QUESTIONS

Conic Section

Circles

One Mark Questions :

1. Define a Circle as the locus of a point.

2. Find the equation of the Circle with

(a) Centre (1, 2) radius 5

(b) Centre (-3, 2), radius $\sqrt{6}$

(c) centre (-5, -6), radius 10

(d) Centre (0, 5), radius 9

(e) Centre (0, 0), radius 4

(f) Centre (0 -6), radius $\sqrt{3}$

(g) Centre (4, 0), radius $\frac{2}{3}$

(h) Centre (1, -1), radius $\frac{3}{\sqrt{2}}$

(i) Centre (-a, b) and radius $\sqrt{a^2 - b^2}$

(j) Centre $\left(\frac{1}{2}, \frac{3}{2}\right)$, radius $\sqrt{8}$

(k) Centre (-2, 3) and radius 4

(l) Centre (-1, -1), radius 5

(m) Centre $\left(\frac{1}{3}, -\frac{1}{2}\right)$ and radius $\frac{1}{\sqrt{6}}$

(n) Centre $\left(\frac{2}{3}, \frac{1}{3}\right)$ and radius $\frac{2}{\sqrt{3}}$

Two Marks Questions :

1. Find the Centre and radius of the following Circles.

(a) $x^2 + y^2 = 16$

(b) $x^2 + y^2 = 1$

(c) $x^2 + y^2 - 8x = 0$

(d) $x^2 + y^2 + 6x = 0$

(e) $x^2 + y^2 - 4x + 2y = 0$

(f) $x^2 + y^2 \; x \; 2x + 2y - 7 = 0$

(g) $x^2 + y^2 - 2x - 4y - 1 = 0$

(h) $x^2 + y^2 - 8 = 0$

(i) $x^2 + y^2 + 10x + 20y + 5 = 0$

(j) $(x-1)^2 + (y + 1)^2 = 4$

(k) $x^2 - (y + 6)^2 = 2$

(l) $(x + 7)^2 + y^2 = 25$

(m) $(x-5)^2 + (y-3)^2 = 100$

(n) $x^2 + (y + 1)^2 = 3$

(o) $(x + 6)^2 + y^2 = 25$

2. Find the equation of the Circle whose centre is (1, -2) and which passes through the point (-3, -5)

3. Find the equation of the Circle whose centre is (3, 6) and which passes through the point (-1, 1).

4. Find the equation of the Circle if the ends of a diameter are (-1, 4) and (3, -8)

5. Find the equation of the Circle if the ends of a diameter are (0, 7) and (-1, 0)

6. Find the equation of the Circle if the circle passes through the points (0, 0), (8, 0) and (0, 9).

7. Find the equation of the Circle whose radius is $\sqrt{7}$ and concentric with the circle $x^2 + y^2 - 8x + 6y - 11 = 0$

8. Find the equation of the Circle If two of it's diameters are $x + 4y = 5$ and $7x - y = 6$, and whose radius is $\sqrt{7}$

9. Find whether (1, 3) lies inside, or outside or on the circle $x^2 + y^2 = 16$.

10. Show that the point (5, 6) lies outside the Circle $x^2 + y^2 = 4$.

11. Show that the point (1, 3) lies outside the circle $x^2 + y^2 - 6x + 2y + 11 = 0$

12. Show that the point (-2, 0) lies inside the Circle $x^2 + y^2 + 6x + 2y - 1 = 0$.

Four Marks Questions

1. Find the equation of the Circle which passes through the points (1, 2), (2, 2), and (4, -1).

2. Find the equation of the Circle which passes through the points (1, 1), (1, 3), and (2, 2).

3. Find the equation of the Circle which passes through the points $(-1, 1)$, $(3, 2)$ and whose Centre lies on the line $x - 2y + 2 = 0$.

4. Find the equation of the Circle which passes through the points $(1, 3)$, $(2, 3)$ and whose centre lies on the line $2x + 3y - 1 = 0$.

Parabola

One Mark Questions

1. Define a Parabola as the Locus of a point.

2. Define latus rectum of a Parabola.

3. Find the focus of the following Parabola.

(i) $y^2 = 12x$ (ii) $y^2 = -16x$ (iii) $x^2 = 20y$

(iv) $x^2 = -10y$ (v) $2y^2 = 3x$ (vi) $4y^2 = -20x$

(vii) $2x^2 = 5y$ (viii) $3x^2 = -2y$

4. Find the axis of the Parabola

(i) $y^2 = x$ (ii) $y^2 = -3x$ (iii) $x^2 = 11y$ (iv) $x^2 = -4y$

5. Find the directrix of the Parabola.

(i) $y^2 = 40x$ (ii) $3y^2 = 10x$ (iii) $y^2 = -36x$

(iv) $2y^2 = -5x$ (v) $x^2 = 100y$ (vi) $4x^2 = y$

(vii) $x^2 = -20y$ (viii) $5x^2 = -2y$

6. Find the length of latus rectum of the Parabola.

(i) $y^2 = 8x$ (ii) $2y^2 = 3x$ (iii) $y^2 = -16x$

(iv) $10y^2 = -3x$ (v) $x^2 = 10y$ (vi) $4x^2 = 6y$

(vii) $x^2 = -10y$ (viii) $5x^2 = -2y$

Two Marks Questions.

1. Show that the line $5x + 2y - 25 = 0$ passes through the focus of the Parabola $y^2 = 20x$.

2. Find the equation of the line which passes through (5, -6) and the focus of the Parabola $x^2 = -12y$.

3. Find the equation of the Parabola with the following data.

   (i) Vertex (0, 0), focus (8, 0) (ii) vertex (0, 0), Focus (-12, 0)

   (iii) Vertex (0, 0), focus (0, 10) (iv) vertex (0, 0), Focus (0, -6)

   (v) Focus (5, 0), directrix $x = -5$ (vi) vertex (0, 0), directrix $y = 11$

4. Prove that the length of the latus rectum of the Parabola $y^2 = 4ax$ is 4a.

Three marks Questions :

1. Derive the equation of the parabola in the standard form $y^2 = 4ax$.

2. If the ends of the latus rectum of a Parabola are (3, 10) and (3, -10) and vertex is (0, 0) find the equation of the Parabola.

3. If the ends of the latus rectum of a Parabola are (16, 8) and (-16, 8), find the equation of the Parabola.

4. Find the equation of the Parabola which is Symmetric about x-axis and passes through the point (6, 4)

5. Find the equation of the parabola which is Symmetric about the y-axis vertex origin and which passes through the point (-3, 5).

6. Find the equation of the Parabola which is symmetric about y-axis, vertex origin and which passes through (6 , -8)

Four Marks Questions :

1. The focus of a Parabolic mirror is at a distance 10mts from the vertex. If the mirror is 40cms. Deep, find the diameter of the outermost circular surface of the mirror.

2. A beam is supported at it's ends by supports which are 20 meters apart. Since the load is concentrated at the centre, there is a deflection of 1 mt. at the centre. How far from the centre is the defection 0.5mtrs.

3. Find the area of the triangle formed by the lines joining vertex of the parabola $y^2 = 12x$, and the ends of it's latus rectum.

Ellipse

One Mark Questions :

1. Find the major axis and minor axis of the following ellipse.

(i) $\frac{x^2}{64}+\frac{y^2}{4}=1$ (ii) $\frac{x^2}{16}+y^2=1$ (iii) $\frac{x^2}{25}+\frac{y^2}{9}=1$

(iv) $\frac{x^2}{16}+\frac{y^2}{64}=1$ (v) $\frac{x^2}{4}+\frac{y^2}{9}=1$ (vi) $x^2+\frac{y^2}{9}=1$

(vii) $25x^2+4y^2=100$ (viii) $9x^2+4y^2=36$ (ix) $4x^2+25y^2=100$

(x) $100x^2+400y^2=40000$ (xi) $16x^2+y^2=16$

2. Find the vertices of the following ellipse.

(i) $2x^2+y^2=2$ (ii) $3x^2+2y^2=6$ (iii) $\frac{x^2}{9}+\frac{y^2}{4}=1$ (iv) $\frac{x^2}{25}+\frac{y^2}{64}=1$

(v) $x^2+\frac{y^2}{9}=1$ (vi) $\frac{x^2}{16}+y^2=1$ (vii) $36x^2 + 4y^2 = 144$

3. Find the latus rectum of the following ellipse.

(i) $25x^2 + 4y^2 = 100$ (ii) $2x^2 + 3y^2 = 6$

(iii) $\frac{x^2}{49}+\frac{y^2}{25}=1$ (iv) $\frac{x^2}{64}+\frac{y^2}{100}=1$

Two Marks Questions :

1. Find the eccentricity of the following ellipse.

   (i) $4x^2 + 9y^2 = 36$ (ii) $9x^2 + 4y^2 = 36$ (iii) $\frac{x^2}{25}+\frac{y^2}{100}=1$

   (iv) $x^2+\frac{y^2}{4}=1$ (v) $\frac{x^2}{49}+y^2=1$ (vi) $9x^2 + 2y^2 = 18$

2. If the major axis of an ellipse is double the minor axis find the eccentricity.

3. If the major axis of an ellipse is three times it's minor axis, find the eccentricity.

4. If the Major axis of an ellipse $bx^2 + a^2y^2 = a^2b^2$ $a > b$ is 40, and the eccentricity is $\frac{1}{3}$, find the 'minor' axis.

5. If the minor axis of an ellipse $a^2x^2 + b^2y^2 = a^2b^2$ $b > a$, is 10 and the eccentricity is $\frac{2}{5}$, find the 'major' axis.

