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DEMO

Population Projection... II

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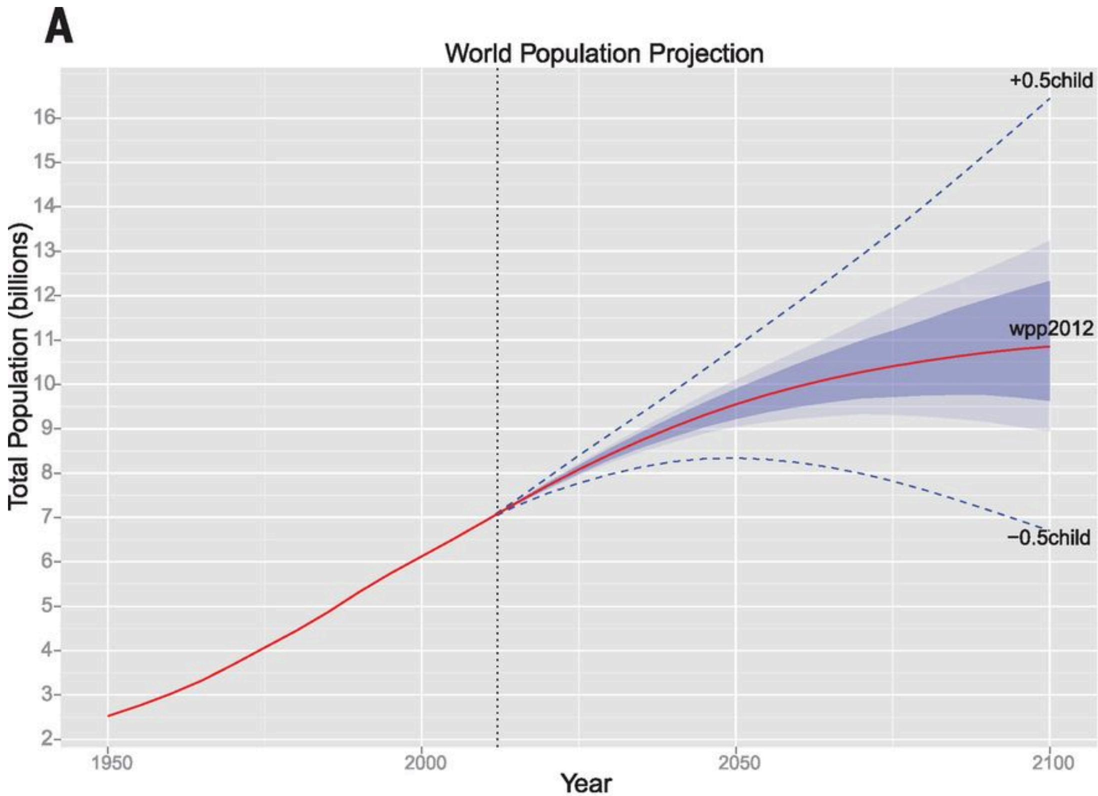
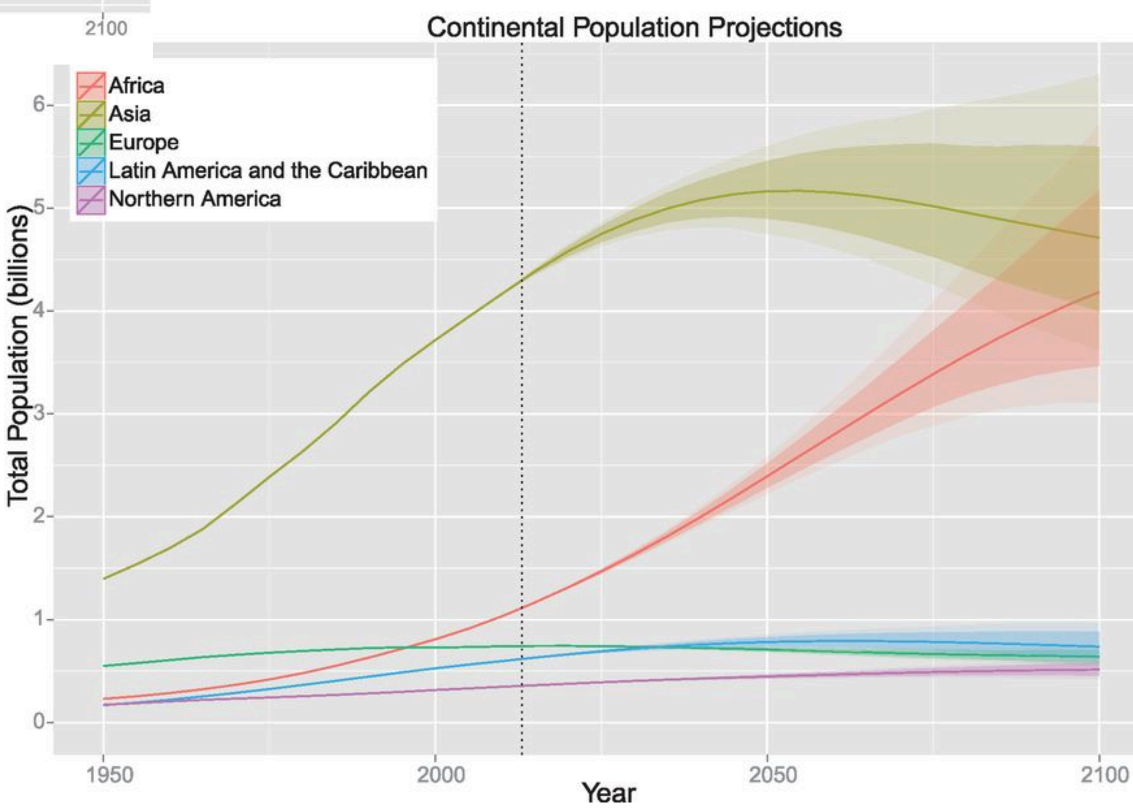
Fig. 1 World and continental population projections.



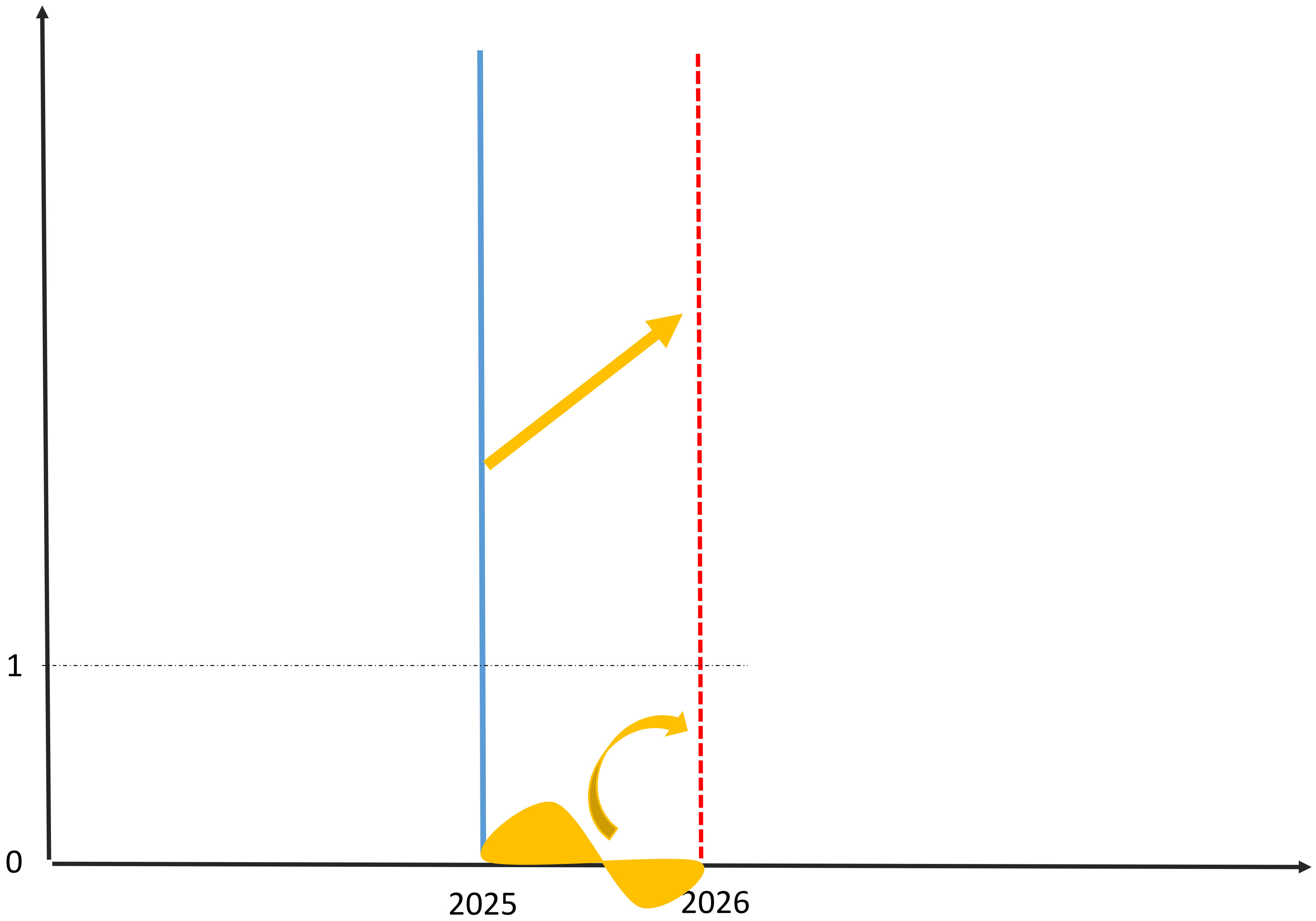


Patrick Gerland et al. Science 2014;346:234-237

OVERVIEW

Projects fertility, mortality and international migration up to the year 2100 for 233 countries.

