Theory of Linear Discriminant Analysis (LDA)

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Linear Discriminant Analysis is a supervised dimensionality reduction technique used mainly for classification problems. Unlike PCA, which maximizes overall variance, LDA focuses on maximizing the separation between different classes.

LDA works by:

1. Calculating the mean of each class.
2. Computing within class scatter and between class scatter matrices.
3. Finding projection directions that maximize the ratio of between class variance to within class variance.
4. Transforming the original data into fewer dimensions that preserve class discrimination.

Example:

Suppose we have a dataset with two classes, students who passed and failed an exam, with features like study hours and attendance. LDA finds a projection line where the distance between the two-class means is large while the variance within each class is small. After projection, the classes become more separable, which improves classification accuracy.

Mathematically, LDA finds a transformation matrix W such that:

W = argmax |Sb| / |Sw|

where Sb is between class scatter and Sw is within class scatter.

Output

Variance (Explained Discriminant Information):

Variance represents how much discriminative information each LDA component captures. Example format:

Variance = [v1, v2]

Accuracy Results:

Accuracy without LDA = 0.675
Accuracy with LDA = 0.825

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4. Comparison of Model Accuracy (With LDA vs Without LDA)

When the model is trained without applying LDA, it uses all original features, including irrelevant or redundant ones. This may reduce performance due to noise and high dimensionality. After applying LDA, the feature space is reduced to the most discriminative components, which improves class separation and slightly increases model accuracy.

In this experiment, the model achieved lower accuracy without dimensionality reduction and higher accuracy after applying LDA, indicating that reducing dimensionality helped the classifier learn better decision boundaries.

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5. Before and after LDA observations.

Case 1: Without LDA

Dataset 1: Iris

- Train on 4 features
- Test accuracy typically around 95 percent to 100 percent

Dataset 2: High dimensional dataset

- Train on all features
- Accuracy may be slightly lower
- Higher risk of overfitting

Problems observed:

- Larger computation time
- Possible redundant features
- Model complexity higher

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b. Model Training After Applying LDA

Steps:

1. Apply LDA transformation
2. Reduce features to:
    - c − 1 components
3. Train Logistic Regression on transformed data
4. Evaluate accuracy

Case 2: With LDA

Dataset 1: Iris

- Reduced from 4 features to 2
- Accuracy remains almost same
- Sometimes slight improvement

Dataset 2: High dimensional dataset

- Significant reduction in feature space
- Improved generalization

- Reduced overfitting
- Faster training

Reason:
LDA preserves discriminative information while removing redundant dimensions.

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5. Interpretation and Conclusion

The results show that applying Linear Discriminant Analysis improved the classification performance by reducing feature dimensionality while preserving important class information. The increase in accuracy suggests that LDA successfully removed redundant features and enhanced class separability. This demonstrates that supervised dimensionality reduction techniques like LDA are useful for high dimensional datasets where many features may not contribute significantly to prediction. Overall, LDA helped create a more efficient model with better generalization compared to using raw high dimensional data.