6. Show that the length of the latus rectum of an

   $$\text{ellipse } b^2x^2 + a^2y^2 = a^2b^2,\ a > b \text{ is } \frac{2b^2}{a}$$

7. Find the Vertices and latus rectum of the following ellipse.

   (i) $4x^2 + 3y^2 = 12$ (ii) $4x^2 + 25y^2 = 100$ (iii) $9x^2 + 4y^2 = 36$

8. Find the focii of the following ellipse.

   (i) $\frac{x^2}{25}+\frac{y^2}{4}=1$ (ii) $\frac{x^2}{16}+\frac{y^2}{25}=1$ (iii) $5x^2 + 2y^2 = 10$

   (iv) $6x^2 + 11y^2 = 66$ (v) $x^2 + 4y^2 = 1$ (vi) $9x^2 + y^2 = 1$

9. If the extremeties of the major axis of an ellipse which is along the x-axis are $(\pm 5, 0)$, find the major axis.

10. If the length of the latus rectum of an ellipse whose major axis is along the y-axis is 50, show that $a = 5\sqrt{b}$.

Three Marks Questions

1. Find the equation of the ellipse in the form $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ a > b if

(i) Major axis = 20, Minor axis = 12 (ii) Major axis = 10, e = ½

(iii) Minor axis = 8 $e = \frac{2}{3}$ (iv) Vertices are ($\pm$8, 0), and (0, $\pm$10)

(v) Vertices are ($\pm$ 12 , 0), (0, $\pm$ 6) (vi) Major axis = 16, focii ($\pm$3, 0)

(vii) Minor axis = 10, focii (0, $\pm$ 4)

(viii)Major axis on x-axis and passes through (4, 6) and (1, 3)

(ix) Major axis on y-axis and passes through (43) and (6, 2)

(x) Major axis on x – axis and passing through (4, 3) & (-1, 4)

2. Show that for the ellipse $b^2x^2 + a^2y^2 = a^2b^2$, a > b, the length of the latus rectum is $\frac{2b^2}{a}$

3. Find the distance between the focii of the ellipse $4x^2 + 25y^2 = 100$

4. If the distance between the focii of an ellipse whose major axis is along x-axis, is 32 and $e = \frac{1}{3}$ find the equation of the ellipse.

5. If the distance between the focii of an ellipse whose major axis is along y-axis, is 20, and $e = \frac{1}{\sqrt{2}}$, find the equation of the ellipse.

Five Marks Questions :

1. Derive the equation of the ellipse in the standard form $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

2. For the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, a > b, the distance between focii is 16 and distance between directrices is 30, find the equation of the ellipse.

3. Find the equation of the ellipse whose focii is (10, 0), directrix is x = 20 and e = $\frac{1}{3}$

4. If P(xy) is any point on the ellipse $b^2x^2 + a^2y^2 = a^2b^2$ a > b, and S_1, S_2 are the focii, then Prove that $PS_1 + PS_2 = 2a$.

5. An arch is in the form of semi ellipse. It is 20cms wide and 8cms high at the centre. Find the height of the arch at the point 10cms from one end.

Hyperbola :

One Mark Questions :

1. Define hyperbola as the locus of a point.

2. Find the transverse axis and conjugate axis of the following hyperbola.

(i) $\frac{x^2}{9}-\frac{y^2}{4}=1$ (ii) $\frac{x^2}{4}-\frac{y^2}{9}=1$ (iii) $\frac{x^2}{25}-\frac{y^2}{16}=1$

(iv) $\frac{x^2}{4}-\frac{y^2}{25}=-1$ (v) $\frac{y^2}{49}-\frac{x^2}{4}=1$ (vi) $x^2 - 4y^2 = 4$

(vii) $25x^2 - y^2 = 25$ (viii) $4x^2 - 25y^2 = -1$

(ix) $16x^2 - 4y^2 = -1$ (x) $3x^2 - y^2 = 3$

3. Find the Vertices of the following hyperbola.

(i) $x^2 - 9y^2 = 1$ (ii) $16x^2 - y^2 = -1$ (iii) $\frac{x^2}{64}-\frac{y^2}{36}=1$

(iv) $9x^2 - 4y^2 = -36$ (v) $\frac{x^2}{100}-\frac{y^2}{49}=1$ (vi) $x^2 - 2y^2 = 2$

4. Define the latus rectum of a hyperbola.

5. Find the latus rectum of the hyperbola

(i) $\frac{x^2}{49}-\frac{y^2}{64}=1$ (ii) $\frac{x^2}{16}-\frac{y^2}{9}=1$ (iii) $4x^2 - 16y^2 = -1$

(iv) $x^2-\frac{y^2}{4}=1$ (v) $9x^2 - y^2 = -1$ v) $3x^2 - 2y^2 = 6$

Two Marks Questions :

1. Find the eccentricity of the hyperbola

(i) $4x^2 - 16y^2 = 64$ (ii) $2x^2 - 3y^2 = -6$

(iii) $\frac{x^2}{36}-\frac{y^2}{9}=1$ (iv) $\frac{x^2}{4}-\frac{y^2}{9}=1$ (v) $\frac{x^2}{9}-\frac{y^2}{4}=-1$

(vi) $6x^2 - 3y^2 = -18$ (vii) $4x^2 - y^2 = 4$

2. If the transverse axis of a hyperbola is double the conjugate axis, find it's eccentricity.

3. If the transverse axis of a hyperbola $bx^2 - a^2y^2 = a^2b^2$ is 20 and e = 2, find the conjugate axis.

4. If the conjugate axis of a hyperbola is 12 and $e = \frac{3}{2}$, find the transverse axis.

5. Show that the length of latus rectum of the hyperbola

$$b^2x^2 - a^2y^2 = a^2b^2 \text{ is } \frac{2b^2}{a}$$

6. Find the Vertices and latus rectum of the following hyperbola.

(i) $\frac{x^2}{25}-\frac{y^2}{4}=1$ (ii) $\frac{x^2}{9}-\frac{y^2}{16}=1$ (iii) $\frac{x^2}{16}-\frac{y^2}{25}=-1$

(iv) $\frac{x^2}{64}-\frac{y^2}{49}=-1$ (v) $\frac{y^2}{4}-\frac{x^2}{2}=1$ (vi) $6x^2 - 3y^2 = -18$

7. Find the focii of the following hyperbola.

(i) $25x^2 - 4y^2 = 100$ (ii) $2x^2 - 5y^2 = -10$

(iii) $6x^2 - 11y^2 = 66$ (iv) $x^2 - 4y^2 = -1$

(v) $\frac{x^2}{100} - \frac{y^2}{64} = 1$ (vi) $\frac{x^2}{64} - \frac{y^2}{49} = -1$

8. If the extremeties of the transverse axis which is along the X-axis are ($\pm$10, 0), find the conjugate X-axis.

9. Define an equilateral hyperbola and find it's eccentricity.

10. If P is any point on the hyperbola $8x^2 - 5y^2 = 40$ whose focii are S_1 and S_2, then find $PS_1 - PS_2$

Three marks Questions :

1. Find the equation of the hyperbola $b^2x^2 - a^2y^2 = a^2b^2$ if

(i) Transverse axis 5 and conjugate axis 3

(ii) Transverse axis 12 and e = 2

(iii) conjugate axis 10 and e = $\frac{3}{2}$

(iv) Vertices ($\pm$2, 0), focii ($\pm$6, 0)

(v) Transverse axis 8 and focii ($\pm$12, 0)

(vi) Transverse axis 6 and latus rectum 10

(vii) Passes through the points (-12) and (4, 6)

(viii) Focii ($\pm$12, 0) and latus rectum 36

(ix) Vertices ($\pm\sqrt{10}$,0), passing through (2, 3)

Five Marks Questions :

1. Derive the standard equation of the hyperbola in the form $b^2x^2 - a^2y^2 = a^2b^2$

CONIC SECTION - SOLUTIONS

Circles

One Mark Questions :

3. Circle is defined as the locus of a point which moves such that it's distance from a fixed point is a Constant.

4. (a) $C \equiv (h\ k) \equiv (1, 2)$, $r = 5$

$\therefore$ the equation of the Circle is

$(x-h)^2 + (y-k)^2 = r^2$

ie. $(x-1)^2 + (y-2)^2 = 25$

(b) $(x+3)^2 + (y-2)^2 = 6$

(c) $(x+5)^2 + (y+6)^2 = 100$

(d) $x^2 + (y-5)^2 = 81$

(e) $x^2 + y^2 = 16$

(f) $x^2 + (y+6)^2 = 3$

(g) $(x-4)^2 + y^2 = \frac{4}{9}$

(h) $(x-1)^2 + (y+1)^2 = \frac{9}{4}$

(i) $(x+a)^2 + (y-b)^2 = a^2 - b^2$

(j) $\left(x-\frac{1}{2}\right)^2 + \left(y-\frac{3}{2}\right)^2 = 8$

(k) $(x+2)^2 + (y-3)^2 = 16$

(l) $(x+1)^2 + (y+1)^2 = 25$

(m) $\left(x+\frac{1}{3}\right)^2 + \left(y+\frac{1}{2}\right)^2 = \frac{1}{6}$

(n) $\left(x-\frac{2}{3}\right)^2 + \left(y-\frac{1}{3}\right)^2 = \frac{4}{9}$

Two Marks Questions :

