MML Assignment 1

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Q2. *Polynomial classifier with one feature.* Generate 200 points $x^{(1)}, \ldots, x^{(200)}$, uniformly spaced in the interval $[-1, 1]$, and take

$$y^{(i)} = \begin{cases} +1 & -0.5 \leq x^{(i)} < 0.1 \text{ or } 0.5 \leq x^{(i)} \\ -1 & \text{otherwise} \end{cases}$$

for $i = 1, \ldots, 200$. Fit polynomial least squares classifiers of degrees $0, \ldots, 8$ to this training data set.

(a) Evaluate the error rate on the training data set. Does the error rate decrease when you increase the degree?

(b) For each degree, plot the polynomial $\tilde{f}(x)$ and the classifier $\hat{f}(x) = \mathbf{sign}(\tilde{f}(x))$.

(c) It is possible to classify this data set perfectly using a classifier $\hat{f}(x) = \mathbf{sign}(\tilde{f}(x))$ and a cubic polynomial

$$\tilde{f}(x) = c(x + 0.5)(x - 0.1)(x - 0.5),$$

for any positive c. Compare this classifier with the least squares classifier of degree 3 that you found and explain why there is a difference.

7 Marks

In [ ]:
```
import numpy as np
import pandas as pd
from matplotlib import pyplot as plt
```

1. Generating the data

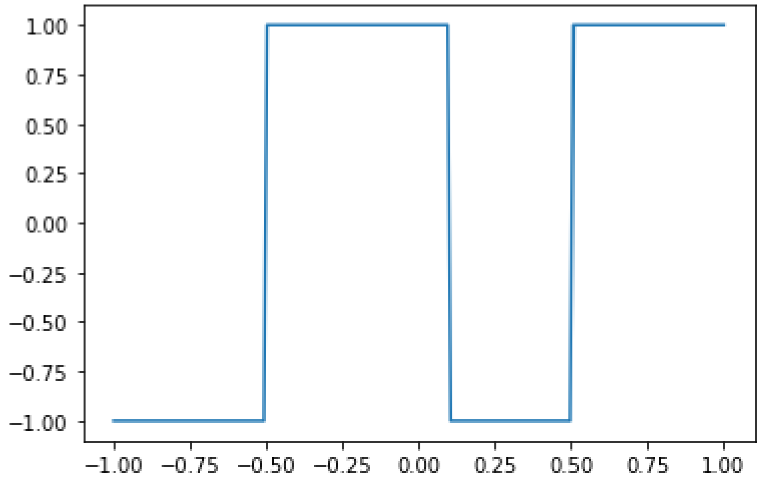
In [ ]:
```
#genrating 200 datapoints uniformly spaced in the interval [-1,1]
X = np.linspace(-1, 1, 200)

#y takes +1 when -0.5<=xi<0.1 or 0.5<=xi
#and -1 otherwise
Y = np.array([1 if (x>=-0.5 and x<0.1)or(x>=0.5) else -1 for x in X])

# print(X, Y)
print(X.shape, Y.shape)
print(type(X), type(Y))
```

```
(200,) (200,)
<class 'numpy.ndarray'> <class 'numpy.ndarray'>
```

plotting the graph

In [ ]:
```
plt.plot(X,Y)
```

Out[ ]:
```
[<matplotlib.lines.Line2D at 0x7efd28783430>]
```



2. Solving the problem

(a) Evaluate the error rate on the training data set. Does the error rate decrease when you increase the degree?

2(a). calculating error rate for each degree using polyfit function

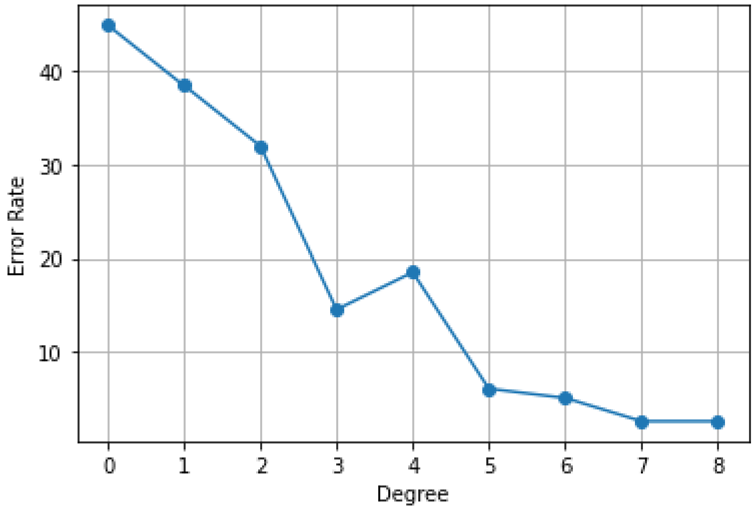
In [ ]:
```
from numpy.polynomial import Polynomial
from sklearn.metrics import accuracy_score
```

