Rice's Theorem (Corrected Proof)

Theorem. Let P be a nontrivial property of recursively enumerable (r.e.) languages. Define

T_P = { | P(L(M)) = T }.

Then T_P is not recursive.

Proof. Since P is nontrivial, there exist r.e. languages B and E with $P(B)=T$ and $P(E)=F$.

Assume without loss of generality that $P(\varnothing)=F$. Choose a Turing machine K that accepts B.

We reduce the Halting Problem HP = { #x | M halts on x } to T_P.

Given input #x, construct a machine M0 that on input y:

1. Simulates M on x.

2. If M halts on x, M0 simulates K on y and accepts iff K accepts y.

3. If M does not halt on x, M0 accepts nothing.

If M halts on x, then $L(M0)=B$, so $P(L(M0))=P(B)=T$.

If M does not halt on x, then $L(M0)=\varnothing$, so $P(L(M0))=P(\varnothing)=F$.

Thus $\#x \in$ HP iff $\in$ T_P. Hence HP $\leq\blacksquare$ T_P. Since HP is undecidable,

T_P is not recursive.

Remark (Symmetric case). If $P(\varnothing)=T$, choose r.e. B with $P(B)=F$ and reverse the construction.