13. (a) By comparing with the standard equation,

$(x-h)^2 + (y-k)^2 = r^2$ we get $h = 0$ $K = 0$, $r = 4$

$\therefore C \equiv (h, k) \equiv (0, 0)$ $r = 4$

(b) $C \equiv (0,0)$ $\quad r = 1$

(c) $(x^2 - 8x) + y^2 = 0$

ie. $(x - 4)^2 - 16 + y^2 = 0 \quad \Rightarrow (x - 4)^2 + y^2 = 16$

$\therefore C \equiv (h, k) \equiv (4, 0), \quad r = 4$

(d) $(x^2 + 6x) + y^2 = 0 \quad \Rightarrow (x + 3)^2 - 9 + y^2 = 0$

$\Rightarrow (x + 3)^2 + y^2 = 9$

$\therefore C \equiv (-3, 0) \quad r = 3$

(e) $(x^2 - 4x) + (y^2 + 2y) = 0$

$\Rightarrow (x - 2)^2 - 4 + (y + 1)^2 - 1 = 0$

$\Rightarrow (x - 2)^2 + (y + 1)^2 = 5$

$\therefore C \equiv (2, -1) \quad r = \sqrt{5}$

(f) $(x^2 + 2x) + (y^2 + 2y) - 7 = 0$

$\Rightarrow (x + 1)^2 - 1 + (y + 1)^2 - 1 - 7 = 0$

$\Rightarrow (x + 2)^2 + (y + 1)^2 = 9$

$\therefore C \equiv (-1, -1) \quad r = 3$

(g) $(x^2 - 2x) + (y^2 - 4y) - 1 = 0$

$\Rightarrow (x - 1)^2 - 1 + (y - 2)^2 - 4 - 1 = 0$

$\Rightarrow (x - 1)^2 + (y - 2)^2 = 6$

$\therefore C \equiv (1, 2) \quad r = \sqrt{6}$

(h) $x^2 + y^2 = 8 \quad \Rightarrow C \equiv (0, 0) \quad r = \sqrt{8}$

(i) $(x^2 + 10x) + (y^2 + 20y) + 5 = 0$

$\Rightarrow (x + 5)^2 - 25 + (y + 10)^2 - 100 + 5 = 0$

$\Rightarrow (x + 5)^2 + (y + 10)^2 = 120$

$\therefore C \equiv (-5, -10) \quad r = \sqrt{120}$

(j) $C \equiv (1, -1)$ $r = 2$ (k) $C \equiv (0, -6)$ $r = \sqrt{2}$

(l) $C \equiv (-7, 0)$ $r = 5$ (m) $C \equiv (5, 3)$ $r = 10$

(n) $C \equiv (0, -1)$ $r = \sqrt{3}$ (o) $C \equiv (-6, 0)$ $r = 5$

14. $C \equiv (h, k) \equiv (1, -2)$ $P \equiv (-3, -5)$

$\therefore$ $r = cp = \sqrt{(1+3)^2+(-2+5)^2} = \sqrt{16+9} = 5$

$\therefore$ equation of the Circle is

$(x - h)^2 + (y - k)^2 = r^2$ => $(x - 1)^2 + (y + 2)^2 = 25$

15. $C \equiv (3, 6)$ $P \equiv (-1, 1)$

$\therefore$ $r = cp = \sqrt{(3+1)^2+(6-1)^2} = \sqrt{16+25} = \sqrt{41}$

=> $(x - 3)^2 + (y - 6)^2 = 41$

16. $P \equiv (-1, 4)$ $Q \equiv (3, -8)$

=> PQ is a diameter

$\therefore$ The centre C is the mid point of AB $\equiv \left(\frac{-1+3}{2}, \frac{4-8}{2}\right) \equiv (1, -2)$

$\therefore$ $r = cp = \sqrt{(-1-1)^2+(4+2)^2} = \sqrt{4+36} = \sqrt{40}$

=> $(x - 1)^2 + (y + 2)^2 = 40$

17. $C \equiv \left(\frac{0-1}{2}, \frac{7-0}{2}\right) \equiv \left(-\frac{1}{2}, \frac{7}{2}\right)$

$\therefore$ r = distance between (0, 7) and $\left(-\frac{1}{2}, \frac{7}{2}\right)$

$$= \sqrt{\left(0+\frac{1}{2}\right)^2+\left(7-\frac{7}{2}\right)^2} = \sqrt{\frac{1}{4}+\frac{49}{4}} = \frac{\sqrt{50}}{2}$$

$$\Rightarrow \left(x+\frac{1}{2}\right)^2 + \left(y-\frac{7}{2}\right)^2 = \frac{50}{4}$$

18. Consider $(x-h)^2 + (y-k)^2 = r^2$ Since it passes through (0, 0), (8, 0) and (0, 9), we have

$h^2 + k^2 = r^2$ -------(1) $\quad$ (1) -(2) => $64 - 16h = 0$ => $h = 4$

$(8-h)^2 + k^2 = r^2$ ---(2) and $\quad$ (2) -(3) => $81 - 18k = 0$ => $k = \frac{9}{2}$

$h^2 + (9 - k)^2 = r^2$ ---(3) $\quad$ from (1) $\quad r = \frac{\sqrt{97}}{2}$

$\therefore$ equation of the Circle is $(x-4)^2 + \left(y - \frac{9}{2}\right)^2 = \frac{97}{4}$

19. $r = \sqrt{7}$, The given Circle is $(x^2 - 8x) + (y^2 + 6y) - 11 = 0$

=> $(x-4)^2 - 16 + (y+3)^2 - 9 - 11 = 0$

=> $(x-4)^2 + (y+3)^2 = 36$

$C \equiv (4, -3)$

Since the required Circle and given Circle are concentric

For the required Circle also $C \equiv (4, -3)$

$\therefore$ equation is $(x-4)^2 + (y+3)^2 = 7$

20. The point of inter section of the diameters is the centre solving the equations of the diameters we get

$x = 1, \quad y = 1 \quad \therefore C \equiv (1\ 1), \quad r = \sqrt{7}$

$\therefore$ The equation of the Circle is $(x-1)^2 + (y-1)^2 = 7$

21. For the Circle $C \equiv (0, 0)$ and $r = 4$

$P \equiv (13) \qquad CP = \sqrt{(1-2)^2 + (3-0)^2} = \sqrt{10} < r$

$\therefore$ P is inside the Circle.

22. $C \equiv (0, 0) \qquad r = 2 \qquad P \equiv (5, 6)$

$CP = \sqrt{(0-5)^2 + (0-6)^2} = \sqrt{61} > 2$

$\therefore$ P lies outside the Circle.

23. The given Circle is $(x^2 - 6x) + (y^2 + 2y) + 1 = 0$

$$\Rightarrow (x-3)^2 - 9 + (y+1)^2 - 1 + 1 = 0$$

$$\Rightarrow (x-3)^2 + (y+1)^2 = 9$$

$\therefore$ $C \equiv (3, 1)$ $r = 3$ and $P \equiv (1, 3)$

$\therefore$ $CP \equiv \sqrt{(3-1)^2+(-1-3)^2} = \sqrt{4+16} = \sqrt{20} > r$

$\therefore$ P lies outside the Circle.

24. The given Circle is $(x^2 + 6x) + (y^2 + 2y) - 1 = 0$

$$\Rightarrow (x+3)^2 - 9 + (y+1)^2 - 1 - 1 = 0$$

$$\Rightarrow (x+3)^2 + (y+1)^2 = 9$$

$\therefore$ $C \equiv (-3, 1)$ $r = 3$ $P \equiv (-2, 0)$

$\therefore$ $CP = \sqrt{(-3+2)^2+(1-0)^2} = \sqrt{(1+1)} = \sqrt{2} < r$

$\therefore$ P lies inside the Circle.

Four Marks Questions

5. The Circle is $(x-h)^2 + (y-k)^2 = r^2$

The given points lie on the Circle

$\therefore$ $(1-h)^2 + (2-k)^2 = r^2$ --------------(1)

$(2-h)^2 + (2-k)^2 = r^2$ --------------(2)

$(4-h)^2 + (-1-k)^2 = r^2$ --------------(3)

$h^2 + k^2 - 2h - 4k + 5 = r^2$ -----------(4)

$h^2 + k^2 - 4h - 4k + 8 = r^2$ -----------(5)

$h^2 + k^2 - 8h + 2k + 17 = r^2$ -----------(6)

(4) - (5) gives $h = \frac{3}{2}$

(5) - (6) gives $4h - 6k - 9 = 0$

$$\Rightarrow 6 - 6k - 9 = 0$$

$$\Rightarrow K = -\frac{1}{3}$$

from (1) wet get $r^2 = \frac{50}{4}$

$\therefore$ The equation of the Circle is $\left(x - \frac{3}{2}\right)^2 + \left(y + \frac{1}{3}\right)^2 = \frac{50}{4}$

6. $(x - h)^2 + (y - k)^2 = r^2$

$\therefore$ $(1 - h)^2 + (1 - k)^2 = r^2$ ----------------(1)

$(1 - h)^2 + (3 - k)^2 = r^2$ ---------------(2)

$(2 - h)^2 + (-2 - k)^2 = r^2$ -------------(3)

Solving we get $h = 7$ $k = 2$ $r^2 = 37$

$\therefore$ The equation is $(x - 7)^2 + (y - 2)^2 = 37$

7. The Circle is $(x - h)^2 + (y - k)^2 = r^2$

Passes through (-1, 1) and (3, 2)

$\therefore$ $(-1 - h)^2 + (1 - k)^2 = r^2$ ----------------(1)

$(3 - h)^2 + (2 - k)^2 = r^2$ ----------------(2)

The centre (h, k) lies on the line $x - 2y + 2 = 0$

$\therefore$ $h - 2k + 2 = 0$ ----------------(3)

(1) - (2) gives $8h + 2k - 11 = 0$ ----- (4)

Solving (3) and (4) we get $h = 1$ $k = \frac{3}{2}$

From (1) $r^2 = \frac{17}{4}$

$\therefore$ The Circle is $(x - 1)^2 + \left(y - \frac{3}{2}\right)^2 = \frac{17}{4}$

8. The Circle is $(x - h)^2 + (y - k)^2 = r^2$

Passes through the points (1, 3) and (2, 3)

$\therefore$ $(1 - h)^2 + (3 - k)^2 = r^2$ ----- (1) and $(2 - h)^2 + (3 - k)^2 = r^2$ -----(2)

The Centre (h, k) lies on $2x + 3y - 1 = 0$ $\therefore$ $2h + 3k - 1 = 0$ ---(3)

Solving we get $h = \frac{3}{2}$, $k = -\frac{2}{3}$, $r^2 = = \frac{109}{36}$

The Circle is $\left(x - \frac{3}{2}\right)^2 + \left(y + \frac{2}{3}\right)^2 = \frac{109}{36}$

Parabola

One Mark Questions:

7. Parabola is defined as the locus of a point which moves that it's distance from a fixed point is equal to it's distance from a Fixed line.

