DEMO2002

Population Projection

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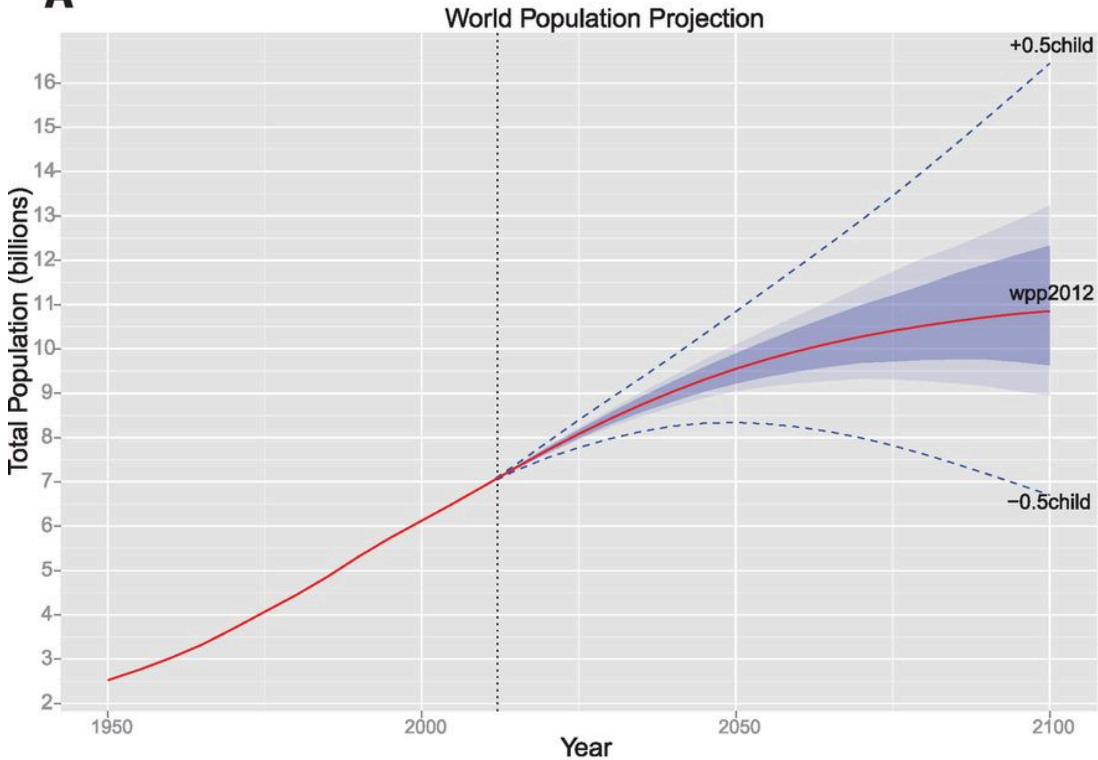
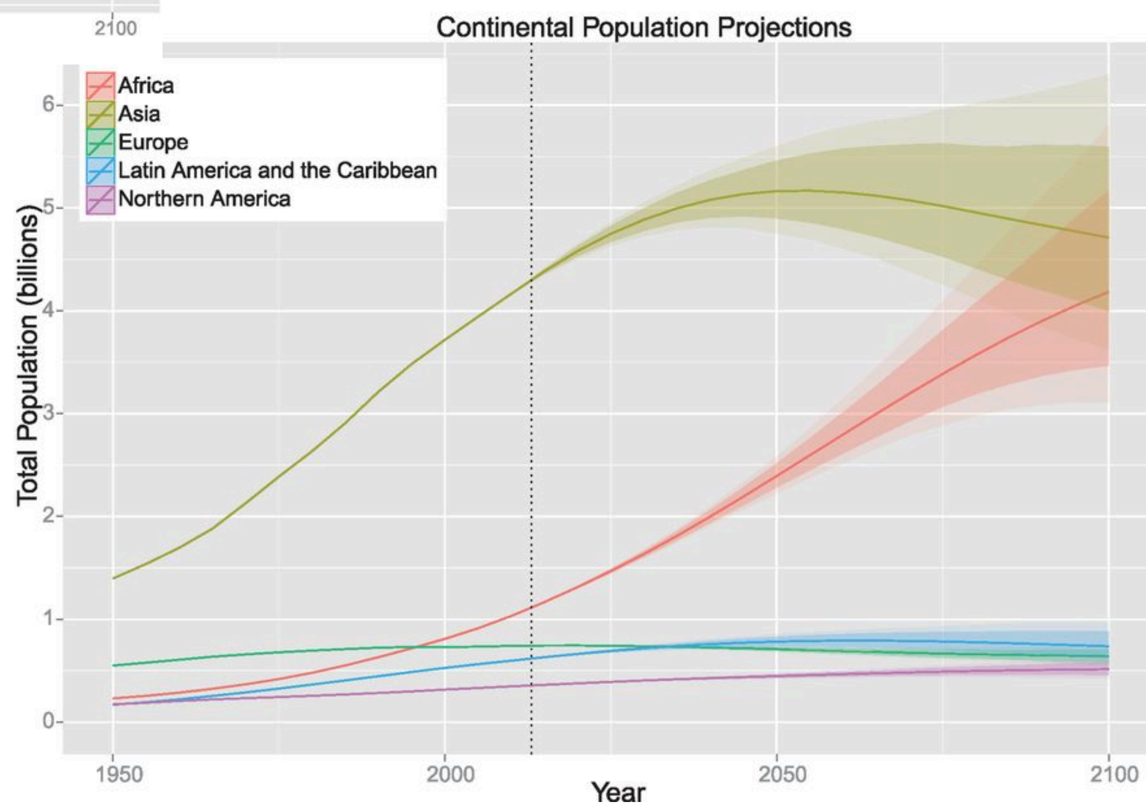


Fig. 1 World and continental population projections.



Patrick Gerland et al. Science 2014;346:234-237

PROJECTIONS

Population projection is one of the most frequently requested outputs from clients:

- Governments: future for roads, schools, medical personnel, etc.
- Businesses: future market size.
- Tax policies.

Also used by demographers to analyse the implications of a certain set of demographic parameters for population size, composition, and growth.

