I PUC – PHYSICS
CHAPTER - 15
Waves

One Mark questions.

1. What is a wave?
2. What is a progressive wave?
3. Does, all the waves requires a material medium for their propogation?
4. Does, a wave carry energy?
5. Name the properties of a medium which are responsible for the propogation of a mechanical wave?
6. What are matter waves?
7. Name the kind of wave that are employed in the working of an electron microscope?
8. Define amplitude of a wave.
9. Define period of a wave.
10. Define frequency of a wave.
11. Define wavelength of a wave.
12. Define wave velocity.
13. Define phase of a vibrating particle?
14. Define propogation constant (or) angular wave number.
15. How is propogation constant related to wavelength of a wave?
16. Name the factors which determine the speed of a propogation of an electromagnetic wave?
17. Name the quantity associated with a wave that remains unchanged when a wave travel from one medium to another?
18. Name the quantities associated with a wave, that changes when a wave travels from one medium to another.
19. What is sound?
20. How is sound produced?
21. Why do we see the flash of lightening before we hear the thunder?
22. What is a stationary wave?
23. How much energy is transported by a stationary wave?
24. What is a node?
25. What is an antinode?
26. What is a segment (or) loop in a stationary wave?
27. What is the length of a loop in a stationary wave in terms of wavelength?
28. How much is the distance between a node and its neighbouring antinode?
29. How much is the distance between a node and its neighbouring node.
30. What happens to a wave, if it meets a rigid boundary?
31. What happens to a wave, if it meets a boundary which is not completely rigid?
32. What is the phase angle between the incident wave and the wave reflected at a rigid boundary?
33. What is the phase angle between the incident wave and the wave reflected at a open boundary?
34. Give the relation between phase difference and path difference.
35. What are normal modes of oscillation in a stationary wave?

36. What is the meaning of the fundamental mode (or) first harmonic of oscillation in a stationary wave?
37. What are harmonics in a stationary wave?
38. What are overtones in a stationary wave?
39. What is resonance?
40. What are beats?
41. What is beat period?
42. What is Doppler effect?
43. Which harmonics are absent in a closed organ pipe?
44. Give the formula for speed of transverse wave on a stretched string.
45. What is the increase in the speed of sound in air when the temperature of the air rises by 1^oC?
46. Why a transverse mechanical wave cannot travel in gases?
47. How does the velocity of sound in air vary with temperature?
48. How does the velocity of sound in air vary with pressure?
49. Give the dimensional formula for propogation constant.
50. The fundamental frequency of a closed pipe is 80Hz. What is the frequency of first overtone.
51. Calculate the wavelength of a wave whose angular wave number is 10π radian m^{-1}?
52. The distance between a node & an next antinode in a stationary wave pattern is 0.08m. What is the wavelength of the wave?
53. How is the frequency of an air column in an open pipe related with the temperature of air?
54. A sound wave has a velocity of 330 ms^{-1} at one atmospheric pressure. What will be its velocity at 4 atmospheric pressure?
55. What happens to the frequency of the wave when it travels from water to air?
56. Give the relation between time period and frequency of a wave.
57. Is Doppler effect observed for sound waves only?
58. What is the distance between two consecutive antinodes in a stationary wave of wavelength 2m?
59. How does speed of a transverse wave on a stretched string vary with its tension?
60. With what velocity does an electro magnetic wave travel in vacuum.

Two Mark questions:

1. What are mechanical waves? Give two examples.
2. What are non-mechanical waves? Give two examples.
3. What are longitudinal waves? Give two examples
4. What are Transverse waves? Give two examples.
5. Obtain the relation connecting v, υ and λ where symbols have their usual meaning.
6. Calculate the period of a wave of wavelength 0.005 m which travels with a speed of 50 $cm.s^{-1}$.
7. The frequency of a tuning fork is 256 Hz and sound travels a distance of 25m while the fork executes 20 vibrations. Calculate the wavelength and velocity of the sound wave.
8. Velocity of sound in air is 340 ms^{-1}. Two sound waves of frequency 1 KHz each interfere to produce stationary wave. What is the distance between two successive nodes?

9. When is the fundamental frequency of the sound emitted by a closed pipe is same as that emitted by an open pipe?
10. A closed pipe & open pipe have same frequency for the first overtone. What is the ratio of their lengths?
11. For what wavelength of waves, does a closed pipe of length 30 cm emit the first overtone?
12. The second overtone of closed pipe of length 1 m is in unison with the third overtone of an open pipe. What is the length of the open pipe?
13. The velocity of a sound wave decreases from 330 ms^{-1} to 220 ms^{-1} on passing from one medium to another. If the wavelength in the first medium is 3m. What is the wavelength in the second medium?
14. Can sound waves of wavelength 33mm be heard in air? Justify.
15. At what temperature will the velocity of sound becomes 1.25 times that at $27^{o}C$?
16. A musical note produces 2 beats per second. When sounded with a tuning fork of frequency 340Hz & 6 beats per second when sounded with a tuning fork of frequency 344 Hz. Find the frequency of the musical note?
17. At which positions (or) locations of the stationary wave, the pressure changes are maximum and minimum.
18. At which positions (or) location of the stationary wave, the displacement is maximum and minimum.
19. Calculate the velocity of sound at $-30^{o}C$ and $30^{o}C$ given the velocity of sound at $0^{o}C$ is 330 ms^{-1}.
20. Give any two applications of Doppler's effect?
21. With what velocity should a sound source travel towards a stationary observer so that the apparent frequency may be double of the actual frequency.
22. A bat emits ultrasonic sound of frequency 1000KHz in air. If sound meets a water surface, what is the wavelength of a) reflected sound b) transmitted sound? (Given speed of sound in air is 340 ms^{-1} & in water 1486 ms^{-1}?
23. The sitar strings A & B playing the note 'Ga' are slightly out of tune & produce beats of frequency 6 Hz. The tension in the string A is slightly reduced & the beat frequency is found to reduce to 3 Hz. If the original frequency of A is 324 Hz. What is the frequency of B?
24. A sinusoidal wave propogating through air has a frequency of 200 Hz. If the wave speed is 300 ms^{-1}, how far apart are the two points (path difference) with a phase of difference of 60^{o}.

4 and 5 marks questions :

1. Give the differences between progressive and stationary waves.
2. Give the differences between mechanical and a non mechanical (electromagnetic) waves.
3. Give the differences between longitudinal and transverse waves.
4. Write Newton's formula for speed of sound in a gas. Discuss Laplace correction & arrive at the formula modified by him.
5. Mention the characteristics of a progressive mechanical wave.
6. Mention the characteristics of a stationary wave.
7. What are beats? Give the theory of beats.
8. What is Doppler effect? Derive an expression for the apparent frequency when a source moves towards a stationary listener.

9. What is Doppler effect? Derive an expression for the apparent frequency when a listener moves towards a stationary source.
10. What is Doppler effect? Derive an expression for the apparent frequency when the source and listener are moving in the same direction.
11. Discuss the effect of pressure, temperature & humidity on the velocity of the sound through air.
12. Discuss different modes of vibration on a stretched string.
13. Discuss different modes of vibration (first three harmonics) produced in a open pipe.
14. Discuss different modes of vibration (first three harmonics) produced in a closed pipe.

5 marks problem.

