2016.

Asanka

Q1.

(a). -

(b).

(i) for sample A;

stem	leaf
3	2 8
4	9 5
5	6 3 0 6 7
6	2 6 5 8
7	5 0

for sample B;

stem	leaf
3	9 4 5 6 7 2
4	2 7 0 2 7 9 1 8
5	0

(ii) for sample A,

$$\text{sample mean } (\bar{x}_A) = \frac{32+38+\dots+75+70}{15}$$

$$= \underline{\underline{56.13}}$$

minimum value = 32

maximum value = 75

sample range = 75 − 32 = $\underline{\underline{43}}$

for sample B,

$$\text{sample mean } (\bar{x}_B) = \frac{39+34+\dots+50}{15}$$

$$= \underline{\underline{41.2667}}$$

minimum value = 32

maximum value = 50

sample range = 50 − 32 = $\underline{\underline{18}}$

(iii). <u>similarities.</u>

* minimum values are equal.
* sample size are equal.

<u>differences.</u>

* sample means, $\bar{x}_A > \bar{x}_B$
* maximum values, $A_{max} > B_{max}$
* range of A larger than B.

(c). Let A_i is the event of system works.
$i = 1, 2, \dots, 6.$

$\therefore P(A_i) = 0.9 \quad ; \quad i = 1, 2, 3, \dots, 6,$

probability of upper subsystem works,

$$
\begin{aligned}
P_r(P_1) &= P(A_1 \cap A_2 \cap A_3) \\
&= P(A_1).P(A_2).P(A_3) \\
&= 0.9 \times 0.9 \times 0.9 \\
&= \underline{\underline{0.9^3}}
\end{aligned}
$$

$\therefore$ due to component works independently.

probability of lower subsystem works,

$$
\begin{aligned}
P_r(P_2) &= P(A_4 \cap A_5 \cap A_6) \\
&= P(A_4).P(A_5).P(A_6). \\
&= 0.9 \times 0.9 \times 0.9 \\
&= \underline{\underline{0.9^3}}
\end{aligned}
$$

$\therefore$ probability of the system works,

$$p = P[(A_1 \cap A_2 \cap A_3) \cup (A_4 \cap A_5 \cap A_6)]$$
$$= P(A_1 \cap A_2 \cap A_3) + P(A_4 \cap A_5 \cap A_6) - P[(A_1 \cap A_2 \cap A_3) \cap (A_4 \cap A_5 \cap A_6)]$$

$$= 0.9^3 + 0.9^3 - 0.9 \times 0.9 \times 0.9 \times 0.9 \times 0.9 \times 0.9$$
$$= 0.926559$$
$$= \underline{\underline{0.9266}}$$

(Q2).

(i). $S = \{RRR, RRL, RLR, RLL, LRR, LRL, LLR, LLL\}$

(ii). $A = \{RLL, LRL, LLR\}$

$B = \{RLL, LRL, LLR, LLL\}$

$C = \{RRR, LLL\}$

(iii). X = number of vehicles taking exit turns left.

$X = \{0, 1, 2, 3\}$.

$P(R) = 0.75$, $P(L) = 0.25$

$\therefore X \sim b(3; 0.25) = {}^3C_x (0.25)^x (0.75)^{3-x}$

$\therefore P(X = x) = {}^3C_x (0.25)^x (0.75)^{3-x}$

$P(X = 0) = {}^3C_0 (0.25)^0 (0.75)^3 = 0.4219$
$P(X = 1) = {}^3C_1 (0.25)^1 (0.75)^2 = 0.4219$
$P(X = 2) = {}^3C_2 (0.25)^2 (0.75)^1 = 0.1406$
$P(X = 3) = {}^3C_3 (0.25)^3 (0.75)^0 = 0.0156$

$\therefore$ probability distribution.

x	0	1	2	3
$P(X=x)$	0.4219	0.4219	0.1406	0.0156

** At most in the probability → means that all the probabilities that are smaller than the given probability.

(b).