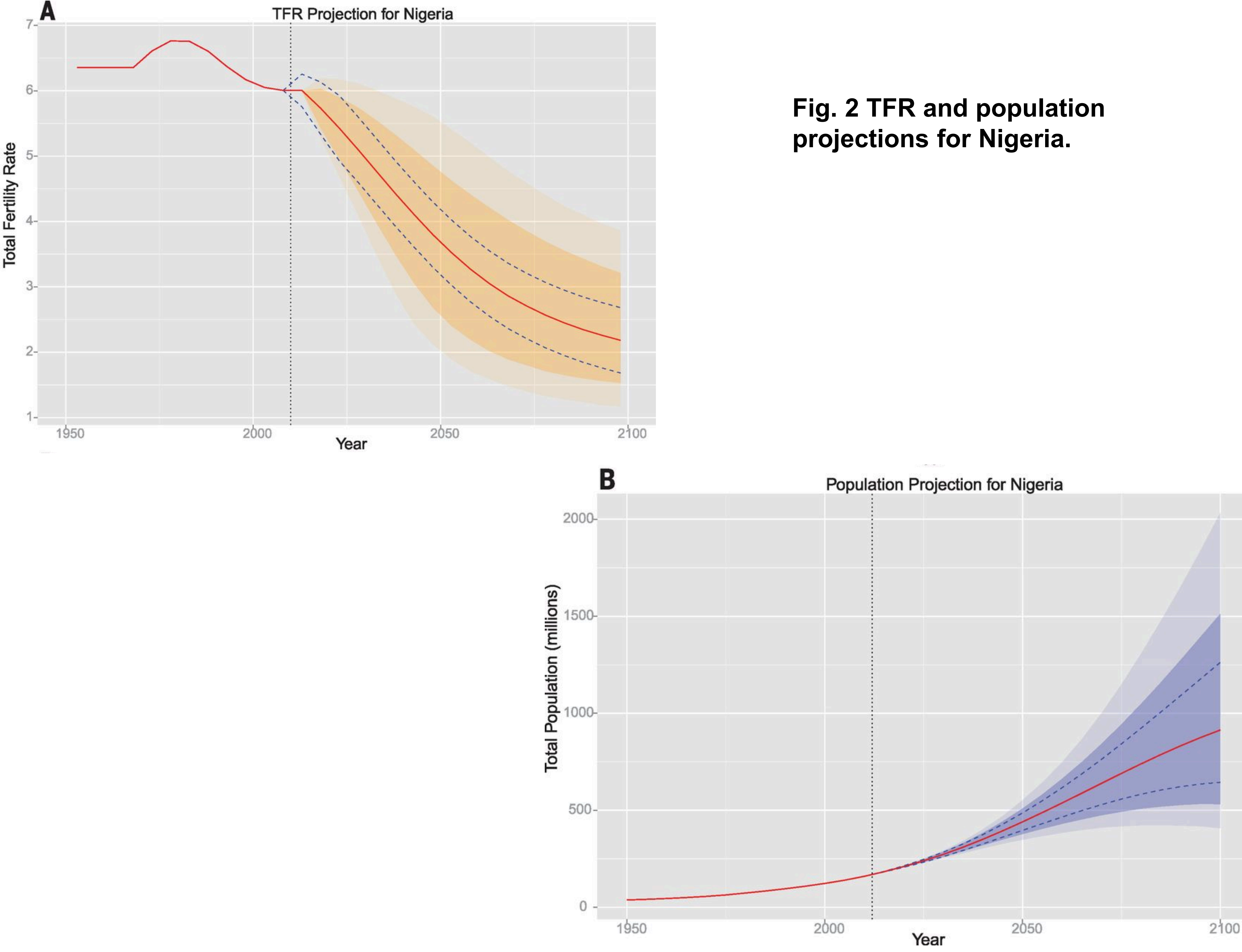


Fig. 2 TFR and population projections for Nigeria.

WORLD POPULATION PROSPECTS

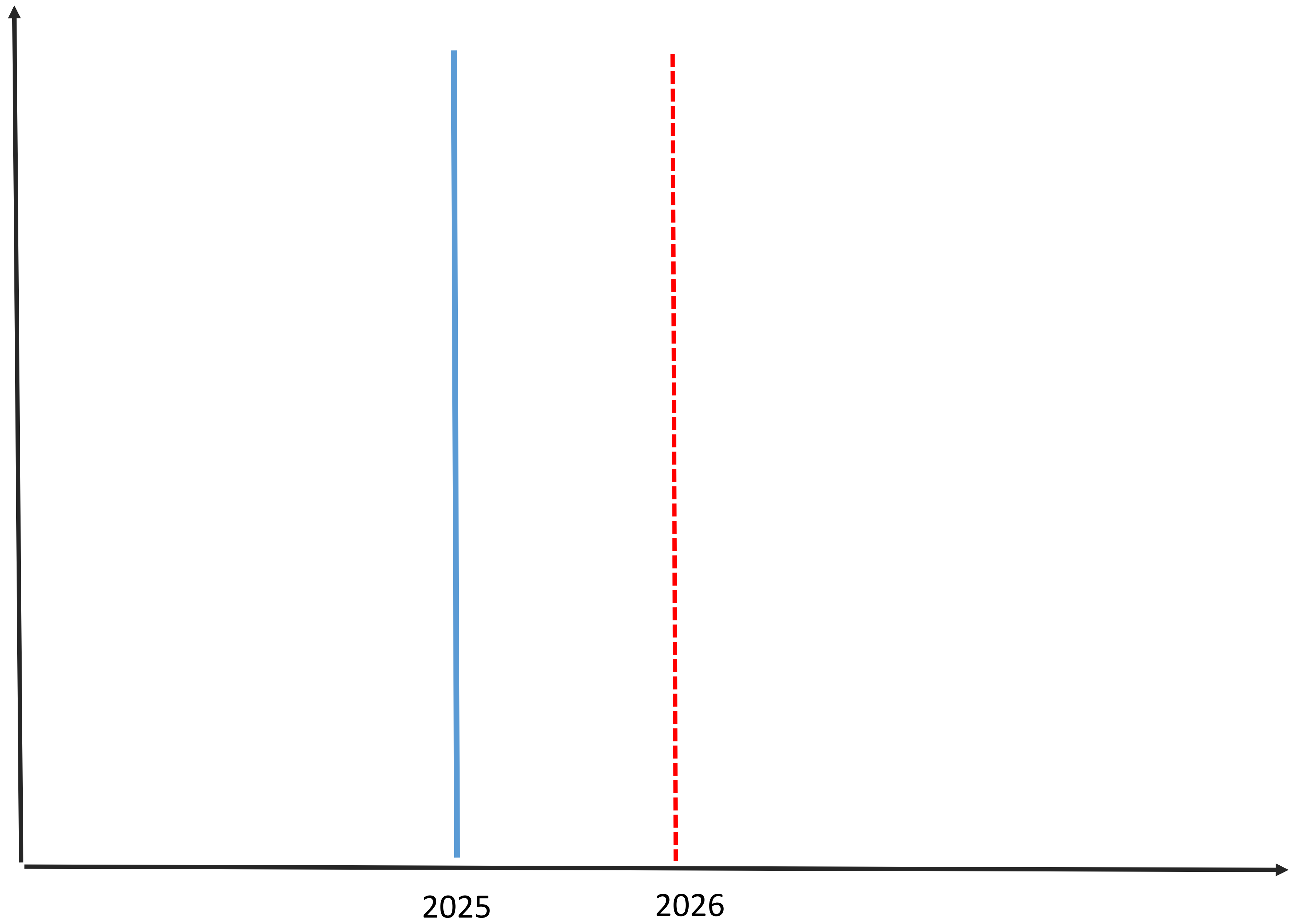
2024 Revision of World Population Prospects