- 150-year time horizon, past (1950-2015) and future (2015-2100).
  - Past: Base population in 1950 advancing in 5-year intervals using the cohort component method. Estimates of components taken from national sources or estimated where partial data available.
  - Future: Base population in 2015.

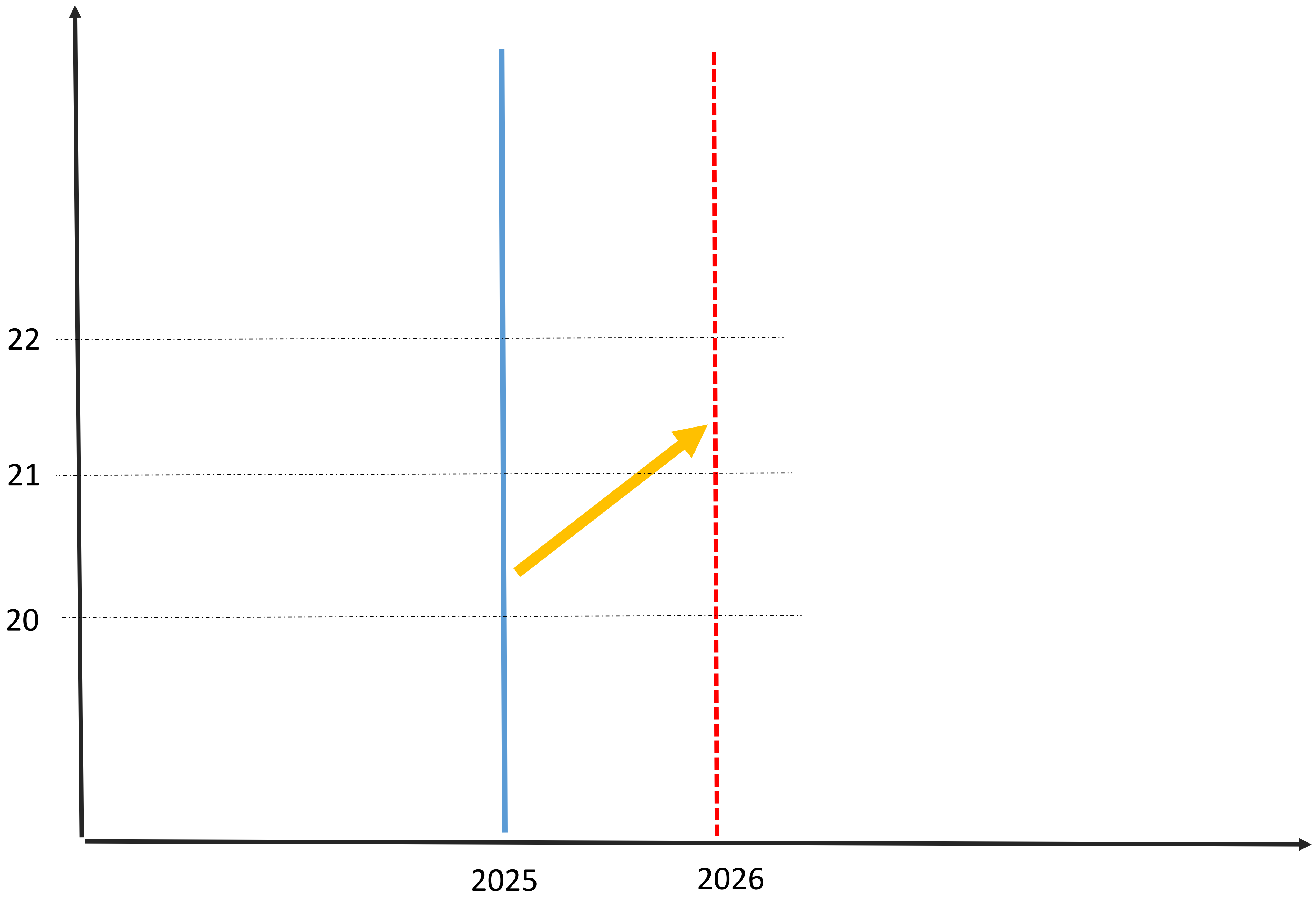


1
0
2025
2026

Cohort Component Method

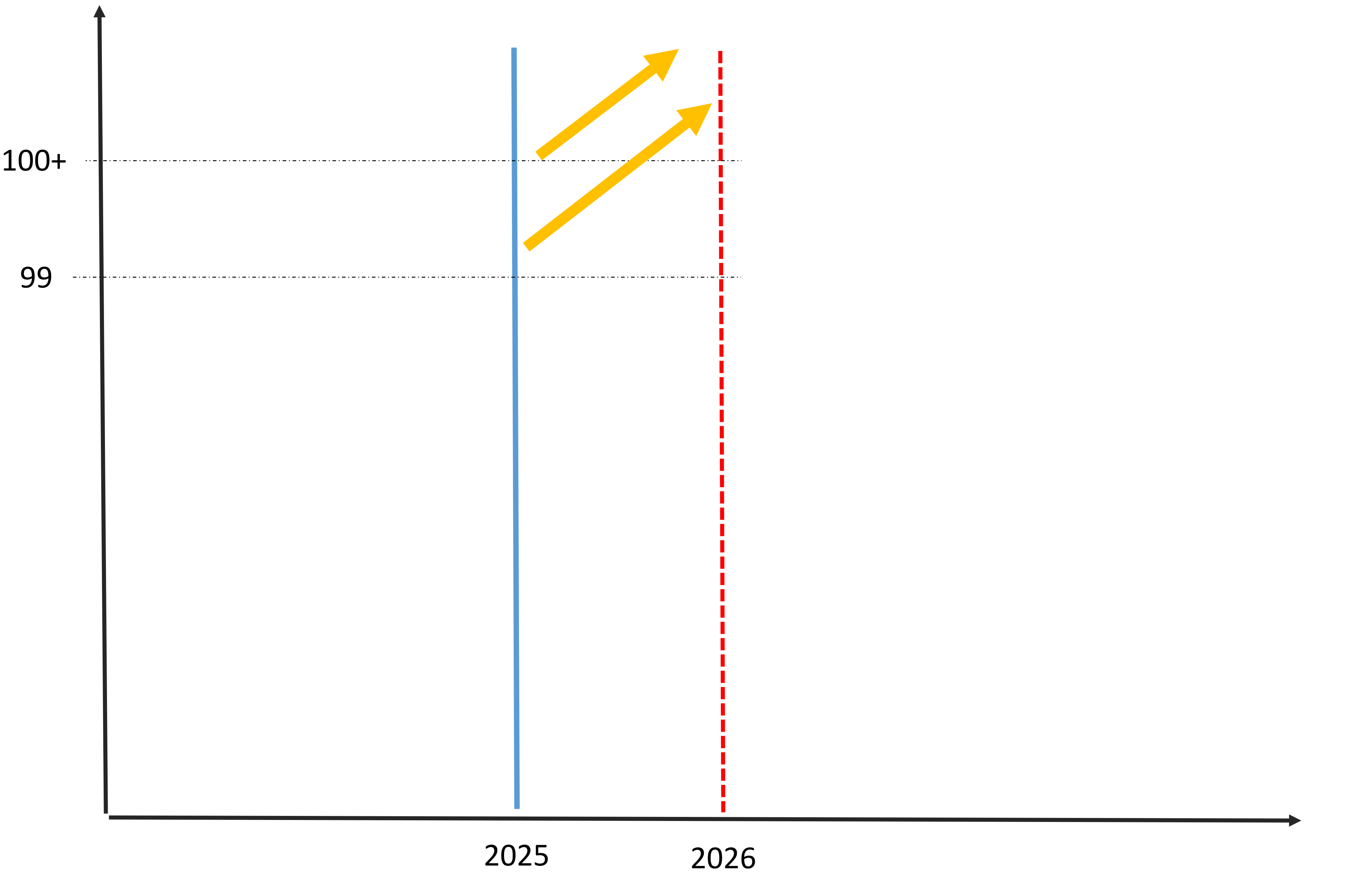
Steps

1. Estimation of survival ratio
2. Estimation of individuals surviving to the end of the projection
3. Estimation of births over the projection period
4. Distribution of the newborn by sex
5. Add migration estimates


22
21
20
2025
2026

More general:

$$P_{x+1}^{t+1} = P_x^t \frac{L_{x+1}}{L_x}$$


100+
99
2025
2026

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Open-Ended Interval

$$P_{\omega}^{t+1} = \left(P_{\omega-1}^{t} + P_{\omega}^{t}\right)\frac{T_{\omega}}{T_{\omega-1}}$$

