1 Question

Pyramids Industries has estimated the aggregate demand, cost, and production data for the next six months as follows:

Cost Data	
Hiring cost/worker	$200.00
Layoff cost/worker	$100.00
Regular time cost/hour	$7.00
Holding cost/unit/month	$8.00
Backorder cost/unit/month	$15.00
Overtime cost/hour	$11.00

Production Data	
Working hours/month	160
Labour hours/unit	4
Beginning Inventory	250
Beginning Backorders	0
Beginning Workforce	20
Planning horizon (months)	6

Monthly Demand Data	
May	500
June	700
July	1,200
Aug.	1,300
Sept.	900
Oct.	800

Develop a linear programming model and solve it using Excel Solver. Assume that overload hours cannot exceed 50% of regular hours capacity in each period and ending inventory must be at least 100 units.

2 Formulas

2.1 Notations

P_t Regular production in Period t;

Q_t Overtime production in Period t;

I_t On hand quantity at the end of Period t;

B_t Backlog quantity at the end of Period t;

W_t Number of workers on payroll at the end of Period t;

H_t Number of workers hired in Period t;

F_t Number of workers fired in Period t;

D_t Aggregate demand for Period t.

2.2 Formulation

Objective function:

$$\textbf{Minimize} \sum_{1 \leq t \leq 6} \{200H_t + 100F_t + 7 * 160W_t + 8I_t + 15B_t + 11 * 4Q_t\} \tag{1}$$

Contraints:
Equality constraints,

$$I_{t-1} - B_{t-1} + P_t + Q_t - I_t + B_t = D_t, t = 1, 2...6 \tag{2}$$

$$W_{t-1} + H_t - F_t - W_t = 0, t = 1, 2...6 \tag{3}$$

Balance constraints,
$$P_t \leq 160W_t/4, t = 1, 2...6 \tag{4}$$

$$Q_t \leq (160/2)W_t/4, t = 1, 2...6 \tag{5}$$

Boundaries constraints
$$I_t \geq 100, t = 1, 2...6 \tag{6}$$

Parameters: $I_0 = 250, B_0 = 0, W_0 = 20$ and $D = 500, 700, 1200, 1300, 900, 800$.

2.3 Answer:

Table 1: Production Plan

Period	Workers	Hiring	Firing	Regular Production	Overtime Production	On-hand Inventory	Backorder
May	9	0	11	360	0	110	0
June	17	8	0	680	10	100	0
July	30	13	0	1200	0	100	0
Aug	32	2	0	1280	0	100	20
Sept	23	0	9	920	0	100	0
Oct	0	0	23	0	0	100	800