1. A stone dropped from the top of tower of height 300 m high splashes into the water of a pond near the base of a tower. When is the splash heard at the top given that the speed of sound in air is 340 ms^{-1}? (given g = 9.8 ms^{-2})
2. A transverse harmonic wave on a string is described by y(x, t) = 3 sin (36 t + 0.018x + $\pi/4$) where x & y are in cm and t is in s.
   i) Is the wave traveling (or) stationary.
   ii) What is the direction of its propogation
   iii) What is its frequency?
   iv) What is its initial phase?
   v) What is the distance between two consecutive crests in the wave?
3. The transverse displacement of a string (clamped at both ends) is given by $y(x,t) = 0.06 \sin\left(\frac{2\pi}{3}\right)x \quad \cos(120\ \pi t)$ where x, y are m & t is in s. The length of the string is 1.5 m & its mass is 3 x 10^{-2} kg.
   i) Does the function represent a traveling wave (or) a stationary wave?
   ii) Interpret the wave as a superposition of two waves traveling in opposite directions what is the wavelength, frequency and speed of each wave?
   iii) Determine the tension in the string.
4. A progressive wave is described by the equation $y = 1.2 \sin \pi \left(\frac{2t}{5} - \frac{x}{4}\right)$ where x & y are in m and t is in s. Determine the amplitude, wavelength, time period & speed of the wave?
5. A closed pipe of length 0.42 m and an open pipe both contain air at 35^{o}C. The frequency of the first overtone of the closed pipe is equal to the fundamental frequency of the open pipe. Calculate the length of the open pipe and the velocity of sound in air at 0^{o}C. Given that the closed pipe is in unison in the fundamental mode with a tuning fork of frequency 210 Hz.
6. Two cars are moving with speeds of 54 $kmhr^{-1}$ & 18 $kmhr^{-1}$ in opposite direction along a straight road. The faster car sounds the horn with a note of frequency of 240 Hz. Calculate the number of waves received per second by a listener sitting in the other car when it (i) approaches (ii) recedes from the listener if the speed of the sound in air is 340 ms^{-1}.
7. y = 1.4 sin π (300t – x) represents a progressive wave where x, y are in m & t is in s. Calculate the wave velocity & the phase difference between oscillatory motion of two points separated by a distance of 0.25 m.

8. Compare the time taken by sound to travel a given distance in air and argon at NTP. Given that the density of air and argon are 1.293 & 1.789 respectively ratio of specific heat capacity for air and organ are 1.402 & 1.667 respectively.
9. The speed of sound in hydrogen is 1270 ms^{-1}. What will be the speed of sound in a mixture of oxygen and hydrogen mixed in a volume ratio 1: 4?
10. A wire stretched between two rigid supports vibrates in its fundamental mode with a frequency of 45 Hz. The mass of the wire is 3.5×10^{-2} kg & its linear density is 4×10^{-2} kg m^{-1} . What is i) the speed of wave on the string. ii) the tension in the string?

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CHAPTER – 15: WAVES

1 mark Answers

1. A wave is a sort of disturbance which is transmitted in a medium without the bulk movement of particles of the medium.

   Or

   A wave is a sort of disturbance when a group of particles of the medium are disturbed, the pattern of disturbance that travels through the medium due to the periodic motion of the particles of the medium about their equilibrium position, with the transfer of energy and momentum and without the transfer of matter(particles) is called a wave.
2. A wave (disturbance) that travels continuously from one point of medium to another is called a progressive wave.
3. No
4. Yes
5. Elastic and Inertial properties of medium
6. The wave associated with moving material particles are called matter waves.
7. Matter waves associated with electrons.
8. Amplitude is the maximum displacement of the particle on either side of the equilibrium position during wave propogation.
9. It is the time taken by a wave to move through a distance of wavelength during wave propagation.
10. It is the number of waves crossing a given cross section per second during wave propagation.
11. The distance between two consecutive particles of medium which are in the same state of vibration (phase) is called as wavelength.
12. Wave velocity is defined as the distance traveled by the wave in one second.
13. The phase of a vibrating particle at a given instant of time is the state of vibration of a particle at that instant of time with reference to its equilibrium position.
14. It is the number of waves that can be accommodated per unit length.
15. $K = \dfrac{2\pi}{\lambda}$
16. Permitivity and permeability of the medium
17. Frequency of the wave
18. Wavelength and velocity of the wave.
19. Sound is a form of energy that produces a sensation of hearing.
20. Vibrating bodies surrounded by a material medium produces sound.
21. Because speed of light is much greater that the speed of sound.
22. When two progressive waves of equal amplitude, frequency and speed traveling in a medium along the same line but in opposite direction superimpose, the resulting waveform appears to be stationary pattern, such a wave is called stationary wave.
23. zero
24. Nodes are certain location (position) in a stationary wave, where the particles of the medium are completely at rest (zero displacement)
25. Antinodes are certain location (position) in a stationary wave, where the particle of the medium vibrate with maximum displacement.

26. The wave form (or) region between two consecutive node in a stationary wave is called a loop (or) segment.
27. $\frac{\lambda}{2}$ (or) half of the wavelength
28. $\frac{\lambda}{4}$
29. $\frac{\lambda}{2}$
30. The waves gets reflected (assuming that there is no absorption of energy by the boundary)
31. A part of the incident wave gets reflected and part of incident wave gets transmitted into the other medium (assuming that there is no absorption of energy by the boundary)
32. π radian (or) 180 degree
33. No phase change (or) zero.
34. Phase difference = $\left(\frac{2\pi}{\lambda}\right)$ path difference

    Where λ is the wavelength of the wave.
35. In a stationary wave, the possible frequencies of oscillation of the system is characterized by a set of natural frequencies called as normal modes of oscillation.
36. In a stationary wave, the oscillation of the system with lowest possible natural frequency is called as fundamental frequency (or) first harmonic.
37. For a vibrating system the frequencies which are integral multiples of fundamental frequency are called harmonics.
38. For a vibrating system, frequencies greater than fundamental frequencies are called overtones.
39. In case of forced vibration, when the frequency of the external agent causing vibration (applied force) becomes equal to natural frequency of the vibrating body. The body vibrates with maximum amplitude. This phenomena is called "Resonance"
40. The periodic waxing (increase (or) rise) and waning (decrease (or) fall) in the intensity of sound due to superposition of two sound waves of nearly same frequencies traveling in same direction are called beats.
41. The time interval between two consecutive waxing (or) waning is called as beat period.
42. The apparent change in the frequency (pitch) of sound heard by the listener due to relative motion between the source producing the sound and the listener is called as Doppler effect.
43. Even harmonics.
44. V = $\sqrt{\frac{T}{\mu}}$ where T is the tension in the string

    μ is the linear mass density
45. The speed of sound increases approximately by 0.61 ms^{-1} per degree centigrade rise in temperature.
46. Shear modulus of elasticity is absent in gaseous medium, which is necessary for the propogation of transverse wave.
47. V α $\sqrt{T}$ i.e. velocity of sound in air is directly proportional to square root of its absolute temperature.

48. Velocity of sound in air is independent of pressure provided temperature remains constant.
49. $[L^{-1}]$
50. Frequency of I overtone in closed pipe = 3 (fundamental frequency)
$$= 3 \times 80 = 240 \text{ Hz}$$
51. $\lambda = \frac{2\pi}{k} = \frac{2\pi}{10\pi} = 0.2$ m
52. The distance between node & an next antinode is $\frac{\lambda}{4}$

$$\therefore \frac{\lambda}{4} = 0.08$$
$$\lambda = 0.32 \text{ m}$$
53. Frequency of air column in an open pipe is directly proportional to square root of its absolute temperature ($v \alpha \sqrt{T}$)
54. 330 ms^{-1}
55. Frequency remains the same
56. Time period = $\frac{1}{frequency}$
57. No
58. 1 m
59. Speed of a transverse wave on a stretched string is directly proportional to square of its tension. i.e. $V \alpha \sqrt{T}$
60. 3×10^8 ms^{-1}

2 Marks

1. The waves that requires material medium for their propogation (transmission) are called as mechanical waves. Eg. Waves on a surface of water, sound waves, seismic waves etc.
2. The waves that do not require material medium for their propogation are called as non-mechanical waves. Eg. Radio waves, light waves, x-rays etc.
3. The waves in which the particles of the medium oscillates parallel (along) to the direction of wave propogation are called longitudinal waves. Eg. Sound waves, waves set up in air column
4. The waves in which the particles of the medium oscillate perpendicular to the direction of wave propogation are called Transverse waves. Eg. Light waves, waves on the surface of water, waves on a string.
5. Consider a wave traveling with a velocity 'V' let υ be its frequency & λ be its wavelength. In a time equal to its time period T, the wave covers a distance equal to its wavelength λ.

   By the definition of wave velocity we have

   Wave velocity V = $\frac{dis\tan cetravelled}{timetaken}$

$$\therefore V = \frac{\lambda}{T}$$

$$V = \upsilon\lambda \quad \text{(because } \frac{1}{T} = \upsilon\text{)}$$

6. Given $\lambda = 0.005$ m ; $V = 50$ cm $s^{-1} = 50 \times 10^{-2}$ ms^{-1}

We have $V = \frac{\lambda}{T}$

$\therefore T = \frac{\lambda}{V} = \frac{0.005}{50\times10^{-2}} = 0.01$ s.