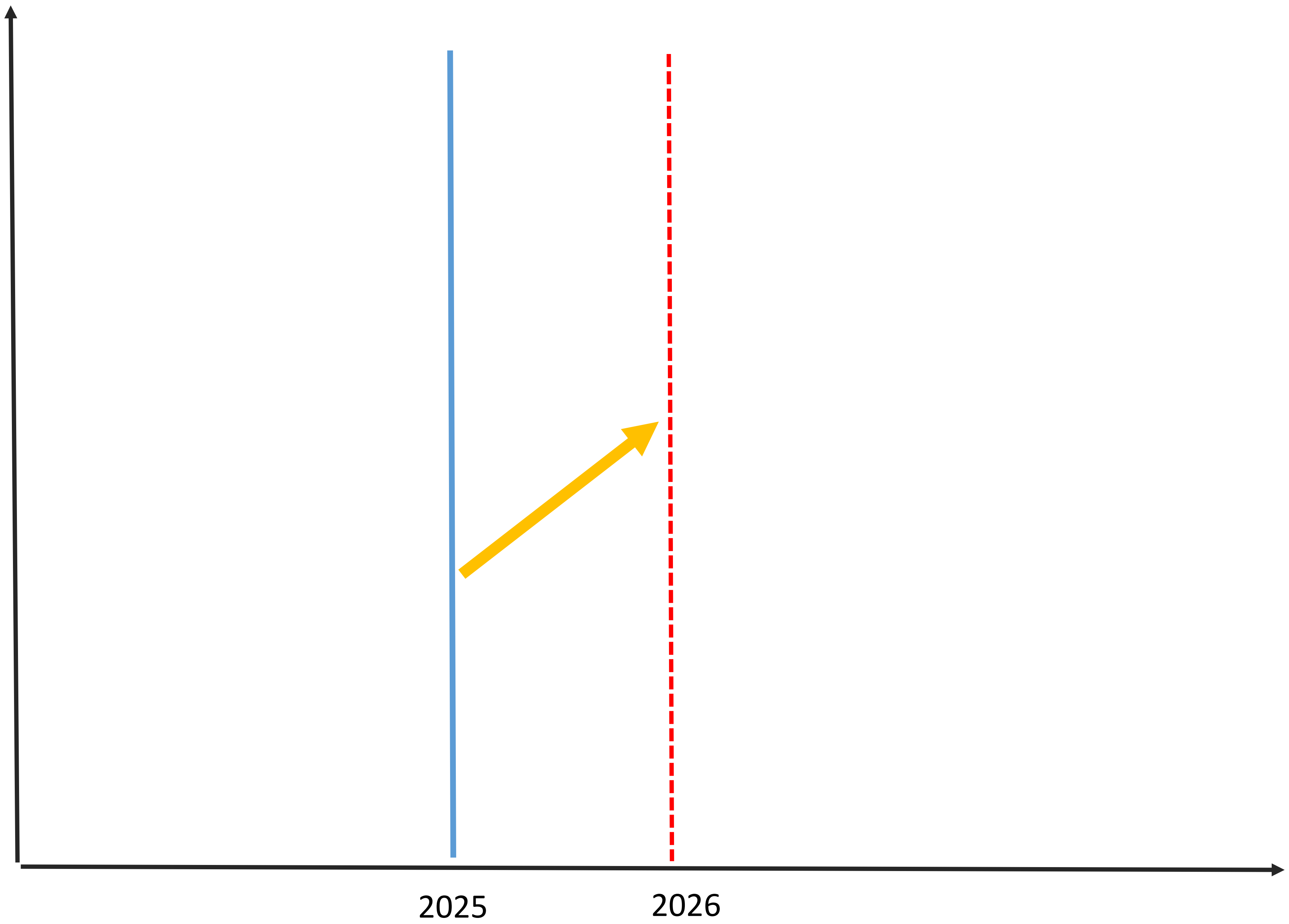
http://esa.un.org/unpd/wpp/

OVERVIEW

Projects fertility, mortality and international migration up to the year 2100 for 233 countries.

- 150-year time horizon, past (1950) and future (2100).
  - Past: Base population in 1950 advancing in 5-year intervals using the cohort component method. Estimates of components taken from national sources or estimated where partial data available.
  - Future: Base population in 2023.