8. Latus rectum of a Parabola is a line segment drawn through the focus, perpendicular to the axis of the parabola having it's extremeties on the parabola.

9. (i) $a = 3$ $\therefore S \equiv (a, 0) \equiv (3, 0)$

(ii) $a = 4$ $\therefore S \equiv (-a, 0) \equiv (-4, 0)$

(iii) $a = 5$ $\therefore S \equiv (0, a) \equiv (0, 5)$

(iv) $a = \frac{5}{2}$ $\therefore S \equiv (0, -a) \equiv \left(0, \frac{5}{2}\right)$

(v) $y^2 = \frac{3}{2}$ => $a = \frac{3}{2(4)} = \frac{3}{8}$

$\therefore S \equiv (a, 0) \equiv \left(\frac{3}{8}, 0\right)$

(vi) $y^2 = -5x$ => $a = \frac{5}{4}$ $\therefore S \equiv (-a, 0) \equiv \left(-\frac{5}{4}, 0\right)$

(vii) $x^2 = \frac{5}{2}y$ => $a = \frac{5}{8}$ $\therefore S \equiv (0, a) \equiv \left(0, \frac{5}{8}\right)$

(viii) $x^2 = -\frac{2}{3}y$ => $a = \frac{1}{6}$ $\therefore S \equiv (0, -a) \equiv \left(0, -\frac{1}{6}\right)$

10. (i) x = axis (ii) x = axis (iii) y = axis (iv) y = axis

11. (i) $y^2 = 40x$ => a = 10 $\therefore$ directrix is x = -a

=> x = - 10 or x + 10 = 0

(ii) $y^2 = \frac{10x}{3}$ => $a = \frac{5}{6}$

$\therefore$ The directrix is $x = - a = -\frac{5}{6}$ or $6x + 5 = 0$

(iii) $y^2 = -36x$ => a = 9

$\therefore$ The directrix is x = a = 9 or x – 9 = 0

(iv) $y^2 = -\frac{5}{2}x$ => $a = \frac{5}{4}$

$\therefore$ The directrix is $x = a = \frac{5}{4}$ or $4x - 5 = 0$

(v) $x^2 = 100y$ => a = 25

$\therefore$ The directrix is y = -a = -25 or y + 25 = 0

(vi) $x^2 = \frac{1}{4}y$ => $a = \frac{1}{16}$

$\therefore$ The directrix is $y = -a = -\frac{1}{16}$ or $16y + 1 = 0$

(vii) $x^2 = -20y$ => a = 5

$\therefore$ The directrix is y = +a = 5 or y - 5 = 0

(viii) $x^2 = -\frac{2}{5}y$ => $a = \frac{1}{10}$

$\therefore$ The directrix is $x = a = \frac{1}{10}$ or $10x - 1 = 0$

12. (i) $y^2 = 8x$ => a = 2 $\therefore$ LR = 4a = 8

(ii) $y^2 = \frac{3}{2}x$ => $L\text{-}R = 4a = \frac{3}{2}$

(iii) $y^2 = -16x$ => LR = 4a = 16

(iv) $y^2 = \frac{3}{10}x$ => $LR = 4a = \frac{3}{10}$

(v) $x^2 = 10y$ => $LR = 4a = 10$

(vi) $x^2 = \frac{3}{2}y$ => $LR = 4a = \frac{3}{2}$

(vii) $x^2 = -10y$ => $LR = 4a = 10$

(viii) $x^2 = \frac{2}{5}y$ => $LR = 4a = \frac{2}{5}$

Two Marks Questions.

5. $y^2 = 20x$ => $a = 5$ $\therefore S \equiv (a, 0) \equiv (5, 0)$

Consider the line $5x + 2y - 25 = 0$. Put $x = 5$ $y = 0$

$\therefore 25 + 0 - 25 = 0$

=> $0 = 0$

$\therefore$ The Line passes through the focus.

6. $x^2 = -12y$. => $a = 3$ $\therefore S \equiv (0, -a) \equiv (0, -3)$

The line passes through (5, -6) and (0, 3). Hence it's equation is

$$\frac{y - y_1}{x - x_1} = \frac{y_1 - y_2}{x_1 - x_2} \Rightarrow \frac{y + 6}{x - 5} = \frac{-6 + 3}{5 - 0} = -\frac{3}{5}$$

=> $5y + 30 = -3x + 15$ or $3x + 5y + 15 = 0$

7. (i) The equation is $y^2 = 4ax$ where $a = 8$ ie $y^2 = 32x$

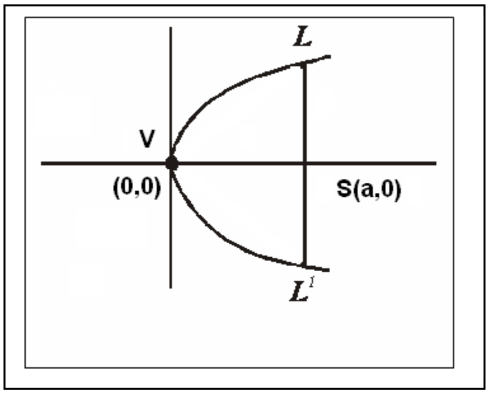
(ii) The equation is $y^2 = -4ax$ where $a = 12$ ie. $y^2 = 48x$

(iii) The equation is $x^2 = 4ay$ where $a = 10$ ie $x^2 = 40y$

(iv) The equation is $x^2 = -4ay$ where $a = 6$ ie $x^2 = 24y$

(v) The equation is $y^2 = 4ax$ where $a= 5$ ie $y^2 = 20x$

(vi) The equation is $x^2 = -4ay$ where $a = 11$ ie. $x^2 = -44y$

8. 

$y^2 = 40ax$

LL^1 is the latus rectum

let $SL = SL^1 = 2t$

$\therefore L \equiv (a, t)$

L lies on $y^2 = 4ax$

$\Rightarrow t^2 = 4a \Rightarrow t = 2a$

$\Rightarrow LL^1 = 2t = 4a$

Three marks Questions :

7. This is a book-work done in the text book.

8. Since the ends of LR are in the 1^{st} & 4^{th} quadrants the LR is Vertical.

Hence the equation of the Parabola is $y^2 = 4ax$.

Also the mid point of LR is the focus

$$\therefore S \equiv (a, 0) \equiv \left(\frac{3+3}{2}, \frac{10-10}{2}\right) \equiv (3,0)$$

$$\Rightarrow a = 3.$$

$\therefore$ The equation is $y^2 = 12x$.

9. Since the ends of LR are in the 1^{st} and 2^{nd} quadrants

The LR is horizontal.

$\therefore$ It's is equation is $x^2 = 4ay$.

The mid point of LR is the focus.

$$\therefore S \equiv (0, a) \equiv \left(\frac{-16+16}{2}, \frac{6+8}{2}\right) \equiv (0,8)$$

$$\Rightarrow a = 8.$$

$\therefore$ The equation of the Parabola is $x^2 = 32y$.

10. By data the equation of the Parabola is

$y^2 = 4ax$ This passes through (6, 4)

$\Rightarrow 16 = 4.\ a \text{-}6 \quad \Rightarrow \quad a = \frac{2}{3}$

$\therefore$ The equation is $y^2 = 4\text{-}\frac{2}{3}x \quad \Rightarrow 3y^2 = 8x$.

11. by data the equation is $x^2 = 4ay$. This passes through (-3, 5)

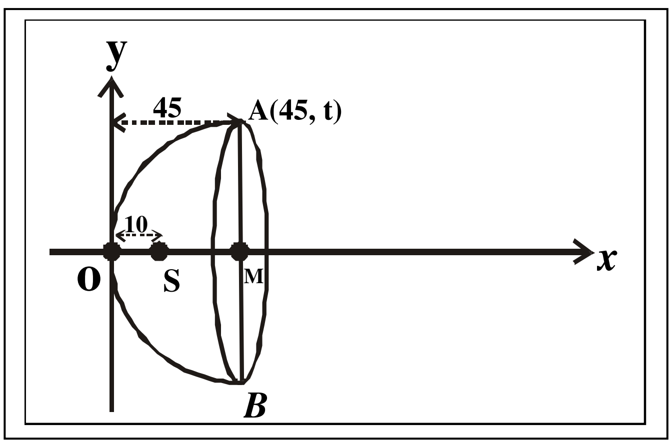
$\therefore 9 = 4.a.5 \Rightarrow a = \frac{9}{20}$

$x^2 = 4\text{-}\frac{9}{20}y \Rightarrow 5x^2 = 9y$.

