Neural Networks

2. The Perceptron

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Types of Learning (recap)

- **supervised** - learning with teacher
- **unsupervised** - self organisation
- **reinforcement** - no immediate feedback

- + hybrid methods

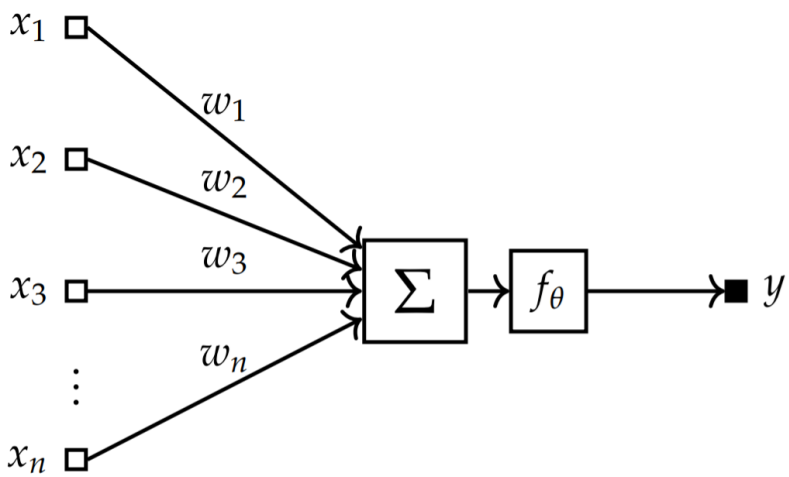
Error-Driven Supervised Learning

General scheme:

- repeat until some criterion (# of epochs, success, ...):
  - for each input $\boldsymbol{x}$ and desired output $\boldsymbol{d}$ (random order):
    - ▷ compute output: $\boldsymbol{y} = ?(\boldsymbol{W}\boldsymbol{x})$
    - ▷ compute error: $\boldsymbol{e} = \boldsymbol{d} - \boldsymbol{y}$
    - ▷ compute adjustment: $\Delta \boldsymbol{W} = ?(\boldsymbol{W}, \boldsymbol{x}, \boldsymbol{y}, \boldsymbol{e})$
    - ▷ adjust weights: $\boldsymbol{W} := \boldsymbol{W} + \alpha \Delta \boldsymbol{W}$

$$\boldsymbol{W}(t+1) = \boldsymbol{W}(t) + \alpha \Delta \boldsymbol{W}$$

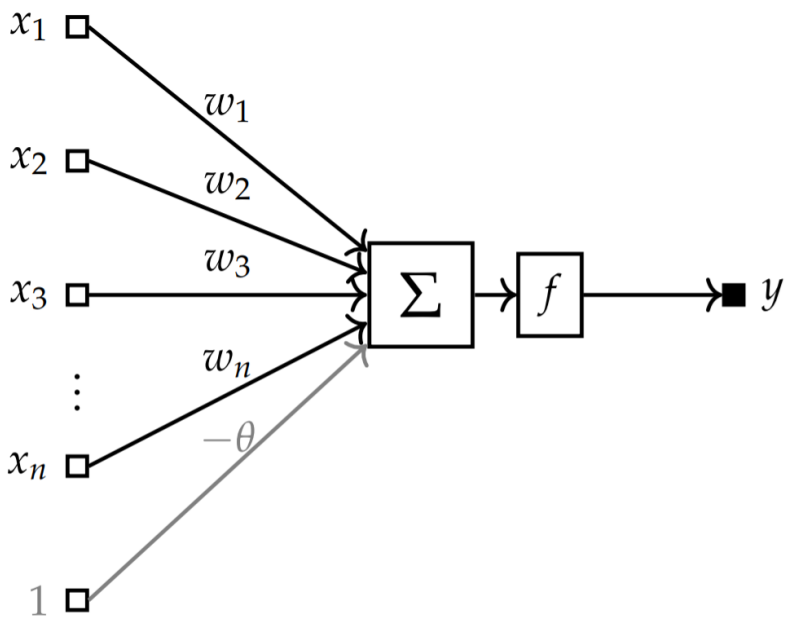
The Perceptron



- net input: $net = \boldsymbol{w} \cdot \boldsymbol{x} = \sum_{i=1}^{n} x_i w_i$
- output: $y = f_\theta(net)$
- activation function for discrete perceptron (θ = treshold)

$$f_\theta(net) = \begin{cases} 1, & net \geq \theta \\ 0, & net < \theta \end{cases}$$

The Perceptron: Threshold "Input" – Bias



- virtual input for the threshold term: $x'_{n+1} = 1$
- threshold as weight: $w'_{n+1} = -\theta$
- simplified activation - "step function":

$$f(net) = \begin{cases} 1, & net \geq 0 \\ 0, & net < 0 \end{cases}$$

The Perceptron: Error-Driven Learning

- input: $\boldsymbol{x} \in \mathbb{R}^n$
  - augmented input $\boldsymbol{x}' \in \mathbb{R}^{n+1}$
  - $\boldsymbol{x}' = \text{add\_bias}(\boldsymbol{x})$
- weights: $\boldsymbol{w} \in \mathbb{R}^n$
  - including threshold $\boldsymbol{w}' \in \mathbb{R}^{n+1}$
  - are initialized randomly
- output $y \in \{0, 1\}$

  $$y = f(\boldsymbol{w}' \cdot \boldsymbol{x}') = f(\sum_{i=1}^{n+1} x_i' w_i')$$
- target $d \in \{0, 1\}$ is given for all data points
- weight adjustment
  - $\boldsymbol{w}'(t+1) = \boldsymbol{w}'(t) + \alpha(d - y)\boldsymbol{x}'$

Task: Linear separation using perceptron

- complete the missing lines in perceptron.py according to theory in these slides.
- C02.py is the main file, algorithm starts after running this file.
- resulting perceptron should be trained to separate two classes of dots with a single line (because the classes are linearly separable).