2025
2026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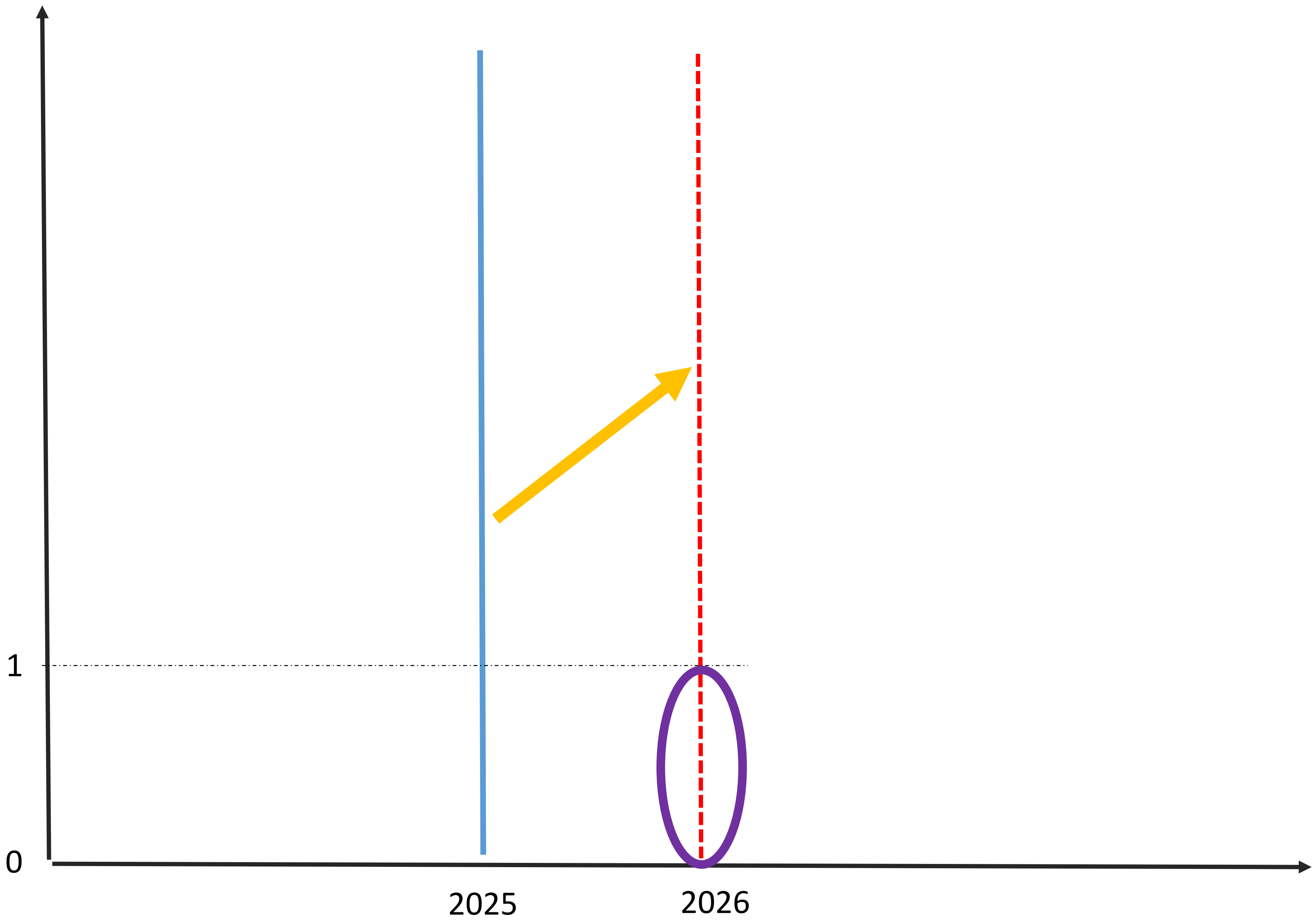
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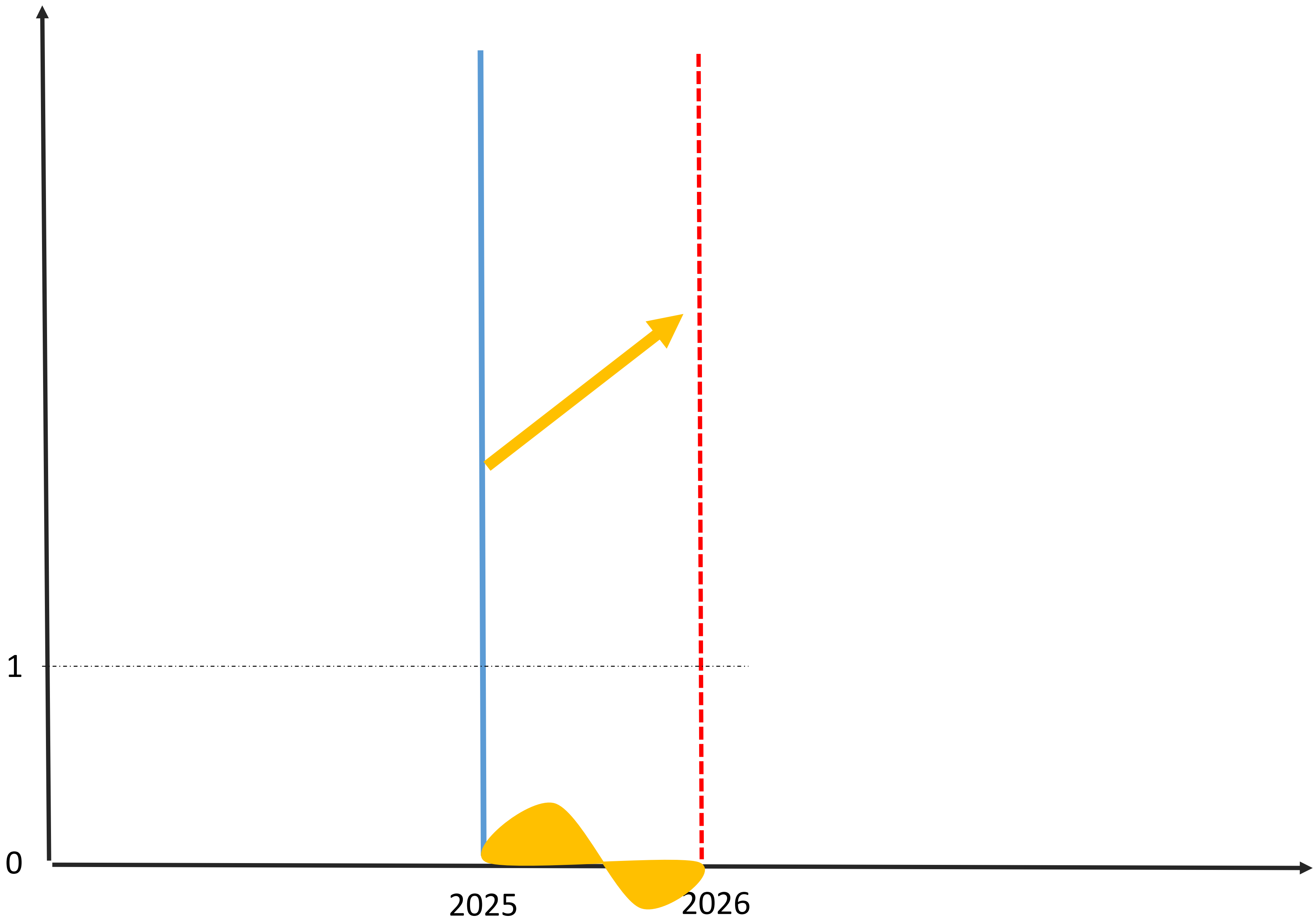
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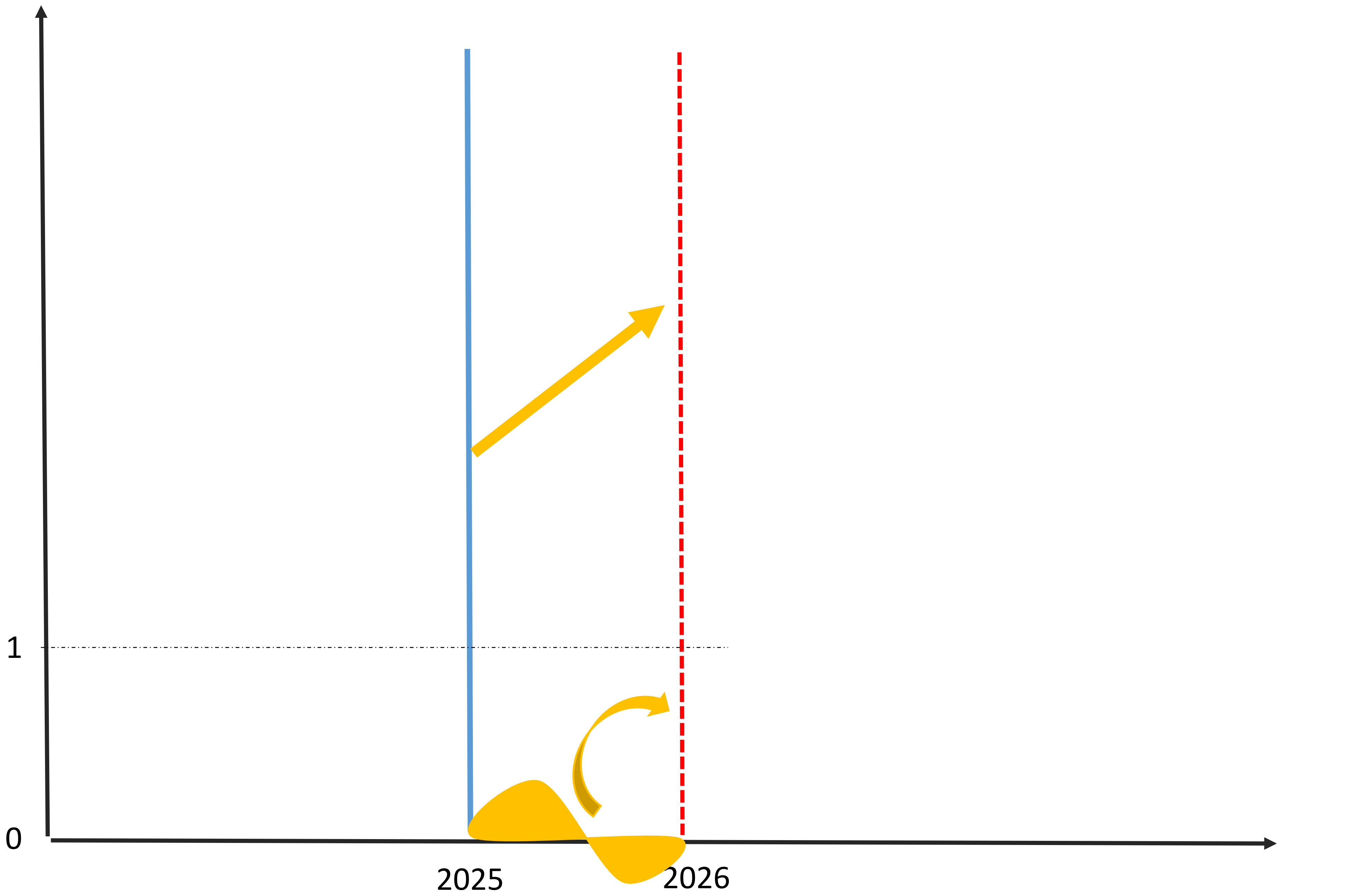