12. By data the equation is $x^2 = 4ay$
This passes through (6, -8)

$\Rightarrow 36 = 4.a\,(-8) \Rightarrow a = -\frac{9}{8} \qquad \therefore\ x^2 = 4\left(-\frac{9}{8}\right)y \quad \Rightarrow 2x^2 = 9y$

Four Marks Questions :



a. From the Figure

$a = 10$

$\therefore$ equation of the Parabola is $y^2 = 40x$

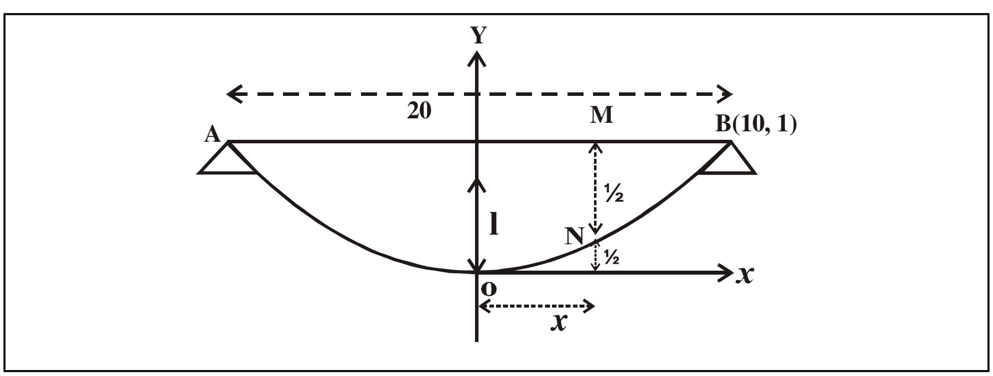
$AM = MB = t$

$\therefore t^2 = 40.45 = 1800$

$t = \sqrt{1800} = 30\sqrt{2}$

$\therefore$ diameter $= AB = 2t = 60\sqrt{2}$

b. The equation of the Parabola is $x^2 = 4ay$



This passes through (10, 1)

$\therefore$ 100 = 4.a.1 $\qquad\qquad$ $\therefore$ a = 25 mts.

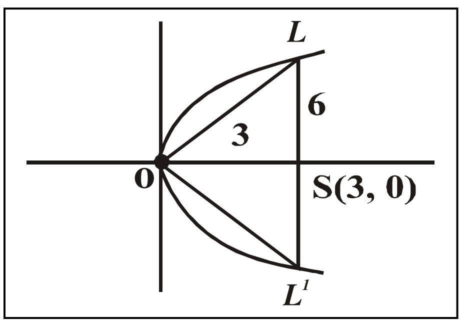
MN = $\frac{1}{2}$ mt is the deflection at M

$$\therefore N \equiv \left(x_1 \frac{1}{2} \right)$$

$x^2 = 4ay \qquad \Rightarrow x^2 = 4.25.\frac{1}{2} = 50$

$\Rightarrow x = \underline{+}\, 5\sqrt{2}$ Mts.

c. $\quad y^2 = 12x \qquad \Rightarrow a = 3$



$\therefore$ a = OS = 3

$LL^1 = 4a = 12$

$\therefore$ SL = 6

$\therefore$ area of the triangle OLS = $\frac{1}{2}$ x 3 x 6 = 9 sq.mt.

$\therefore$ area of the triangle L O L^1 = 2(9) = 18 sq.units.

ELLIPSE

One Mark Questions :

1. (i) a = 6 $\quad$ b = 2 $\quad$ => $\quad$ Major axis = 2a = 12

a > b $\qquad$ Minor axis = 2b = 4

(ii) a = 4 $\quad$ b = 1 $\quad$ => $\quad$ 2a = 8, $\quad$ 2b = 6 $\quad$ a > b

(iii) a = 5 $\quad$ b = 3 $\quad$ => 2a = 10, $\quad$ 2b = 6 $\quad$ a > b

(iv) $a = 4$ $b = 8$ => Major axis = $2b = 16$

$b > a$ Minor axis = $2a = 8$

(v) $a = 2$ $b = 3$ => $2b = 6$, $2a = 4$ $b > a$

(vi) $a = 2$ $b = 3$ => $2b = 6$, $2a = 2$ $b > a$

(vii) $\frac{x^2}{4} + \frac{y^2}{25} = 1$ => $a = 2$, $b = 5$ $b > a$

=> $2b = 10$ $2a = 4$

(viii) $\frac{x^2}{4} + \frac{y^2}{9} = 1$ => $a = 2$, $b = 3$ $b > a$

$\therefore$ $2b = 6$ $2a = 4$

(ix) $\frac{x^2}{400} + \frac{y^2}{100} = 1$ => $a = 20$, $b = 10$ $a > b$

=> $2a = 40$ $2b = 20$

(x) $x^2 + \frac{y^2}{16} = 1$ => $a = 1$, $b = 4$ $b > a$

$\therefore$ $2b = 8$ $2a = 2$

2. (i) $x^2 + \frac{y^2}{2} = 1$ => 1, $a = 1$ $b = \sqrt{2}$ $b > a$

$\therefore$ Vertices are $(0, \pm \sqrt{2})$

(ii) $\frac{x^2}{2} + \frac{y^2}{3} = 1$ => $a = \sqrt{2}$ $b = \sqrt{3}$ $b > a$

$\therefore$ Vertices are $(0, \pm \sqrt{3})$

(iii) $\frac{x^2}{9} + \frac{y^2}{4} = 1$ => $a = 3$ $b = 2$ $a > 2$

$\therefore$ Vertices are $(\pm 3, 0)$

(iv) $\frac{x^2}{25}+\frac{y^2}{64}=1$ $\Rightarrow a = 5$ $b = 8$ $b > a$

$\therefore$ Vertices are $(0, \pm 8)$

(v) $a = 1$ $b = 3$ $b > a$ $\Rightarrow 2b = 6$ $2a = 2$

(vi) $a = 4$ $b = 1$ $a > b$ $\Rightarrow 2a = 8$ $2b = 1$

(vii) $\frac{x^2}{4}+\frac{y^2}{36}=1$ $a = 2$ $b = 6$ $b > a$

$\Rightarrow 2b = 12$ $2a = 8$

3. (i) $\frac{x^2}{4}+\frac{y^2}{25}=1$ $a = 2$ $b = 5$ $b > a$

$$\therefore LR = \frac{2a^2}{b} = \frac{2.4}{5} = \frac{8}{5}$$

(ii) $\frac{x^2}{3}+\frac{y^2}{2}=1$ $\Rightarrow a = \sqrt{3}$ $b = \sqrt{2}$ $a > b$

$$\therefore LR = \frac{2b^2}{a} = \frac{2.3}{\sqrt{2}} = 3\sqrt{2}$$

(iii) $a = 7$ $b = 5$ $a > b$

$$\therefore LR = \frac{2b^2}{a} = \frac{2.25}{7} = \frac{50}{7}$$

(iv) $a = 8$ $b = 10$ $b > a$

$$\therefore LR = \frac{2a^2}{b} = \frac{2.64}{10} = \frac{64}{5}$$

Two Marks Questions :

1. (i) $\frac{x^2}{9}+\frac{y^2}{4}=1$ $\Rightarrow a = 3,$ $b = 2$ $a > b$

$$\therefore e = \frac{\sqrt{a^2 - b^2}}{a} = \frac{\sqrt{9-4}}{3} = \frac{\sqrt{5}}{3}$$

(ii) $\frac{x^2}{4} + \frac{y^2}{9} = 1$ => $a = 2$, $b = 3$ $b > a$

$$\therefore e = \frac{\sqrt{b^2 - a^2}}{b} = \frac{\sqrt{9 - 4}}{3} = \frac{\sqrt{5}}{3}$$

(iii) $a = 5$ $b = 10$ $b > a$

$$\therefore e = \frac{\sqrt{b^2 - a^2}}{b} = \frac{\sqrt{100 - 25}}{10} = \frac{5\sqrt{3}}{10} = \frac{\sqrt{3}}{2}$$

(iv) $a = 1$ $b = 2$ $b > a$

$$\therefore e = \frac{\sqrt{b^2 - a^2}}{b} = \frac{\sqrt{4 - 1}}{2} = \frac{\sqrt{3}}{2}$$

(v) $a = 7$ $b = 1$ $a > 1$

$$\therefore e = \frac{\sqrt{a^2 - b^2}}{a} = \frac{\sqrt{49 - 1}}{7} = \frac{4\sqrt{3}}{7}$$

(vi) $\frac{x^2}{2} + \frac{y^2}{9} = 1$ => $a = \sqrt{2}$, $b = 3$ $b > a$

$$\therefore e = \frac{\sqrt{b^2 - a^2}}{b} = \frac{\sqrt{9 - 2}}{3} = \frac{\sqrt{7}}{3}$$

2. By data $2a = 2(2b)$ => $a = 2b$

$$\therefore b^2 = a^2 (1 - e^2) \quad => b^2 = 4b^2 (1 - e^2)$$

$$=> 1 - e^2 = \frac{1}{4} \quad => e^2 = 1 - \frac{1}{4} = \frac{3}{4} \qquad \therefore e = \frac{\sqrt{3}}{2}$$

3. $2a = 3(2b)$ => $a = 3b$

$$\therefore b^2 = a^2 (1 - e^2) \quad => b^2 = 9b^2 (1 - e^2) \quad => 1 - e^2 = \frac{1}{9}$$

$$\therefore e^2 = 1 - \frac{1}{9} = \frac{8}{9} \qquad => e = \frac{2\sqrt{2}}{3}$$