The function NumPy.polyfit() helps us by finding the least square polynomial fit. This means finding the best fitting curve to a given set of points by minimizing the sum of squares. It takes 3 different inputs from the user, namely X, Y, and the polynomial degree.

for reference on what polyfit and polyval does https://www.mathworks.com/help/matlab/ref/polyfit.html

In [ ]:
```
error = []
predictions = []
sign_predictions = []
y_coeff_for_deg3 = np.zeros(9)

for i in range(0,9):
    p = np.polyfit(X,Y,i)
    # print(p)

    ################################
    #storing the coefficients when deg = 3 to be used later
    if i==3:
        y_coeff_for_deg3 = p
    ################################

    y = np.polyval(p,X)
    predictions.append(y)
    #sign(f(x)) for sign predictions
    sign_predictions.append(np.sign(y))

    # calculating error
    error_rate = 1 - accuracy_score(Y, np.sign(y)) # calcualte error rate
    error.append(error_rate*100)
```

In [ ]:
```
plt.plot(error, marker="o")
plt.xlabel("Degree")
plt.ylabel("Error Rate")
plt.grid()
```



Answer to question 2a

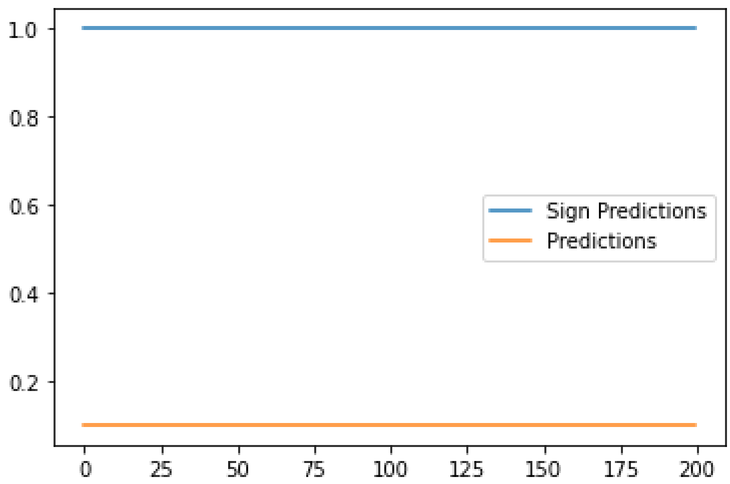
Yes, the error rate decreases when we increase the degree but
at degree 3 and 4 this trend is not followed , that is error increases when we increase the degree.

(b) For each degree, plot the polynomial $\tilde{f}(x)$ and the classifier $\hat{f}(x) = \mathbf{sign}(\tilde{f}(x))$.

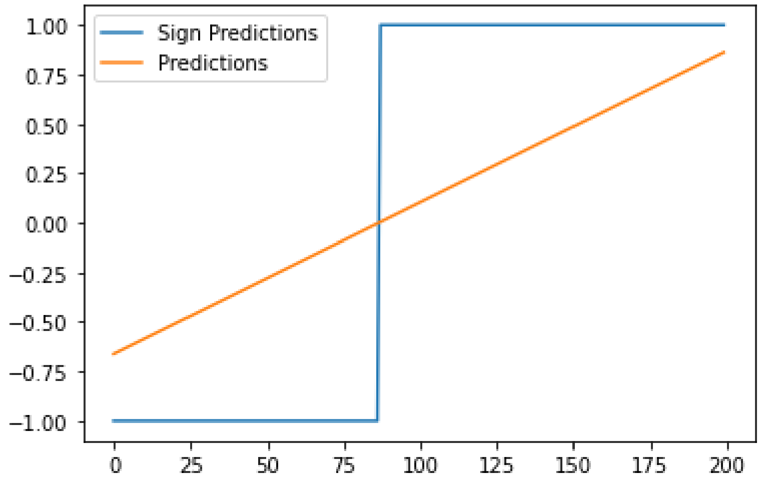
2(b). For each degree, plotting f(x) and sign(f(x))