(i) $E(X) = \sum_{\forall x} x.p(x)$

$= 13.5 \times 0.2 + 15.9 \times 0.5 + 19.1 \times 0.3$

$= \underline{16.38}$

$E(X^2) = \sum_{\forall x} x^2.p(x)$

$= 13.5^2 \times 0.2 + 15.9^2 \times 0.5 + 19.1^2 \times 0.3$

$= \underline{272.298}$

$Var(X) = E(X^2) - [E(X)]^2$

$= 272.298 - 16.38^2$

$= \underline{3.9936}$

(ii). $E(25X - 8.5) = \sum_{\forall x} (25X - 8.5).p(x)$

$= 25 \sum x.p(x) - 8.5 \sum p(x)$

$= 25E(X) - 8.5(1)$

$$= 25(16.38) - 8.5 \times 1$$
$$= \underline{\underline{401}}$$

(iii) $\text{var}(25X - 8.5) = 25^2 . \text{var}(X) - 0$
$$= 25^2 (3.9936)$$
$$= \underline{\underline{2496}}$$

(iv). $E[h(x)] = E[X - 0.01X^2]$
$$= E(X) - 0.01\, E(X^2)$$
$$= 272.298 - 0.01 (272.298)$$
$$= \underline{\underline{269.575}}$$

(c).

(i). $X \sim N(\mu, \sigma^2)$.

$\Rightarrow \quad E(X) = \mu \quad , \quad \text{var}(X) = \sigma^2$

$\therefore \quad E(Y) = E(ax + b)$
$$= a\,E(x) + b.$$
$$= \underline{\underline{a\mu + b}}$$

$\text{var}(Y) = \text{var}(ax + b)$
$$= a^2 \text{var}(x) + 0.$$
$$= \underline{\underline{a^2 . \sigma^2}}$$

$\therefore \quad Y \sim N(a\mu + b, a^2\sigma^2)$

(ii) X = diameter of the ball bearing.

$X \sim N(3, 0.005^2)$

to accept the diameter should be between $3+0.01$ and $3-0.01$

3.01 and 2.99

$\therefore$ accepted probability of diameter $= P(2.99 < X < 3.01)$

$$= P\left(\frac{2.99-3}{0.005} < \frac{X-\mu}{\sigma} < \frac{3.01-3}{0.005}\right)$$

$$= P(-2 < Z < 2)$$

$$= P(Z < 2) - P(Z < -2)$$

$$= 0.97725 - 0.02275$$

$$= \underline{\underline{0.9545}}$$

$\therefore$ the probability of bearing will be scrapped $= 1 - 0.9545$

$$= \underline{\underline{0.0455}}$$

$\therefore$ 4.55% manufactured ball-bearing will be scrapped.

(Q3)
(a).
(i). $X_i \sim N(\mu, \sigma^2)$: n - sample size.

$\therefore E(X_i) = \mu$ and $var(X_i) = \sigma^2$

$$E(\bar{X}) = E\left[\frac{\Sigma X_i}{n}\right] = \frac{1}{n} \Sigma E(X_i)$$

$$= \frac{1}{n} . \Sigma . \mu$$

$$= \frac{1}{n} . n\mu$$

$$= \mu$$

$$var(\bar{X}) = var\left[\frac{\Sigma X_i}{n}\right]$$

$$= \frac{1}{n^2} var(\Sigma X_i)$$

$$= \frac{1}{n^2} \Sigma var(X_i)$$

$$= \frac{1}{n^2} . \Sigma \sigma^2$$

$$= \frac{1}{n^2} . n\sigma^2$$

$$= \frac{\sigma^2}{n}$$

$\therefore$ distribution of sample mean.

$$\bar{X} \sim N(\mu, \frac{\sigma^2}{n})$$

(ii) X = length of life.

