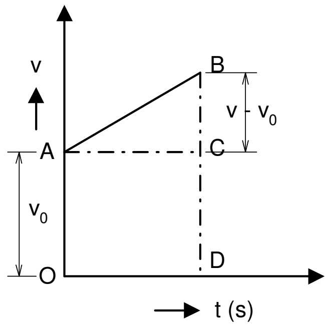
$\therefore\ v - v_0 = at$

Substituting this in equation (1) we have

$x = v_0 t + \frac{1}{2}at^2$

2. Derive the equations of motion (i) $v = v_0 + at$ and (ii) $v^2 = v_0^2 + 2ax$ from v - t graph.

Consider an object moving with an initial velocity $v_0$ under constant acceleration 'a'. After 't' second, 'v' be its velocity and 'x' the displacement. Let AB represent the velocity - time graph. Here OA represents $v_0$, DB represents 'v' and OD represents 't'.




(i) to derive $v = v_0 + at$

The slope of velocity - time graph represents uniform acceleration 'a'.

$\therefore$ Acceleration = slope = $\dfrac{BC}{AC}$

$$\therefore\ a = \frac{v - v_0}{t}$$

$$\therefore v - v_0 = at$$

$$\therefore v = v_0 + at \;\ldots\ldots\ldots\ldots\ (1)$$

(ii) The object has traveled distance 'x' in time 't' with average velocity $\bar{a}$ given as

$$\bar{a} = \frac{v + v_0}{2}$$

$$x = \bar{a}\,t$$

$$= \left(\frac{v + v_0}{2}\right)t \;\ldots\ldots\ldots\ldots(2)$$

From equation (1) $t = \dfrac{v - v_0}{a}$

Substituting this in equation (1) we have

$$x = \left(\frac{v + v_0}{2}\right)\left(\frac{v - v_0}{a}\right)$$

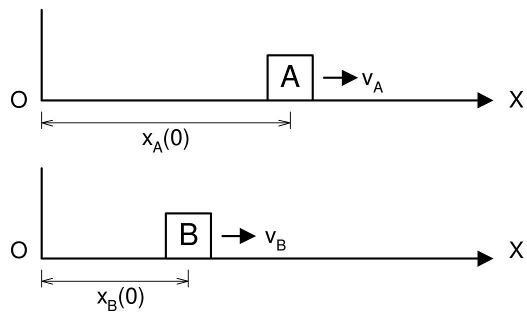
$$= \frac{v^2 - v_0^2}{2a}$$

$$\therefore\ v^2 - v_0^2 = 2ax$$

$$\therefore\ v^2 = v_0^2 + 2ax \;\ldots\ldots\ldots\ldots\ (3)$$

3. Derive an expression for relative velocity between two moving objects.

Consider two objects A and B moving along x - axis uniformly with average velocities $v_A$ and $v_B$ respectively as shown in the figure.




Let $x_A(0)$ and $x_B(0)$ be their displacements from the origin 0 at t = 0
Let $x_A(t)$ and $x_B(t)$ be their displacements at time 't'.

$\therefore v_A = \dfrac{x_A(t) - x_A(0)}{t}$ and $v_A = \dfrac{x_B(t) - x_b(0)}{t}$

$\therefore x_A(t) - x_B(t) = [x_A(0) - x_B(0)] + [v_A(t) - v_B(t)]$

$= [x_A(0) - x_B(0)] + (v_A - v_B)t$ ……….. (1)

Change in displacement between the 2 bodies in time t

$= [x_A(t) - x_B(t)] - [x_A(0) - x_B(0)]$

$= [(x_A(0) - x_B(0)) + (v_A - v_B)t] - [(x_A(0) - x_B(0)]$ (because of equation (1))

$= (v_A - v_B)t$

Therefore relative velocity of A w.r.t B is

$$v_{AB} = \frac{\text{change in displacement}}{\text{time}} = \frac{(v_A - v_B)t}{t}$$

$\therefore v_{AB} = v_A - v_B$

Similarly we can show that relative velocity of B w.r.t A is $v_{BA} = v_B - v_A$

**Five mark problem**

1. **A car moving along a straight highway with speed of 126 km $h^{-1}$ to brought to stop within a distance of 200 m. What is the retardation of the car and how long does it take for the car to stop?**

$$v_0 = 126\text{kmh}^{-1}, = \frac{126 \quad 1000}{3600} = 35 \ \text{ms}^{-1}, \ v = 0, \ x = 200 \text{ m}$$