In [ ]:
```
for i in range(0,9):
    plt.plot(sign_predictions[i], label="Sign Predictions")
    plt.plot(predictions[i], label = "Predictions")
    print(f'---------- for degree = {i} ----------')
    plt.legend()
    plt.show()
```

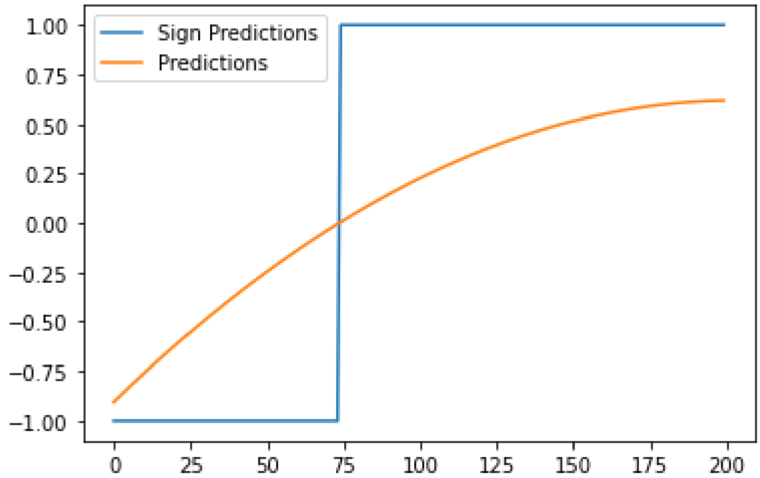
```
---------- for degree = 0 ----------
```



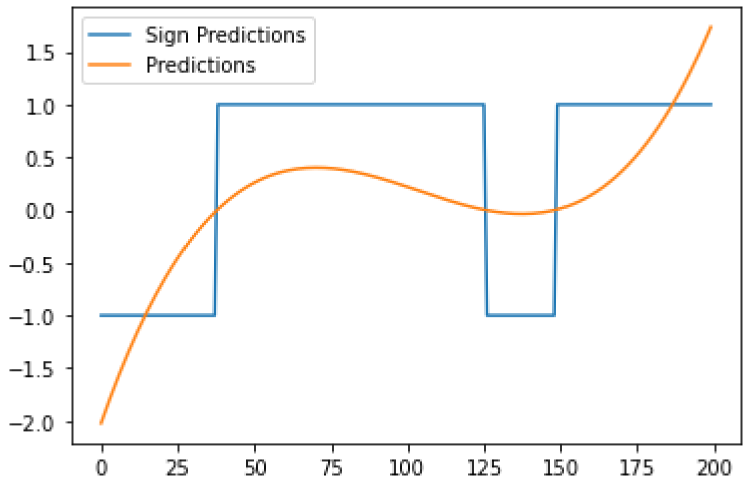

```
---------- for degree = 1 ----------
```



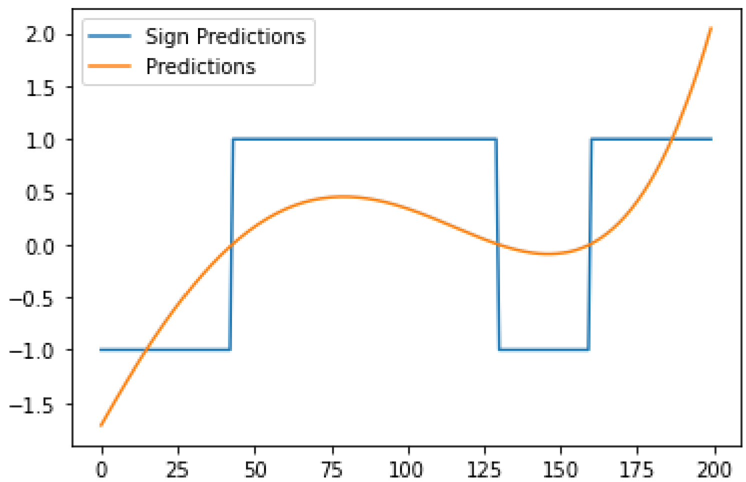

```
---------- for degree = 2 ----------
```