$X \sim N(800, 40^2)$

$n = 16$, $\bar{X} \sim N(800, \frac{40^2}{16})$

$$\therefore Pr(\bar{X} < 775) = Pr\left(\frac{\bar{X}-\mu}{\sigma/\sqrt{n}} < \frac{775-800}{40/4}\right)$$

$$= Pr(Z < -2.5)$$

$$= \underline{\underline{0.00621}}$$

(b).

(i) probability mass function for poisson distribution,

$$f(x;\lambda) = \frac{e^{-\lambda} \cdot \lambda^{x}}{x!} \quad ; \quad x = 0, 1, \ldots$$

$$L(x_1, x_2, \ldots, x_n; \lambda) = \prod_{i=1}^{n} \frac{e^{-\lambda} \cdot \lambda^{x}}{x!}$$

$$= \frac{e^{-\lambda} \cdot \lambda^{x_1}}{x_1!} \cdot \frac{e^{-\lambda} \lambda^{x_2}}{x_2!} \cdot \frac{e^{-\lambda} \lambda^{x_3}}{x_3!} \cdots\cdots \frac{e^{-\lambda} \cdot \lambda^{x_n}}{x_n!}$$

$$= \frac{e^{-\lambda n} \cdot \lambda^{(x_1 + x_2 + \cdots + x_n)}}{x_1!\, x_2!\, x_3! \ldots x_n!}$$

$$= \frac{e^{-\lambda n} \cdot \lambda^{\Sigma x_i}}{\prod_{i=1}^{n} x_i!}$$

$$\ln L(x_i;\lambda) = \ln\left(\frac{e^{-\lambda n}\cdot \lambda^{\sum x_i}}{\prod_{i=1}^{n} x_i!}\right)$$

$$= \ln e^{-\lambda n} + \ln \lambda^{\sum x_i} - \ln \prod_{i=1}^{n} x_i!$$

$$= -\lambda n . \ln e + (\sum x_i).\ln \lambda - \ln \prod_{i=1}^{n} x_i$$

$$= -\lambda n + (\sum x_i).\ln \lambda - \ln \prod_{i=1}^{n} x_i$$

$$\frac{\partial \ln L(x_i;\lambda)}{\partial \lambda} = -n + \frac{1}{\lambda}.\sum x_i - 0$$

when $\frac{\partial \ln L(x_i;\lambda)}{\partial \lambda} = 0$, $\lambda = \hat{\lambda}$

$$\therefore \Rightarrow -n + \frac{1}{\hat{\lambda}} \sum x_i = 0$$

$$\underline{\underline{\hat{\lambda} = \frac{\sum x_i}{n}}}$$

(ii) $X \sim$ poisson $(x;\lambda)$

$E(x) = \lambda.$

$$E(\hat{\lambda}) = E\left(\frac{\sum x_i}{n}\right) = \frac{1}{n}\sum E(x_i)$$

$$= \frac{1}{n}.\sum \lambda$$

$$= \frac{1}{n}\lambda n$$

$$= \underline{\underline{\lambda}}$$

$\therefore$ estimator of λ is unbiased estimator.

(iii). $\hat{\lambda} = \bar{x}$

$$= \frac{0 \times 18 + 1 \times 37 + 2 \times 42 + \cdots + 7 \times 1}{150}$$

$$= \underline{\underline{2.113}}$$

(iv) for a poisson distribution,

$E(X) = var(X) = \lambda.$

$var(\bar{x}) = \frac{\hat{\lambda}}{n}$

$\therefore$ standard error, $\sigma = \sqrt{\frac{\hat{\lambda}}{n}}$

$$= \sqrt{\frac{2.113}{150}} = \underline{\underline{0.1187}}$$

(Q4).

(a). pre-compaction.

$n_1 = 10$

$\bar{x}_1 = 0.413$

$s_1 = 0.0324$

post-compaction.

$n_2 = 20$

$\bar{x}_2 = 0.340$

$s_2 = 0.0469$

(i) Let, μ_1 = population mean for porosity before compaction.