1
0
2025
2026

Population Projection

$$P_0^{t+1} =$$

$$P_1^{t+1} =$$

$$P_2^{t+1} =$$

$$P_3^{t+1} =$$

$$P_4^{t+1} =$$

Population Projection

$$P_0^{t+1} =$$

$$P_1^{t+1} =$$

$$P_2^{t+1} =$$

$$P_3^{t+1} =$$

$$P_4^{t+1} =$$

$$f_1$$

$$f_2$$

$$f_3$$

Population Projection

$$P_0^{t+1} = P_1^t f_1 + P_2^t f_2 + P_3^t f_3$$

$$P_1^{t+1} =$$

$$P_2^{t+1} =$$

$$P_3^{t+1} =$$

$$P_4^{t+1} =$$

Population Projection

$$P_0^{t+1} = P_1^t f_1 + P_2^t f_2 + P_3^t f_3$$

$$P_1^{t+1} = P_0^t S_0$$

$$P_2^{t+1} = P_1^t S_1$$

$$P_3^{t+1} = P_2^t S_2$$

$$P_4^{t+1} = P_3^t S_3$$

Matrix Representation

Patrick Holt Leslie

England (1900–1974)

The Leslie matrix is a discrete model of population growth in time as a function of the mortality and fertility schedules of a population

Leslie PH. 1945. “On the use of matrices in certain population mathematics.” *Biometrika* 33: 183–212.

Matrix Representation

$$
\begin{aligned}
P_0^{t+1} &= P_1^t f_1 + P_2^t f_2 + P_3^t f_3 \\
P_1^{t+1} &= P_0^t S_0 \\
P_2^{t+1} &= P_1^t S_1 \\
P_3^{t+1} &= P_2^t S_2 \\
P_4^{t+1} &= P_3^t S_3
\end{aligned}
$$

Matrix Representation

$$
\begin{aligned}
P_0^{t+1} &= P_0^t\, 0 + P_1^t f_1 + P_2^t f_2 + P_3^t f_3 \\
P_1^{t+1} &= P_0^t S_0 \\
P_2^{t+1} &= P_1^t S_1 \\
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P_4^{t+1} &= P_3^t S_3
\end{aligned}
$$

Matrix Representation

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$$P_1^{t+1} = P_0^t S_0$$

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$$P_3^{t+1} = P_0^t\,0 + P_2^t S_2$$

$$P_4^{t+1} = P_0^t\,0 + P_3^t S_3$$

Matrix Representation

$$P_0^{t+1} = P_0^t\, 0 + P_1^t f_1 + P_2^t f_2 + P_3^t f_3$$

$$P_1^{t+1} = P_0^t S_0 + P_1^t\, 0 +$$

$$P_2^{t+1} = P_0^t\, 0 + P_1^t S_1$$

$$P_3^{t+1} = P_0^t\, 0 + P_1^t\, 0 + P_2^t S_2$$

$$P_4^{t+1} = P_0^t\, 0 + P_1^t\, 0 \qquad P_3^t S_3$$

Matrix Representation

$$P_0^{t+1} = P_0^t\, 0 + P_1^t f_1 + P_2^t f_2 + P_3^t f_3$$

$$P_1^{t+1} = P_0^t S_0 + P_1^t\, 0 + P_2^t\, 0 +$$

$$P_2^{t+1} = P_0^t\, 0 + P_1^t S_1 + P_2^t\, 0 +$$

$$P_3^{t+1} = P_0^t\, 0 + P_1^t\, 0 + P_2^t S_2 +$$

$$P_4^{t+1} = P_0^t\, 0 + P_1^t\, 0 + P_2^t\, 0 + P_3^t S_3$$

Matrix Representation

$$\begin{pmatrix} P_0^{t+1} \\ P_1^{t+1} \\ P_2^{t+1} \\ P_3^{t+1} \\ P_4^{t+1} \end{pmatrix} = \begin{array}{c} \begin{array}{ccccc} P_0^t & P_1^t & P_2^t & P_3^t & P_4^t \end{array} \\ \begin{pmatrix} 0 & f_1 & f_2 & f_3 & 0 \\ S_0 & 0 & 0 & 0 & 0 \\ 0 & S_1 & 0 & 0 & 0 \\ 0 & 0 & S_2 & 0 & 0 \\ 0 & 0 & 0 & S_3 & 0 \end{pmatrix} \end{array}$$

Matrix Representation

$$\begin{pmatrix} P_0^{t+1} \\ P_1^{t+1} \\ P_2^{t+1} \\ P_3^{t+1} \\ P_4^{t+1} \end{pmatrix} = \begin{pmatrix} 0 & f_1 & f_2 & f_3 & 0 \\ S_0 & 0 & 0 & 0 & 0 \\ 0 & S_1 & 0 & 0 & 0 \\ 0 & 0 & S_2 & 0 & 0 \\ 0 & 0 & 0 & S_3 & 0 \end{pmatrix} \begin{pmatrix} P_0^{t} \\ P_1^{t} \\ P_2^{t} \\ P_3^{t} \\ P_4^{t} \end{pmatrix}$$

