EENG 5320

PROJECT Report

Submitted by

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① Given,

Step input function $e_{in}(t) = R\,u(t)\,v$

Value is initially at state equilibrium $(z=0)$

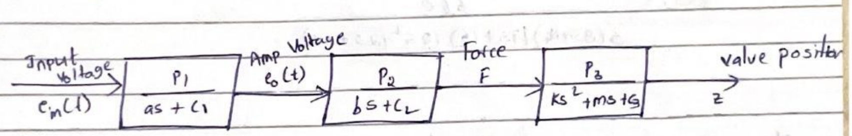
ID Number $= 11722528$

$R = 7,\ P_1 = 2,\ P_2 = 5,\ P_3 = 8,\ a = 8,\ b = 7,\ k = 2,\ m = 2$

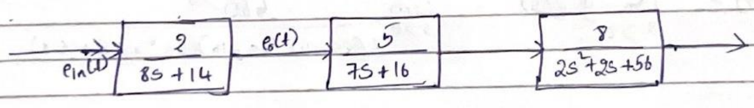
$c_1 = R \times k = 7 \times 2 = 14$

$c_2 = a \times P_1 = 8 \times 2 = 16$

$c_3 = R \times P_3 = 7 \times 8 = 56$



From given value



$e_{in}(t) = R\,u(t) = 7\,u(t)$

By laplace transform

$E_{in}(s) = \frac{7}{s}$

19. Steady state position of the spool valve

$$\frac{Z(s)}{E_{in}(s)} = \frac{E_o(s)}{E_{in}(s)} \cdot \frac{F(s)}{E_o(s)} \cdot \frac{Z(s)}{F(s)}$$

By Cascading:-

$$\frac{Z(s)}{E_{in}(s)} = \frac{2 \times 5 \times 8}{(8s+14)(7s+16)(2s^2+2s+56)}$$

$$Z(s) = \frac{80}{(8s+14)(7s+16)(2s^2+2s+56)} \times \frac{7}{s}$$

$$Z(s) = \frac{560}{s(8s+14)(7s+16)(2s^2+2s+56)}$$

Now, By using final value theorem:-

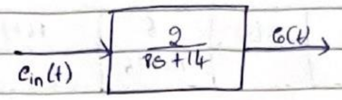
$$z(\infty) = \lim_{s \to 0} s\, Z(s) = \lim_{s \to 0} \frac{560\cancel{s}}{\cancel{s}(8s+14)(7s+16)(2s^2+2s+56)}$$

$$Z(\infty) = \frac{560}{(14)(16)(56)}$$

$$= 0.045$$

$$\boxed{Z(\infty) = 0.045\ m}$$

b. Determine amplier voltage e_o response for step input



$$E_{in}(t) = 7\,u(t)$$

$$E_{in}(s) = \frac{7}{s}$$

$$E_o(t) = \left(\frac{2}{8s+14}\right) E_{in}(t)$$

$$E_o(s) = \frac{2}{8s+14} \times \frac{7}{s} = \frac{14}{s(8s+14)} = \frac{\cancel{14}\,1.75}{\cancel{8}s(s+1.75)}$$

$$E_o(s) = \frac{1.75}{s(s+1.75)}$$

By Partial fraction:-

$$E_o(s) = \frac{1.75}{s(s+1.75)} = \frac{a}{s} + \frac{b}{s+1.75}$$

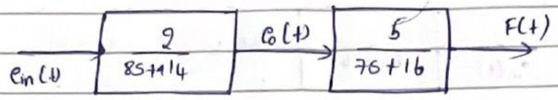
$$a = \text{ at } s=0 \quad \frac{1.75}{s+1.75}\ (s=0) = \frac{1.75}{1.75} = 1$$

$$b = \text{ at } s=-1.75 \quad \frac{1.75}{s} = \frac{1.75}{-1.75} = -1$$

Now, $E_o(s) = \frac{1.75}{s(s+1.75)} = \frac{1}{s} - \frac{1}{s+1.75}$

$$E_o(t) = 1 - 1e^{-1.75t}\ V$$

c, Force responce of the solenoid F(t) for step i/p



$$\frac{F(t)}{e(t)} = \frac{10}{(7s+16)(8s+14)}$$

$$F(t) = \frac{10}{(7s+16)(8s+14)} \cdot e_{in}(t)$$

$$= \frac{10 \times 7}{s(7s+16)(8s+14)}$$

$$= \frac{70}{8 \times 7 \times s(s+2.286)(s+1.75)}$$

$$F(s) = \frac{1.25}{s(s+2.286)(s+1.75)}$$

By partial fractions:-

$$F(s) = \frac{1.25}{s(s+2.286)(s+1.75)} = \frac{a}{s} + \frac{b}{s+2.286} + \frac{c}{s+1.75}$$

$$a = s \cdot F(s), \quad \left.\frac{1.25}{(s+2.286)(s+1.75)}\right|_{s=0} = \frac{1.25}{(2.286)(1.75)} = 0.312$$

$b = (s+2.286)F(s), \; s = -2.286 \quad = \dfrac{1.25}{(-2.28)(-2.286+1.75)} = 1.020$

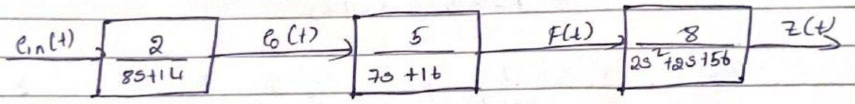
$C = (s+1.75)\,F(s), \; s = -1.75 \quad = \dfrac{1.25}{(-1.75)(-1.75+2.286)} = -1.333$

$$F(s) = \frac{0.312}{s} + \frac{1.020}{s+2.286} - \frac{1.333}{s+1.75} \quad \text{--- } (1)$$

Apply inverse laplace transform to equation (1)

$$F(t) = 0.312 + 1.020\,e^{-2.286t} - 1.333\,e^{-1.75t} \; N$$

d Determine the spool value position $z(t)$, for the step input.



By Cascading:-

$$\frac{Z(t)}{E_{in}(t)} = \left(\frac{8}{2s^2+2s+56}\right) \times \left(\frac{5}{7s+16}\right) \times \left(\frac{2}{8s+14}\right)$$

$$z(t) = \frac{80}{(8s+14)(7s+16)(2s^2+2s+56)} \times \frac{7}{s}$$

$$\frac{560}{s(8s+14)(7s+16)(2s^2+2s+56)}$$

$$= \frac{\overset{10^5}{\cancel{560}}}{\cancel{8}\times\cancel{7}\times\cancel{2}\times s\ (s+1.75)\ (s+2.286)\ (s^2+s+28)}$$

$$Z(s) = \frac{5}{s\ (s+1.75)\ (s+2.286)\ (s^2+s+28)}$$

By Partial fractions :-

$$Z(s) = \frac{a}{s} + \frac{b}{s+1.75} + \frac{c}{s+2.286} + \frac{ds+e}{s^2+s+28}$$

Now poles of s^2+s+28

$$s = -0.5 \pm 5.267j$$

$$Z(s) = \frac{a}{s} + \frac{b}{s+1.75} + \frac{c}{s+2.286} + \frac{d}{s-0.5+5.267j} + \frac{e}{s-0.5 \pm 5.267j}$$

$$a = s\,Z(s)\Big|_{s=0} = \frac{5}{1.75\times 2.286\times 28} = 0.045$$

$$b = (s+1.75)\Big|_{s=-1.75} = \frac{5}{(-1.75)(-1.75+2.286)\left((-1.75)^2 \pm 1.75 + 28\right)} = -0.182$$

$$c = (s+2.286)\Big|_{s=-2.286} = \frac{5}{(-2.286)(-2.286+1.75)\left((-2.286)^2 - 2.286 + 28\right)} = 0.132$$

$$d = (-0.5+5.267j)\,Z(s)\Big|_{s=-0.5-5.267j}$$

$$= \frac{5}{(-0.5-5.267j)(1.25-5.267j)(1.786-5.267j)(0.0087)}$$

$$= -1.618-3.224j \quad = 3.608 \angle{-116.65^\circ}$$

$$e = \frac{5}{(-0.5+5.267j)(1.25+5.267j)(1.786+5.267j)(0.0087)}$$

$$= -1.618+3.224j \quad = 3.608 \angle{116.65^\circ}$$

$$Z(s) = \frac{0.045}{s} - \frac{1.0182}{s+1.75} + \frac{0.132}{s+2.286} + \frac{3.608\angle{-116.65^\circ}}{s-0.5+5.267j} + \frac{3.608\angle{118.65^\circ}}{s-0.5-5.267j}$$