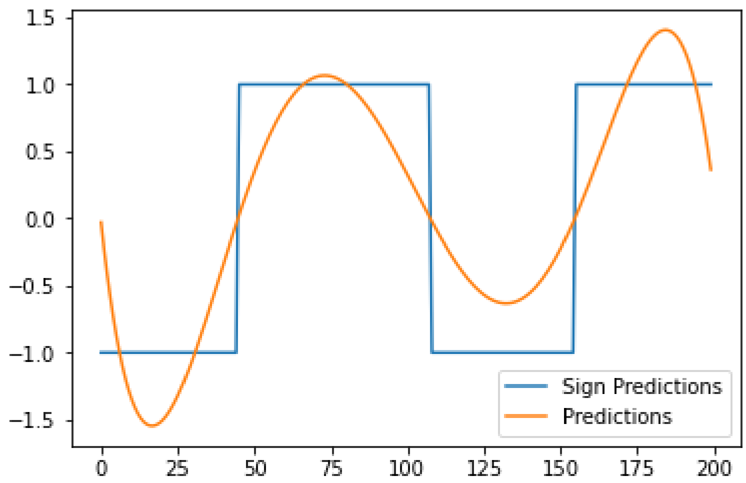

```
---------- for degree = 3 ----------
```



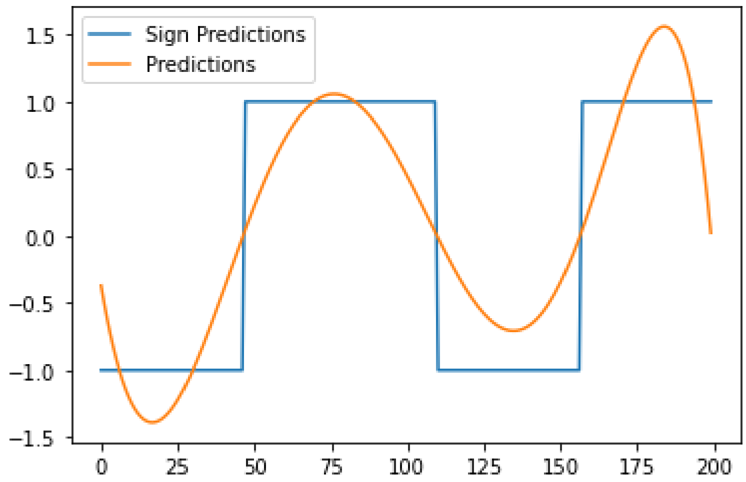

```
---------- for degree = 4 ----------
```



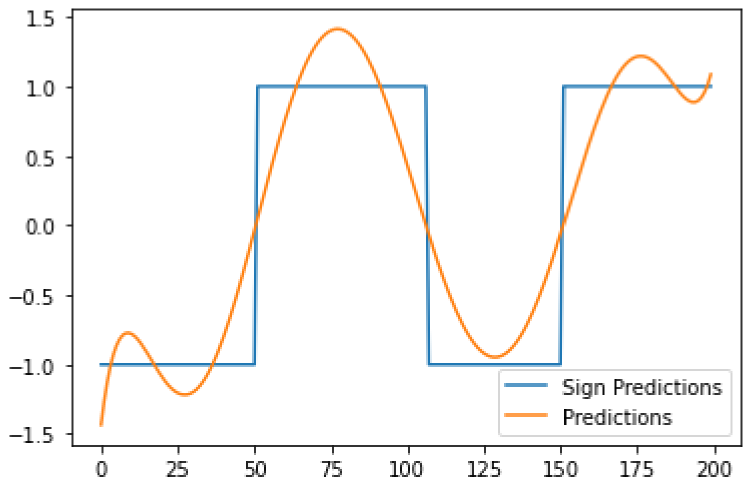

```
---------- for degree = 5 ----------
```



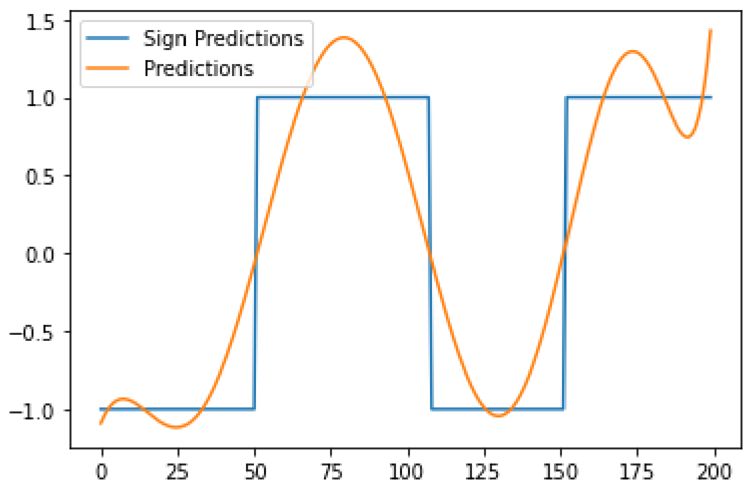
---------- for degree = 6 ----------



---------- for degree = 7 ----------



---------- for degree = 8 ----------



(c) It is possible to classify this data set perfectly using a classifier $\hat{f}(x) = \textbf{sign}(\tilde{f}(x))$ and a cubic polynomial