Applying $\quad v^2 = v_0^2 + 2ax$

$$0 = 35^2 + 2a \ \text{x} \ 200$$

$$\therefore \ 0 = -\frac{35^2}{200 \ \text{x} \ 2} = -3.06 \ \text{ms}^{-2}$$

Applying $\quad v = v_0 + at$

$$0 = 35 - 3.06 \ \text{x} \ t$$

$$t = \frac{35}{3.06} = 11.44 \text{ second}$$

2. **The displacement (in metre) of a particle moving along x - axis is given by $x = At^2 + B$, where A = 2m and B = 3m. Calculate (i) average velocity between t = 3s and t = 5s. (ii) instantaneous velocity at t = 5s and (iii) instantaneous acceleration.**

**(i) Average velocity**

At $t_1 = 3s$, the displacement of the particle is

$$x_1 = 2.3^2 + 3 = 21 \ \text{m}$$

At $t_2 = 5s$, the displacement of the particle is

$$x_2 = 2.5^2 + 3 = 53 \ \text{m}$$

Average velocity $\bar{v} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{53 - 21}{5 - 3} = \frac{32}{2} = 16 \ \text{ms}^{-1}$

**(ii) Instantaneous velocity**

$$v = \frac{dx}{dt} = \frac{d}{dt}\left(At^2 + B\right) = 2At$$

At t = 5s, v = 2 x 2 x 5 = $20 \ ms^{-1}$

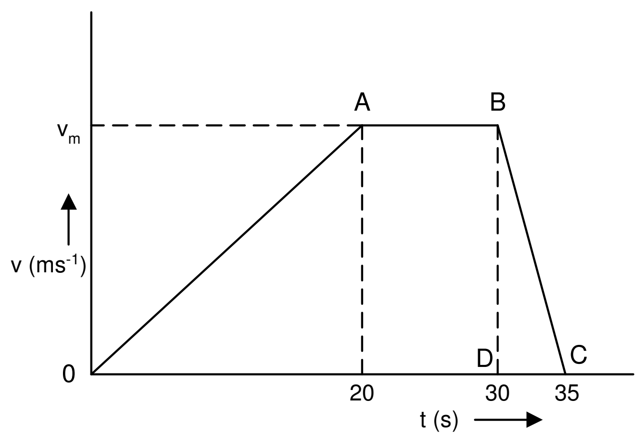
**(iii) Instantaneous acceleration**

$$a = \frac{dv}{dt} = \frac{d}{dt}(2At) = 2A$$

= 2 x 2 = $4 \ ms^{-2}$

**3. A car starts from rest and accelerates uniformly at a rate of 2 $ms^{-2}$ for 20 second. It then maintains a constant velocity for 10 second. The brakes are then applied and the car is uniformly retarded and comes to rest in 5 second. Draw the velocity - time graph for the motion and find : (i) the maximum velocity (ii) the retardation in the last 5 second (iii) total distance traveled and (iv) average velocity.**

The velocity - time graph for the motion of the car is shown below




**(i) Maximum velocity**

$a = 2 \ ms^{-2}, \ v_0 = 0$ , t = 20 s

$v_m = v_0 + at$

= 0 + 2 x 20

= $40 \ ms^{-1}$

**(ii) Retardation**

Retardation is equal to the slope of the line BC

$$= -\frac{BC}{DC} = -\frac{40}{5} = 8 \ \ ms^{-2}$$

**(iii) Total distance traveled S = Area of trapezium OABC**

$$= \frac{1}{2}(AB + OC)BD$$

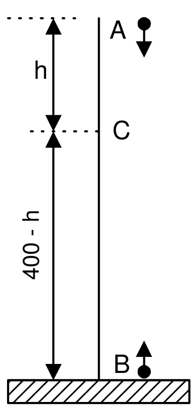
$$= \frac{1}{2}(10 + 35)40 \ = 900 \text{ m}$$

**(iv) Average velocity**

$$= \frac{S}{t} = \frac{900}{35} = 25.71 \ \ ms^{-1}$$

**4. A stone is dropped from the top of a tower 400 m high and at the same time another stone is projected upward vertically from the ground with a velocity of 100 $ms^{-1}$. Find where and when the two stones will meet.**

Suppose the two stones meet after 't' second, when the stone from the top of the tower has covered a distance 'h' as shown in the figure. We have the equation $x = v_0 t + \frac{1}{2}at^2$.




**For downward motion,** $v_0 = 0$**,** $g = 9.8 \ \ ms^{-2}$

$$h = v_0 t + \frac{1}{2}gt^2$$

$$h = 0 + \frac{1}{2} \text{ x } 9.8t^2$$

$$h = 4.9 \ \ t^2 \ \ \ldots\ldots\ldots. (1)$$

**For upward motion,** $v_0 = 100 \ ms^{-1}$**,** $g = -9.8 \ ms^{-2}$

$$400 - h = 100t - \frac{1}{2} \times 9.8t^2$$

$$400 - h = 100t - 4.9 \ t^2$$

Substituting the values of h from equation (1) we have

$$400 - 49t^2 = 100t - 49t^2$$

$\therefore$ 400 = 100 t

$\Rightarrow$ t = 4 second

Substituting t = 4 s in equation (1) we have

$$h = 4.9 \times 4^2$$

= 78.4 m

Therefore the two stones meet at 78.4 m below the top of the tower after 4 second.