Matrix Representation

- To get the population at a later date

$$P^{t+1} = L\,P^{t}$$

Matrix Representation

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Matrix Representation

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$$P^{t+2} = L\, P^{t+1}$$

Matrix Representation

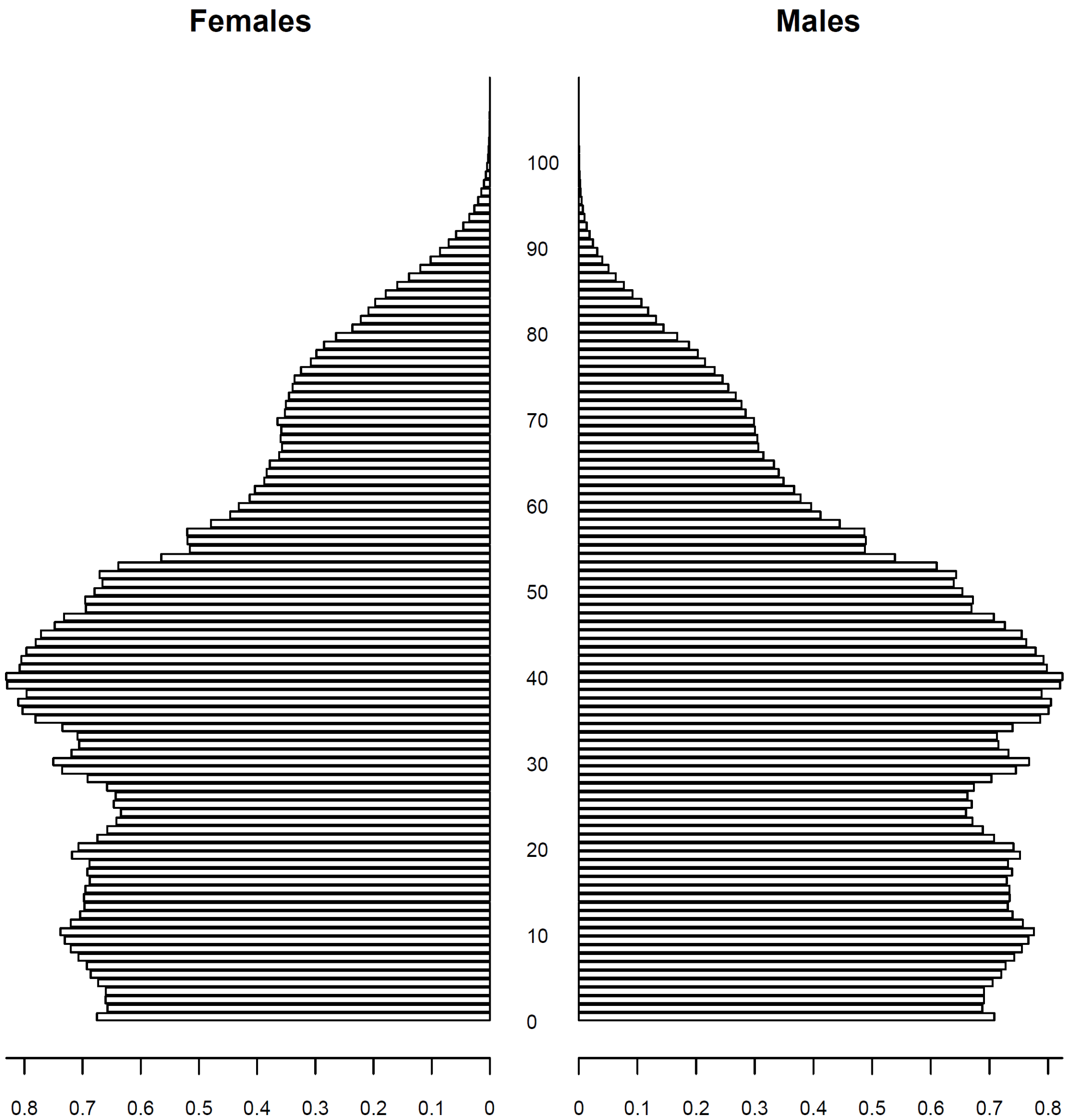
- To get the population at a later date

$$P^{t+1} = L\,P^{t}$$

$$P^{t+2} = L\,P^{t+1} = L^{2}P^{t}$$

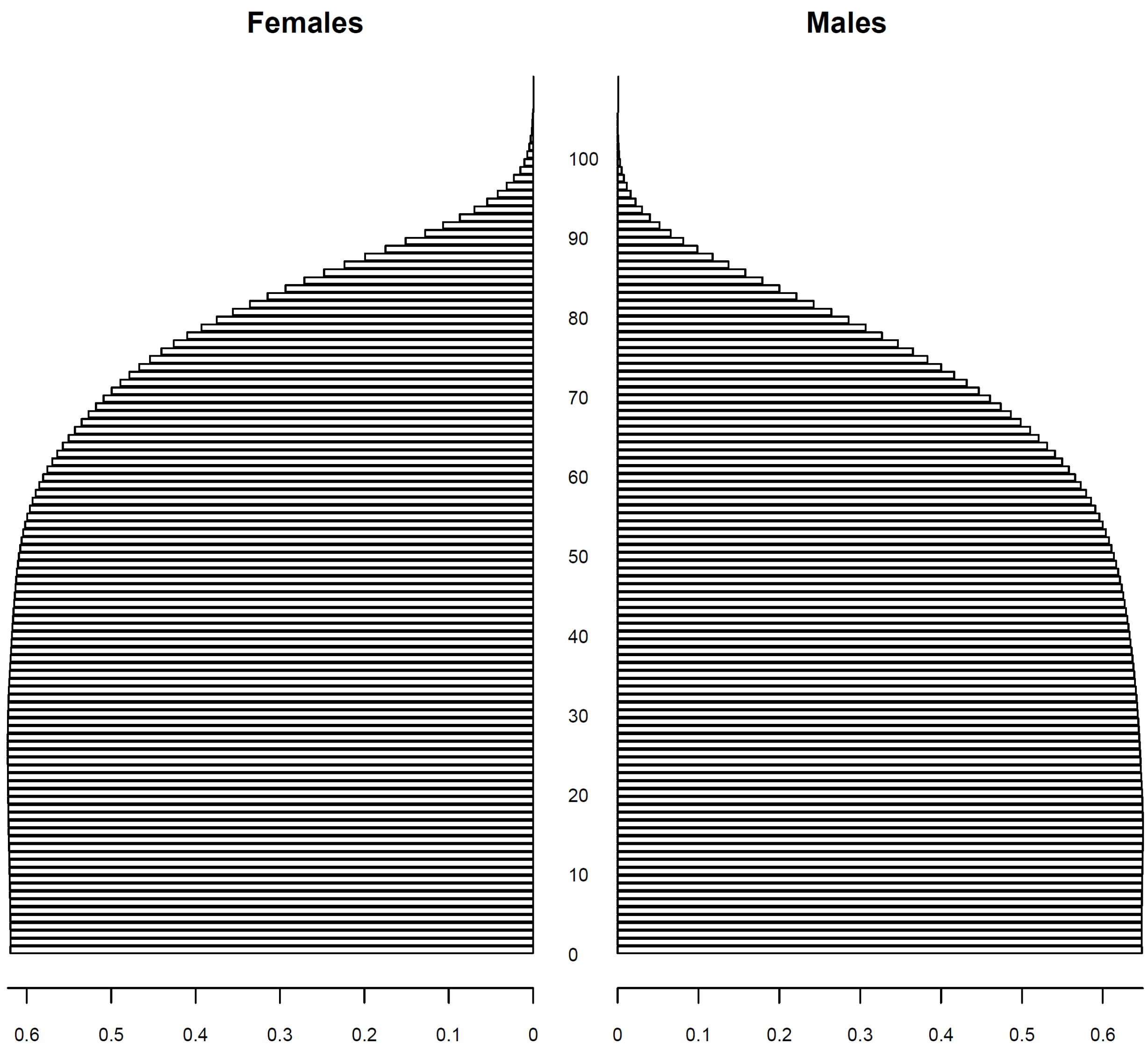
Age Distribution & Rates

Population Pyramid for the United States of America, 2000



Age Distribution & Rates

Population Pyramid for the United States of America, 2100



Population Projection

- To get the population at a later date

$$P^{t+n} = L^{n} P^{t}$$

Cohort Component Method

- Has become the standard methodology for projection
- Makes explicit the assumptions regarding the components of population growth—mortality, fertility, and net migration
- Gives insight into the way population changes