μ_2 = population mean of porosity after compaction.

assuming population variances are equal.

$$s_p^2 = \frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1+n_2-2}$$

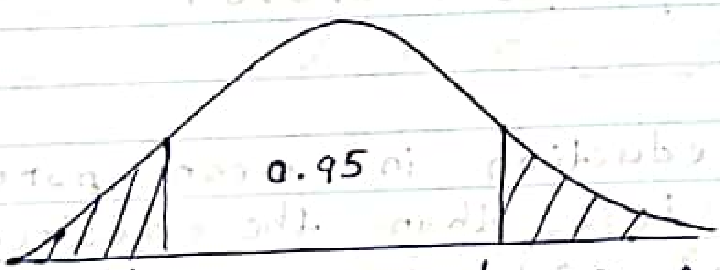
$$= \frac{9 \times 0.0324^2 + 19 \times 0.0469^2}{28}$$

$$= 1.83 \times 10^{-3}$$

$$\therefore s_p = \underline{\underline{0.0428}}$$

degrees of freedom $= 10 + 20 - 2$
$= \underline{\underline{28}}$

significance level, $\alpha = 5\%$



$$t_{0.025,28} = \underline{\underline{2.048}}$$

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{s_p \cdot \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

$$P_r(-t_{\alpha/2} < t < t_{\alpha/2}) = 1 - \alpha$$

$$\therefore -t_{\alpha/2} < \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{s_p \cdot \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} < t_{\alpha/2}$$

$$-t_{\alpha/2} \cdot s_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}} < (\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2) < t_{\alpha/2} \cdot \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$$

$$(\bar{x}_1-\bar{x}_2)-t_{\alpha/2}.s_p\sqrt{\frac{1}{n_1}+\frac{1}{n_2}} < \mu_1-\mu_2 < (\bar{x}_1-\bar{x}_2) + t_{\alpha/2}.\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}$$

substiduting corresponding value,

$$0.073-(0.2048)(0.0428)\sqrt{\frac{1}{10}+\frac{1}{20}} < \mu_1-\mu_2 < 0.073+(0.2048)(0.0428)\sqrt{\frac{1}{10}+\frac{1}{20}}$$

$$0.0696 < \mu_1-\mu_2 < 0.0764$$

(ii). yes, the reduction in mean porosity significantly less than the desired reduction of 0.1, because the value of 0.1 does not lies between the 0.0696 and 0.0764.

(b).

	Accident	No accident	Total
cellular phone	22	278	300
No phone.	26	374	400
Total	48	652	700.

Let,
H_0; having cellular phone in a car and accident are independent
H_1; having cellular phone in a car and accident are not independent.

$$e_{ij} = \frac{(i^{th} \text{ row total}) \times (j^{th} \text{ column total})}{\text{grand total.}}$$

$$e_{11} = \frac{300 \times 48}{700} = 20.5714$$

$$e_{12} = \frac{300 \times 652}{700} = 279.4285$$

$$e_{21} = \frac{400 \times 48}{700} = 27.4286$$

$$e_{22} = \frac{400 \times 652}{700} = 372.5714$$

$$\therefore \chi^2 = \sum \frac{(o_i - e_i)^2}{e_i}$$

$$= \frac{(22 - 20.5714)^2}{20.5714} + \frac{(278 - 279.4285)^2}{279.4285} + \frac{(26 - 27.4286)^2}{27.4286} + \frac{(374 - 372.5714)^2}{372.5714}$$