~~At steady state position all exponential terms~~

By Applying inverse laplace transform

$$Z(t) = 0.045 - 1.0182\,e^{-1.75t} + 0.132\,e^{-2.286t} - 1.618 + i\left(e^{0.5t}\sin(-5.267t) - 3.224\,e^{0.5t}\cos(-5.267t)\right)$$

$$- 1.618 + i\left(e^{0.5t}\sin(5.267t) + 3.224\,e^{0.5t}\cos(5.267t)\right)$$

At steady state position all exponental terms becomes zero

$$\therefore Z(\infty) = 0.045\,m$$

② Given

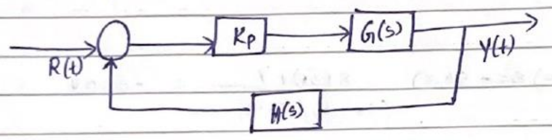
$$G(s) = \frac{s+1}{C_1 + bs + as^2}$$

$$H(s) = \frac{1}{C_1 + as^2}$$

$a = a \Rightarrow a = 8, \; b = b \Rightarrow b = 7, \; C_1 = C_1 \Rightarrow C_1 = 14, \; C_2 = C_2 \Rightarrow C_2 = 16$

$P_1 = \frac{P_1}{10} \Rightarrow P_1 = 0.2, \; P_2 = \frac{P_2}{10} \Rightarrow P_2 = 0.5$

a, $K_p = P_1 = \frac{P_1}{10} = 0.2$



Transfer Function:-

$$\frac{Y(t)}{R(t)} = \frac{K_p \cdot G(s)}{1 + K_p \cdot G(s) \cdot H(s)}$$

$$Y(s) = \frac{\dfrac{(0.2)(s+1)}{14 + 7s + 8s^2}}{1 + \dfrac{(0.2)(s+1)}{8s^2 + 7s + 14} \times \dfrac{1}{8s^2 + 14}}$$

$$Y(s) = \frac{0.2)(s+1)}{(8s^2 + 7s + 14)(8s^2 + 14) + 0.2s + 0.2}$$

Consider the denominator :-

$\Rightarrow (8s^2+7s+14)(8s^2+14) + 0.2s + 0.2$

$\Rightarrow 64s^4 + 112s^2 + 56s^3 + 98s + 112s^2 + 56 + 0.2s + 0.2$

$\Rightarrow 64s^4 + 56s^3 + 224s^2 + 98.2s + 56.2$

$\Rightarrow s^4 + 0.875s^3 + 3.5s^2 + 1.534s + 0.878$

By applying Routh hurwitz table:

s^4	1	3.5	0.878
s^3	0.875	1.534	0
s^2	3.5	0.878	0
s	1.534	0	0
s^0	0.878		

$\therefore$ Since, the first column is positive the system is stable

Q.2 b Given

$$G'(s) = \frac{1}{(s+8)(s^2+7s+14)}$$

$$H(s) = \frac{1}{s^2+7s+16}$$

$$T(s) = \frac{1}{(s+8)(s^2+7s+14)(s^2+7s+16)}$$

Characteristic Equation:-

$$(s+8)(s^2+7s+14)(s^2+7s+16) + K_p = 0$$

Damping Ratio $= \xi = \dfrac{-\text{Real part of Pole}}{\omega_n}$

$$s^2 + 2\xi\omega_n s + \omega_n^2$$

$2\xi\omega_n = 7$, $\omega_n^2 = 15$

$\omega_n = \sqrt{15}$

$\xi\omega_n = 3.5$

$\xi = \dfrac{3.5}{\omega_n} = \dfrac{3.5}{\sqrt{15}}$

using $\xi \approx 0.404$ solve for new ω_n

$2 \times 0.404 \times \omega_n = 7$

$\omega_n = \dfrac{7}{2 \times 0.404} = 8.663 \Rightarrow \omega_n^2 = 75$

$$K_p\,(s^2+7s+15) = s^2+2\xi\omega_n s+\omega_n^2$$

$$K_p = \frac{\omega_n^2}{15} = \frac{75}{15} = 5$$

$$T_s = \frac{4}{\xi\omega_n} = \frac{4}{0.404\times 8.663} \approx 1.15 \text{ seconds}$$

2) c, $G(s) = \dfrac{1}{(s+8)(s^2+7s+14)}$

$$H(s) = \frac{1}{s^2+7s+16}$$

$$K_p = 0.5$$

Transfer function:

$$T(s) = \frac{(0.5)\times\dfrac{1}{(s^2+7s+16)(s+8)(s^2+7s+14)}}{1+0.5\times\dfrac{1}{(s^2+7s+16)(s+8)(s^2+7s+14)}}$$

$$T(s) = \frac{0.5}{0.5+(s+8)(s^2+7s+16)(s^2+7s+14)}$$

Characteristic equation:

$$0.5+(s+8)(s^2+7s+16)(s^2+7s+14) = 0$$

Assuming k_p scales the quadratic terms, the charateristic eq becomes

$$s^2 + (7 + k_p \times 7)s + (15 + k_p \times 15)$$

For $k_p = 0.5$

$$s^2 + 10.5s + 22.5$$

$$2\xi\omega_n = 10.5, \quad \omega_n^2 = 22.5$$

$$\omega_n = \sqrt{22.5}$$

$$\xi = \frac{10.5}{2\omega_n}$$

$$= \frac{10.5}{2\sqrt{22.5}}$$

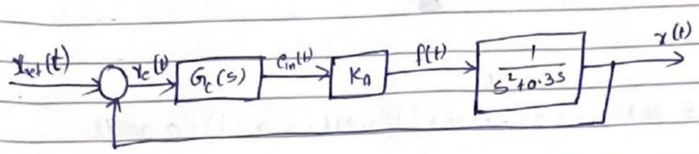
$$\xi = 1.106$$

Settling time $(T_s) = \dfrac{4}{\xi\omega_n}$

$$T_s = \frac{4}{1.106 \times 4.74}$$

$$\underline{T_s = 0.76 \text{ seconds}}$$

(2) Given,

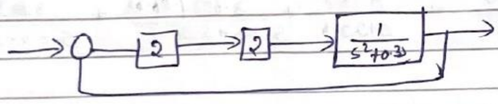


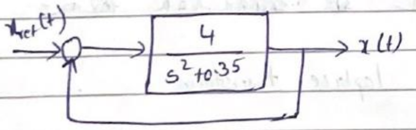
$$x_{ref}(t) = 0.1\, u(t)$$

$$K_A = 2\ N/v$$

$G_c(s)$ is equal to K_p.

a. Given, $K_p = 2$, $G_c = 2$





Transfer function:-

$$\frac{x(t)}{x_{ref}(t)} = \frac{\frac{4}{s^2+0.35}}{1+\frac{4}{s^2+0.35}}$$

$$= \frac{4}{s^2+0.35+4}$$

Characteristic Equation :-

$$s^2 + 0.3s + 4$$

$$s^2 + 2\xi\omega_n s + \omega_n^2$$

$$2\xi\omega_n = 0.3 \quad , \quad \omega_n^2 = 4 \Rightarrow \boxed{\omega_n = 2}$$

$$\xi = \frac{0.3}{2\omega_n}$$

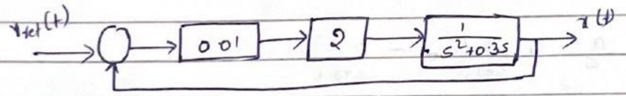
$$= \frac{0.3}{2 \times 2} = \frac{0.3}{4}$$

$$\xi = 0.075$$

b. Given, $K_p = 0.01$

$$x_{ref}(t) = 0.1\, U(t)$$

$$K_A = 2\, N/V$$



Transfer function :-

$$\frac{x(t)}{x_{ref}(t)} = \frac{\dfrac{0.02}{s^2+0.3s}}{1 + \dfrac{0.02}{s^2+0.3s}}$$