Cohort Component Method

- Allows the user to estimate the effect of alternative levels of fertility, mortality, or migration on population growth
- Used to obtain projections of age-sex structure

Australian National University

General Principles

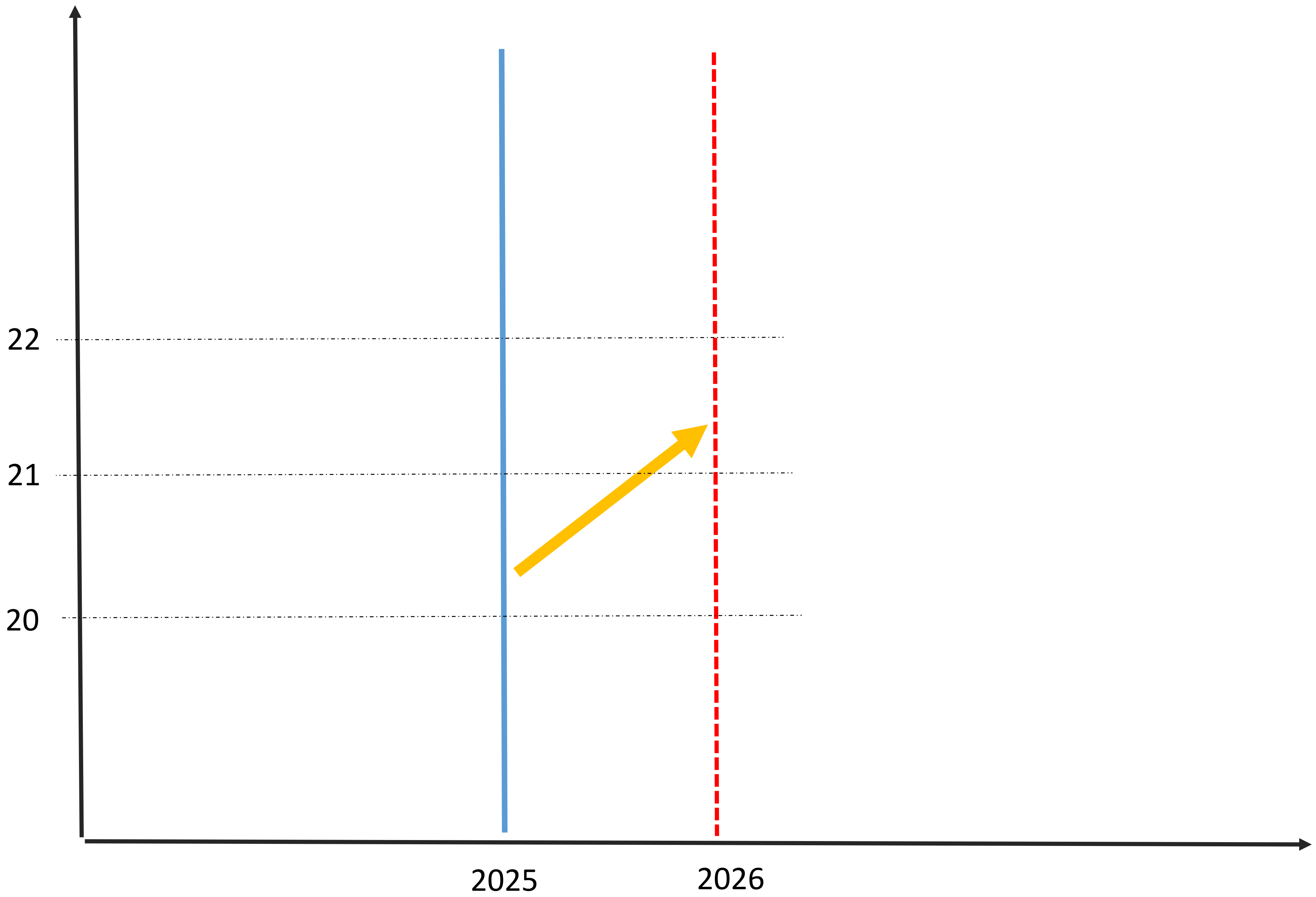
- Start with the population distributed by age and sex at base date
- Apply assumed survival rates and age-sex specific fertility rates to obtain number of persons alive at the end of a unit of time

General Principles

- Make allowance for net migration by age and sex, if desired
- Projection interval must be integer multiple of age interval

Steps

1. Estimation of survival ratio
2. Estimation of individuals surviving to the end of the projection
3. Estimation of births over the projection period
4. Distribution of the newborn by sex
5. Add migration estimates


22
21
20
2025
2026

Population Projection

- Projection of the population aged 20 to 21 from 2025 to 2026

$$P_{21}^{2026} = P_{20}^{2025} \frac{L_{21}}{L_{20}}$$

Survival ratio

– Let x = Age 0,1, 2, …, ω

L_x = life table number of persons at age x

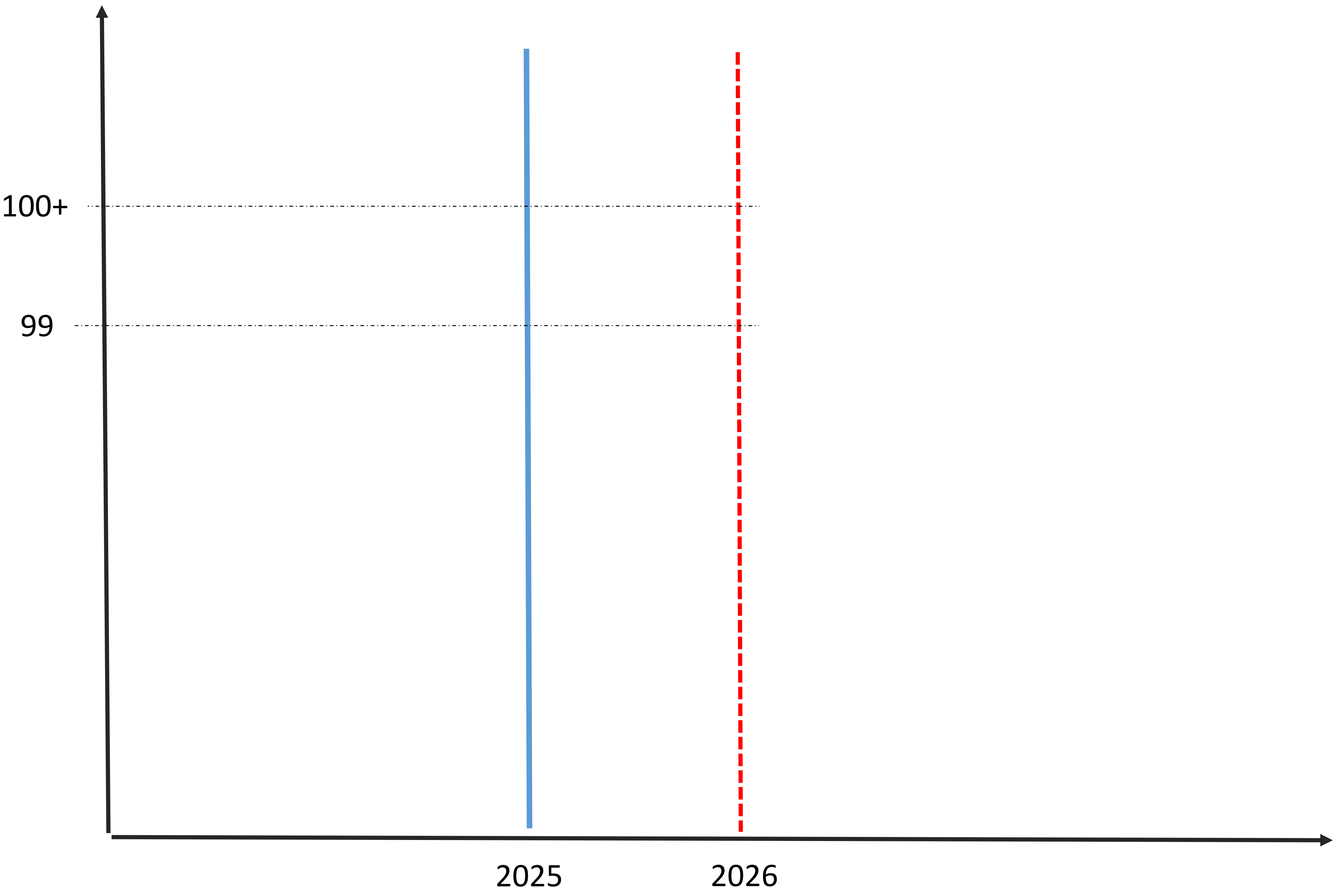
$\frac{L_{x+1}}{L_x}$ = Survival ratio from age x to x+1