7. Wavelength $\lambda = \frac{25}{20} = 1.25$ m

$V = \upsilon\lambda$

$V = 256 \times 1.25$

$V = 320$ ms^{-1}

8. Given $V = 340$ ms^{-1} ; $\upsilon = 1$ KHz = 1000 Hz.

We have $\lambda = \frac{V}{\upsilon}$

$$\lambda = \frac{340}{1000} = 0.34m$$

$\therefore$ Distance between two successive node $= \frac{\lambda}{2} = \frac{0.34}{2} = 0.17$ m

9. Let ℓ_o & ℓ_c be the length of open & closed pipe respectively

Given $(\upsilon_{\text{fundamental}})_{\text{Open pipe}} = (\upsilon_{\text{fundamental}})_{\text{closed pipe}}$

$$\frac{V}{2\ell_o} = \frac{V}{4\ell_c}$$

$$\frac{\ell_c}{\ell_o} = \frac{1}{2}$$

$$\therefore \ell_c = \frac{\ell_o}{2}$$

10. Let ℓ_o & ℓ_c be the length of open and closed pipe respectively

Given & $(\upsilon_{\text{first overtone}})_{\text{closed pipe}} = (\upsilon_{\text{first overtone}})_{\text{open pipe}}$

$$\frac{3V}{4\ell_c} = \frac{2V}{2\ell_o}$$

$$\frac{\ell_c}{\ell_o} = \frac{3}{4}$$

11. The frequency of the first overtone in a closed pipe is given by

$$\upsilon = \frac{3V}{4\ell} = \frac{3V}{4\times30} = \frac{V}{40}$$

But $V = \upsilon\lambda$ $\quad \therefore \upsilon = \frac{\upsilon\lambda}{40}$

$\therefore \lambda = 40$ cm

12. Given $(\upsilon_{\text{II overtone}})_{\text{Closed pipe}} = (\upsilon_{\text{III overtone}})_{\text{open pipe}}$

$$\frac{5V}{4\ell_c} = \frac{4V}{2\ell_o}$$

$$\ell_o = \frac{8}{5}\,\ell_c$$

$$\ell_o = \frac{8}{5}\,(1) = 1.6 \text{ m}$$

13. We have $V = \upsilon\lambda$

$$\therefore \frac{V_1}{V_2} = \frac{\cancel{\upsilon}\lambda_1}{\cancel{\upsilon}\lambda_2}$$

Where V_1 & V_2 are velocity of sound in first and second medium respectively and λ_1 & λ_2 are the corresponding wavelength.

$$\lambda_2 = \frac{V_2}{V_1}\lambda_1$$

$$\lambda_2 = \left(\frac{220}{330}\right)\times 3$$

$$\lambda_2 = 2\text{m}$$

14. frequency (υ) = $\dfrac{V}{\lambda}$

$$\upsilon = \frac{330}{33\times10^{-3}} = 10 \text{ KHz}$$

Since this wave belong to audible range of sound they can be heard.

15. We have $V \;\alpha\; \sqrt{T} = \sqrt{t+273}$

Given $V_{t^oC} = 1.25\, V_{27^oC}$

$$\therefore \frac{V_{t^oC}}{V_{27^oC}} = \frac{\sqrt{t+273}}{\sqrt{27+273}}$$

$$\frac{1.25V_{27^oC}}{V_{27^oC}} = \frac{\sqrt{t+273}}{\sqrt{300}}$$

$\therefore$ t = 195.75°C

16. We have $\upsilon_b = \upsilon_1 \sim \upsilon_2$

Given I case υ_1 = 340 Hz & υ_b = 2 beats per second

$\therefore$ possible values of υ_2 = 342 Hz (or) 338 Hz.

II case υ_1 = 344 Hz & υ_b = 6 beats per second

$\therefore$ possible values of υ_2 = 338 Hz (or) 350 Hz

$\therefore$ frequency of the musical note is υ_2 = 338 Hz.

17. Pressure changes are maximum at Node and pressure changes are minimum at antinode.
18. Displacement is maximum antinode and displacement is minimum at node.
19. We have $V \;\alpha\; \sqrt{T}$

$$\therefore \frac{V_{-30^oC}}{V_{0^oC}} = \frac{\sqrt{-30+273}}{\sqrt{0+273}}$$

$V_{-30^oC} = 311.3 \text{ ms}^{-1}$

$$\frac{V_{30^o C}}{V_{0^o C}} = \frac{\sqrt{30+273}}{\sqrt{0+273}}$$

$V_{30^o C} = 347.7\ ms^{-1}$

20. Sonography, echocardiogram and speed of vehicles.

21. When a source travel towards a stationary listener, apparent, frequency is given by

$$\upsilon = \upsilon_o \left(\frac{V}{V - V_S}\right)$$

$$2\upsilon_o = \upsilon_o \left(\frac{V}{V - V_S}\right)$$

$$\therefore\ V_S = \frac{V}{2}$$

22. $\lambda_{\text{reflected sound}} = \frac{V_{air}}{\upsilon}$

$$\lambda_\upsilon = \frac{340}{1000 \times 10^3}$$

$$\lambda_\upsilon = 3.4 \times 10^{-4}\ m$$

$$\lambda_{\text{transmitted sound}} = \frac{V_{water}}{\upsilon}$$

$$\lambda_t = \frac{1486}{1000 \times 10^3}$$

$$\lambda_t = 1.486 \times 10^{-3}\ m$$

23. We have $\upsilon_b = \upsilon_A - \upsilon_B$

I case $\upsilon_b = 6$ Hz, $\upsilon_A = 324$ Hz

$\therefore$ possible values of $\upsilon_B = 318$ Hz (or) 330 Hz

II case when tension in the string A is reduced, its frequency (υ_A) also decreases, the new beat frequency is given to be 3 Hz. This is possible only if $\upsilon_B = 318$ Hz.

24. Given $\upsilon = 200$ Hz ; $V = 300\ ms^{-1}$

Phase difference = 60^o (or) $\frac{\pi}{3}$ radian

We have $\lambda = \frac{V}{\upsilon} = \frac{300}{200} = 1.5$ m

Path difference = $\frac{\lambda}{2\pi}$ phase difference

$\therefore$ path difference = $\frac{1.5}{2\cancel{\pi}} \times \frac{\cancel{\pi}}{3}$

Path difference = 0.25m

4 and 5 Marks

1.

Progressive Wave	Stationary wave
a) The wave travel continuously with certain velocity called wave velocity	The waves does not move. It remains localized.
b) The propogation of the disturbance from particle due to elastic properties of the medium give rise to a progressive wave.	The superposition of two identical waves traveling in opposite direction along the same line results in a stationary wave.
c) Amplitude of vibration is the same for every particle of the medium along the wave	The amplitude of vibration varies from zero at node & maximum at antinode.
d) Different particles over a distance λ different phases at a given instant of time.	All particles lying in a loop have same phase at a given instant of time.
e) No particles in the medium is completely at rest.	The particles at node are permanently at rest.
f) There is a net transfer of energy in the direction of propogation of wave.	There is no net transfer of energy across any section of the medium.
g) The wave equation is of the form $y(x, t) = A \sin(wt-kx)$ i.e , $y(x, t) = f(x, t)$	The wave equation is of the form. $y(x,t) = 2A \cos(Kx) \sin wt$. i.e. $y(x,t) = f(x). g(t)$

2.

Mechanical wave	Non-mechanical wave
a) Requires a material medium for their propogation	Do not require a material medium for their propogation.
b) Particles of the medium oscillate.	Electric and magnetic field oscillate.
c) Can be longitudinal (or) transverse in nature.	Are always transverse in nature.
d) Travels at relatively lower speed in a medium.	Travels at relatively higher speed in a medium
e) Doppler effect is asymmetric Eg. Sound waves	Doppler effect is symmetric Eg.Light waves.

3.