$$x(t) = \frac{0.02}{s^2 + 0.3s + 0.02} \times \frac{0.1}{s}$$

$$X(s) = \frac{0.002}{s^3 + 0.3s^2 + 0.02s}$$

poles of the denominator are:-

$s = -0.2 ; -0.1 ; 0$

$$X(s) = \frac{0.002}{(s+0.2)(s+0.1)s}$$

By partial fraction:-

$$X(s) = \frac{a}{s} + \frac{b}{s+0.2} + \frac{c}{s+0.1}$$

$a = 0.1$

$b = 0.1$

$c = -0.2$

$$X(s) = \frac{0.1}{s} + \frac{0.1}{s+0.2} - \frac{0.2}{s+0.1}$$

$\therefore$ By inverse laplace transform:-

$$x(t) = 0.1 + 0.1e^{-0.2t} - 0.2e^{-0.1t}$$

4. Given

$$Y(s) = \frac{as+b}{(s+p)(s+q)(s+r)}$$

$a = a \Rightarrow a = 8$, $b = b \Rightarrow b = 7$, $P = c_3 \Rightarrow P = 56$, $q = c_2 \Rightarrow q = 16$,
$r = k \Rightarrow r = 2$

$$Y(s) = \frac{8s+7}{(s+56)(s+16)(s+2)}$$

By partial fractions:

$$Y(s) = \frac{a}{s+56} + \frac{b}{s+16} + \frac{c}{s+2}$$

$$a = (s+56)\,Y(s) \Rightarrow s = -56 \Rightarrow \frac{8(-56)+7}{(-56+16)(-56+2)} = -0.204$$

$$b = (s+16)\,Y(s) \Rightarrow s = -16 \Rightarrow \frac{8(-16)+7}{(-16+56)(-16+2)} = 0.216$$

$$c = (s+2)\,Y(s) \Rightarrow s = -2 \Rightarrow \frac{8(-2)+7}{(-2+56)(-2+16)} = -0.0119$$

$$Y(s) = \frac{-0.204}{s+56} + \frac{0.216}{s+16} - \frac{0.0119}{s+2}$$

By inverse laplase transform:-

$$y(t) = -0.204\,e^{-56t} + 0.216\,e^{-16t} - 0.0119\,e^{-2t}$$

Question 5

Motor 1

⑤ Given $G(s) = \frac{\omega(s)}{E_a(s)} = \frac{K_t}{(L_a s + R_a)(Js + B) + K_b K_t}$

i. Motor 1

$R_a = 2$, $L_a = 0.5$, $J = 0.02$, $B = 0.2$, $K_t = 0.015$

$K_b = 0.01$

$$G(s) = \frac{0.015}{(0.5s + 2)((0.02)s + 0.2) + 0.01 \times 0.015}$$

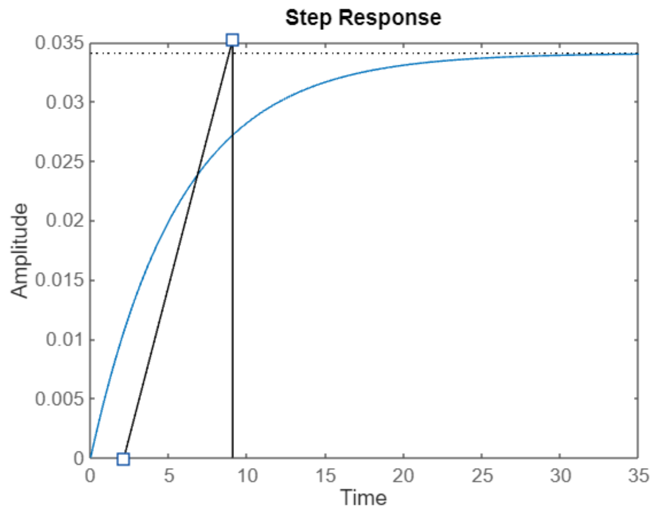
$$= \frac{0.015}{0.01s^2 + 0.1s + 0.04s + 0.4 + 0.00015}$$

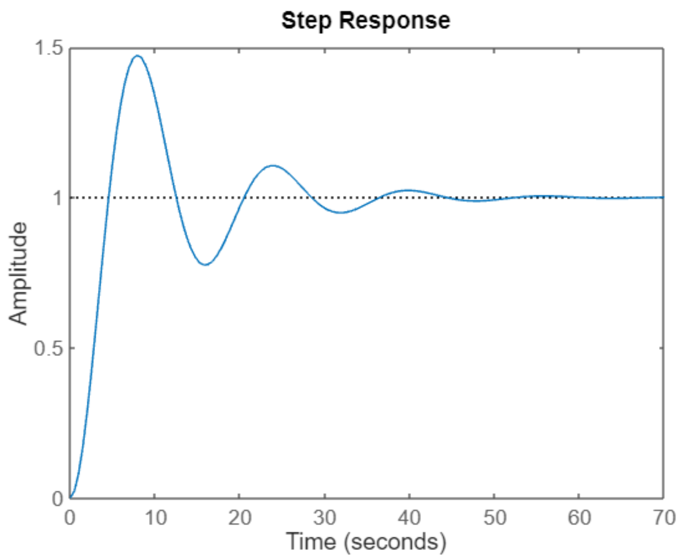
$$= \frac{0.015}{0.01s^2 + 2.14s + 0.4[illegible]}$$

$$= \frac{0.015}{0.01s^2 + 2.5s + 0.44}$$

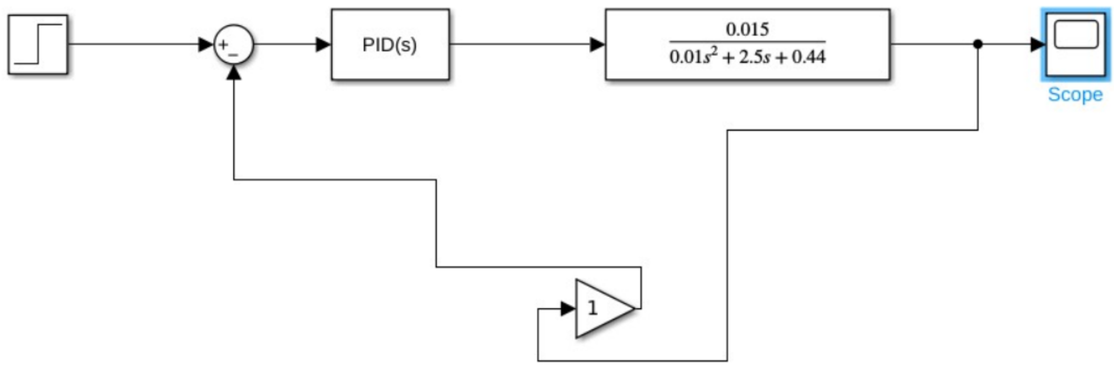
Code:

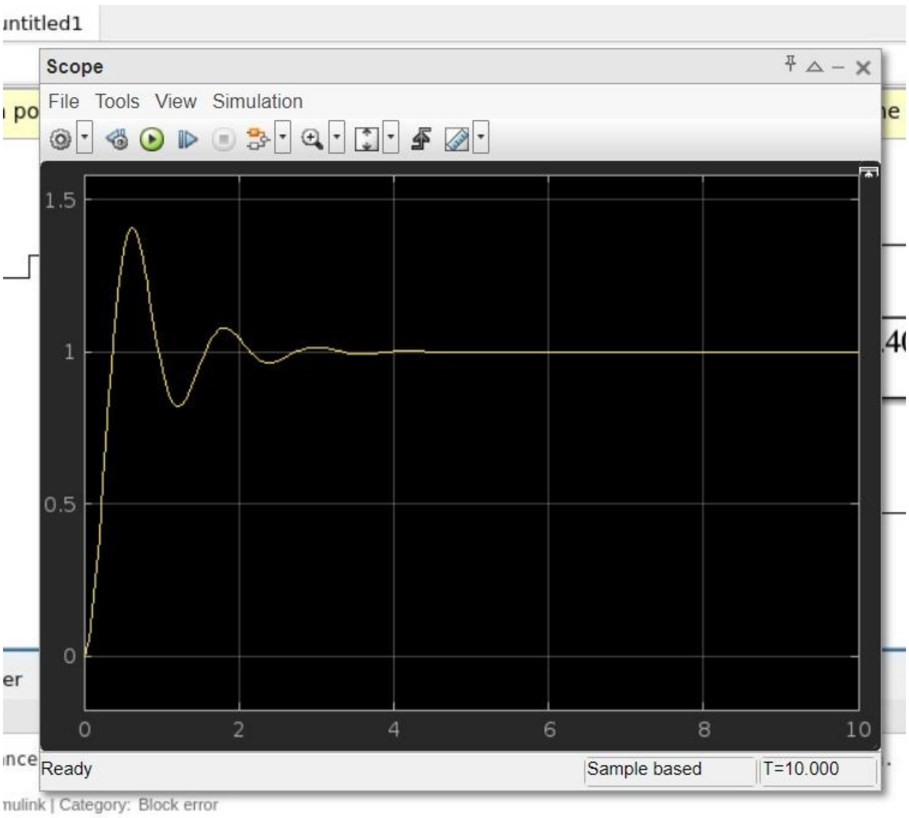
```
s = tf('s');
sys=0.015/(0.01*s^2+2.5*s+0.44);
step(sys)

k=0.01; %steady state value
L = 0.036; % delay time
T = 0.095 - L; %constant time
Ti = 2*L;
Td = L/2;
Kp = 1.2*T/L;
Ki = Kp/Ti;
Kd = Kp*Td;
disp(Kp);
disp(Ki);
disp(Kd);
cont = pid(Kp, Ki, Kd);
figure
step(feedback(cont*sys,1))
```