4. $2a = 40 \quad \Rightarrow \quad a = 20 \qquad e = \dfrac{1}{3}$

$$b^2 = a^2(1 - e^2) = 400\left(1 - \frac{1}{9}\right) = \frac{3200}{9} \qquad \therefore\ 2b = \frac{2\sqrt{3200}}{3}$$

$$\therefore\ \frac{x^2}{400} + \frac{9y^2}{3200} = 1$$

5. $2b = 10 \quad \Rightarrow \quad b = 5 \qquad e = \dfrac{2}{5}$

$$b^2 = a^2(1 - e^2) \Rightarrow \qquad 25 = a^2\left(1 - \frac{4}{25}\right) = \frac{21a^2}{25}$$

$$\therefore\ a^2 = \frac{625}{21} \qquad \therefore\ 2a = \frac{2\sqrt{625}}{\sqrt{21}}$$

6. This is a book work done in the book.

7. (i) $\dfrac{x^2}{3} + \dfrac{y^2}{4} = 1 \quad \Rightarrow \quad a = \sqrt{3} \quad b = 2 \qquad b > a$

$\therefore$ Vertices are $(0, \pm b) \equiv (0, \pm 2)$

$$LR = \frac{2a^2}{b} = \frac{2.3}{2} = 3$$

(ii) $\dfrac{x^2}{25} + \dfrac{y^2}{4} = 1 \quad \Rightarrow \quad a = 5 \quad b = 2 \qquad a > b$

Vertices are $(\pm a, 0) \equiv (\pm 5, 0)$

$$LR = \frac{2b^2}{a} = \frac{2.4}{5} = \frac{8}{5}$$

(iii) $\dfrac{x^2}{4} + \dfrac{y^2}{9} = 1 \quad \Rightarrow \quad a = 2 \quad b = 3 \qquad b > a$

Vertices are $(0, \pm b) \equiv (0, \pm 3)$

$$LR = \frac{2a^2}{b} = \frac{2.4}{3} = \frac{8}{3}$$

8. (i) $\frac{x^2}{25} + \frac{y^2}{4} = 1 \quad \Rightarrow a = 5, \quad b = 2 \quad a > b$

$$e = \frac{\sqrt{a^2 - b^2}}{a} = \frac{\sqrt{25-4}}{5} = \frac{\sqrt{21}}{5}$$

$\therefore$ Focii $\equiv (\pm ae, 0) \equiv (\pm\sqrt{21}, 0)$

(ii) $\frac{x^2}{16} + \frac{y^2}{25} = 1 \quad \Rightarrow a = 4, \quad b = 5 \quad b > a$

$$\therefore e = \frac{\sqrt{b^2 - a^2}}{b} = \frac{\sqrt{25-16}}{5} = \frac{3}{5}$$

$\therefore$ Focii $\equiv (0, \pm be) \equiv (0, \pm 3)$

(iii) $\frac{x^2}{2} + \frac{y^2}{5} = 1 \quad \Rightarrow a = \sqrt{2}, \quad b = \sqrt{5} \quad b > a$

$$\therefore e = \frac{\sqrt{5-2}}{\sqrt{5}} = \sqrt{\frac{3}{5}}$$

$\therefore$ Focii $\equiv (0, \pm be) \equiv (0, \pm \sqrt{3})$

(iv) $\frac{x^2}{11} + \frac{y^2}{6} = 1 \quad \Rightarrow a = \sqrt{11}, \quad b = \sqrt{6} \quad a > b$

$$e = \frac{\sqrt{11-6}}{\sqrt{11}} = \frac{\sqrt{5}}{\sqrt{11}}$$

$\therefore$ Focii $\equiv (\pm\sqrt{5}, 0)$

(v) $\frac{x^2}{4} + y^2 = 1 \quad \Rightarrow a = 2, \quad b = 1 \quad a > b$

$$e = \frac{\sqrt{4-1}}{2} = \frac{\sqrt{3}}{2}$$

$\therefore$ Focii $\equiv (\pm\sqrt{3}, 0)$

(vi) $x^2 + \frac{y^2}{9} = 1$ $\Rightarrow a = 1,\quad b = 3 \qquad b > a$

$$e = \frac{\sqrt{9-1}}{3} = \frac{2\sqrt{2}}{3}$$

$\therefore$ Focii $\equiv (0, \pm 2\sqrt{2})$

9. Vertices are $(\pm a, 0) \equiv (\pm 5, 0) \quad \Rightarrow a = 5$

$\therefore$ Major axis $= 2a = 10$

10. L.R. $= \frac{2a^2}{b} = 50$

$$\Rightarrow a^2 = 50b \qquad \Rightarrow \qquad a = 5\sqrt{2b}$$

Three Marks Questions

1. (i) $2a = 20 \qquad b^2 = 12 \qquad \Rightarrow a = 10 \qquad b = 6$

$$\frac{x^2}{100} + \frac{y^2}{36} = 1$$

(ii) $2a = 10 \qquad \Rightarrow a = 5 \qquad e = \frac{1}{2}$

$$b^2 = a^2(1 - e^2) = 25\left(1 - \frac{1}{4}\right) = \frac{75}{4}$$

$$\therefore \frac{x^2}{25} + \frac{y^2}{\frac{75}{4}} = 1 \qquad \text{or} \qquad \frac{x^2}{25} + \frac{4y^2}{75} = 1$$

(iii) $2b = 8 \qquad \Rightarrow \quad b = 4, \qquad e = \frac{2}{3}$

$$b^2 = a^2(1 - e^2) \qquad \Rightarrow \quad 16a^2\left(1 - \frac{4}{9}\right) = \frac{5a^2}{9}$$

$$\Rightarrow \quad a^2 = \frac{144}{5} \qquad \therefore \frac{x^2}{\frac{144}{5}} + \frac{y^2}{16} = 1$$

(iv) Vertices are $(\pm a, 0) \equiv (\pm 8, 0)$

$(0, \pm b) \equiv (0, \pm 7)$

$\therefore a = 8 \qquad b = 7$

$$\therefore \frac{x^2}{64} + \frac{y^2}{49} = 1$$

(v) Vertices are $(\pm 12, 0) \equiv (\pm a, 0) \Rightarrow a = 12$

$$b^2 = a^2 (1 - e)^2 = 144\left(1 - \frac{1}{9}\right) = 128 \qquad \therefore \frac{x^2}{144} + \frac{y^2}{128} = 1$$

(vi) $2a = 16 \Rightarrow a = 8$ focii $\equiv (\pm ae, 0) \equiv (\pm 3, 0)$

$$= ae = 3 \Rightarrow 8e = 3 \quad \therefore e = \frac{3}{8}$$

$$\therefore b^2 = a^2 (1 - e^2) = 64\left(1 - \frac{9}{64}\right) = 55 \qquad \therefore \frac{x^2}{64} + \frac{y^2}{55} = 1$$

(vii) $2b = 10 \Rightarrow b = 5$

$\therefore$ focii are $(\pm ae, 0) \equiv (\pm 4, 0) \Rightarrow ae = 4$

$b^2 = a^2 (1 - e^2) \Rightarrow 25 = a^2 - a^2e^2 = a^2 - 16$

$$\Rightarrow a^2 = 41 \qquad \therefore \frac{x^2}{41} + \frac{y^2}{25} = 1$$

(viii) $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, passes through (4, 3) and (6, 2)

$$\therefore \frac{16}{a^2} + \frac{9}{b^2} = 1 \qquad \text{and } \frac{36}{a^2} + \frac{4}{b^2} = 1$$

take $\frac{1}{a^2} = p$ & $\frac{1}{b^2} = q$

$\therefore 16p + 9q = 1$ and $36p + 4q = 1$

Solving use get $\frac{1}{a^2} = p = \frac{1}{52}$ and $\frac{1}{b^2} = q = \frac{1}{13}$

$$\therefore a^2 = 52 \text{ \& } b^2 = 13 \qquad \therefore \frac{x^2}{52} + \frac{y^2}{13} = 1$$

(ix) $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, passes through (1, 4) and (-6, 1)

$\therefore \frac{1}{a^2} + \frac{16}{b^2} = 1$ and $\frac{36}{a^2} + \frac{1}{b^2} = 1$

Solving we get $a^2 = \frac{578}{18}$ and $b^2 = \frac{578}{35}$

(x) $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, passes through (4, 3) and (-1, 4)

$\therefore \frac{16}{a^2} + \frac{9}{b^2} = 1$ and $\frac{1}{a^2} + \frac{16}{b^2} = 1$

Solving we get $a^2 = \frac{247}{7}$ and $b^2 = \frac{247}{15}$

2. This is a book work done in the book.

3. $\frac{x^2}{25} + \frac{y^2}{4} = 1$ => $a = 5$ $b = 2$ $a > b$

$\therefore e = \frac{\sqrt{a^2 - b^2}}{a} = \frac{\sqrt{25-4}}{5} = \frac{\sqrt{21}}{5}$

distance between focii = $2ac = 2.5\ \frac{\sqrt{21}}{5} = 2\sqrt{21}$

4. $2ac = 32$ and $e = \frac{1}{3}$

$\therefore 2a\left(\frac{1}{3}\right) = 32$ $\therefore a = 48$

$b^2 = a^2 (1 - e^2) = 2274\left(1 - \frac{1}{9}\right) = 2274 \text{ x } \frac{8}{9} = \frac{6064}{3}$ $\therefore \frac{x^2}{2274} + \frac{3y^2}{6064} = 1$