Longitudinal wave	Transverse wave
a) The particles of the medium oscillate along (parallel to) the direction of propogation of the wave.	The particles of the medium (electric & magnetic fields) oscillate at right angles (perpendicular) to the direction of the propogation of the waves.

b) These waves travels in alternate compression & rarefaction. (compression are the region of higher density & rarefaction are the regions of lower density).	These waves travels in alternate crests and troughs. (crests are highly raised portions of a wave & troughs are highly depressed portion of a wave).
c) Are always mechanical wave.	They are either mechanical (or) non mechanical wave.
d) The pressure and density varies as the wave propogates	The pressure and density donot vary as the wave propogates.
e) They can travel in solids, liquids and gases.	They can travel in solids and on the Surface of liquids, if the waves are mechanical.
f) These waves cannot be polarized.	These waves can be polarized.
g) Velocity of a longitudinal wave in a gas is given by v = $\sqrt{\frac{B}{\rho}}$ where B is the bulk modulus and ρ is the density. Eg. Sound waves.	Velocity of a transverse wave on a stretched string is given by v = $\sqrt{\frac{T}{\mu}}$ Where T is the tension & μ is linear Density. Eg. Waves on a string. Light wave.

4. Velocity of sound waves is determined by the elastic and inertial properties of the medium and in a given medium sound travels as a longitudinal wave.

Newton's formula : According to Newton, velocity of sound wave (longitudinal wave) in any medium is given by V = $\sqrt{\frac{E}{\rho}}$ ➔ (1) where E is the modulus of elasticity and ρ is the density of the medium.

For solids , E = Y (Young's modulus) and for a gaseous medium E = B (bulk modulus). Therefore, according to newton velocity of sound in a gaseous medium is given by v = $\sqrt{\frac{B}{\rho}}$ ➔ (2) [from equation (1)]

The Bulk modulus of the medium is defined by B = - $\frac{\Delta P}{\left(\frac{\Delta V}{V}\right)}$ ➔ (3) where the changes in pressure ΔP produces a volumetric strain $\frac{\Delta V}{V}$

When sound waves travel through a gas, alternate compression and rarefaction are produced.

In a compressed region, volume decreases & pressure increases and in a rarefied region, volume increases and hence pressure decreases. Thus, propogation of sound waves in a gaseous medium is accompanied by continuous changes in pressure and volume and newton assumed that these changes in pressure and volume takes place under ISOTHERMAL condition that is at constant temperature.

Further, for an isothermal process, the relation between pressure (p) & volume(v) is given by pv = constant

Differenting ; $\Delta p.v + p\,.\,\Delta v = 0$

Or $P = \frac{-\Delta p}{\left(\Delta V / V\right)}$

And hence from equation (3) we have B = P

$\therefore$ equation (2) becomes $V = \sqrt{\frac{P}{\rho}}$ ➔ (4)

Equation(4) is called as newton's formula for velocity of sound wave in a gaseous medium

For air at STP, $P = 1.013 \times 10^5$ Pascal (Nm^{-2})

$\rho = 1.293$ kg m^{-3}

$\therefore$ velocity of sound in air at STP from equation (4) is $V = \sqrt{\frac{1.013 \times 10^5}{1.293}} = 280\ ms^{-1}$

But, the experimental value of velocity of sound in air at STP is found to be 331 ms^{-1}. Thus, the value of velocity of sound in gas obtain by newton's formula does not agree with the experimental value and this has a discrepancy of about 16%. Hence Newton's formula was discarded. However, newton's formula needs a correction and this correction was given by Laplace.

Laplace correction (or) Newton-Laplace formula

According to Laplace the condition prevailing to the compression and rarefaction is ADIABATIC & not ISOTHERMAL. This is because, the vibration (compression & rarefaction) of layer of air are so rapid, that there is hardly any time for the exchange of heat between the layers and also air is a bad conductor of heat.

Under adiabatic condition, the relation between pressure(p) & volume (v) is given by pv^{γ} = constant ➔ (5) where γ is the ratio of specific heat capacities at constant pressure to constant volume i.e $\gamma = \frac{Cp}{Cv}$

Differentiating equation (5) we get, $p\gamma v^{\gamma-1} = \frac{-\Delta p}{\left(\Delta v / v\right)}$

And hence from equation (3) we get $B = \gamma p$.

$\therefore$equation (2) becomes $V = \sqrt{\frac{\gamma p}{\delta}}$ ➔ (6)

Equation (6) is called Newton's laplace formula.

For air $\gamma = \frac{7}{5} = 1.4$

$\therefore$ from equation (6) velocity of sound in air at STP is given by $v = \sqrt{\frac{1.4 \times 1.013 \times 10^5}{1.293}} = 331.3\ ms^{-1}$

This is in close agreement with the experimental value.

5. Characteristics of a progressive mechanical wave.

- The waves (disturbance) produced at any point in a medium is propogated by continuous periodic oscillation of the particles about their mean positions.
- The elastic and inertial properties of the medium are responsible for wave propogation.
- There is a transfer of energy and momentum from the source of vibration away from it in the form of disturbance. However, the particles of the matter themselves do not move away and they perform only simple harmonic motion about their mean position.
- Waves in a homogeneous medium travel with constant velocity at a given temperature. However, particle velocity is different.
- Every particle along the wave vibrate with same frequency and amplitude about their mean position but the phase of the different particles are different at given instant of time.
- The waves undergo reflection, refraction, interference and diffraction.
- Longitudinal waves do not show polarization whereas the transverse waves show polarization.
- Wave propogation is longitudinal inside the liquids and gases. Wave propogation can be transverse on a liquid surface and on the strings. Wave propogation can be either longitudinal (or) transverse inside a solid.

6. Characteristics of a stationary wave.
    - Stationary wave remain localized between two fixed points. i.e., waves do not move in a medium
    - In a stationary wave, there exist certain points called Nodes and antinodes. At a Node, the amplitude of vibration is zero and at an antinode, the amplitude of vibration is maximum.
    - Nodes and antinode are equally spaced. The distance two consecutive nodes (or) antinodes is equal to half the wavelength (λ) [i.e., $\frac{\lambda}{2}$]
    - Always a node exist between two successive antinodes and vice-versa. The distance between a node and next antinode is $\left(\frac{1}{4}\right)^{th}$ the wavelength (λ) i.e., $\frac{\lambda}{4}$
    - The amplitude of vibration increases from zero to maximum between a node and an neighbouring antinode.
    - Except at nodes, all the points of the medium in a segment (or) loop vibrate with the same phase, but the points in the adjacent segment vibrate in opposite phase.
    - There is no net transfer of energy across any segment of the stationary wave.

7. This is a phenomena based on the principle of superposition of waves. The periodic waxing (rise) and waning (fall) in the intensity of sound due to superposition of two sound waves of nearly the same frequencies (but not equal) traveling in the same direction are called beats.

Consider two sound waves of same amplitude (a) and nearly equal angular frequencies ω_1 &. ω_2 such that $\omega_1 > \omega_2$

Let S_1 & S_2 be the longitudinal displacement of the particles of the medium at a time 't' due to these two waves and hence these two waves can be represented as

$S_1 = a \cos \omega_1 t$ and $S_2 = a \cos \omega_2 t$

According to principle of superposition of waves, the resultant displacement is given by

$S = S_1 + S_2$

$\therefore S = a \cos \omega_1 t + a \cos \omega_2 t$

$S = a (\cos \omega_1 t + \cos \omega_2 t)$

Using $\cos A + \cos B = 2 \cos \left(\frac{A+B}{2}\right) . \cos \left(\frac{A-B}{2}\right)$ we get,

$S = 2 a \cos \left(\frac{\omega_1 - \omega_2}{2}\right) t . \cos \left(\frac{\omega_1 - \omega_2}{2}\right) t$

This can be rewritten as

$$S = 2a \cos \omega_b t . \cos w_a t \text{ ; where } \omega_b = \frac{\omega_1 - \omega_2}{2} \text{ \& } \omega_a = \frac{\omega_1 + \omega_2}{2}$$

This equation is similar to $S = R_A \cos \omega t$.

$\therefore$ Here $R_A = 2a \cos \omega_b t$ is the amplitude of the resultant wave and $\omega = \omega_a$ is the average angular frequency of the resultant wave.

If $|\omega_1 - \omega_2| << \omega_1$, which means $\omega_a >> \omega_b$.