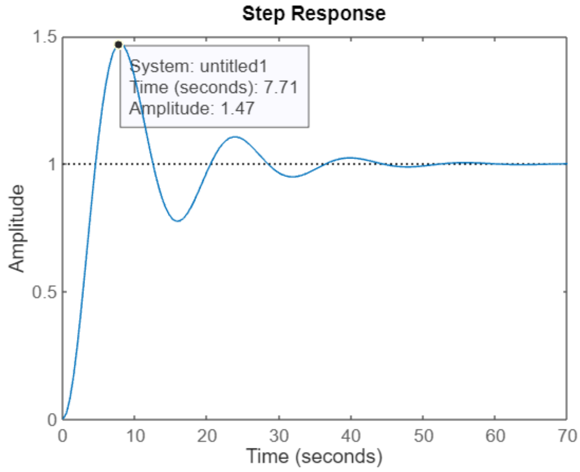


Schematic Diagram:

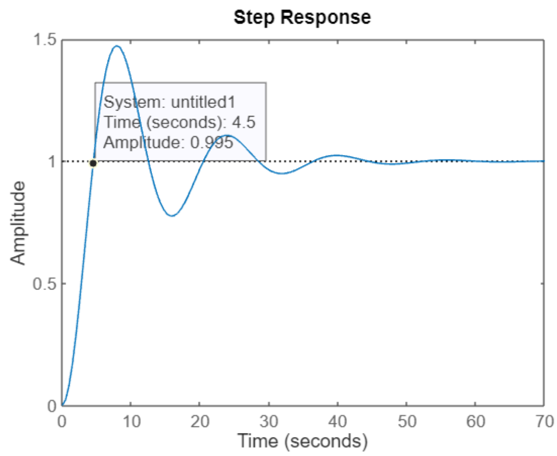




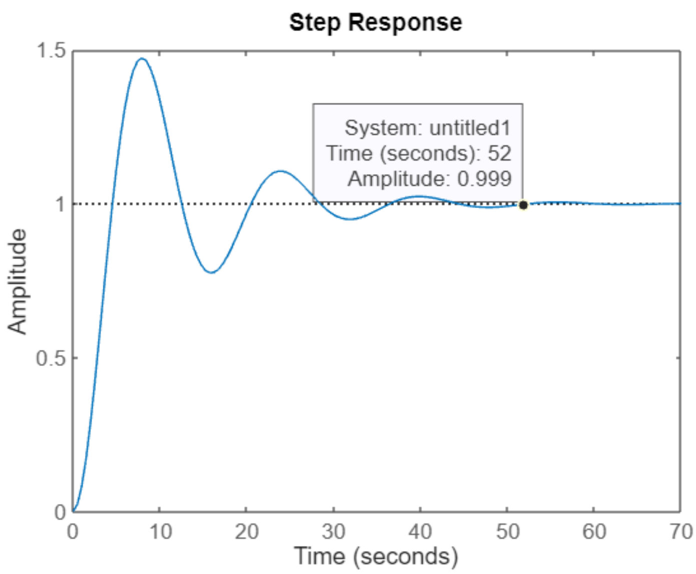
Over Shoot:-



Rising Time:-



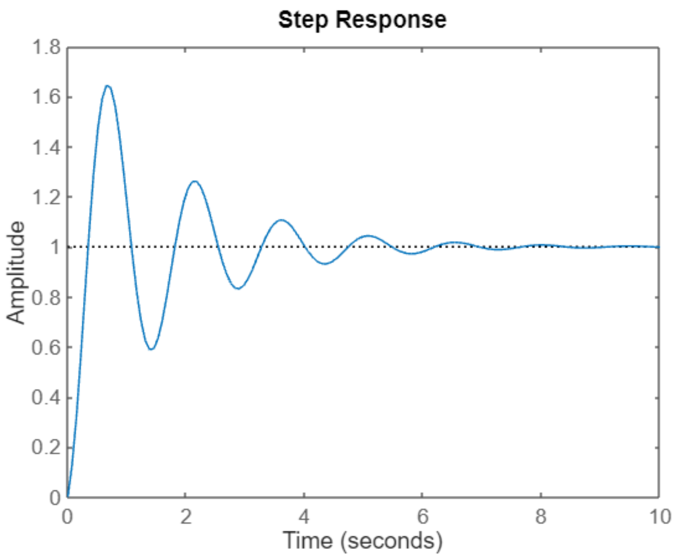
Settling Time:-



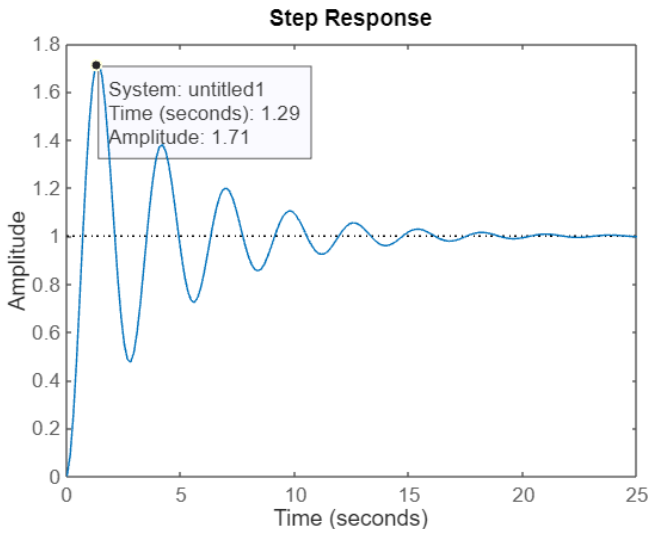
Motor 1 (Modified):-

Code:-

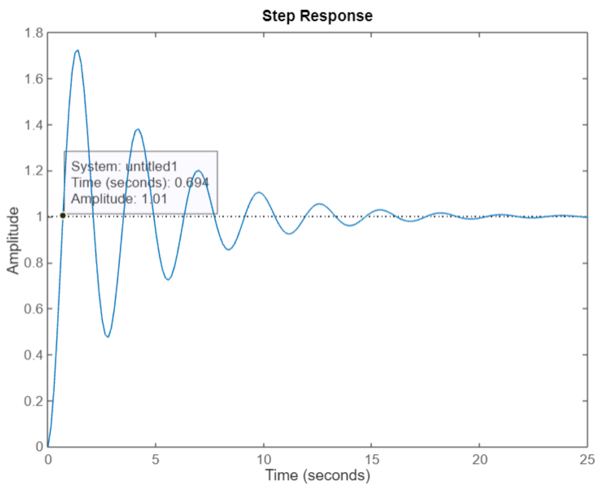
```
s = tf('s');

sys=0.015/(0.01*s^2+2.5*s+0.44);
step(sys)

k=0.01; %steady state value
L = 0.030; % delay time
T = 0.090 - L; %constant time

a= k*L/T;
Ti = 2*L;
Td = L/2;

Kp = 0.95/a;
Ki = Kp/Ti;
Kd = Kp*Td;
disp(Kp);
disp(Ki);
disp(Kd);
cont = pid(Kp, Ki, Kd);
figure
step(feedback(cont*sys,1))
```