5. $2be = 20$ and $e = \frac{1}{\sqrt{2}}$

$\therefore 2b\left(\frac{1}{\sqrt{2}}\right) = 20 \quad => \quad b = 10\sqrt{2}$

$a^2 = b^2(1 - e^2) = 200\left(1 - \frac{1}{2}\right) = 100 \qquad \therefore \frac{x^2}{100} + \frac{y^2}{200} = 1$

Five Marks Questions :

6. This is a book work done in the text book.

7. $2ac = 16$ ------(1) and $\frac{2a}{e} = 30$ -------(2)

(1) x (2) gives $2ae \times \frac{2a}{e} = 16 \times 30$

$=> a^2 = 120 \quad => a = 2\sqrt{30}$

From (1) $2.2\sqrt{30}$ e= 16 $=> e = \frac{4}{\sqrt{30}} = \frac{4}{\sqrt{30}}$

$b^2 = a^2(l\text{-}e^2) = 120\left(1 - \frac{16}{30}\right) = 56 \qquad \therefore \frac{x^2}{120} + \frac{y^2}{56} = 1$

8. focus $\equiv (ae, 0) \equiv (10, 0)$ $=>$ ae = 10 -----(1)

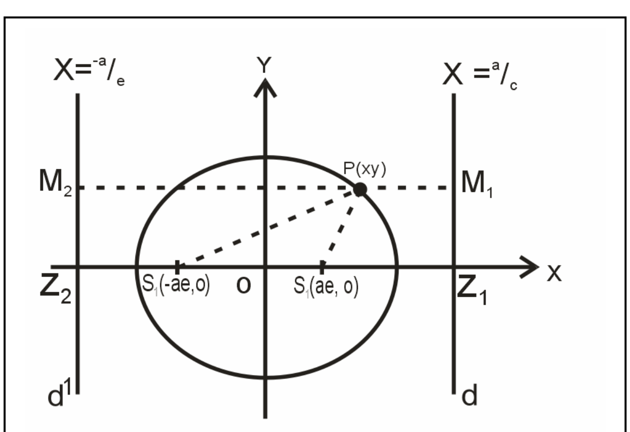
directrix is $x = \frac{a}{e} = 20 \quad => \quad \frac{a}{e} = 20$ -----(2)

(1) x (2) gives $ae\,\frac{a}{e} = 200 \quad => a^2 = 200 => a = 10\sqrt{2}$

from (1) $10\sqrt{2}\,.\,e = 10 \quad => e = \frac{1}{\sqrt{2}}$

$b^2 = a^2(l\text{-}e^2) = 200\left(1 - \frac{1}{2}\right) = 100 \qquad \therefore \frac{x^2}{200} + \frac{y^2}{100} = 1$

9. p(xy) is any point on the ellipse



focii are $S_1 \equiv (ae, 0)$ $s_2 = (-ae, 0)$

directrix are $d_1 : x = \frac{a}{e}$ $d_2 : x = -\frac{a}{e}$

we know that

$PS_1 = e.\ PM_1$ and $PS_2 = e.PM_2$

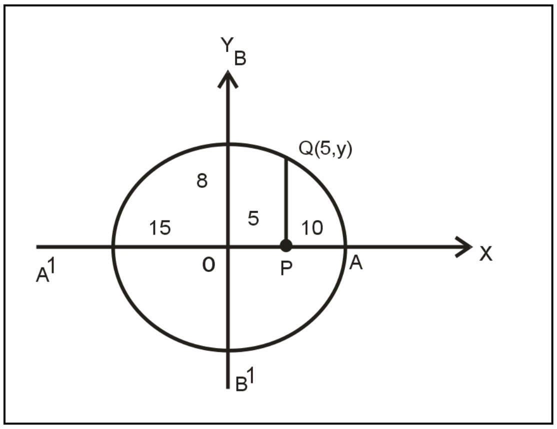
adding we get $PS_1 + PS_2 =$

$e\ (PM_1 + PM_2) = e\ (M_1\ M_2)$

$= e$ (distance between directricies)

$= e \left(\frac{2a}{e}\right) = 2a$

10. The equation of the ellipse is



$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$OA = OA = 15 = a$

$OB = OB^1 = 8 = b$

$$\therefore \frac{x^2}{225} + \frac{y^2}{64} = 1$$

Q(5, y) lies on the ellipse

$$\therefore \frac{25}{225} + \frac{y^2}{64} = 1 \qquad \Rightarrow \therefore \frac{y^2}{64} = 1 - \frac{1}{9} = \frac{8}{9} \Rightarrow y^2 = \frac{512}{9}$$

$$\therefore\ y = \frac{\sqrt{512}}{3} \text{ cms}$$

Hyperbola :

One Mark Questions :

6. Hyperbola is the locus of a point which moves such that the difference between it's distances from two fixed points is a constant.

7. (i) $a = 3,\quad b = 2$

   $TA = 2a = 6,\qquad CA = 2b = 4$

   (ii) $a = 2\qquad b = 3$

   $TA = 2a, = 4,\qquad CA = 2b = 6$

(iii) $a = 5, \quad b = 4$

$TA = 2a = 10, \quad CA = 2b = 8$

(iv) $a = 2, \quad b = 5$

$TA = 2b = 10 \quad CA = 2a = 4$

(v) $a = 2, \quad b = 7$

$TA = 2b = 14 \quad CA = 2a = 4$

(vi) $\frac{x^2}{4} - y^2 = 1$

$\Rightarrow a = 2, \quad b = 1$

$TA = 2a = 4 \quad CA = 2b = 2$

(vii) $\frac{x^2}{1} - \frac{y^2}{25} = 1 \quad \Rightarrow a = 1, \quad b = 5$

$TA = 2a = 2 \quad CA = 2b = 10$

(viii) $\frac{x^2}{25} - \frac{y^2}{4} = -1 \quad \Rightarrow a = 5, \quad b = 2$

$TA = 2b = 4, \quad CA = 2a = 10$

(ix) $\frac{x^2}{4} - \frac{y^2}{16} = -1 \quad \Rightarrow a = 2, \quad b = 4$

$\therefore TA = 2b = 8 \quad CA = 2a = 4$

(x) $x^2 - \frac{y^2}{3} = 1 \quad \Rightarrow a = 1, \quad b = \sqrt{3}$

$\therefore TA = 2a = 2 \quad CA = 2b = 2\sqrt{3}$

8. (i) $\frac{x^2}{9} - y^2 = 1$ => $a = 3$, $b = 1$

Vertices are $V \equiv (\pm a, 0) \equiv (\pm 3, 0)$

(ii) $x^2 - \frac{y^2}{16} = -1$ => $a = 1$, $b = 4$

Vertices are $V \equiv (0, \pm b) \equiv (0, \pm 4)$

(iii) $a = 2$ $b = 6$

Vertices are $V \equiv (\pm a, 0) \equiv (\pm 2, 0)$

(iv) $\frac{x^2}{4} - \frac{y^2}{9} = -1$ => $a = 2$, $b = 3$

Vertices are $V \equiv (0, \pm b) \equiv (0, \pm 3)$

(v) $a = 10$, $b = 7$ Vertices are $V \equiv (\pm 10, 0)$

(vi) $\frac{x^2}{2} - y^2 = 1$ => $a = \sqrt{2}$ $b = 1$

$\therefore V \equiv (\pm \sqrt{2}, 0)$

9. Latum rectum of a hyperbola, is a line segment through a focus, perpendicular to the transverse axis, having the extremeties on the hyperbola.

10. (i) $a = 7$ $b = 8$

$$LR = \frac{2b^2}{a} - \frac{2.64}{7} = \frac{128}{7}$$

(ii) $a = 4$ $b = 3$ $LR = \frac{2.9}{4} = \frac{9}{2}$

(iii) $\frac{x^2}{16} - \frac{y^2}{4} = -1$ => a = 4, b = 2

$$LR = \frac{2a^2}{b} - \frac{2.16}{2} = 16$$

(iv) a = 1 b = 2 $LR = \frac{2b^2}{a} = \frac{2-4}{1} = 8$

(v) $x^2 - \frac{y^2}{9} = -1$ => a = 1, b = 3 $LR = \frac{2a^2}{b} = \frac{2.1}{3} = \frac{2}{3}$

vi) $\frac{x^2}{2} - \frac{y2}{3} = 1$ => $a = \sqrt{2}$, $b = \sqrt{3}$ $LR = \frac{2b^2}{a} = \frac{2.3}{\sqrt{2}} = 3\sqrt{2}$

Two Marks Questions :

11. (i) $\frac{x^2}{16} - \frac{y2}{4} = 1$ => a = 4, b = 2

$$e = \frac{\sqrt{a^2 + b^2}}{a} = \frac{\sqrt{16+4}}{4} = \frac{2\sqrt{5}}{4} = \frac{\sqrt{5}}{2}$$

(ii) $\frac{x^2}{3} - \frac{y2}{2} = -1$ => $a = \sqrt{3}$, $b = \sqrt{2}$

$$e = \frac{\sqrt{a^2 + b^2}}{b} = \frac{\sqrt{3+2}}{\sqrt{2}} = \sqrt{\frac{5}{2}}$$

(iii) a = 6, b = 3 $e = \frac{\sqrt{36+9}}{6} = \frac{3\sqrt{5}}{6} = \frac{\sqrt{5}}{2}$