- Where ω refers to the beginning age of the oldest age group

Continued

Matrix Representation

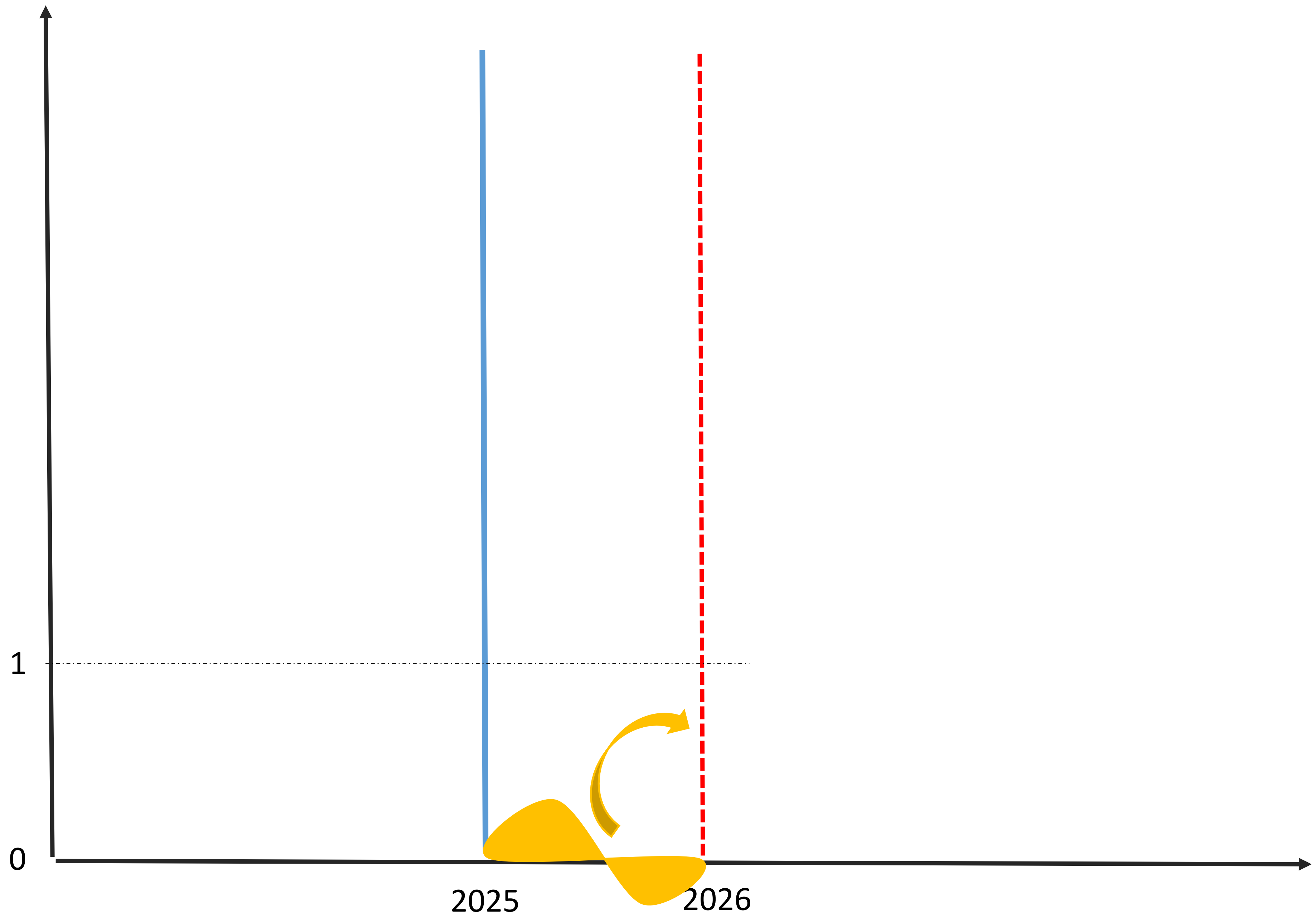
Where the survival ratios are calculated from a life table for males

$$
L_m = \begin{bmatrix}
0 & 0 & 0 & 0 & \ldots & 0 & 0 \\
\frac{L_1}{L_0} & 0 & 0 & 0 & \ldots & 0 & 0 \\
0 & \frac{L_2}{L_1} & 0 & 0 & \ldots & 0 & 0 \\
\vdots & & & & & & \\
0 & 0 & 0 & 0 & \ldots & \frac{T_\omega}{T_{\omega-1}} & \frac{T_\omega}{T_{\omega-1}}
\end{bmatrix}
$$