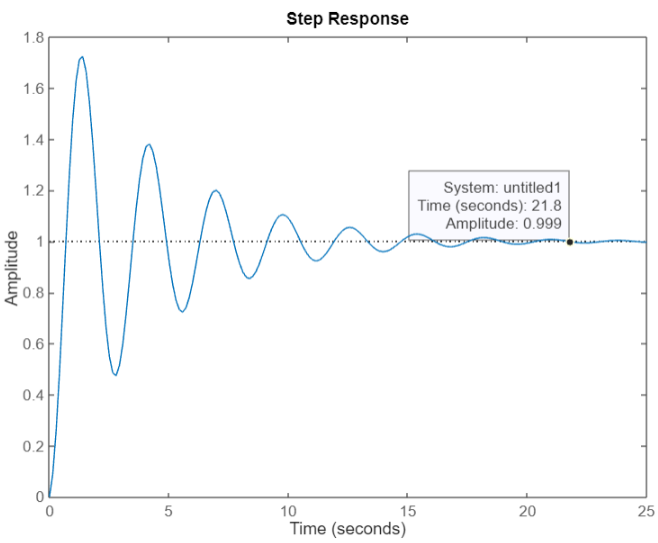
Over Shoot:-



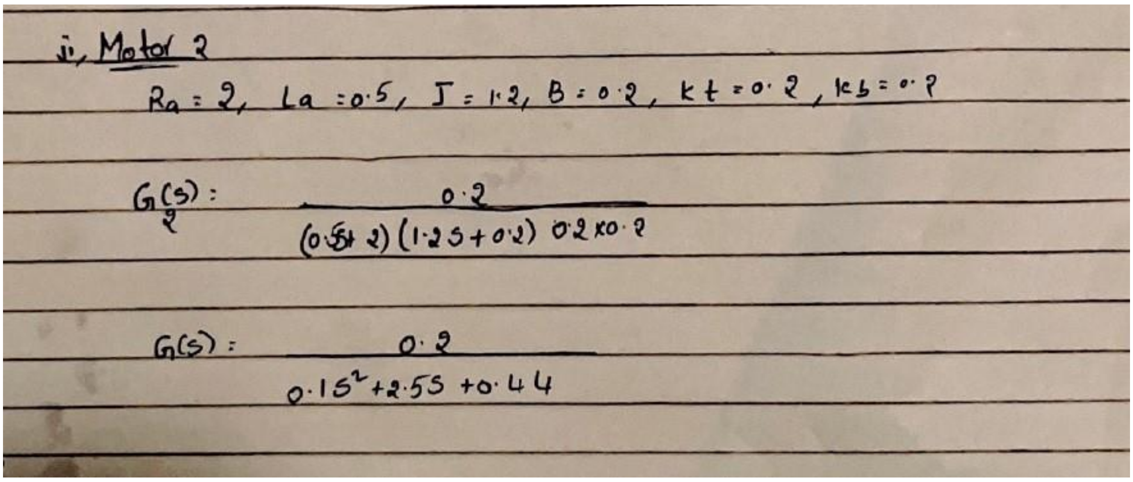
Rising Time:-



Settling Time:-



Motor 2



ii, Motor 2

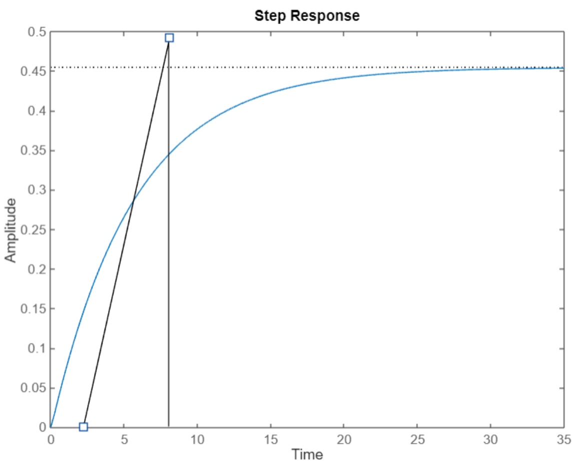
$R_a = 2,\ L_a = 0.5,\ J = 1.2,\ B = 0.2,\ k_t = 0.2,\ k_b = 0.2$

$$G_2(s) = \frac{0.2}{(0.5s + 2)(1.2s + 0.2)\ 0.2 \times 0.2}$$

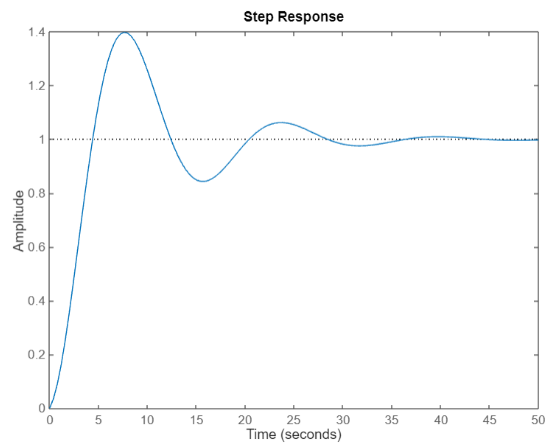
$$G(s) = \frac{0.2}{0.1s^2 + 2.5s + 0.44}$$

Code:

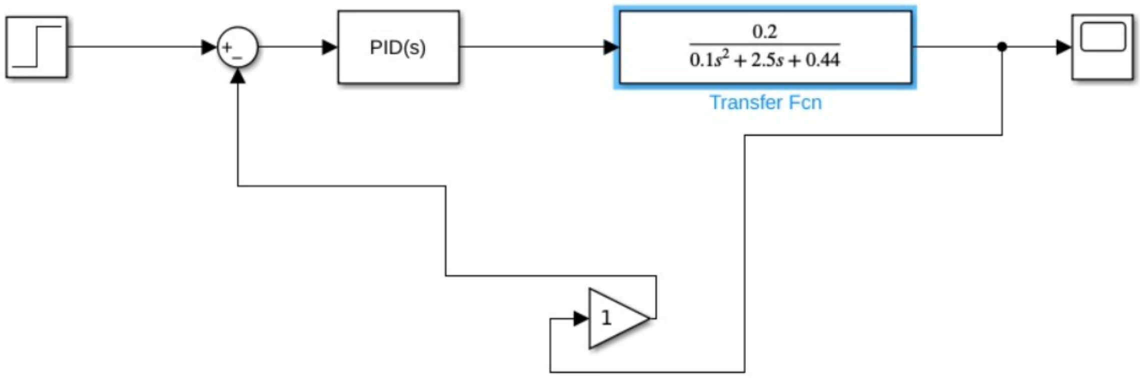
```
s = tf('s');

sys=0.2/(0.1*s^2+2.5*s+0.44);
step(sys)

k=0.01; %steady state value
L = 0.185; % delay time
T = 0.304 - L; %constant time


Ti = 2*L;
Td = L/2;

Kp = 1.2*T/L;
Ki = Kp/Ti;
Kd = Kp*Td;
cont = pid(Kp, Ki, Kd);
figure
step(feedback(cont*sys,1))
```

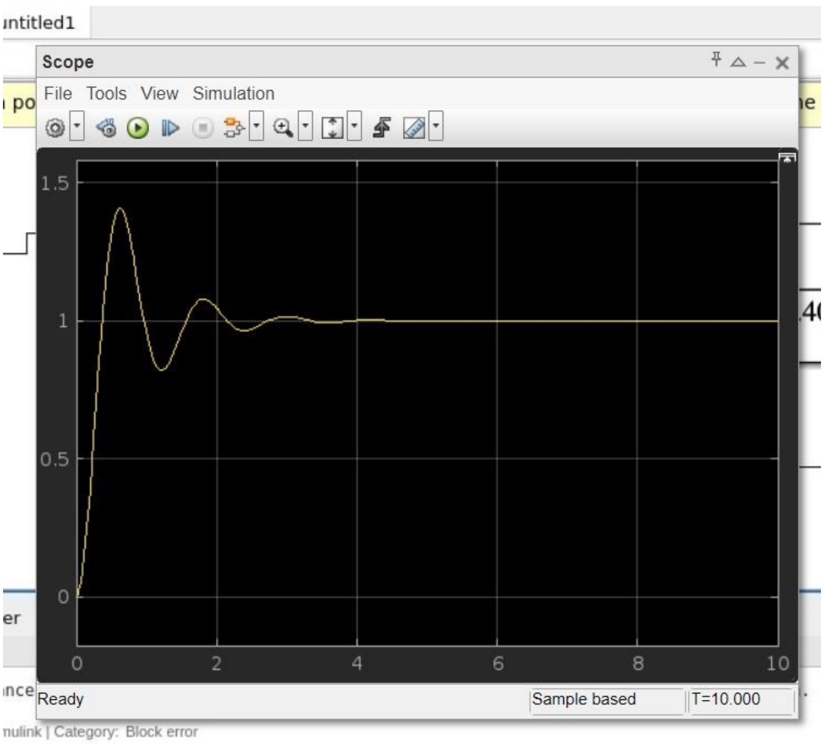


```
s = tf('s');

sys=0.2/(0.1*s^2+2.5*s+0.44);
step(sys)

k=0.01; %steady state value
L = 0.185; % delay time
T = 0.304 - L; %constant time


Ti = 2*L;
Td = L/2;

Kp = 1.2*T/L;
Ki = Kp/Ti;
Kd = Kp*Td;
cont = pid(Kp, Ki, Kd);
figure
step(feedback(cont*sys,1))
```

