

Report of the Phase 1 of the Machine Learning Project (Tomography)

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The code to generate our data is presented in ML_Project2_Phase 1_File2-Haarpure.py

The code for analyzing our data is in Statistical_Analysis_Phase1_Project_Jupyter.ipynb

The code to generate the data with removed measurements is in the last cell of Statistical_Analysis_failure_Phase1_Project_Jupyter.ipynb

1 Report

The data is produced through a simulation. We simulated the experiment of tomography of 2 photonic qubits. We first produced 10^5 random matrices from the Ginibre ensemble and then constructed from that our density operators as proposed in [1]. Furthermore, we made 0.1 of our produced states using Haar measure to produce unitary matrices and then their first column to produce pure states as proposed in [1]. Our aim was to make a machine to be able to predict pure states as well, which otherwise might have been less powerful in doing so, since a random density operator is not pure with a very high probability.

For the simulation of the measurement process, we followed [2] and performed measurements along different polarization directions of each qubit, and then converted the results to make the density operator as proposed in [2]. We added 3 different noise types (comparing to the measured values in the measurements, one is a highly noisy measurement and the other two are less noisy) to the measured values for 3 different datasets in order to see how the machine will deal with noise comparing to classical methods later. Moreover, one of our datasets is comprised of 3 random missing measurements for each density operator, simulating the situation that with a certain probability ($1/5$) our measurement apparatus does not work well and give nothing for the outcome (but we are not aware of that) as in [1]. Finally, to save density operators we did not save the whole matrix, whose elements, for sure, are not all independent. We saved the free parameters of each density operator instead, whose values are independent of each other. For our data, we saved the measurement results (after noise and possible removing of some measurements) as the features to be given to the machine later. Our labels comprise of the free parameters of the

original density matrices which are to be predicted.

2 Sample Data (From the highly Noisy case)

X = [0.25234228, 0.45036674, 0.46333274, 0.37211634, 0.15630952, 0.19391162,
0.26026577, 0.13797004, 0.1167762, 0.02634066, 0.12345927, -0.02967077, 0.20363065,
0.40761875, 0.48656565, 0.2471791]

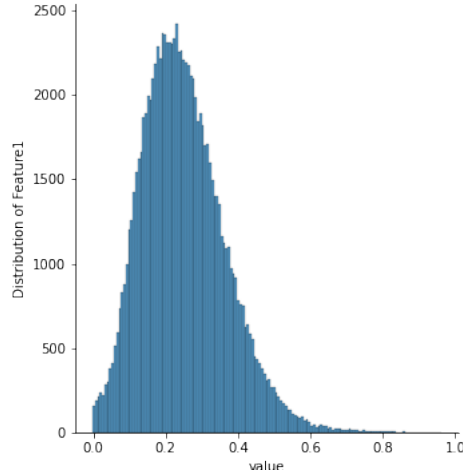
Y = [0.19425208, 0.50409521, 0.43835288, 0.18061491, 0.06024363, 0.24140908,
0.14010594, 0.21822987, 0.19210194, 0.16684037, 0.25768243, 0.12534765, 0.1290543,
0.38609412, 0.41834356, 0.25392471]

3 Statistical Analysis

We provide the results here. The file containing the codes is provided in our github page.

3.1 Distribution of Features

The distribution of feature 1, as an example, is as follows:



The distributions of other features are more or less the same, as we expect from our domain knowledge and the way we produced our data.

3.2 Variance of Features

The variances of different features (in order) are as follows:

0.013619432967901267, 0.013590675479461896, 0.013677388167263113, 0.01361235944114115,
0.013781686732859226, 0.0137870340409087, 0.013684350497896944, 0.013629322326157677,
0.013694537946997506, 0.013611272475921518, 0.013728683944742493, 0.013577678314940328,
0.013802239435494178, 0.013614748469204815, 0.01372581419341496, 0.013693800449181195

As we see, all the features have almost the same variance, which is of course, what we expected since our data was generated randomly and the features are the free parameters of the density operators. So, we may not filter features based on their variance and we had better keep all of them.

3.3 Feature Importance Based on How Important They Are in Predicting Labels

These are the feature importance for the linear regression model based on the code:

```
[0.07911382, 0.08368352, 0.09562909, 0.09797733, 0.08182275, 0.08187128,
0.09720486, 0.09777472, 0.09719424, 0.09815005, 0.0959644 , 0.09783426, 0.09933344,
0.10050695, 0.0967515 , 0.09768291]
```

As we can see, there is not a significant difference in the importance of features, which is, again, to be expected because of the way we have generated and saved our data.

3.4 Correlation Between Different Features

The correlation matrix is produced in the code. Although some features have correlation with each other, that is somewhat what we expected based on the way we generated our data. However, using our domain knowledge, we know that we must not change the features since they are the independent parameters of the matrices.

So, using this and also our domain knowledge we may not use feature transformation and PCA.

4 Conclusion

All in all, as can be inferred from our statistical analysis and also our domain knowledge and specifically the way we produced our data and saved our features, we might not have to manipulate our data before the training. The main reason is that our matrices were randomly generated. Moreover, we have kind of done this "feature engineering" before, since our features are the independent parameters of these matrices, which are expected to be independent from each other.

5 References

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