Matrix Representation

- To get the population at a later date

$$P^{t+1} = L\, P^{t}$$


1
0
2025
2026

Getting P_{0-1} at Time t+1

- Let

L_x = Number of survivors between ages x and x+1

F_x = Age-specific fertility rate

Getting P_{0-1} at Time t+1

- Also let

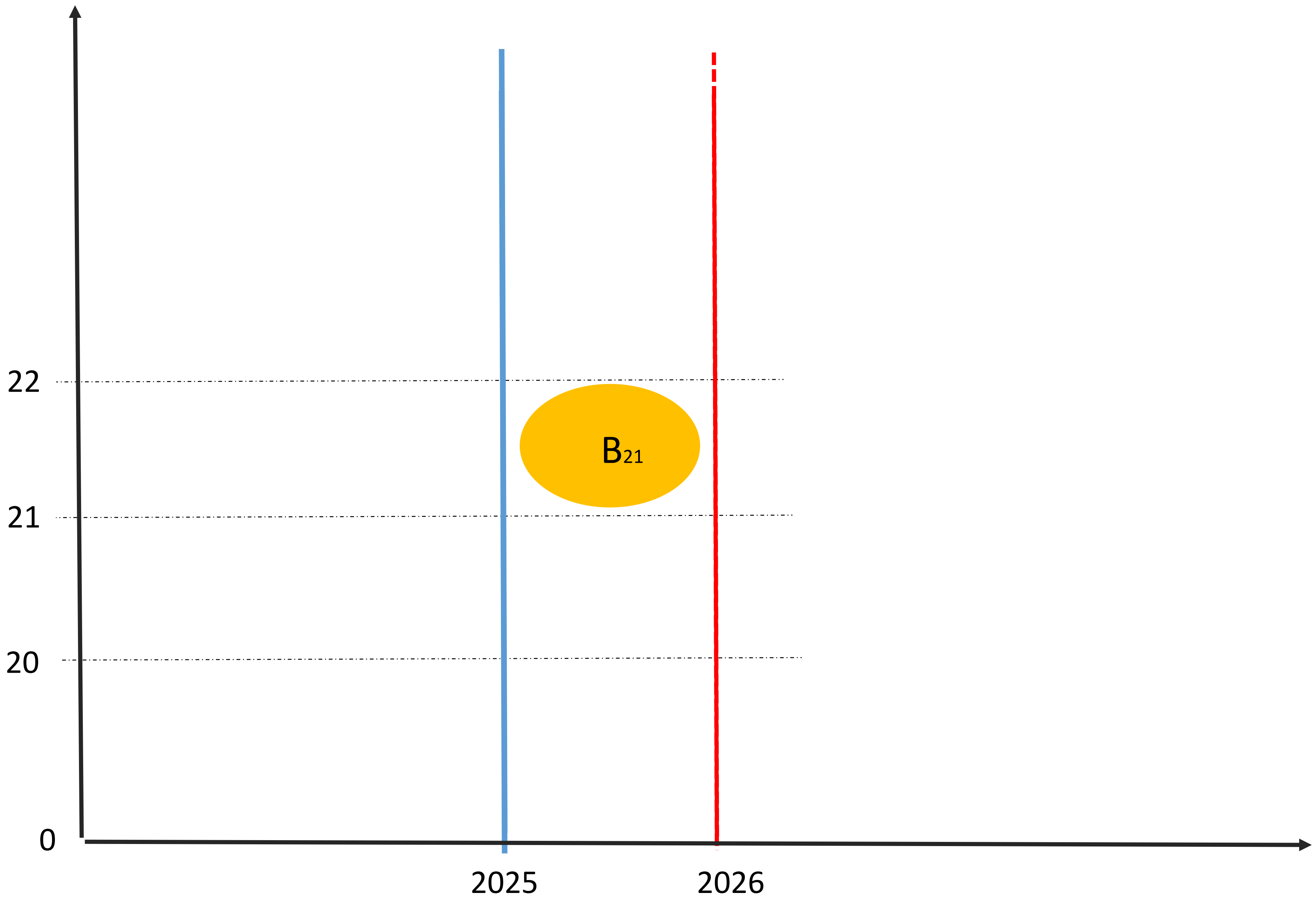
α = Beginning age of reproduction

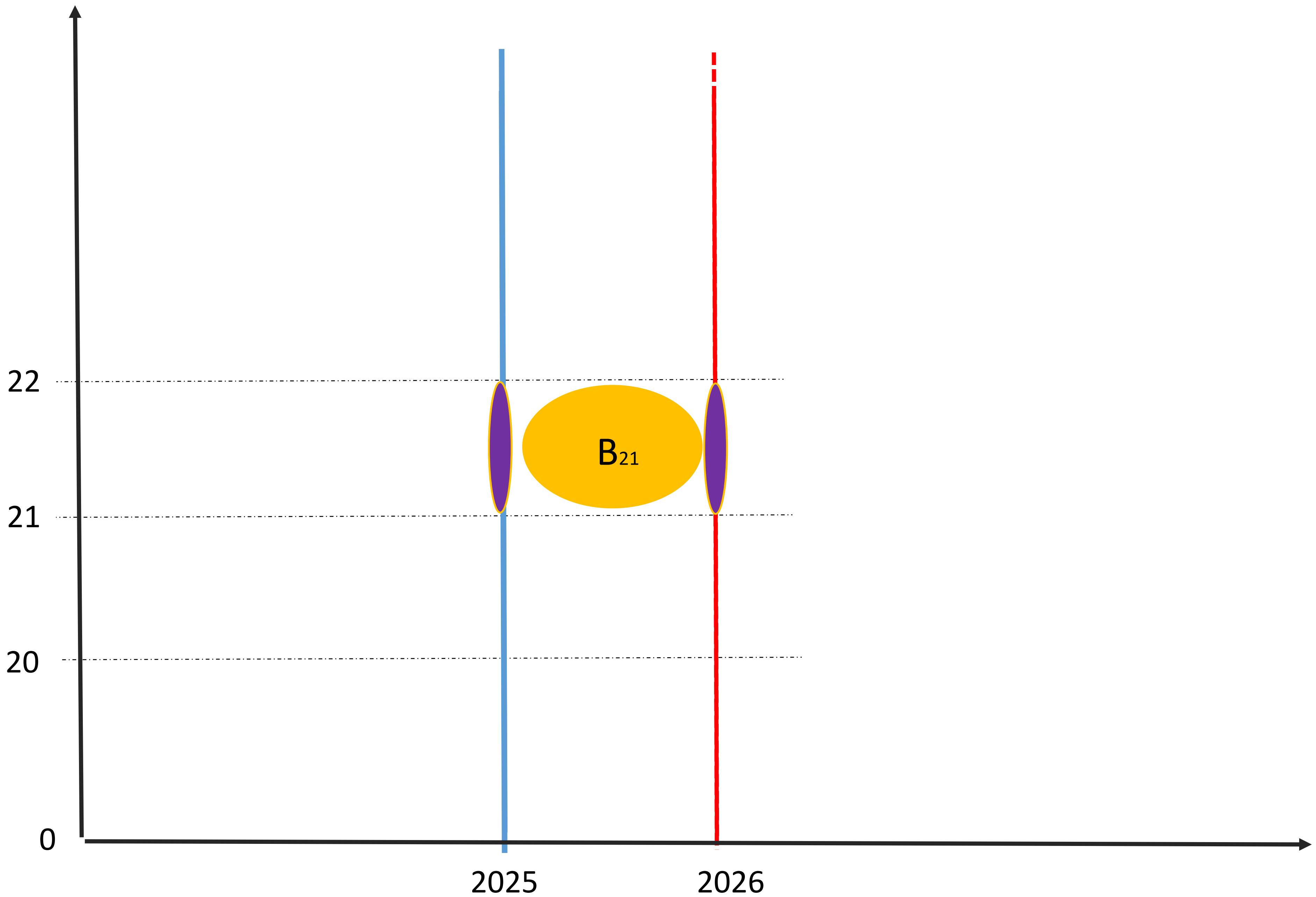
β = End of reproductive age

Getting $P_{0\text{-}1}$ at Time t+1

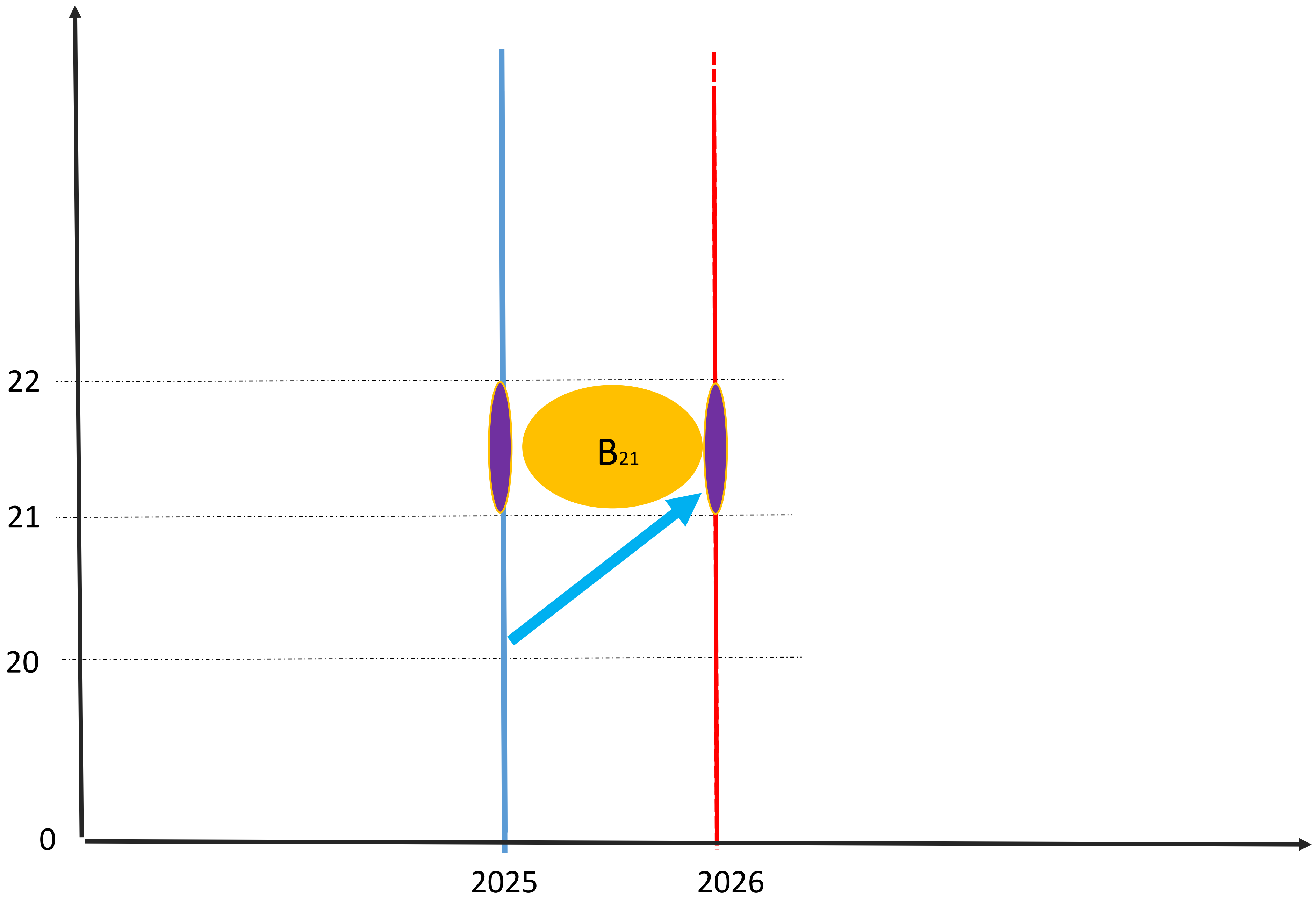
${}_1P_0^{t+1}$ = Survivors to time *t+1* of births that occurred between *t* and *t+1*

$$P_0^{t+1}$$


22
21
20
0
B21
2025
2026



22
21
20
0
B_{21}
2025
2026



22
21
20
0
B21
2025
2026

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Getting P_{0-1} at Time t+1

$$F_x\left(\frac{P_x^{t}+P_x^{t+1}}{2}\right)$$

Step 1. First babies for one age-group

Getting P_{0-1} at Time t+1

$$F_x\left(\frac{P_x^{t}+P_x^{t+1}}{2}\right)$$

Step 1. First babies for one age-group

Getting P_{0-1} at Time t+1

$$F_x\left(\frac{P_x^t + P_{x-1}^t\dfrac{L_x}{L_{x-1}}}{2}\right)$$

Getting P_{0-1} at Time t+1

$$B_t^b = \sum_{x=\alpha}^{\beta} \frac{1}{2} \quad F_x \left(\quad P_x^t + \; P_{x-1}^t \frac{L_x}{L_{x-1}} \right)$$