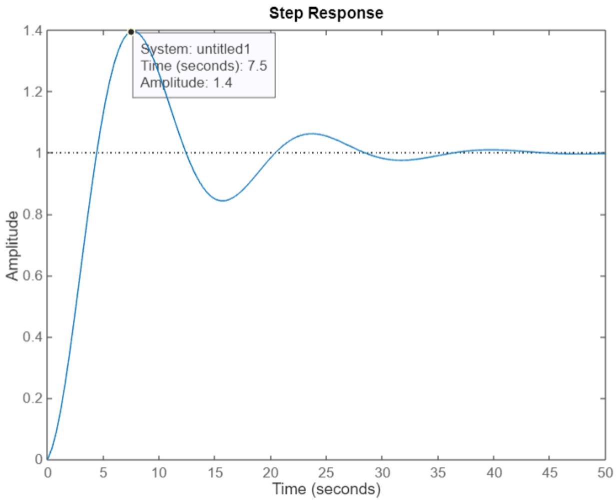


Schematic Diagram:

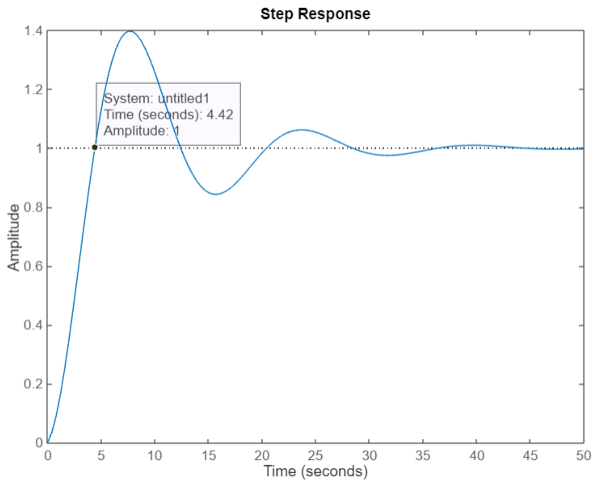




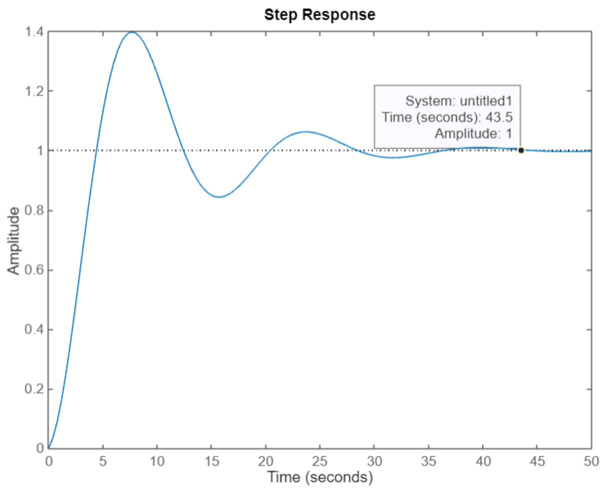
Over Shoot:-



Rising Time:-



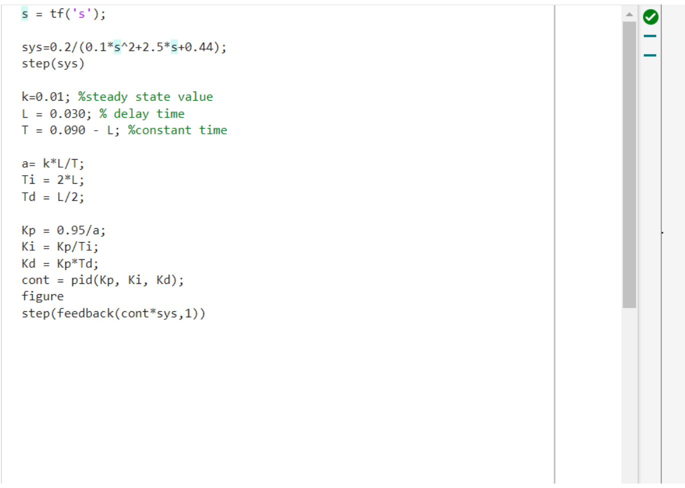
Settling Time:-

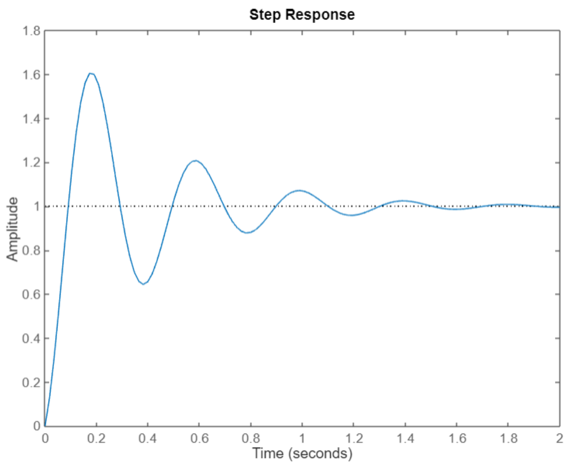


Motor 2 (Modified):-

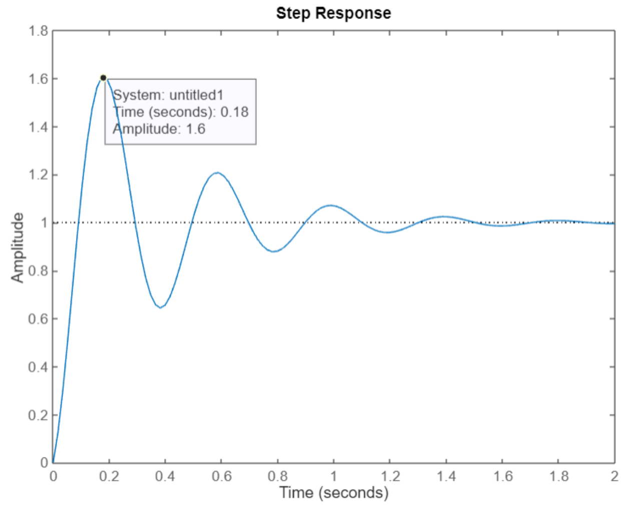
Code:-

```
s = tf('s');

sys=0.2/(0.1*s^2+2.5*s+0.44);
step(sys)

k=0.01; %steady state value
L = 0.030; % delay time
T = 0.090 - L; %constant time

a= k*L/T;
Ti = 2*L;
Td = L/2;

Kp = 0.95/a;
Ki = Kp/Ti;
Kd = Kp*Td;
cont = pid(Kp, Ki, Kd);
figure
step(feedback(cont*sys,1))
```