**5. Two trains are moving in opposite directions. Train A moves east with a speed of 10** $ms^{-1}$ **and train B moves west with a speed of 20** $ms^{-1}$**. What is the (i) relative velocity of B w.r.t A and (ii) the relative velocity of ground w.r.t B. (iii) A dog is running on the roof of train A against its motion with a velocity of 5** $ms^{-1}$ **w.r.t train A. What is the velocity of the dog as observed by a man standing on the ground?**

Let the direction of travel from west to the east be considered as positive direction.

Speed of train A w.r.t earth, $v_{AE} = 10 \ ms^{-1}$

Speed of train B w.r.t earth, $v_{BE} = -20 \ ms^{-1}$

(i) **Relative velocity of train B w.r.t train A** $= v_{BA} = v_{BE} + v_{EA}$

$= v_{BE} - v_{AE}$

$= -20 - 10$

$= -30 \ ms^{-1}$ from east to west

(ii) **Relative velocity of ground (i.e. earth) w.r.t B** $= v_{EB} = v_{EE} + v_{EB}$

$= v_{EE} - v_{BE}$

$= 0 - (-20)$

$= 20\ \text{ms}^{-1}$ from west to east

(iii) **Velocity of dog w.r.t train A is** $v_{DA} = -5\ \text{ms}^{-1}$

$v_{DA} = v_{DE} + v_{EA}$

$= v_{DE} - v_{AE}$

$\therefore\ v_{DE} = v_{DA} + v_{AE}$

$= -5 + 10$

$= 5\ \text{ms}^{-1}$ from west to east

6. **Two trains A and B of length 200 m each are moving on two parallel tracks with a uniform speed of 10 $\text{ms}^{-1}$ in the same direction, with the train A ahead of B. The driver of train B decides to overtake train A and accelerates by 1 $\text{ms}^{-2}$. If after 50 s, the guard of train B brushes past the driver of train A, what was the original distance between the two trains?**

Let x be the original distance between the trains.

**For train A**: $v_0 = 10\ \text{ms}^{-1}$, t = 50 s, a = 0

Distance traveled $= x_A = v_0 t + \frac{1}{2}at^2$

$= 10 \times 50 + 0$

$= 500$ m

**For train B**: $v_0 = 10\ \text{ms}^{-1}$, t = 50 s, a = 1 $\text{ms}^{-2}$

Distance traveled $= x_B = v_0 t + \frac{1}{2}at^2$

$= 10 \times 50 + \frac{1}{2} \times 1 \times 50^2$

$= 500 + 1250$

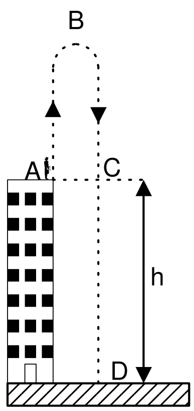
$= 1750$ m

$x_B = x + x_A$ + length of train A + length of train B

$\therefore\ 1750 = x + 500 + 200 + 200$

$\therefore\ x = 1750 - 900 = 850$ m

**7. A body is thrown vertically up from the top of a building with a velocity of 10 $ms^{-1}$. It reaches the ground in 5 s. Find the height of the building and the velocity with which the body reaches the ground. (g = 10 $ms^{-2}$).**




Let the body be thrown vertically up with a velocity 10 $ms^{-1}$ from the top of a building at a point A. It reaches the point B where its velocity is zero. Now $v_0 = 10\ ms^{-1}$, v = 0, $g = -10\ ms^{-2}$, x = AB.

Using $v^2 = v_0^2 + 2ax$

$$0^2 = 10^2 - 2 \times 10.AB$$

$$\therefore\ -20AB = 0^2 - 10^2$$

$$\therefore\ AB = \frac{100}{20} = 5\ m.$$

Let t be the time taken by the body to go from A to B.

Using $v = v_0 + at$

$$0 = 10 - 10\ t$$

$$\therefore\ t = \frac{10}{10} = 1\ s$$

Total time taken by body to reach the ground is 5 s.

Therefore the time taken by body to fall from B to D = 5 – 1 = 4 s.

BD = BC + CD = AB + CD = $5^{th}$

Considering the body falling from B to D, velocity of the body at B is $v_0 = 0$, $g = -10 \ ms^{-2}$

Using equation of motion $x = v_0 t + \frac{1}{2}at^2$

$5^{th} = 0 + ½ \times 10 \ (4)^2$

$= 80$

$\therefore \ h = 80 - 5 = 75$ m

The velocity with which the body reaches the ground is given by $v = v_0 + at$

$v = 0 + 10 \times 4$

$v = 40 \ ms^{-1}$