Optimization

Efthymios Ioannis Kavour

2022-06-24

<u>Exercise 1</u>:

Assume that :

$$A = \begin{bmatrix} 3 & 0 & 3 \\ 0 & 1 & -2 \\ 3 & -2 & 8 \end{bmatrix} \quad B = \begin{bmatrix} 2 & -1 & b \\ -1 & 2 & -1 \\ b & -1 & 2 \end{bmatrix}$$

Is **A** positively definite? Are there values of b that matrix **B** is positively definite? If yes what are those values?

<u>Solution</u>:

We know that a symmetrical matrix $M_{k \times k} \quad k = 1, \ldots, n$ is positively definite if

1. All the eigen values are positive.
2. All the n upper left determinants of a $M_{k \times k}$ are positive
3. All the drivers $d_i, \quad \forall i = 1, \ldots, n$ in the Gaussian Elimination without interchanging of lines are positive.

In order to check whether A is positive definite, we are going to calculate all the upper left determinants. As a result we have the following observations. The first term $m_{1 \times 1}$ is positive.

$$3 > 0$$

Furthermore, we can see that the next *left-upper* determinant is the determinant of the 2×2 submatrix, as shown below, which is positive as well.

$$\begin{vmatrix} 3 & 0 \\ 0 & 1 \end{vmatrix} = 3 > 0$$

Finally what is left is the determinant of the full 3×3 matrix A which as shown below, positive.

$$\begin{vmatrix} 3 & 0 & 3 \\ 0 & 1 & -2 \\ 3 & -2 & 8 \end{vmatrix} =$$
$$3 \times 1 \times 8 + 0 \times (-2) \times 3 + 3 \times 0 \times (-2) - 3 \times 1 \times 3 - (-2) \times (-2) \times 3 - 8 \times 0 \times 0 =$$
$$24 + 0 + 0 - 9 - 12 - 0 = 3 > 0$$

In conclusion, we can see that matrix A is **positive define**.