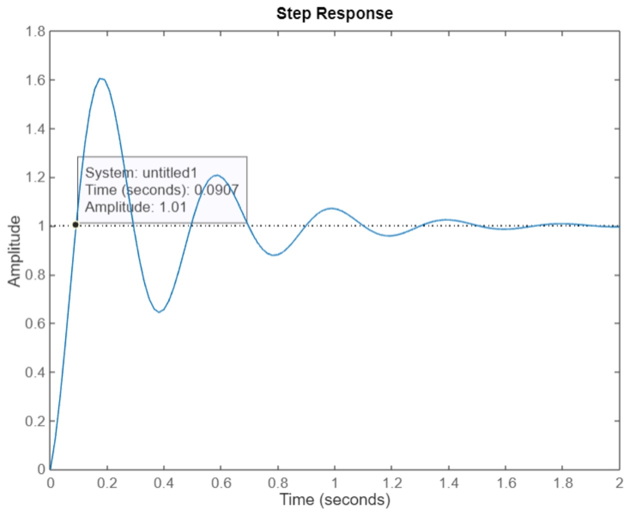




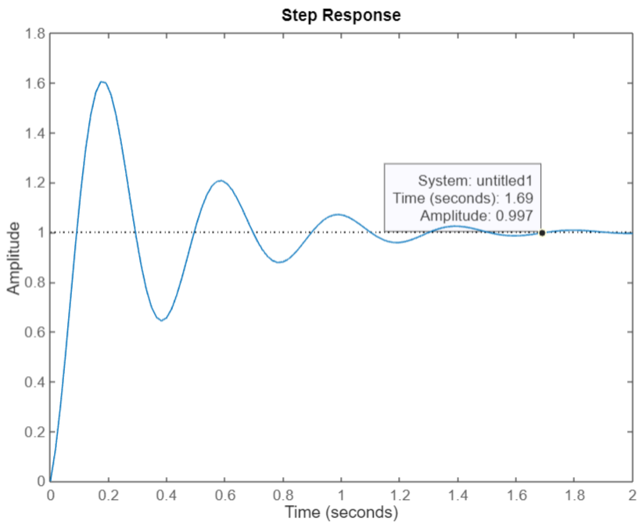
Over Shoot:-



Rising Time:-



Settling Time:-



Motor 3

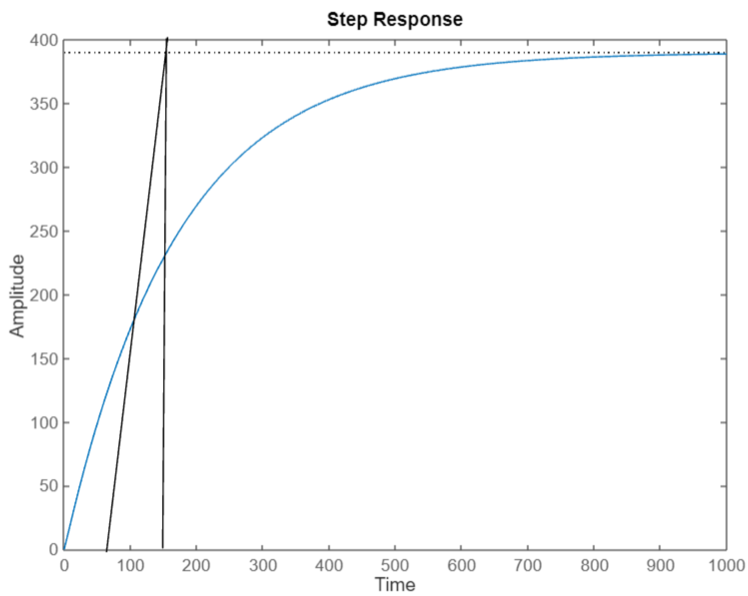
iii, Motor 3:

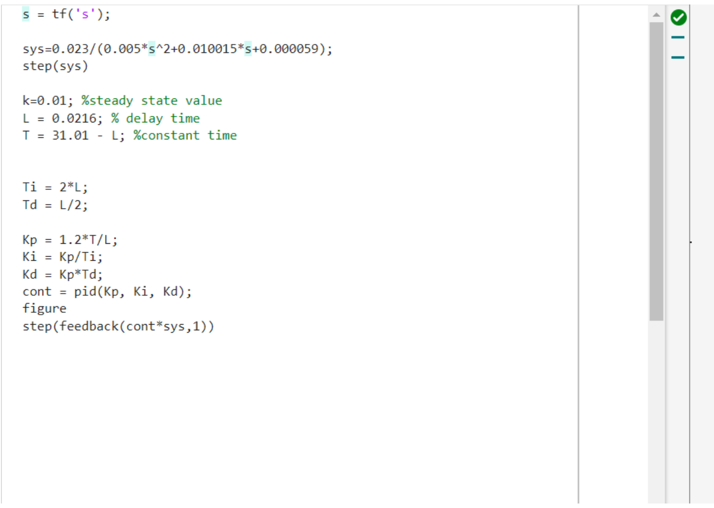
$R_a = 1$, $L_a = 0.5$, $J = 0.01$, $B = 0.00003$, $K_t = 0.023$, $K_b = 0.023$

$$G_3(s) = \frac{0.023}{0.005s^2 + 0.010015s + 0.000059}$$

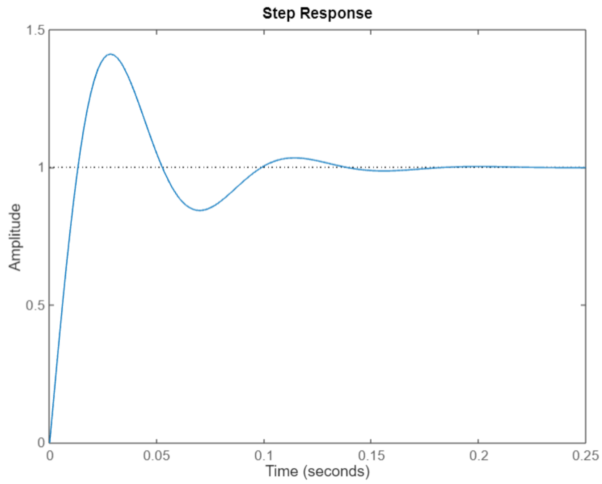
Code:

```
s = tf('s');

sys=0.023/(0.005*s^2+0.010015*s+0.000059);
step(sys)

k=0.01; %steady state value
L = 0.0216; % delay time
T = 31.01 - L; %constant time
Ti = 2*L;
Td = L/2;
Kp = 1.2*T/L;
Ki = Kp/Ti;
Kd = Kp*Td;
cont = pid(Kp, Ki, Kd);
figure
step(feedback(cont*sys,1))
```

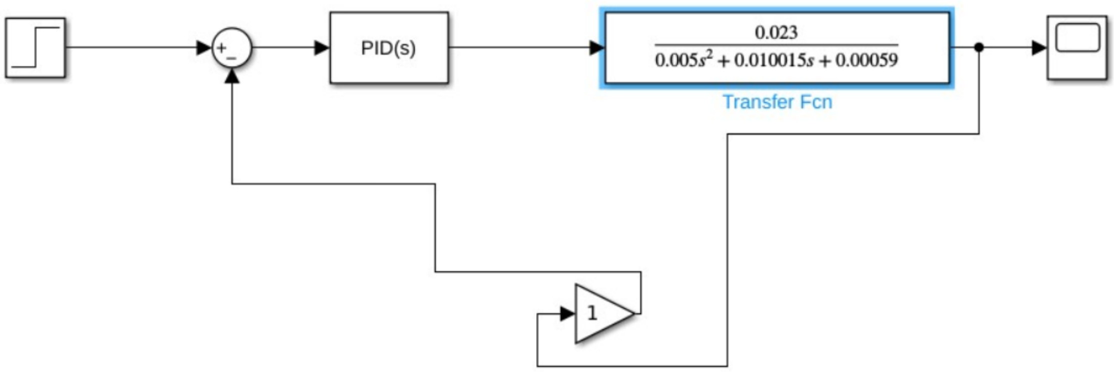


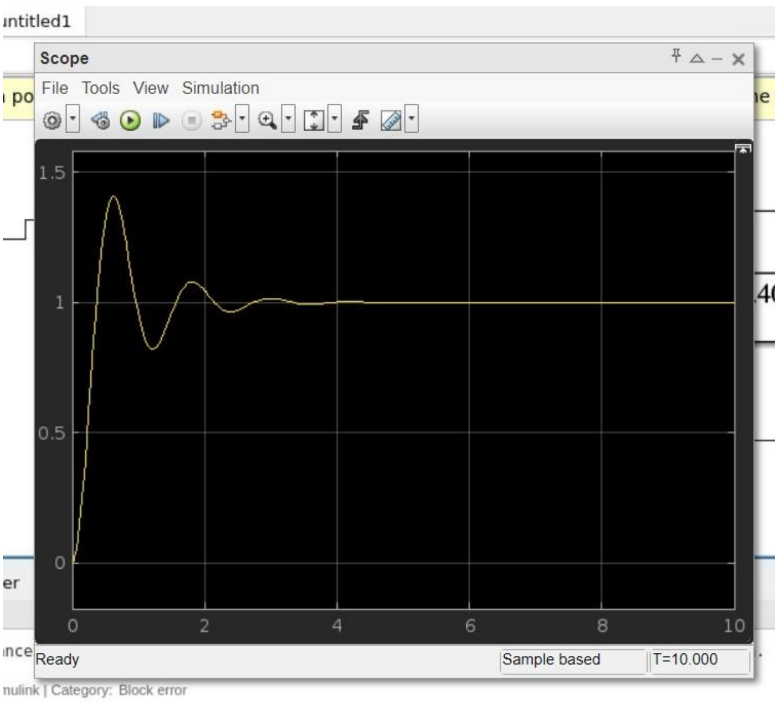


```
s = tf('s');

sys=0.023/(0.005*s^2+0.010015*s+0.000059);
step(sys)

k=0.01; %steady state value
L = 0.0216; % delay time
T = 31.01 - L; %constant time


Ti = 2*L;
Td = L/2;

Kp = 1.2*T/L;
Ki = Kp/Ti;
Kd = Kp*Td;
cont = pid(Kp, Ki, Kd);
figure
step(feedback(cont*sys,1))
```