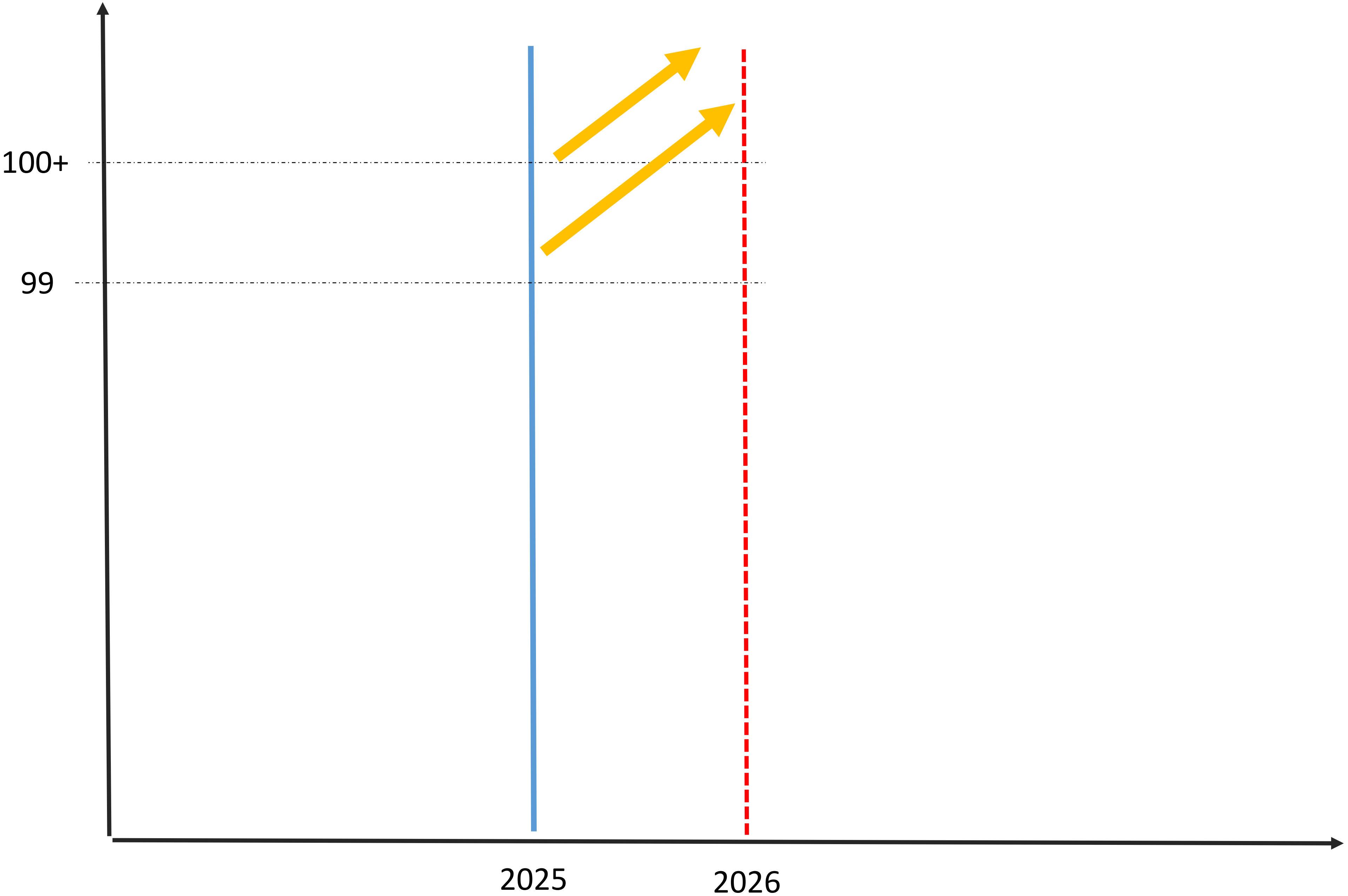
More general:

$$P_{x+1}^{t+1} =$$

More general:

$$P_{x+1}^{t+1} = P_x^t \frac{L_{x+1}}{L_x}$$


100+
99
2025
2026


100+
99
2025
2026

Open-Ended Interval

$$P_{\omega}^{t+1} = \left(P_{\omega-1}^{t} + P_{\omega}^{t}\right)\frac{T_{\omega}}{T_{\omega-1}}$$

- Where ω refers to the beginning age of the oldest age group

Continued

Matrix Representation

$$
L = \begin{bmatrix}
0 & 0 & 0 & 0 & \ldots & 0 & 0 \\
\frac{L_1}{L_0} & 0 & 0 & 0 & \ldots & 0 & 0 \\
0 & \frac{L_2}{L_1} & 0 & 0 & \ldots & 0 & 0 \\
\vdots & & & & & & \\
0 & 0 & 0 & 0 & \ldots & \frac{T_\omega}{T_{\omega-1}} & \frac{T_\omega}{T_{\omega-1}}
\end{bmatrix}
$$

Steps

1. Estimation of survival ratio
2. Estimation of individuals surviving to the end of the projection
3. Estimation of births over the projection period
4. Distribution of the newborn by sex
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Recommended Reading

- Preston et al. (2001). Chapter 6.
- PAPP101 - S10: Population projectionsPAPP101- S10