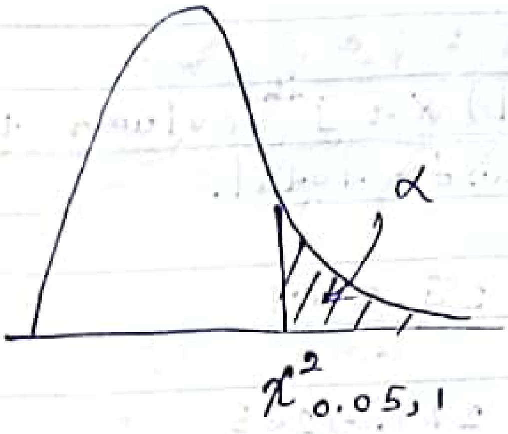
$$= 0.0992 + 0.0073 + 0.0744 + 0.0055$$

$$= \underline{\underline{0.1864}}$$

degrees of freedom $= (c-1)(r-1)$

$= (2-1)(2-1)$

$= \underline{\underline{1}}$



from chi-squared table.

$$\chi^2_{0.05,1} = 3.841,$$

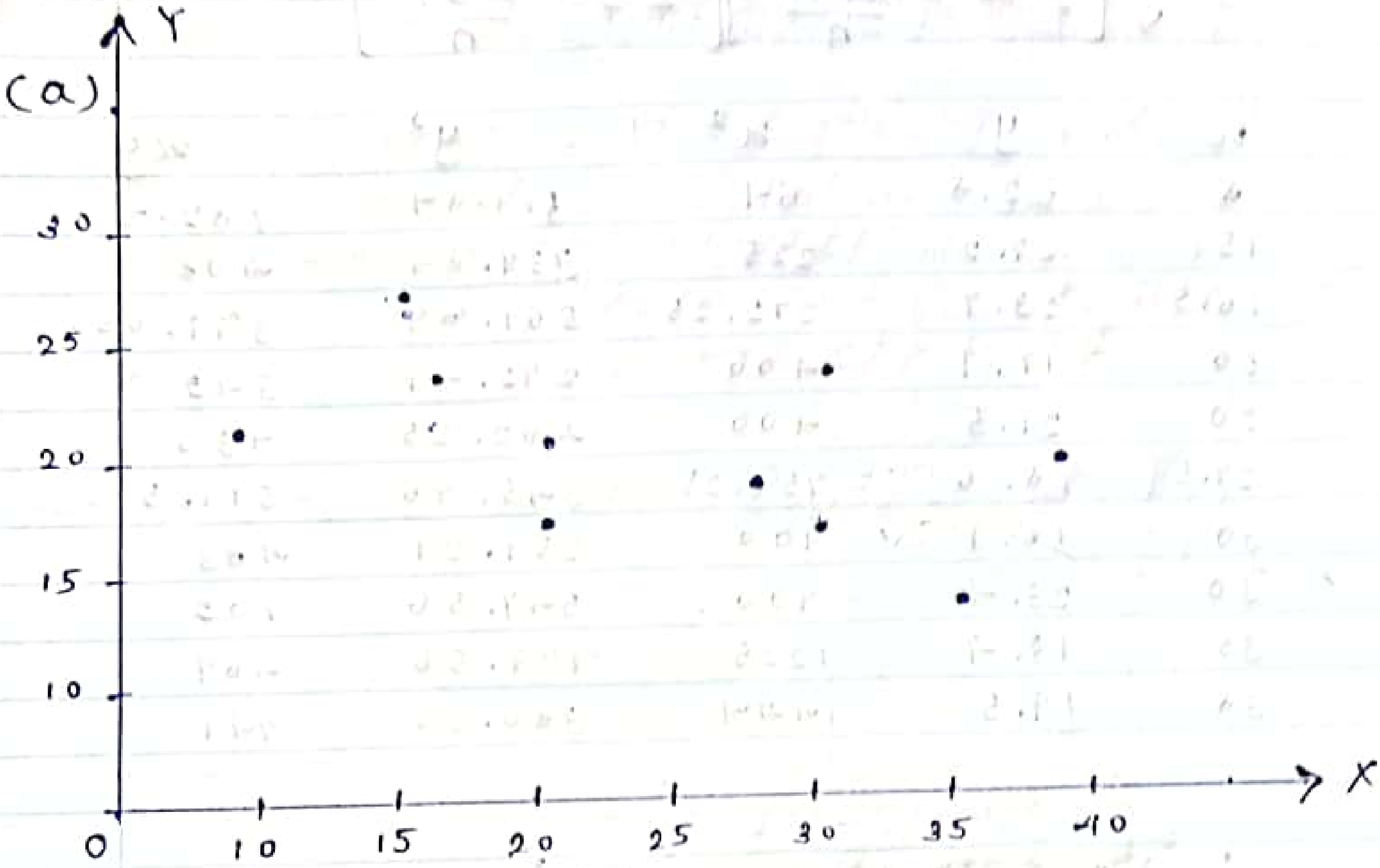
$\therefore \chi^2_{observed} = 0.1864 < \chi^2_{0.05,1} = 3.841$