Here the amplitude and hence intensity of resultant wave is maximum (longest) when the term $\cos \omega_b t = t_1$ (or) – 1

i.e., the intensity of the resultant wave waxes and wanes with a frequency which is $2\omega_b = \omega_1 - \omega_2$

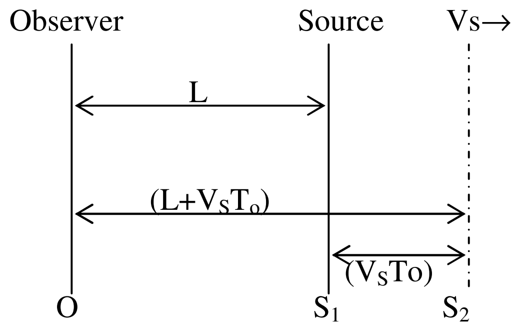
since $\omega = 2\pi\upsilon$

the beat frequency is given by $\upsilon_{beat} = \upsilon_1 - \upsilon_2$

8. The apparent change in the frequency of the sound heard by the listener due to relative motion between the source producing the sound and the listener is called as Doppler effect.

(convention – Take the direction from the observer to the source as the positive direction of velocity)

Consider a source S producing the sound waves. Let the speed of the waves of frequency υ and period T_o, both measured by an observer at rest with respect to the medium be V and we assume that the observer has a detector that counts everytime a crest reaches it.



As shown in figure at time t = 0, the source is at S_1 located at a distance of L from an observer at rest, emits a crest & this crest reaches the observer at a time $t_1 = \frac{L}{V}$

At a time $t = T_o$, the source has moved through a distance $V_S T_o$ and is at a point S_2 located at a distance of $(L + V_S T_o)$ from an observer.

At S_2, the source emits the second crest this crest reaches the observer at $t_2 = T_o + \frac{(L + V_S T_o)}{V}$.

Similarly at a time $t = nT_o$, the source emits $(n+1)^{th}$ crest and this crest reaches the observer at a time

$$t_{n+1} = nT_o + \frac{(L + V_S T_o)}{V}$$

Hence in a time interval of $(t_{n+1} - t_1)$ observers detector counts n crest and this time interval is given by $\left[nT_o + \frac{(L + nV_S T_o)}{V} - \frac{L}{V} \right]$

Therefore, observer records the time period of the wave as T given by

$$T = \frac{\left[nT_o + \frac{(L + nV_S T_o)}{V} - \frac{L}{V} \right]}{n}$$

$$T = T_o + \frac{V_S T_o}{V}$$

$$T = To \left(1 + \frac{V_S}{V} \right)$$

This equation can be rewritten in terms of the frequency υ_o that would be measured if the source and observer at rest and the frequency υ observed when the source moves with a speed V_S as

$$\upsilon = \upsilon_o \left(1 + \frac{V_S}{V} \right)^{-1}$$

If V_S is small compared to the speed of the wave V, taking binomial expansion to terms in first order in $\frac{V_S}{V}$ & neglecting higher power we get, $\upsilon = \upsilon_o \left(1 - \frac{V_S}{V} \right)$.

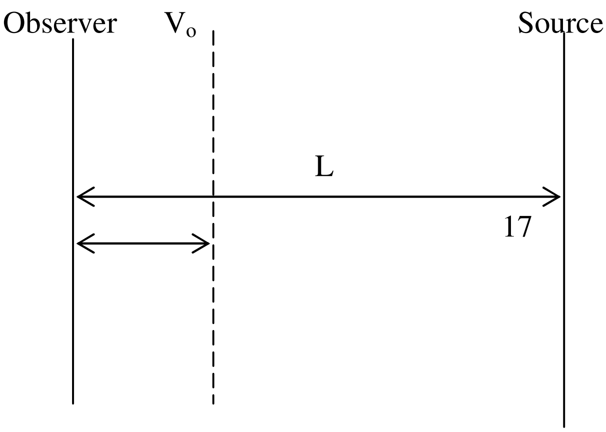
Note: For a source approaching the observer, we get

$\upsilon = \upsilon_o \left(1 - \frac{V_S}{V} \right)$. (that is by replacing V_S by $-V_S$)

9. The apparent change in the frequency of the sound heard by the listener due to relative motion between the source producing the sound and the listener is called as Doppler effect.

(convention – Take the direction from the observer to the source as the positive direction of velocity)

Consider a source S producing the sound waves. Let the speed of the waves of frequency υ and period T_o, both measured by an observer at rest with respect to the medium be V and we assume that the observer has a detector that counts everytime a crest reaches it.



V_oT_o

$(L - V_oT_o)$ $\longleftrightarrow$

O $\qquad$ O_1 $\qquad\qquad$ S

When the observer is moving with a velocity 'Vo' towards a stationary source, the source and medium are approaching at a speed of Vo and the speed with which the wave approaches is Vo+V.

As shown in figure at time t = 0, the source is at S located at a distance of L from an observer, emits the first crest, now since the observer is moving the velocity of the wave relative to the observer is $V+V_o$ and therefore the first crest reaches the observer at a time $t_1 = \frac{L}{V+V_o}$

At a time t = To, the observer has moved through a distance V_oT_o and is at a position O_1 located at a distance of $(L\text{-}V_oT_o)$ from the stationary source, now the source emits the second crest, and this crest reaches the observer at $t_2 = T_o + \frac{(L-V_oT_o)}{(V+V_o)}$

Similarly, at a time $t = nT_o$, the source emits $(n+1)^{th}$ crest and this crest reaches the observer at a time

$$t_{n+1} = \left[nT_o + \frac{(L - nV_oT_o)}{(V+V_o)} \right]$$

Hence in a time interval of $(t_{n+1} - t_1)$ observer's detector counts n crest and this time interval is given by

$$\left[nT_o + \frac{(L - nV_oT_o)}{(V+V_o)} \right] - \left(\frac{L}{V+V_o} \right)$$

Therefore, observer records the time period of the waves as T given by

$$T = \frac{\left[nTo + \frac{(L - nVoTo}{(V+V_o)} \right] - \left(\frac{L}{V+V_o} \right)}{n}$$

$$T = T_o - \frac{V_oT_o}{V+V_o}$$

$$T = T_o \left(1 - \frac{V_o}{V+V_o} \right)$$

$$T = T_o \left(1 + \frac{V_o}{V} \right)^{-1}$$

This can be rewritten in terms of frequency υ_o that would be measured if the source and observer at rest and the frequency υ, observed when the observer is moving with a velocity of V_o as

$$\upsilon = \upsilon_o \left(1 + \frac{V_o}{V} \right)$$

Note: For an observer moves away from the source at rest we get $\upsilon = \upsilon_o \left(1 - \frac{V_o}{V}\right)$

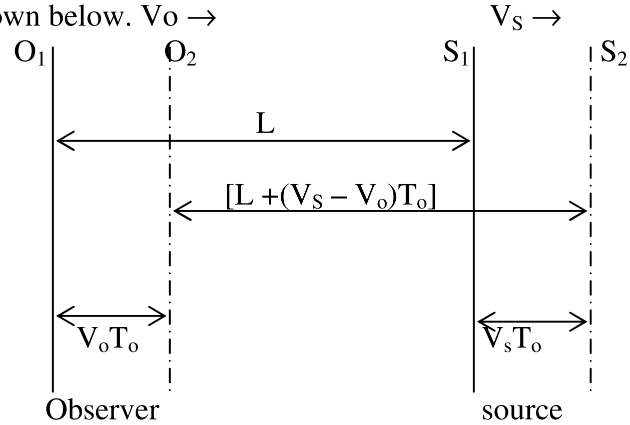
(that is by replacing V_o by $- V_o$)

10. The apparent change in the frequency of the sound heard by the listener due to relative motion between the source producing the sound and the listener is called as Doppler effect.

(convention – Take the direction from the observer to the source as the positive direction of velocity)

Consider a source S producing the sound waves. Let the speed of the waves of frequency υ and period T_o, both measured by an observer at rest with respect to the medium be V and we assume that the observer has a detector that counts everytime a crest reaches it.

Let the source and the observer be moving with a speed of VS & Vo respectively as shown below.