Step 2. Now all the babies

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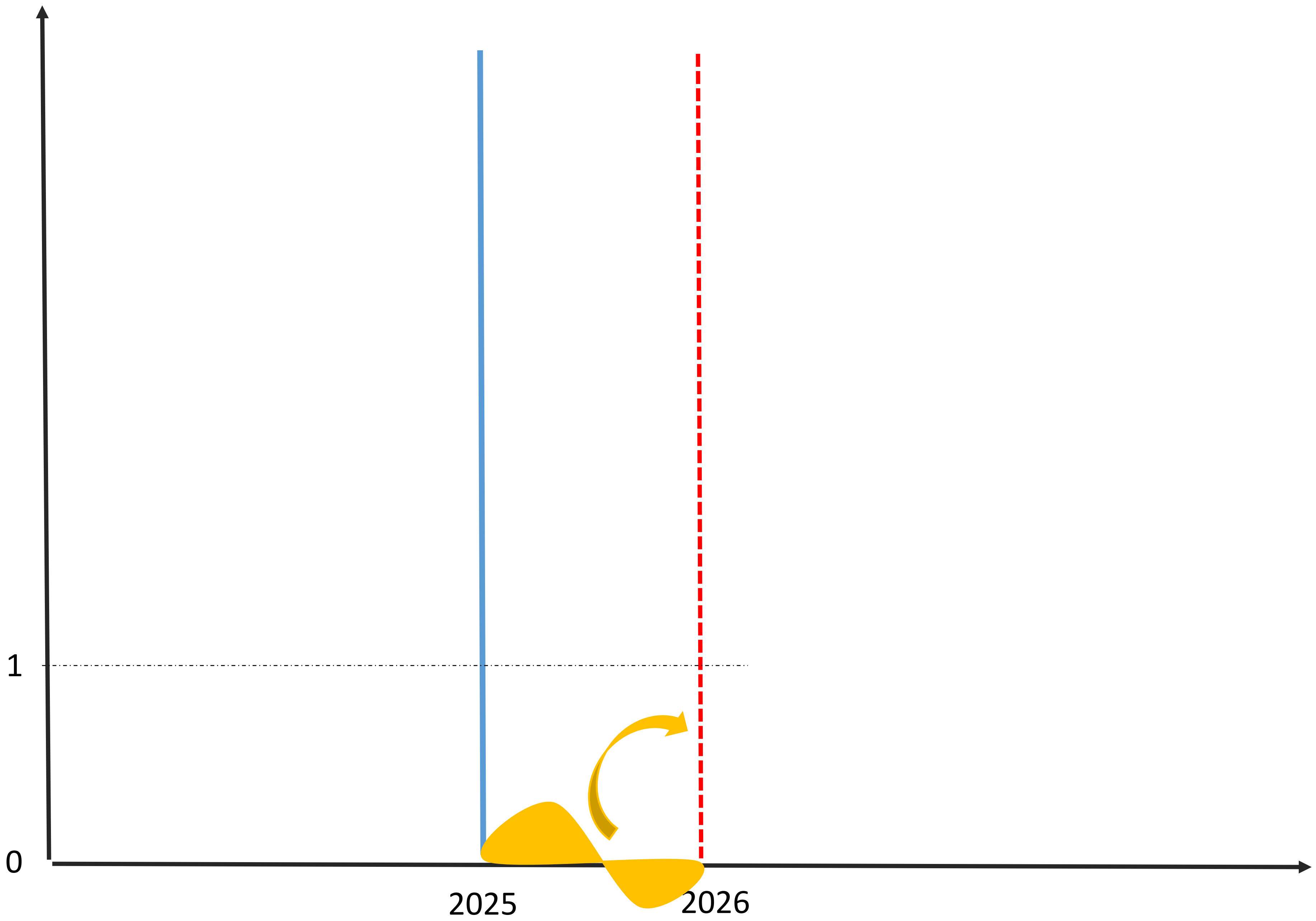
Getting P_{0-1} at Time t+1

$$B_t = \frac{B_t^b}{1 + SRB}$$

Step 3. Now female babies


1
0
2025
2026

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Getting $P_{0\text{-}1}$ at Time t+1

$$P_0^{t+1} = B_t \frac{L_0}{\ell_0}$$

Step 4. Now survive those female babies to time *t+1*

Getting P_{0-1} at Time t+1

$$P_0^{t+1} =$$

$$\left(\frac{L_0}{2\,\ell_0}\right)\left(\frac{1}{1+SRB}\right)\left[\sum_{x=\alpha}^{\beta-1} F_x\left(P_x + P_{x-1}\frac{L_x}{L_{x-1}}\right)\right]$$

Matrix Representation

$$P_0^{t+1} =$$

$$k\left[\sum_{x=\alpha}^{\beta-1} F_x\left(P_x + P_{x-1}\frac{L_x}{L_{x-1}}\right)\right]$$

Matrix Representation

- 1st row represents fertility rates and 0–1 survival
- Sub-diagonal represents survival ratios from one age group to another
- Note: The rest of the matrix (L) has zeros

Matrix Representation

$$L = \begin{bmatrix} 0 & 0 & \left(\frac{L_{12}}{L_{11}} F_{12}\right) & \left(F_{12} + \frac{L_{13}}{L_{12}} F_{13}\right) & \dots & 0 & 0 \\ \frac{L_1}{L_0} & 0 & 0 & 0 & \dots & 0 & 0 \\ 0 & \frac{L_2}{L_1} & 0 & 0 & \dots & 0 & 0 \\ \vdots & & & & & & \\ 0 & 0 & 0 & 0 & \dots & \frac{T_\omega}{T_{\omega\text{-}1}} & \frac{T_\omega}{T_{\omega\text{-}1}} \end{bmatrix}$$

Population Projection

- To get the population at a later date

$$P^{t+1} = L\,P^{\mathrm{t}}$$

Population Projection

- To project the total population at a later date
  - First, use the female projection and fertility rates for both boy and girl births
  - Then, put the male births into male projection
  - Note: The male matrix only contains survival ratios

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Population Projection

$$B_t^m = B_t^b \frac{SRB}{1+SRB}$$

Step 3. Now male babies

Getting P_{0-1} at Time t+1

$$P_0^{t+1} = B_t^m \frac{L_0^m}{\ell_0^m}$$

Step 4. Now survive those male babies to time *t+1*

Getting $P_{0\text{-}1}$ at Time t+1

$$P_0^{t+1,m} =$$

$$\left(\frac{L_0^m}{2\,\ell_0^m}\right)\left(\frac{SRB}{1+SRB}\right)\left[\sum_{x=\alpha}^{\beta-1} F_x\left(P_x + P_{x-1}\frac{L_x}{L_{x-1}}\right)\right]$$

Getting P_{0-1} at Time t+1

$$P_0^{t+1,m} =$$

$$k_m \left[\sum_{x=\alpha}^{\beta-1} F_x \left(P_x + P_{x-1} \frac{L_x}{L_{x-1}} \right) \right]$$

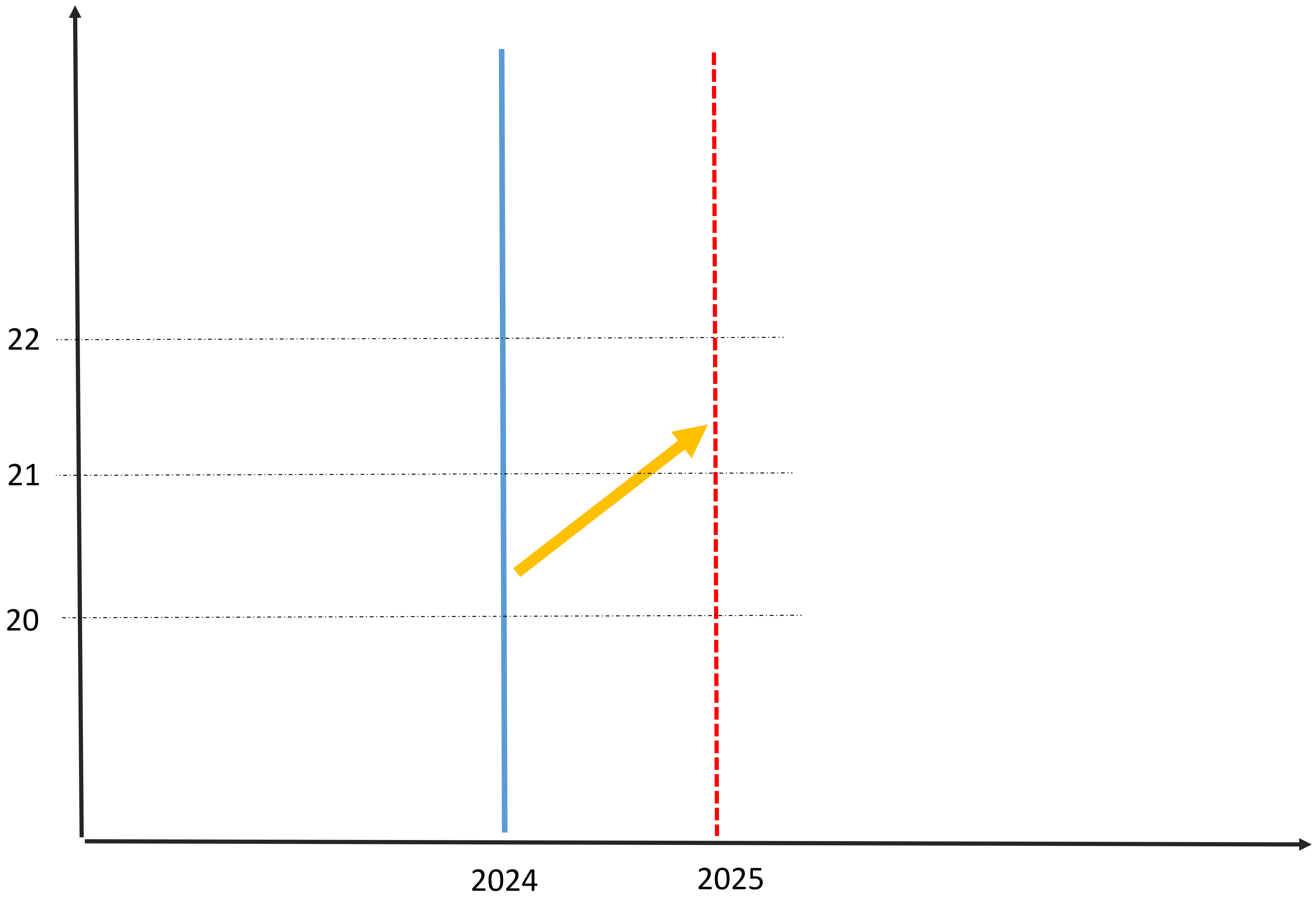
Adjusting for Net Migration (NM)