observed value lies in the acceptable region. therefore we can't reject the null hypothesis.

$\therefore$ having a cellular phone in a car and being involved in an accident are independent.

(Q5)

(a)



yes, in the above graph, there appear a linear relationship between strength and carbonation depth, so the scatter plot of the data supports the choice of simple linear regression model.

(b). correlation coefficient,

$$r = \frac{\sum (x-\bar{x})(y-\bar{y})}{\sqrt{\sum (x-\bar{x})^2 \sum (y-\bar{y})^2}}$$

$$= \frac{\sum xy - n\bar{x}\bar{y}}{\sqrt{(\sum x^2 - n\bar{x}^2)(\sum y^2 - n\bar{y}^2)}}$$

$$= \frac{\sum xy - (\sum x \sum y)/n}{\sqrt{\left[\sum x^2 - \frac{(\sum x)^2}{n}\right]\left[\sum y^2 - \frac{(\sum y)^2}{n}\right]}}$$

x	y	x^2	y^2	xy
8	22.8	64	519.84	182.4
15	27.2	225	739.84	408
16.5	23.7	272.25	561.69	391.05
20	17.1	400	292.41	342
20	21.5	400	462.25	430
27.5	18.6	756.25	345.96	511.5
30	16.1	900	259.21	483
30	23.4	900	547.56	702
35	13.4	1225	179.56	469
38	19.5	1444	380.25	741

$\therefore \sum x = 240.$ $\qquad \sum x^2 = 6586.5$

$\sum y = 203.3$ $\qquad \sum y^2 = 4285.57$

$\sum xy = 4659.95$ $\qquad n = 10$

$$\therefore r = \frac{4659.95 - (240 \times 203.3)/10}{\sqrt{6586.5 - \frac{240^2}{10}}\sqrt{4285.57 - \frac{203.3^2}{10}}}$$

$$= -\underline{\underline{0.6176}}$$

r is minus value. therefore strength and carbonation depth have negative correlation. that means strength decreases with carbonation depth increases.

(c).

(i). regression line is given by

$$\hat{y} = \hat{b}_0 + \hat{b}_1 x$$

$$\hat{b}_1 = \frac{S_{xy}}{S_{xx}} = \frac{\sum (x-\bar{x})(y-\bar{y})}{\sum (x-\bar{x})^2}.$$

$$= \frac{\sum xy - n\bar{x}\bar{y}}{\sum x^2 - n\bar{x}^2}$$

$$= \frac{\sum xy - (\sum x \sum y)/n}{\sum x^2 - (\sum x)^2/n}$$

$$= \frac{4659.95 - (240 \times 203.3)/10}{6586.5 - (240)^2/10}$$

$$= -\underline{\underline{0.2653}}$$

$$\hat{b}_0 = \frac{\sum Y_i}{n} - \hat{b}_1 \frac{\sum x_i}{n}$$

$$= \frac{203.3}{10} - (-0.2653)\frac{(240)}{10}$$

$$= \underline{\underline{26.6972}}$$

$\therefore \hat{y} = 26.6972 - 0.2653x$

(ii) when carbonate depth 45

strength $= 26.6972 - 0.2653(45)$

$= \underline{\underline{14.7587}}$

(iii).

Scanned with CamScanner