At a time t = 0, the source is at S_1 located at a distance of L from an observer at O_1, emits the first crest. Now, since the observer is moving, the velocity of the wave relative to the observer is $(V+V_o)$ and therefore the first crest reaches the observer at a time $t_1 = \frac{L}{(V + Vo)}$.

At a time $t = T_o$, both the observer and source has moved to other new position O_2 and S_2 respectively. The new distance between the observer and source O_2S_2 is given by $[L+(V_S-V_o)T_o]$.

At S_2, the source emits second crest, this crest reaches the observer at a time

$$t_2 = T_o + \frac{[L + (V_S - V_o)T_o]}{V + V_o}$$

At a time $t = nT_o$, the source emits $(n+1)^{th}$ crest and this reaches the observer at time

$$t_{n+1} = nT_o + \frac{[L + n(V_S - V_o)T_o]}{(V + V_o)}$$

Hence, in a time interval of $(t_{n+1} - t_1)$ observer's detector counts 'n' crests and this time interval is given by

$$nT_o + \frac{[L + n(V_S - V_o)T_o]}{(V + V_o)} - \frac{L}{(V + Vo)}$$

Therefore, observer records the time period of the wave as T given by

$$T = \frac{nT_o + \frac{[L + n(V_S - V_o)T_o}{(V+V_o)} - \frac{L}{(V+V_o)}}{n}$$

$$T = T_o + \frac{(V_S - V_o)T_o}{(V+V_o)}$$

$$T = T_o\left[1 + \frac{(V_S - V_o)}{(V+V_o)}\right]$$

$$T = T_o\left[\frac{V+V_S}{V+V_o}\right]$$

Thus the frequency υ observed by an observer is given by $\upsilon = \upsilon_o\left[\frac{V+V_0}{V+V_S}\right]$

11. Pressure: Let p be the pressure and v be the volume of a given mass 'm' of the gas. If ρ is the density of a gas (air) then $v = \frac{m}{\rho}$. If the temperature of the gas remains constant, then, according to Boyle's law, we have pv = constant

$$\therefore \frac{pm}{\rho} = \text{constant}$$

Since m is constant we get $\frac{p}{\rho}$ = constant

But, by newton's laplace formula, velocity of sound in gas is given by $v = \sqrt{\frac{\gamma p}{\rho}}$

As γ is constant and $\frac{p}{\rho}$ is constant, then $\frac{\gamma p}{\rho}$ is also constant. Therefore velocity of sound remains constant. Hence, the velocity of sound in a gas, is independent of pressure provided temperature remains constant (same)

Temperature: Consider a given mass 'm' of the gas at a pressure p, volume v, and temperature T. Then, from perfect gas equation we have $pv = RT \rightarrow (1)$

If 'ρ' is the density of gas, then $v = \frac{m}{\rho}$

$$\therefore (1) \text{ becomes } \frac{pm}{\rho} = RT$$

$$\frac{p}{\rho} = \frac{RT}{m}$$

$$\frac{\gamma p}{\rho} = \frac{\gamma RT}{m} \rightarrow (2)$$

By newton's laplace formula, the velocity of sound in gas is given by $V = \sqrt{\frac{\gamma p}{\rho}}$

$\therefore V = \sqrt{\frac{\gamma RT}{m}}$ (from equation (2))

Since γ, R and m are constant we get, v α $\sqrt{T}$. Hence, the velocity of sound in gas is directly proportional to the square root of the absolute temperature.

Humidity : Humidity is a measure of water content in air. Under same conditions of temperature and pressure, experimental observation shows that humid air is less denser than dry air. In otherwords, the density of humid air (ρ_H) is less than that of dry air (ρ_d)

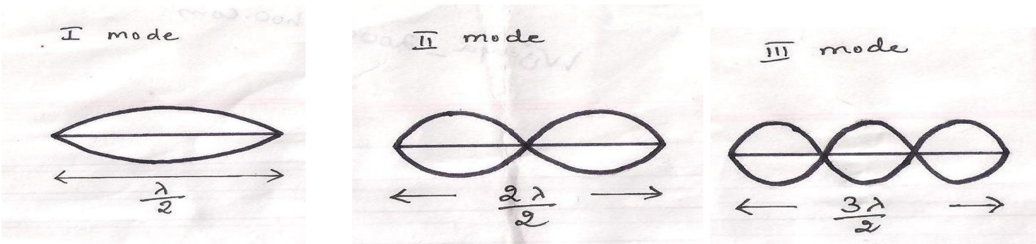
[i.e., $\rho_H < \rho_d$]

But, by Newton laplace formula, we have v = velocity of sound in gas is given by

v = $\sqrt{\frac{\gamma p}{\rho}}$.

$\therefore$ As the humidity increases, density decreases and hence velocity of sound increases and vice-versa. Hence, the velocity of sound in gas depends directly on humidity.

12. Consider a string of length 'L' fixed at either ends A & B. when such a string is plucked at any part of its length, the transverse wave of velocity V, frequency υ, & wavelength λ travel towards each end of the wire and gets reflected at the fixed ends, this reflected wave superpose with the incident wave, forming a stationary wave, such that always at the fixed ends, nodes are formed and the string oscillates in such a way that it is divided into an integral number of equal loops which is characterized by a set of natural frequencies called as normal nodes of oscillations 3 modes are as shown below.



These stationary wave formed is given by

y = 2a sin Kx cos ωt where 2a sin Kx gives its amplitude, therefore the positions of nodes are given by sin Kx = 0

and hence Kx = nπ where n = 0, 1, 2, 3,………………

since' K = $\frac{2\pi}{\lambda}$; we get , x = $\frac{n\lambda}{2}$ where n = 0, 1, 2, 3,……………

In the same way, the positions of the antinodes are given by

Sin Kx = 1

& hence Kx = $\left(n+\frac{1}{2}\right)\pi$ where n = 0, 1, 2, 3, …………….

Since K = $\frac{2\pi}{\lambda}$; we get, x = $\left(n+\frac{1}{2}\right)\frac{\lambda}{2}$, where n = 0, 1, 2, 3,………………

Taking x = 0 and x = L as the positions of node, the condition for node at x = 0 is already satisfied & at x = L, the condition for node requires that the length L is related to wavelength λ by

$L = \frac{n\lambda}{2}$

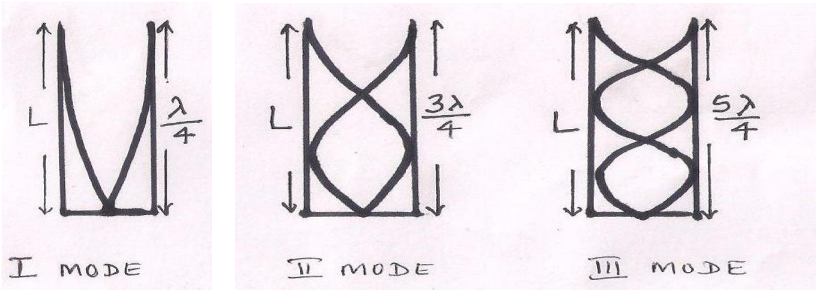
Thus the possible wavelengths of stationary wave, formed in different modes of oscillations are given by $\lambda = \frac{2L}{n}$ where n = 1, 2, 3,…………… with corresponding frequencies given by $\upsilon = \frac{nV}{2L}$ where n = 1, 2, 3,………

Thus, for fundamental mode of oscillation (or) I harmonic, n = 1 and corresponding frequency is given by $\upsilon_1 = \frac{V}{2L}$;

For second mode of oscillation n = 2 and corresponding frequency is given by $\upsilon_2 = \frac{2V}{2L}$; this mode is called as I overtone (or) II harmonic. Similarly, for third mode of oscillation n =3 & corresponding frequency is given by $\upsilon_3 = \frac{3V}{2L}$; this is called as II overtone (or) III harmonic and so on……….. Therefore, $\upsilon_1 : \upsilon_2 : \upsilon_3 : \ldots$ ..= 1 : 2 : 3........

Closed pipe ➔ A pipe which is closed at one end, such that always a node is formed at the closed end and an antinode is formed at the open end is called as a closed pipe.

