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MML Assignment 1

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Q1) Given,

$N \times n$ matrix P which is a path matrix

$$P_{ij} = \begin{cases} 1 ; \text{link } j \text{ is on path } i \\ 0 ; \text{ otherwise} \end{cases}$$

$\vec{d}$ is an n-dimensional vector which gives link delays

$\vec{t}$ be an N-dimensional vector which gives travel time

Such that

$$\underset{(N \times n)}{P} \cdot \underset{(n \times 1)}{\vec{d}} = \underset{(N \times 1)}{\vec{t}}$$

$$\underset{(N \times n)}{\begin{bmatrix} P_{11} & P_{12} & \cdots & P_{1n} \\ P_{21} & P_{22} & \cdots & P_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ P_{N1} & P_{N2} & \cdots & P_{Nn} \end{bmatrix}} \underset{(n \times 1)}{\begin{bmatrix} d_1 \\ d_2 \\ \vdots \\ d_n \end{bmatrix}} = \underset{(N \times 1)}{\begin{bmatrix} t_1 \\ t_2 \\ \vdots \\ t_N \end{bmatrix}}$$

Now since the P matrix is of dimension $N \times m$ & given that $(N > n)$, it implies that it has more rows that columns i.e, P is a tall matrix

Now beause of tall matrix we try to get the best solution by minimizing RMS (root mean square error)

$$RMS = \left(\sqrt{\|P\vec{d} - \vec{t}\|^2}\right) * \frac{1}{N}$$

Assumptions

- Given, links are labelled $1, 2, \ldots, n$ which means columns are linearly independent
  i.e P has left inverse
- If P has left inverse, then $P^T P$ is invertible

Minimizing RMS $\Rightarrow$ minimize $\frac{\|P.\vec{d} - \vec{t}\|^2}{N}$

- we want a $\vec{d}$ such that the residual $(P\vec{d} - \vec{t})$ is minimal
- if $\|P.\hat{d} - \vec{t}\| < \|P\vec{d} - \vec{t}\|$, then $\hat{d}$ is our solution

Loss function, L can be written as $(Pd - t)^T (Pd - t)$

$$L = (d^T P^T - t^T)(Pd - t)$$

$$L = d^T P^T P d - d^T P^T t - t^T P d + t^T t$$

To minimize, differentiate L w.r.t d

$$\frac{\partial L}{\partial \vec{d}} = \frac{d}{\partial \vec{d}}\left(d^T P^T P \vec{d} - d^T P^T t - t^T P d + t^T t\right)$$

$$= 2P^T P d - 2P^T t + 0$$

equating with 0 to find $\hat{d}$

$$2P^T P \hat{d} - 2P^T t = 0$$

$$\boxed{\hat{d} = (P^T P)^{-1} P^T t}$$

where, $(P^T P)^{-1} P^T$ is our pseudo inverse