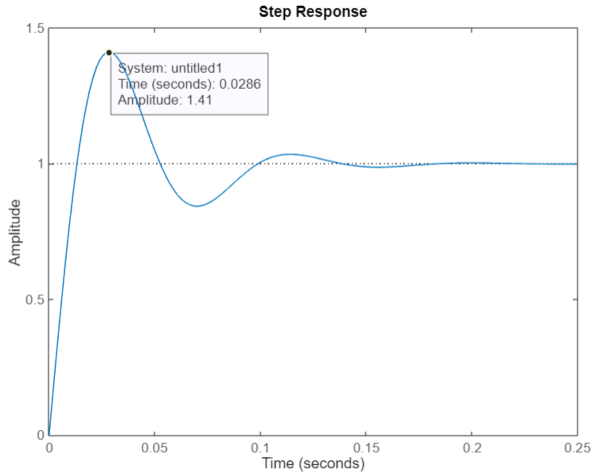


Schematic Diagram:

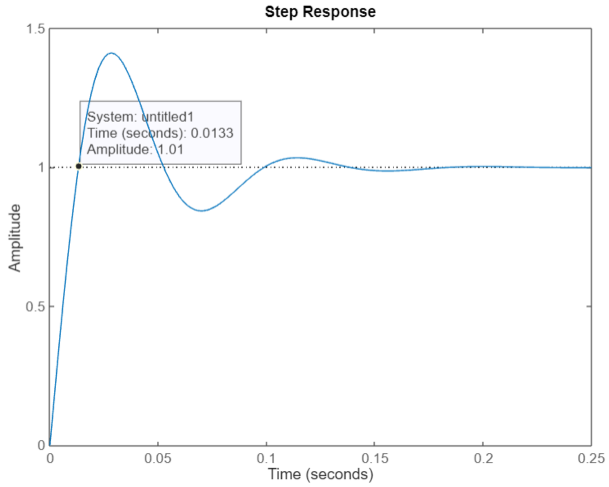




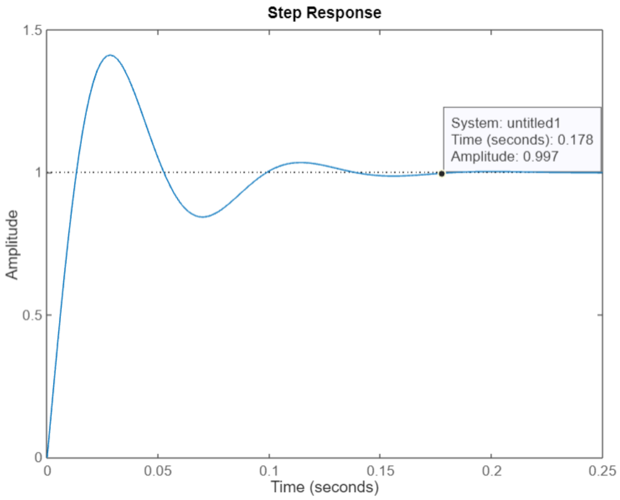
Over Shoot:-



Rising Time:-



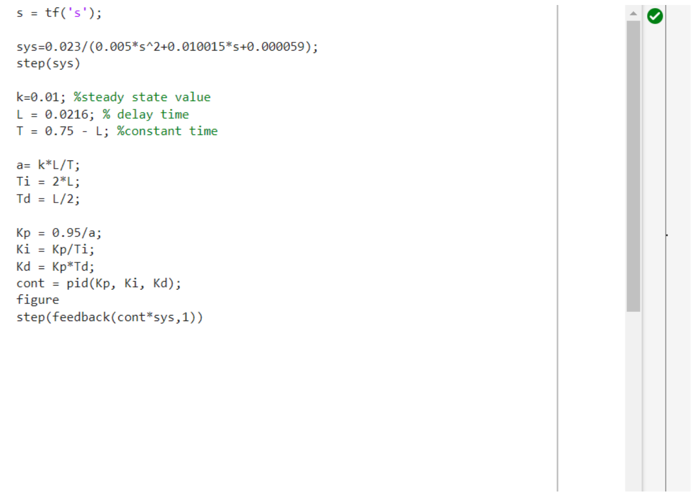
Settling Time:-

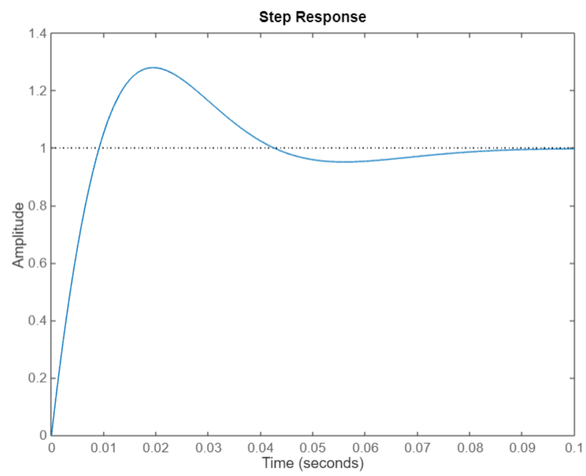


Motor 3 (Modified):-

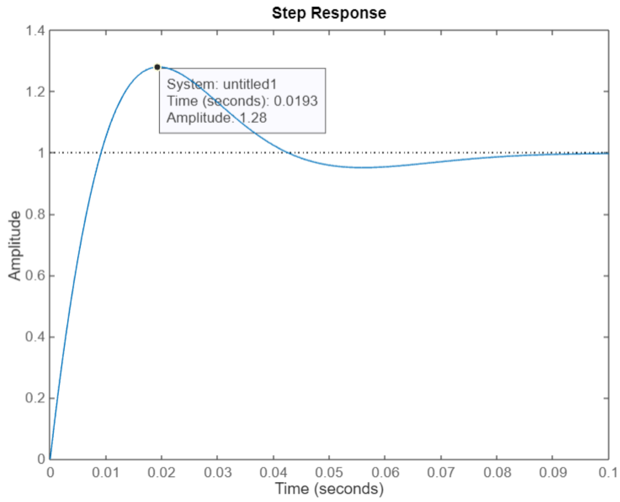
Code:-

```
s = tf('s');

sys=0.023/(0.005*s^2+0.010015*s+0.000059);
step(sys)

k=0.01; %steady state value
L = 0.0216; % delay time
T = 0.75 - L; %constant time

a= k*L/T;
Ti = 2*L;
Td = L/2;

Kp = 0.95/a;
Ki = Kp/Ti;
Kd = Kp*Td;
cont = pid(Kp, Ki, Kd);
figure
step(feedback(cont*sys,1))
```

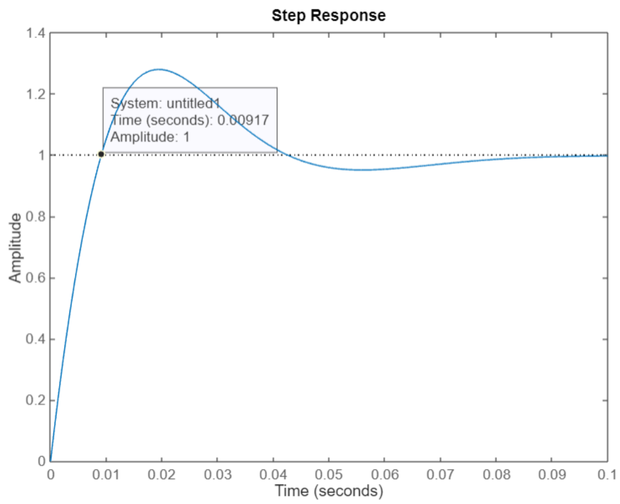




Over Shoot:-



Rising Time:-



Settling Time:-