13. Consider a closed pipe of length L, which encloses certain specific amount of air called as air column. This air column is set into vibrations by holding a vibrating tuning fork near its mouth. These longitudinal waves of frequency υ & wavelength λ travel with a velocity of v through the pipe and gets reflected at the other end, because the other end acts like a boundary and this reflected wave traveling in opposite direction superpose with incident wave forming a stationary wave such that a node is formed at the closed end and an antinode at the open end, which is characterized by a set of natural frequencies called as normal modes of oscillation. The first 3 nodes of oscillations are as shown below.



The stationary waves formed is given by
y = 2a sin kx cos ωt where 2a sin Kx gives its amplitude, therefore the positions of nodes are given by sin Kx = 0

and hence Kx = nπ where n = 0, 1, 2, 3,………………

since' $K = \frac{2\pi}{\lambda}$; we get , $x = \frac{n\lambda}{2}$ where n = 0, 1, 2, 3,……………

In the same way, the positions of the antinodes are given by

Sin Kx = 1

& hence Kx = $\left(n+\frac{1}{2}\right)\pi$ where n = 0, 1, 2, 3, …………….

Since K = $\frac{2\pi}{\lambda}$; we get, x = $\left(n+\frac{1}{2}\right)\frac{\lambda}{2}$, where n = 0, 1, 2, 3,………………

Taking the closed end of the pipe to be x = 0, the condition for node is satisfied and the other end of the pipe to be x = L where an antinode is formed, requires that the length L to be related to wavelength λ given by L = $\left(n+\frac{1}{2}\right)\frac{\lambda}{2}$ for n = 0, 1, 2, 3………

Thus, the possible wavelength's of stationary wave formed in different modes of oscillation are given by

$\lambda = \frac{2L}{(n+1/2)}$ where n = 0, 1, 2, 3, ………..and corresponding frequencies are given by $\upsilon = \left(n+\frac{1}{2}\right)\frac{V}{2L}$ where n = 0, 1, 2, 3,…………

Thus, for fundamental mode of oscillation (or) I harmonic n = 0, and corresponding frequency is given by

$$\upsilon_1 = \frac{V}{4L}$$

For second mode of oscillation n =1, corresponding frequency is given by $\upsilon_2 = \frac{3V}{4L}$; this is called as I overtone (or) III harmonic.

Similarly, for third mode of oscillation n =2, corresponding frequency is given by $\upsilon_3 = \frac{5V}{4L}$; this is called as II overtone (or) V harmonic and so on…………

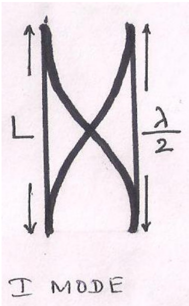
Therefore, $\upsilon_1 : \upsilon_2 : \upsilon_3 :$ …………= 1 : 3 : 5:………

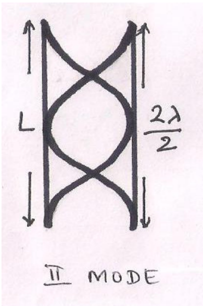
Thus, a closed pipe produces odd harmonics i.e., the ratio of frequencies of overtone to that of fundamental frequency are odd natural numbers.

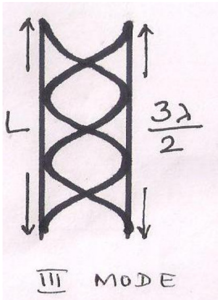
14. Open pipe – A pipe which is open at both ends such that always antinodes are formed at their open ends is called as open pipe.

Consider a open pipe of length L, which encloses a certain specific amount of air called as air column. This air column is set into oscillation by holding a vibrating tuning fork at one of its ends. These longitudinal waves of frequency υ & wavelength λ travel with a velocity of V through the pipe and gets reflected at the other end, because the other end acts like a boundary and this reflected wave traveling in the opposite direction superpose with the incident wave forming a stationary wave, such that always antinodes are formed at their open ends which is characterized by a set of natural frequencies called as normal modes of oscillation.

The first three nodes of oscillations are as shown below.







The stationary waves formed is given by
y = 2a sin Kx cos wt where 2a sin Kx gives its amplitude, therefore the positions of nodes are given by sin Kx = 0

and hence Kx = nπ where n = 0, 1, 2, 3,………………

since' K = $\frac{2\pi}{\lambda}$; we get , x = $\frac{n\lambda}{2}$ where n = 0, 1, 2, 3,……………

In the same way, the positions of the antinodes are given by

Sin Kx = 1

& hence Kx = $\left(n+\frac{1}{2}\right)\pi$ where n = 0, 1, 2, 3, …………….

Since K = $\frac{2\pi}{\lambda}$; we get, x = $\left(n+\frac{1}{2}\right)\frac{\lambda}{2}$, where n = 0, 1, 2, 3,………………

Since at both the ends, antinodes are formed in different modes of oscillation, the possible wavelengths of stationary waves formed indifferent modes of oscillation is given by λ = $\frac{2L}{n}$ where n = 1, 2, 3, 4,…….& corresponding frequencies are given by υ = $\frac{nV}{2L}$ where n = 1, 2, 3,…………..

Thus, for I mode of oscillation (or) I harmonic n =1 & the corresponding frequency is given by $\upsilon_1 = \frac{V}{2L}$.

For II mode of oscillation, n =2 & the corresponding frequency is given by $\upsilon_2 = \frac{2V}{2L}$; this is called as I overtone (or) II harmonic and similarly,

for III mode of oscillation n = 3 and the corresponding frequency is given by $\upsilon_3 = \frac{3V}{2L}$; this is called as II overtone (or) III harmonic and so on……..

Therefore, $\upsilon_1 : \upsilon_2 : \upsilon_3$:…………= 1 : 2 : 3 : …….

Thus, open pipe produces both odd & even harmonics. i.e the ratio of frequencies of the overtones to that of fundamental frequency are both even and odd natural numbers.

5 marks problems answers :

1. Let time taken by stone to reach the surface of water dropped from top of the tower of height 300 m be t_1

   ∴ we have x = $\upsilon_o t_1 + \frac{1}{2} a t_1^2$

   -300 = o (t_1) + $\frac{1}{2}$(-9.8) t_1^2

   ∴ t_1 = 7.82 s

Let the time taken by sound to reach the person on the top of tower be t_2.

$\therefore\ t_2 = \dfrac{distance\ \ covered\ \ by\ \ sound\ \ wave}{velocity\ \ of\ \ sound\ \ wave}$

$t_2 = \dfrac{300}{340} = 0.88$ s

$\therefore$ The splash of sound is heard after a time (t) equal $t_1 + t_2$.

i.e., $t = t_1 + t_2$

$= 7.82 + 0.88$

$T = 8.7$ s

2. Given equation is $y =(x, t) = 3 \sin (36t + 0.018x + \frac{\pi}{4}) \rightarrow$ (1)

This equation is of form $y (x, t) = a \sin (\omega t + kx + \phi) \rightarrow$ (2)

i) It is a traveling wave

ii) It travels from right to left (i.e, along the negative x direction)

iii) Comparing equation (1) & (2) we get

$\omega = 36$

$2\pi\upsilon = 36$

$\upsilon = \dfrac{36}{2\pi}$

$\upsilon = 5.73$ Hz

iv) Initial phase $= \dfrac{\pi}{4}$ radian

v) Distance between two consecutive crests is its wavelength (λ)

Comparing (1) & (2) we have

$K = \dfrac{2\pi}{\lambda} = 0.018$

$\therefore \lambda = \dfrac{2\pi}{0.018}$

$\lambda = 349$ cm

$\therefore \lambda = 3.49$ m

3. Given equation is $y(x, t) = 0.06 \sin \left(\dfrac{2\pi}{3}\right) x. \cos(120\ \pi t) \rightarrow$ (1)

Given equation is of form $y(x, t) = 2a \sin kx \cos \omega t \rightarrow$ (2)

i) It is a stationary wave

ii) Comparing equation (1) and (2); we get $K = \dfrac{2\pi}{\lambda} = \dfrac{2\pi}{3}$

$\therefore \lambda = 3$m

$\omega = 2\pi\upsilon = 120\pi$

$\therefore \upsilon = 60$ Hz

$V = \upsilon\lambda = 60 \times 3 = 180\ ms^{-1}$

$V = 180\ ms^{-1}$

iii) Given $m = 3 \times 10^{-2}$ kg

$\ell = 1.5$ m

$\therefore$ linear density $\mu = \dfrac{m}{\ell} = \dfrac{3\times10^{-2}}{1.5} = 2 \times 10^{-2}\ kg\ m^{-1}$

We have $v = \sqrt{\frac{T}{\mu}}$

$\therefore T = V^2\mu$

$T = (180)^2 \times 2 \times 10^{-2}$

$T = 648$ N

4. Given equation is $y = 1.2 \sin\pi \left(\frac{2t}{5} - \frac{x}{4}\right)$

   It can be rewritten as $y = 1.2 \sin \left(\frac{2\pi t}{5} - \frac{\pi x}{4}\right) \rightarrow$ (1)

   This equation is of the form $y = a \sin(\omega t - kx) \rightarrow$ (2)

   Comparing equation (1) & (2)

   We get, a = 1.2 m

   $K = \frac{2\pi}{\lambda} = \frac{\pi}{4}$

   $\therefore \lambda = 8$ m

   $\omega = \frac{2\pi}{T} = \frac{2\pi}{5}$

   $\therefore T = 5$ s.