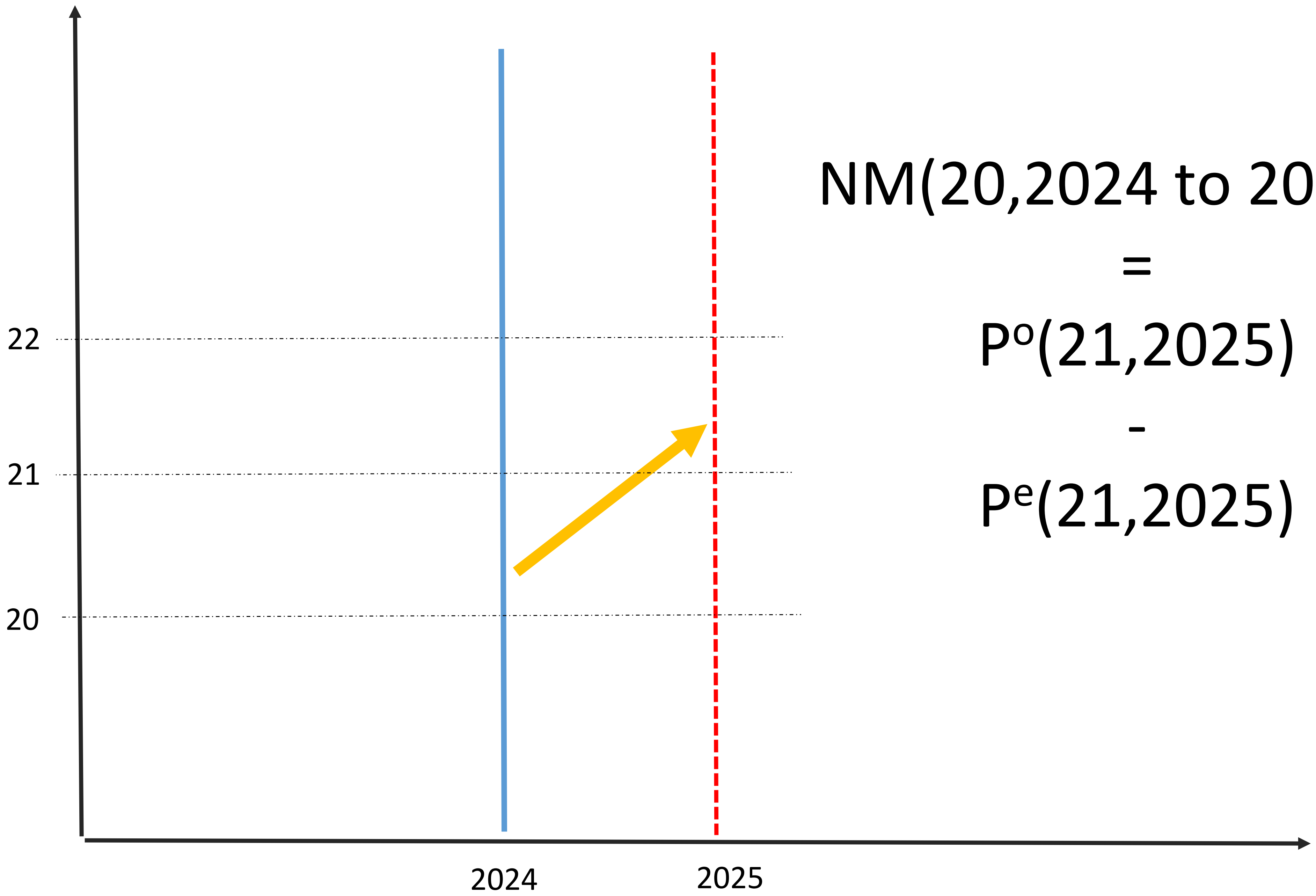
$$P^{t+1} = L\,P^{t} + NM$$

- Where NM = Net migration
- It is easiest to take net migration into account at the end of each time interval

Adjusting for Net Migration (NM)

$$P^{t+1} = L\left(P^{t} + \frac{NM}{2}\right) + \frac{NM}{2}$$


22
21
20
2024
2025



$$NM(20, 2024 \text{ to } 2025) = P^{o}(21,2025) - P^{e}(21,2025)$$

Population Projection

Estimated:

$$P_{21}^{2025} = P_{20}^{2024} \frac{L_{21}}{L_{20}}$$

Net migration

$$\Delta P_{20} = P_{21}^{2025} - P_{20}^{2024} \frac{L_{21}}{L_{20}}$$

Net migration

$$NM_{20} = \frac{\Delta P_{20} + \Delta P_{19}}{2}$$

Summary

Population projection

Cohort component method: survival ratios for all ages, and special fertility and survival calculations for the first age group and for the last.

Leslie matrix: female and male matrices.

Cohort Component Method

Steps

1. Estimation of survival ratio
2. Estimation of individuals surviving to the end of the projection
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Sources of errors