$$\tilde{f}(x) = c(x + 0.5)(x - 0.1)(x - 0.5),$$

for any positive c. Compare this classifier with the least squares classifier of degree 3 that you found and explain why there is a difference.

for this we need to check the values of c

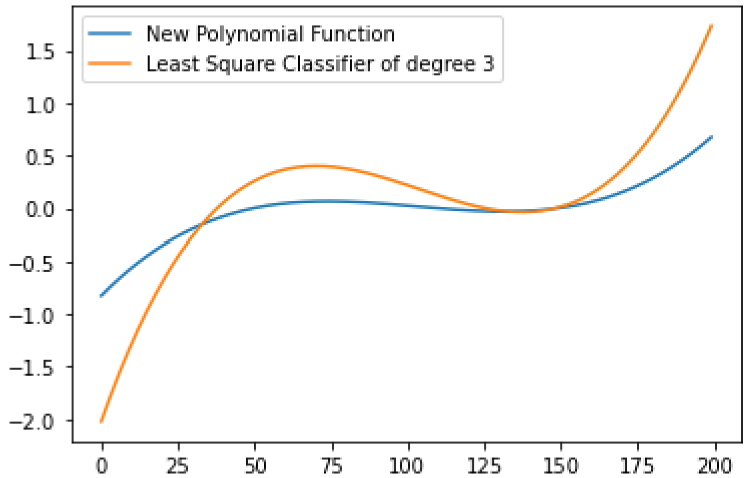
new classifier given is:

f(x) = c(x+0.5)(x-0.1)(x-0.5)

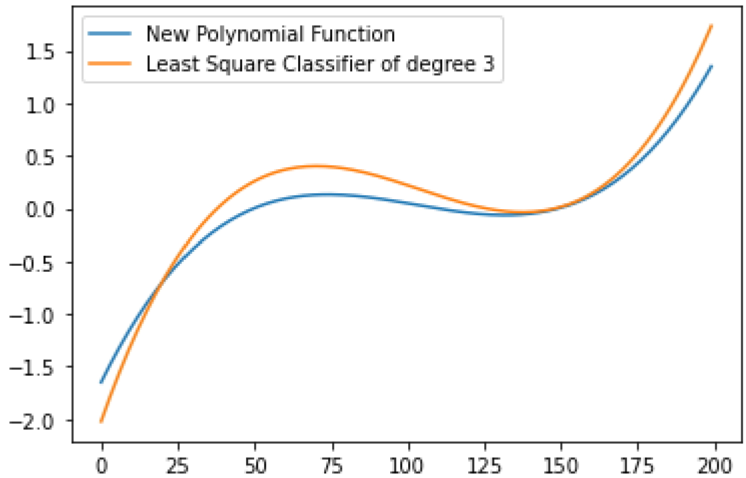
In [ ]:

```
#predictions for our new polynomial
print(len(X))
for c in range(1,5):
    pred=[]
    for x in X:
        y_new = c*(x+0.5)*(x-0.1)*(x-0.5)
        pred.append(y_new)
        # print(len(pred))

    print(f'---------- for the value of c = {c} ----------\n')
    plt.plot(pred, label="New Polynomial Function")
    #predictions store all the predictions but predictions[3] stores the cubic predictions
    plt.plot(predictions[3], label="Least Square Classifier of degree 3")
    plt.legend()
    plt.show()
```