   $V = \frac{\lambda}{T}$

   $V = \frac{8}{5} = 1.6\ ms^{-1}$

   $V = 1.6\ ms^{-1}$

5. Let ℓ_c & ℓ_o be the length of closed pipe and open pipe respectively

   Given $\ell_c = 0.42$m t = 35°C

   (frequency of first overtone) = (fundamental frequency)

   Closed pipe open pipe

   i.e., $\frac{3V_{35^oC}}{4\ell_c} = \frac{V_{35^oC}}{2\ell_o} \rightarrow$ (1)

   Fundamental frequency of closed pipe = 210 Hz

   i.e., $\frac{V_{35^oC}}{4\ell_c} = 210$ Hz $\rightarrow$ (2)

   from (1) we get, $\ell_o = \frac{4\ell c}{6}$

   $\ell_o = \frac{4(0.42)}{6}$

   $\ell_o = 0.28$ m

   From (2) we get $V_{35^oC} = 210 \times 4\ \ell_c$

   $= 210 \times 4 \times 0.42$

   $V_{35^oC} = 352.8\ ms^{-1}$

   We have $V \propto \sqrt{T}$

$$\therefore \frac{V_{0^oC}}{V_{35^oC}} = \frac{\sqrt{0+273}}{\sqrt{35+273}}$$

$$\therefore V_{0^oC} = \sqrt{\frac{273}{308}} \times V_{35^oC}$$

$$V_{0^oC} = \sqrt{\frac{273}{308}} \times 352.8$$

$$V_{0^oC} = 332.15 \text{ ms}^{-1}$$

6. The frequency (υ) as heard by an observer due to motion of source and observer is given by

$$\upsilon = \upsilon_o \left(\frac{V+V_o}{V+Vs}\right) \rightarrow (1)$$

$V_o = V_{\text{second Car}} = 18 \text{ km h}^{-1}$

$$= \frac{18 \times 7000}{3600}$$

$V_o = 5 \text{ ms}^{-1}$

$V_S = V_{\text{first car}} = 54 \text{ km h}^{-1}$

$$= \frac{54 \times 1000}{3600} = VS = 15 \text{ ms}^{-1}$$

i) $\bullet\!\longrightarrow$ O $\qquad$ S $\longleftarrow\!\bullet$

$V_o = 5 \text{ ms}^{-1}$ $\qquad$ $V_S = 15 \text{ ms}^{-1}$

$$\upsilon = \upsilon_o \left(\frac{V+V_o}{V+Vs}\right)$$

Here V_o = +ve
& V_S = -ve

$\therefore$ equation (1) becomes $\upsilon = \upsilon_o \left(\frac{V+V_o}{V-Vs}\right)$

$$\upsilon = 240 \left[\frac{340+5}{340-15}\right]$$

υ = 254.76Hz

ii) $\bullet\!\longrightarrow$ O $\qquad$ $\bullet\!\longrightarrow$ S

Here V_o = - ve
V_s = +ve

$\therefore$ equation (1) becomes $\upsilon = \upsilon_o \left(\frac{V-V_o}{V+Vs}\right)$

$$\upsilon = 240 \left[\frac{340-5}{340+15}\right]$$

υ = 226.47Hz

7. Given equation is $y = 1.4 \sin \pi (300\, t - x)$

This can be rewritten as $y = 1.4 \sin (300\, \pi t - \pi x) \rightarrow (1)$

This equation is of the form $y = a \sin (\omega t - kx) \rightarrow (2)$

We have $V = \upsilon\lambda \rightarrow (3)$

Comparing equation (1) & (2)

We get, $\omega = 2\pi\upsilon = 300\,\pi$

$\therefore \upsilon = 150$ Hz

$K = \dfrac{2\pi}{\lambda} = \pi$

$\therefore \lambda = 2m$

$\therefore$ from (3) $V = 150 \times 2$

$V = 300\ ms^{-1}$

We have phase difference = $\dfrac{2\pi}{\lambda}$ (path difference)

$$\Delta\phi = \frac{2\pi}{\lambda}(0.25)$$

$$= \frac{2\pi}{2}(0.25)$$

$$\Delta\phi = \frac{\pi}{4} \text{ radian}$$

8. Given distance traveled by sound in air = distance traveled by sound in argon.

i.e., $d_{air} = d_{argon}$

$$\frac{\gamma_{air}}{\gamma_{argon}} = \frac{1.402}{1.667} \quad \& \quad \frac{\rho_{air}}{\rho_{argon}} = \frac{1.293}{1.789}$$

& we know that $V_{sound} = \sqrt{\dfrac{\gamma p}{\rho}} \rightarrow (1)$

We have $\dfrac{V_{air}}{V_{argon}} = \dfrac{d_{air} \times t_{argon}}{t_{air} \times d_{argon}}$

$$\therefore \frac{t_{air}}{t_{argon}} = \frac{V_{argon}}{V_{air}} = \sqrt{\frac{\gamma_{argon} \times p \times \rho_{air}}{\gamma_{argon} \times \rho_{air} \times p}} \qquad \text{(from (1))}$$

$$\frac{t_{air}}{t_{argon}} = \sqrt{\frac{\gamma_{argon} \times \rho_{air}}{\rho_{argon} \times \gamma_{air}}}$$

$$\therefore \frac{t_{air}}{t_{argon}} = \sqrt{\frac{1.667}{1.402} \times \frac{1.293}{1.789}}$$

$$\frac{t_{air}}{t_{argon}} = 0.927$$

9. If 'm' is the molecular mass of the given gas then, we have velocity of sound in gas is given by

$$V = \sqrt{\frac{\gamma RT}{m}}$$

$$\therefore \frac{V_{mixture}}{V_{hydrogen}} = \sqrt{\frac{\gamma RT}{m_{mixture}} \times \frac{m_{hydrogen}}{\gamma RT}}$$

$$\therefore \frac{V_{mixture}}{V_{hydrogen}} = \sqrt{\frac{m_{hydrogen}}{m_{mixture}}} \qquad \rightarrow (1)$$

$$m_{mixture} = \frac{(32\times1)+(2\times4)}{1+4}$$

$m_{mixture} = 8$

$\therefore$ equation (1) becomes $\frac{V_{mixture}}{V_{hydrogen}} = \sqrt{\frac{2}{8}}$

$$\therefore V_{mixture} = \frac{1}{2}\times V_{hydrogen}$$

$$= \frac{1270}{2}$$

$V_{mixture} = 635\ ms^{-1}$

10. Length of the wire $L = \frac{mass\ \ of\ \ the\ \ wire}{Linear\ \ density}$

$$L = \frac{3.5\times10^{-2}}{4\times10^{-2}}$$

$L = 0.875$ m

When the wire vibrates in its fundamental mode then

$L = \frac{\lambda}{2}$ ➔ $\lambda = 2 \times 0.875$

$\lambda = 1.75$ m

$\therefore$ speed of the transverse wave on the string is

Given $V = \upsilon\lambda = 45 \times 1.75$

$V = 78.75\ ms^{-1}$

ii) $V = \sqrt{\frac{T}{\mu}}$

$\therefore T = V^2 \mu$

$T = (78.75)^2 \times 4 \times 10^{-2}$

$T = 248$ N

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