(iv) a = 2 b = 3 $e = \frac{\sqrt{4+9}}{2} = \frac{\sqrt{13}}{2}$

(v) a = 3 b = 2 $e = \frac{\sqrt{9+4}}{2} = \frac{\sqrt{13}}{2}$

(vi) $\frac{x^2}{3} - \frac{y^2}{6} = 1$ => $a = \sqrt{3}$ $b = \sqrt{6}$ $\therefore e = \frac{\sqrt{3+6}}{6} = \frac{\sqrt{3}}{\sqrt{2}}$

(vii) $\frac{x^2}{1} - \frac{y^2}{4} = 1$ $\Rightarrow a = 1 \quad b = 2$ $e = \frac{\sqrt{1+4}}{1} = \sqrt{5}$

12. By data $2a = 2(2b) \Rightarrow a = 2b$

$b^2 = a^2 (e^2 - 1) \Rightarrow b^2 = 4b^2 (e^2 - 1)$

$\Rightarrow e^2 - 1 = \frac{1}{4}$ or $e = \sqrt{1+\frac{1}{4}} = \frac{\sqrt{5}}{2}$

13. $2a = 20 \Rightarrow a = 10, \quad e = 2$

$b^2 = a^2 (e^2 - 1) = 100 (4 - 1) = 300$

$b = 10\sqrt{3}$ $\therefore CA = 2b = 20\sqrt{3}$

14. $2b = 12 \Rightarrow b = 6, \quad e = \frac{3}{2}$

$b^2 = a^2 (e^2 - 1) \Rightarrow 36 = a^2 \left(\frac{9}{4} - 1\right) = a^2\left(\frac{5}{4}\right)$

$\therefore a^2 = \frac{4.36}{5} \Rightarrow a = \frac{12}{\sqrt{5}}$ $\therefore TA = 2a = \frac{24}{\sqrt{5}}$

15. This is a book – work done in the text book.

16. (i) $a = 5 \quad b = 2$

$V \equiv (\underline{+}a, 0) \equiv (\underline{+}5, 0)$ and

$LR = \frac{2b^2}{a} = \frac{2-4}{5} = \frac{8}{5}$

(ii) $a = 3, \quad b = 4$ $V \equiv (\underline{+}3, 0)$ $LR = \frac{2.16}{3} = \frac{32}{3}$

(iii) $a = 4 \quad b = 5$

$V \equiv (0, \underline{+}b) \equiv (0, \underline{+}5)$

$LR = \frac{2a^2}{b} = \frac{2.16}{5} = \frac{32}{5}$

(iv) $a = 8$ $\quad b = 7$

$$V \equiv (0, \pm 7) \qquad LR = \frac{2.64}{7} = \frac{128}{7}$$

(v) $\frac{x^2}{2} - \frac{y^2}{4} = -1 \quad \Rightarrow a = \sqrt{2} \quad b = 2$

$$V \equiv (0, \pm 2) \qquad LR = \frac{2.2}{2} = 2$$

(vi) $\frac{x^2}{3} - \frac{y^2}{6} = -1 \quad \Rightarrow a = \sqrt{3} \quad b = \sqrt{6}$

$$\therefore V \equiv (0, \pm\sqrt{6}) \qquad LR = \frac{2.3}{\sqrt{6}} = \sqrt{6}$$

7. (i) $\frac{x^2}{4} - \frac{y^2}{25} = -1 \quad \Rightarrow a = 2 \quad b = 5$

$$e = \frac{\sqrt{a^2 + b^2}}{a} = \frac{\sqrt{4+25}}{4} = \frac{\sqrt{29}}{4}$$

$\therefore$ focii are $S \equiv (\pm ae, 0) \quad \equiv (\pm\sqrt{29}, 0)$

(ii) $\frac{x^2}{5} - \frac{y^2}{2} = -1 \quad \Rightarrow a = \sqrt{5} \quad b = \sqrt{2}$

$$e = \frac{\sqrt{a^2 + b^2}}{b} = \frac{\sqrt{5+2}}{\sqrt{2}} = \frac{\sqrt{7}}{\sqrt{2}}$$

$S \equiv (0, \pm be) \quad \equiv \quad (0, \pm\sqrt{7})$

(iii) $\frac{x^2}{11} - \frac{y^2}{6} = 1 \quad \Rightarrow a = \sqrt{11} \quad b = \sqrt{6}$

$$e = \frac{\sqrt{11+6}}{\sqrt{11}} = \frac{\sqrt{17}}{\sqrt{11}} \qquad S \equiv (\pm\sqrt{17}, 0)$$

(iv) $\frac{x^2}{4} - y^2 = -1$ => $a = 2$ $b = 1$

$$e = \frac{\sqrt{4+1}}{1} = \sqrt{5} \qquad S \equiv (0, \pm\sqrt{5})$$

(v) $a = 10$ $b = 8$

$$e = \frac{\sqrt{100+64}}{10} = \frac{\sqrt{164}}{10} \qquad S \equiv (\pm\sqrt{164}, 0)$$

(vi) $a = 8$ $b = 7$

$$e = \frac{\sqrt{64+49}}{7} = \frac{\sqrt{113}}{7} \qquad \therefore \; S \equiv (0, \pm\sqrt{113})$$

8. Vertices $\equiv (\pm a, 0) \equiv (\pm 10, 0)$ and $e = 2$

$b^2 = a^2(e^2 - 1) = 100\,(4 - 1) = 300$

$b = 10\sqrt{3}$ $\therefore$ CA $= 20\sqrt{3}$

9. An equilateral hyperbola is defined as the hyperbola, in which transverse axis and conjugate axis are of equal length.

$\therefore 2a = 2b$ => $a = b$

$\therefore b^2 = a^2\,(e^2 - 1)$ => $e^2 - 1 = 1$ => $e = \sqrt{2}$

10. $\frac{x^2}{5} - \frac{y^2}{8} = 1$ => $a = \sqrt{5}$ $b = \sqrt{8}$

$\therefore PS_1 - PS_2 = 2a = 2\sqrt{5}$

Three marks Questions :

(i) $2a = 5$ and $2b = 3$

$$\Rightarrow a = \frac{5}{2}, \qquad b = \frac{3}{2}$$

$$\therefore \frac{x^2}{\frac{25}{4}} - \frac{y^2}{\frac{9}{4}} = 1 \qquad \Rightarrow \frac{4x^2}{25} - \frac{4y^2}{9} = 1$$

(ii) $2a = 12 \Rightarrow a = 6 \qquad e = 2$

$$b^2 = a^2 (e^2 - 1) = 36 (4 - 1) = 108$$

$$\therefore \frac{x^2}{36} - \frac{y^2}{108} = 1$$

(iii) $2b= 10 \Rightarrow b= 5 \qquad e = \frac{3}{2}$

$$b^2 = a^2 (e^2 - 1) = 25 = a^2 \left(\frac{9}{4} - 1\right) = \frac{5a^2}{4}$$

$$a^2 = \frac{100}{5} = 20 \qquad \therefore \frac{x^2}{20} - \frac{y^2}{25} = 1$$

(iv) Vertices $\equiv (\pm a, 0), \equiv (\pm 2, 0) \Rightarrow a = 2$

$S \equiv (\pm ae, 0) \equiv (\pm 6, 0), \Rightarrow ae = 6 \Rightarrow e = 3$

$b^2 = a^2(e^2 - 1) = 4 (9 -1) = 32.$

$$\therefore \frac{x^2}{4} - \frac{y^2}{32} = 1$$

(v) $TA = 2a = 8 \Rightarrow a = 4$

$S \equiv (\pm ae, 0) \Rightarrow ae = 12 \Rightarrow e = 3$

$\therefore$ $b^2 = a^2 (e^2 - 1) = 16 (9 - 1) = 128$

$$\therefore \frac{x^2}{16} - \frac{y^2}{128} = 1$$

(vi) $2a = 6$ $\Rightarrow$ $a = 3$ and

$LR = \frac{2b^2}{a} = 10$ $\Rightarrow$ $\frac{2b^2}{3} = 10$ $\Rightarrow$ $b^2 = 15$

$$\therefore \frac{x^2}{9} - \frac{y^2}{15} = 1$$

(vii) $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ Passes through (3, -2) and (2, -1)

$\therefore \frac{9}{a^2} - \frac{4}{b^2} = 1$ and $\frac{4}{a^2} - \frac{1}{b^2} = 1$

Solving we get $a^2 = \frac{7}{3}$, $b^2 = \frac{7}{5}$

$$\therefore \frac{3x^2}{7} - \frac{5y^2}{7} = 1$$

(viii) $(\pm ae, 0) \equiv (\pm 12, 0)$ $\Rightarrow$ $ae = 12$

$\frac{2b^2}{a} = 36$ $\Rightarrow$ $b^2 = 18a$

$b^2 = a^2 (e^2 - 1) = a^2e^2 - a^2$ $\Rightarrow$

$18a = 144 - a^2$ $\Rightarrow$ $a^2 + 18a - 144 = 0$

Solving we get $a = -9 + 10\sqrt{2}$

$\therefore$ $b^2 = 18 (-9 + 10\sqrt{2})$ $\therefore \frac{x^2}{(-9+10\sqrt{2})^2} - \frac{y^2}{18(-9+10\sqrt{2})} = 1$

(ix) $V \equiv (\pm a, 0) \equiv (\pm\sqrt{10}, 0)$ $\Rightarrow a = \sqrt{10}$ $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

passes through (4, 3) $\Rightarrow \frac{16}{10} - \frac{9}{b^2} = 1$

$\Rightarrow \frac{9}{b^2} = \frac{8}{5} - 1 = \frac{3}{5}$ $\Rightarrow b^2 = 15$ $\therefore \frac{x^2}{10} - \frac{y^2}{15} = 1$

Five Marks Questions :

2. This is a book work done in the text book.

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