Actual and Projected Life Expectancy at Birth, Males, UK

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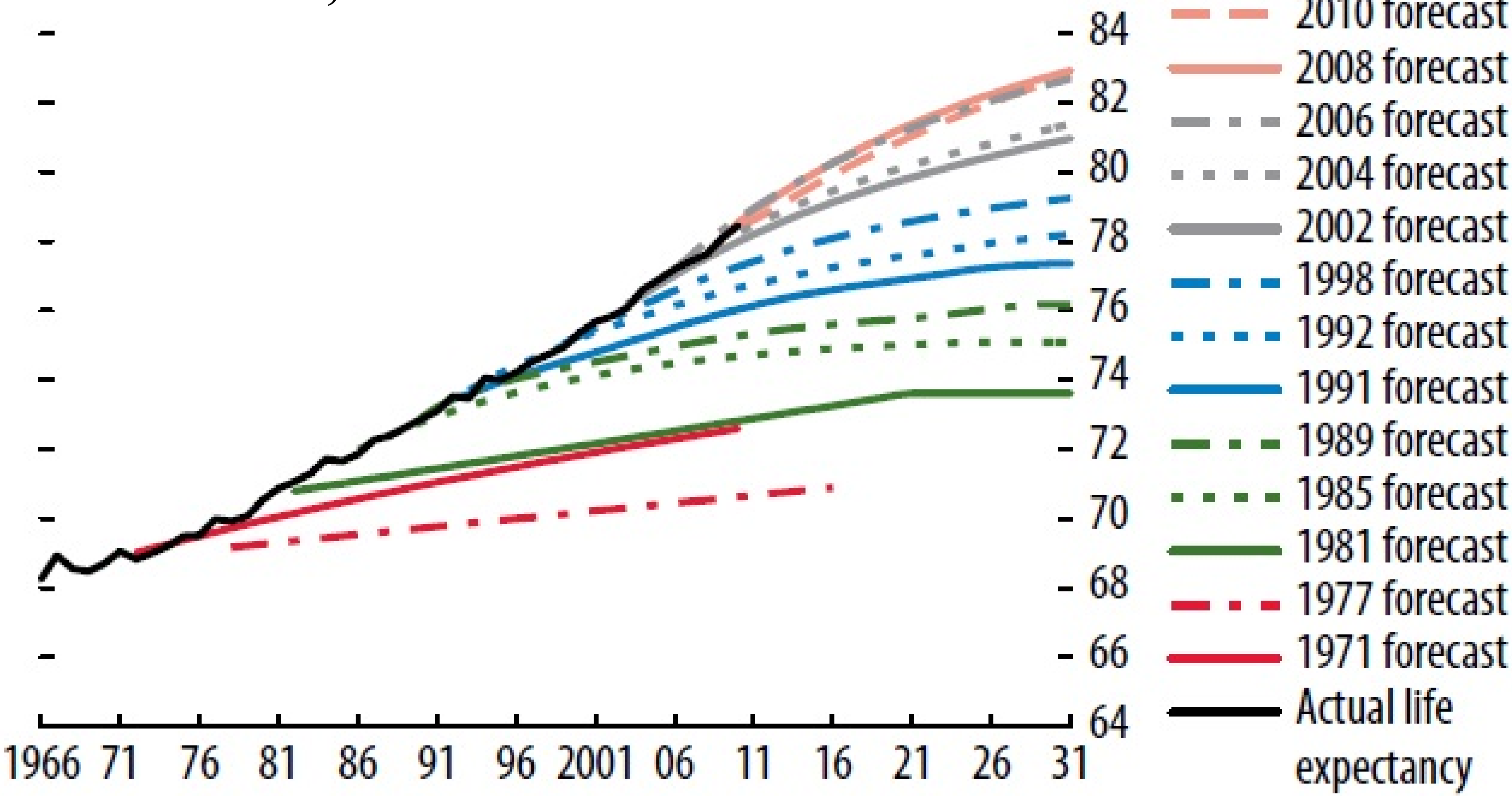
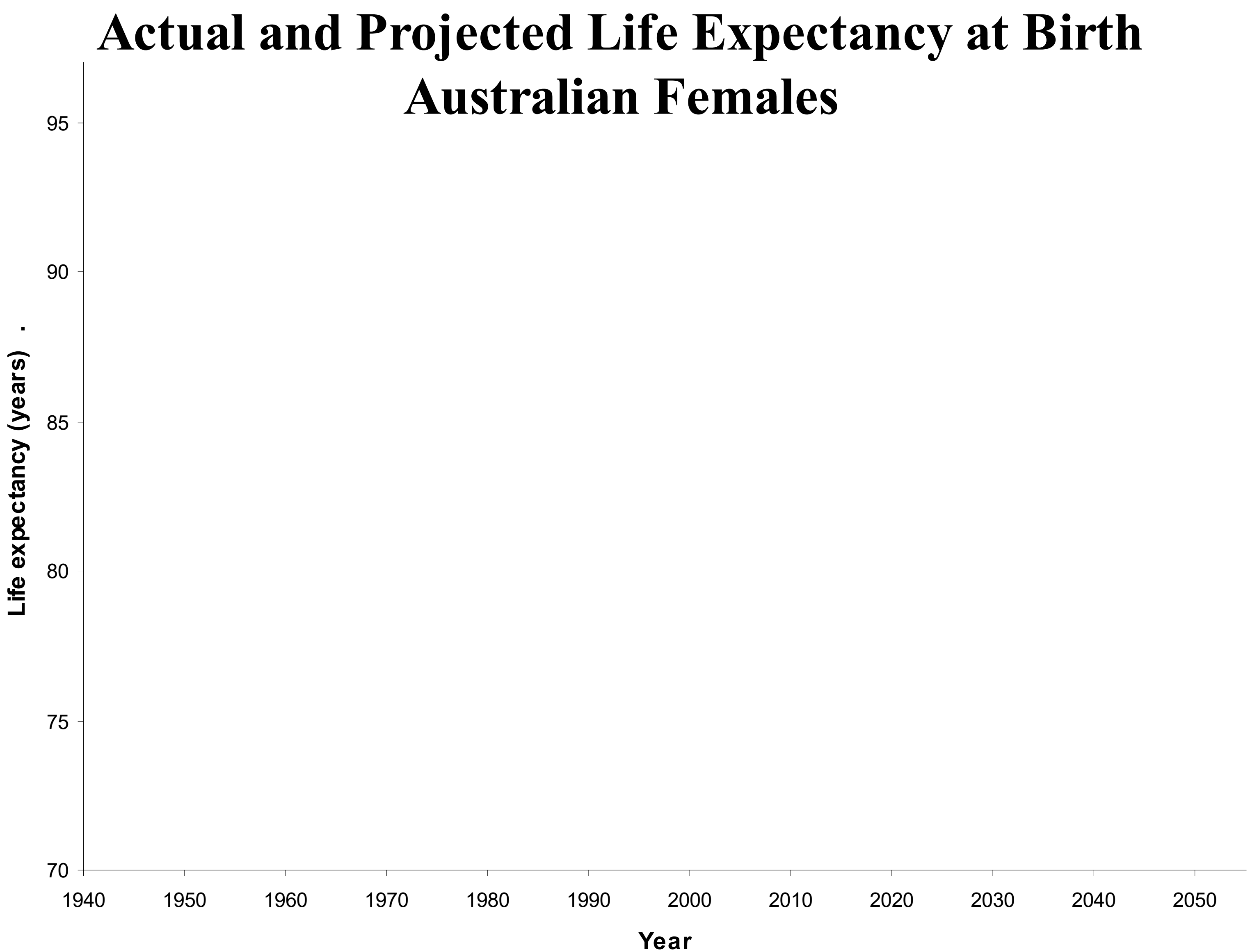
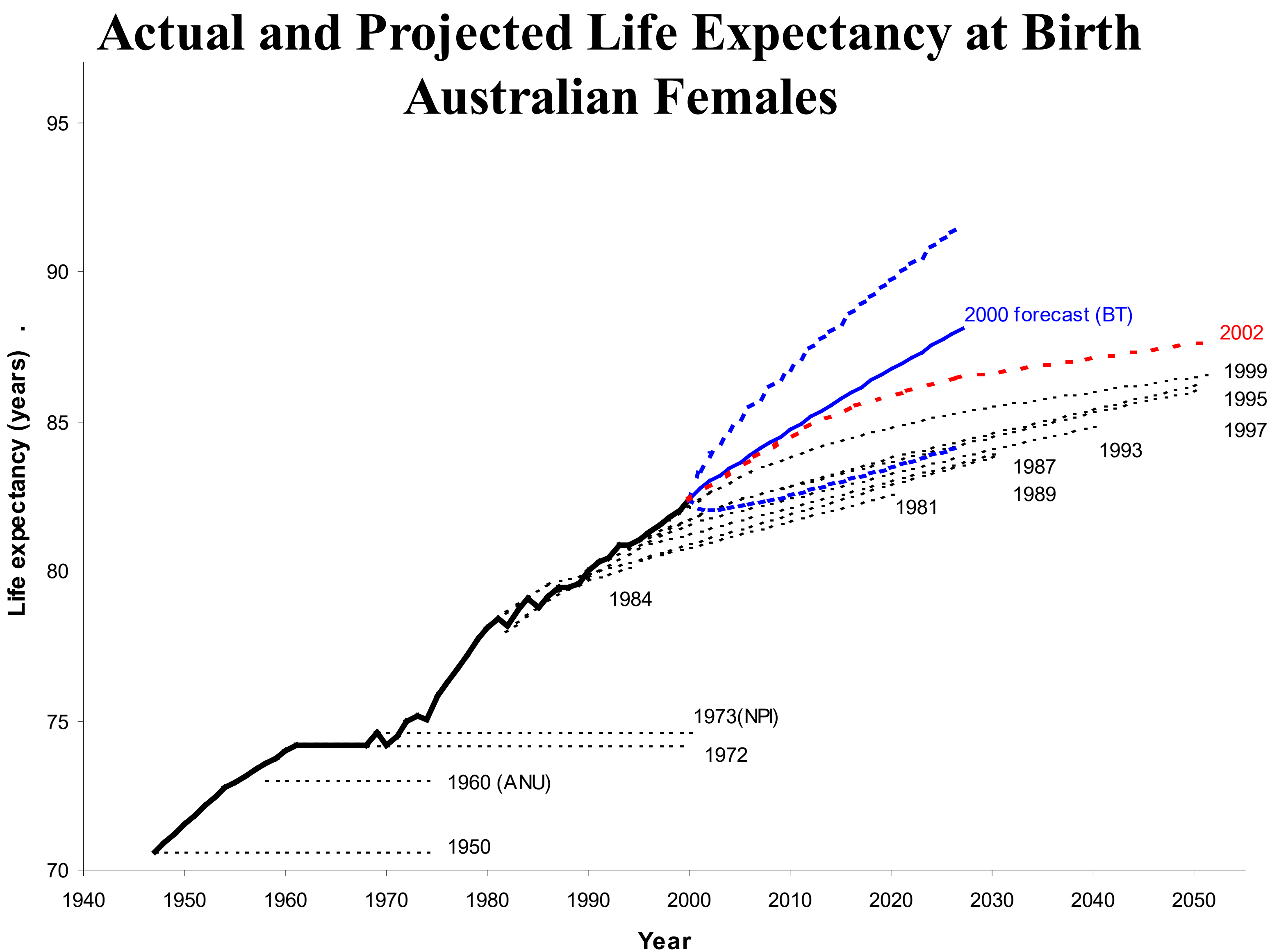


Source: Office of National Statistics.

Sources of errors

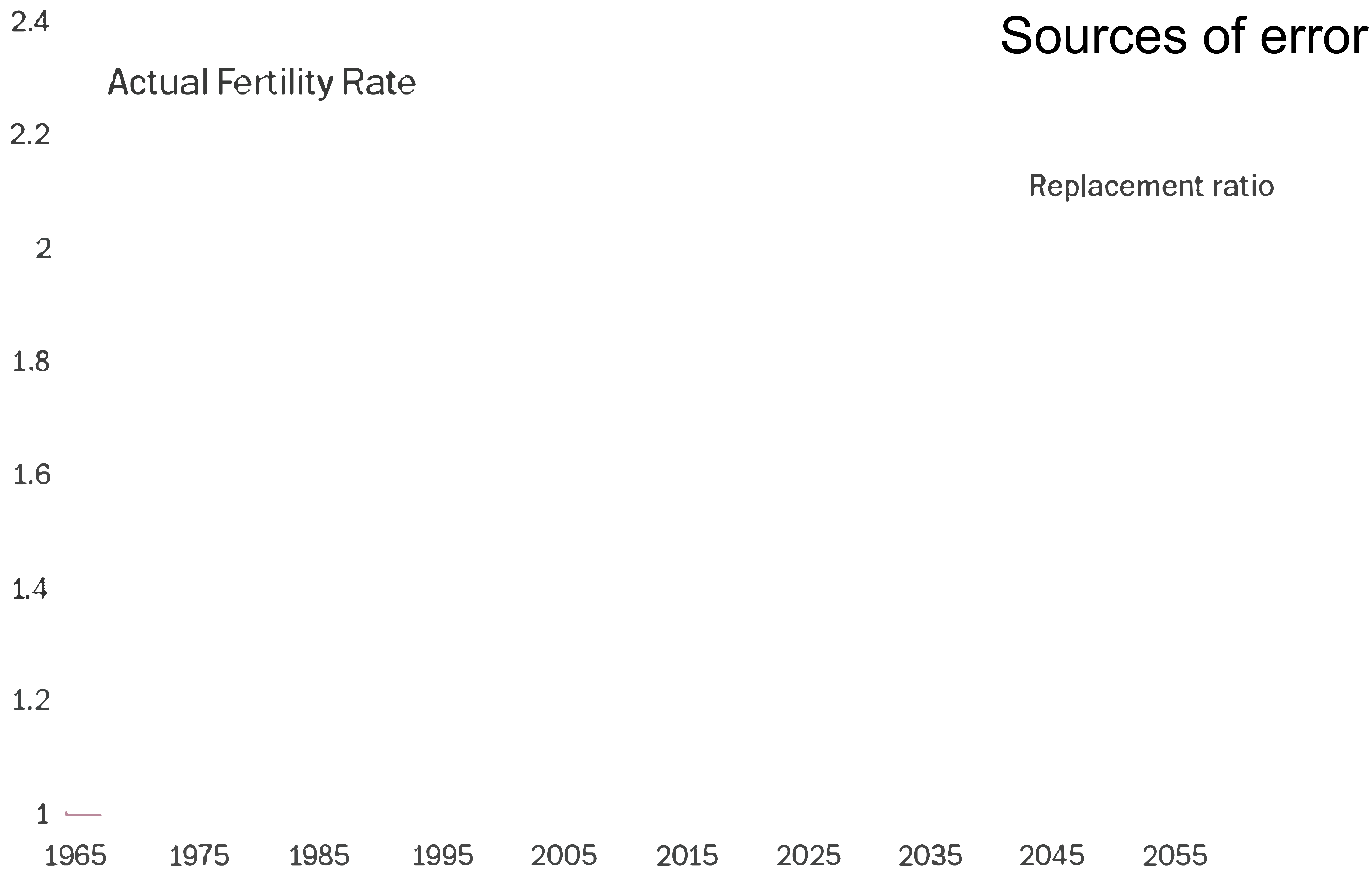


Sources of errors



Japan has been way too optimistic about its birth rate

Actual vs. projected fertility rate, 1965-2055



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SOURCE: Ministry of Health, Labour and Welfare, National Institute of Population and Social Security Research