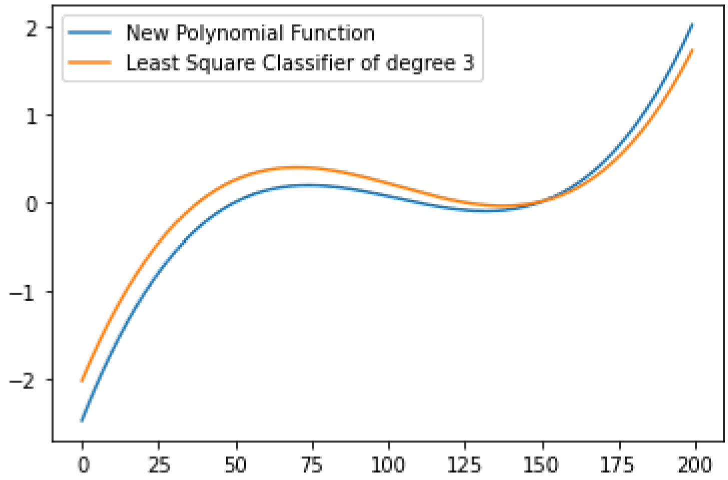
```
200
---------- for the value of c = 1 ----------
```



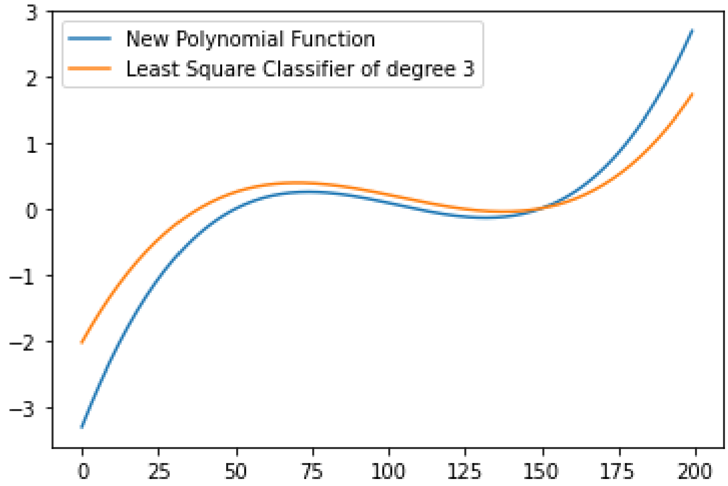

```
---------- for the value of c = 2 ----------
```



```
---------- for the value of c = 3 ----------
```




```
---------- for the value of c = 4 ----------
```



From the above graphs it is clearly visible that for value of c=3 we get the closest graphs

printing the most optimal weights for degree 3 of our least square classifier

In [ ]:
```
print(y_coeff_for_deg3)
# y_coeff_for_deg3
```

```
[ 2.83580921 -0.36759273 -0.95733457  0.22376238]
```

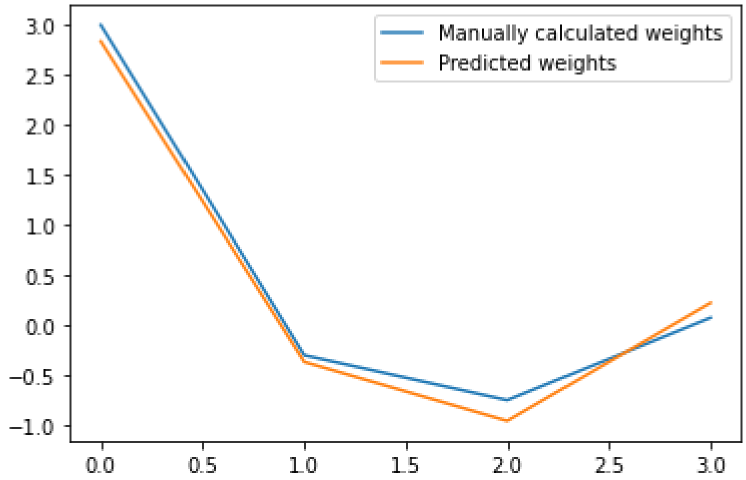
Answer 2c

The most optimal weights for degree 3 are
2.83580921, -0.36759273, -0.95733457, 0.22376238
but when we calculate the coefficiants manually with c=3 we get values
0.075, -0.75, -0.3, 3

So we can see that there is a slight difference in the values.

we can verify the above statement by plotting them

In [ ]:
```
W = [3, -0.3, -0.75, 0.075]
#now plotting the weight difference
plt.plot(W, label="Manually calculated weights")
plt.plot(y_coeff_for_deg3, label="Predicted weights")
plt.legend()
plt.show()
```



Hence it is not possible to classify the data using the given classifier perfectly

3. Answers

- (2a) Yes, the error rate decreases when we increase the degree but

at degree 3 and 4 this trend is not followed , that is error increases when we increase the degree.

- (2b) Plotted the graph
- (2c) It is not possible to classify the data using the given classifier perfectly