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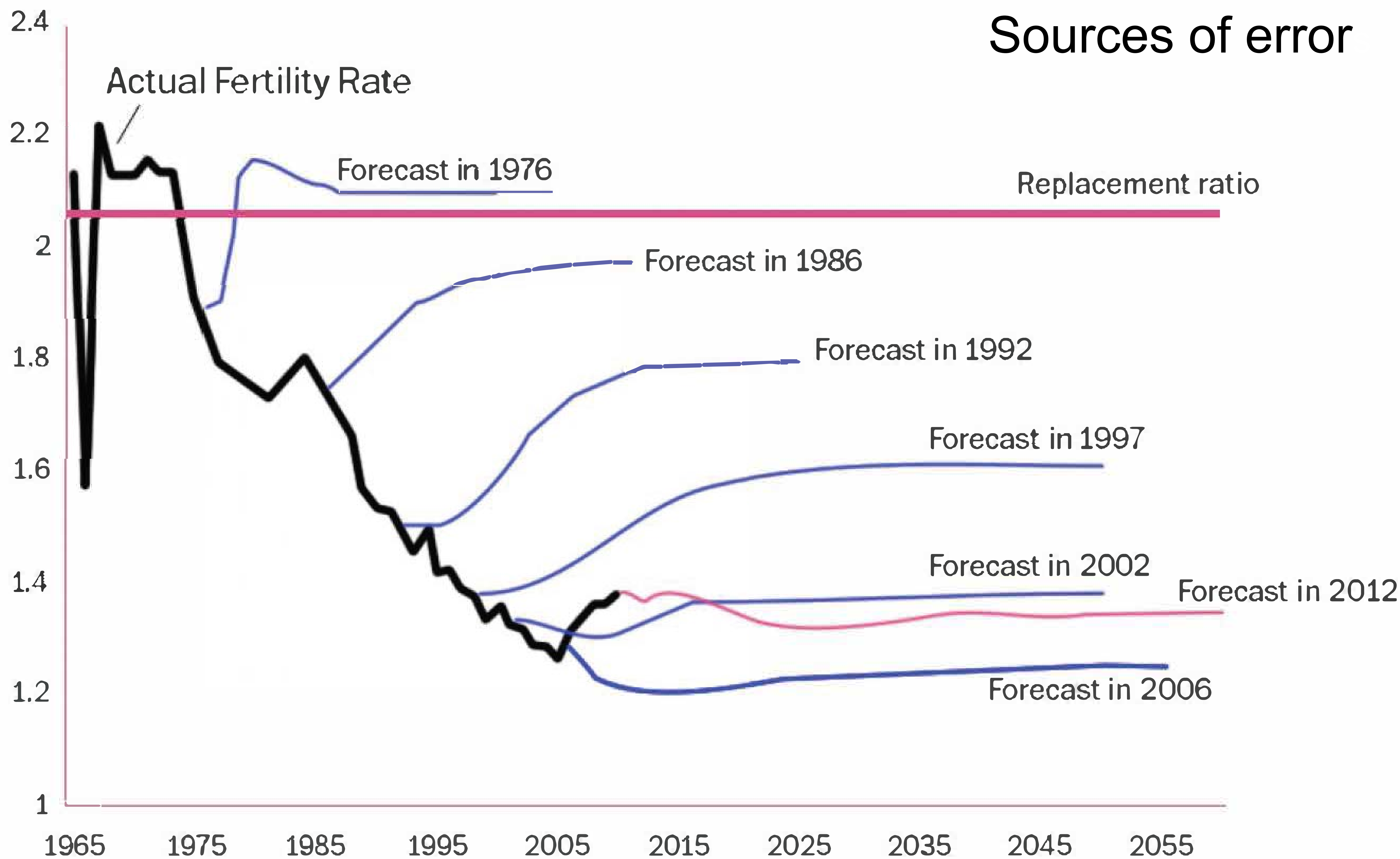

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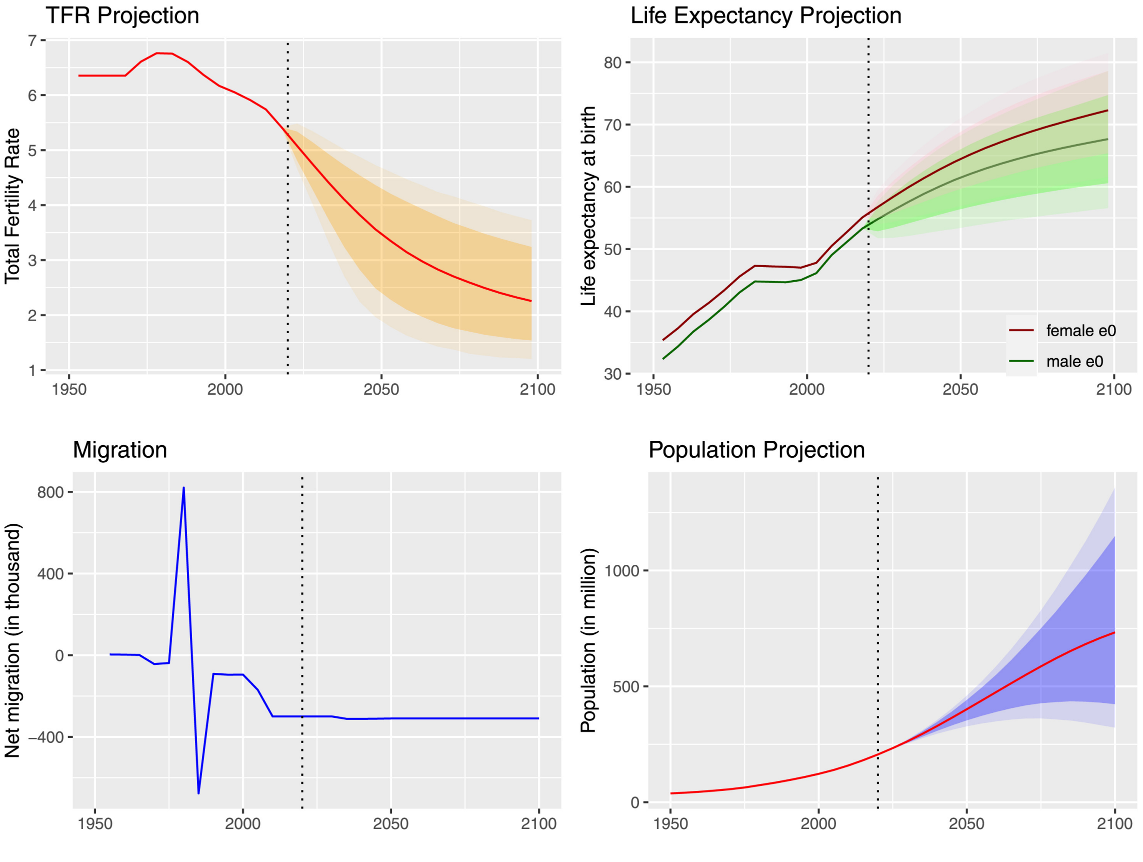
SOURCE: Ministry of Health, Labour and Welfare, National Institute of Population and Social Security Research

Nigeria



TFR Projection
Total Fertility Rate
1
2
3
4
5
6
7
1950
2000
2050
2100
Life Expectancy Projection
Life expectancy at birth
30
40
50
60
70
80
1950
2000
2050
2100
female e0
male e0
Migration
Net migration (in thousand)
−400
0
400
800
1950
2000
2050
2100
Population Projection
Population (in million)
0
500
1000
1950
2000
2050
2100

Population Projection

- To get the population at a later date n

$$P^{t+n} = L^n P^t$$

Changes in Fertility and Mortality

Let

- L^0 = Projection matrix for first time interval
-
-
- L^i = Projection matrix for i^{th} time interval

Population Projection

- At time t+2

$$P^{t+2} = L^{1}\left(L^{0}\ P^{t}\right)$$

- Or more generally

$$P^{t+n} = L^{n-1} L^{n-2} \ldots\ L^{0} P^{t}$$

- Where L^i = Projection matrix for i^{th} time interval
- Recall: matrix multiplication is not commutative

Recommended Reading

- Preston et al. (2001). Chapter 6.
- PAPP101 - S10: Population projectionsPAPP101- S10