Calculus & Optimization

Module 1.3

carlos.chavez@immune.institute

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Example:

Let's suppose we are ask to build the following function:

Create a function that given two parameters ***a*** and ***b***, calculates the probability $P(x = a) = \dfrac{a}{b}$

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input: a,b

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            return 'Error, b has to be non-zero'
        else
            if a > b
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              p = a/b
  output  p
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function restrictions

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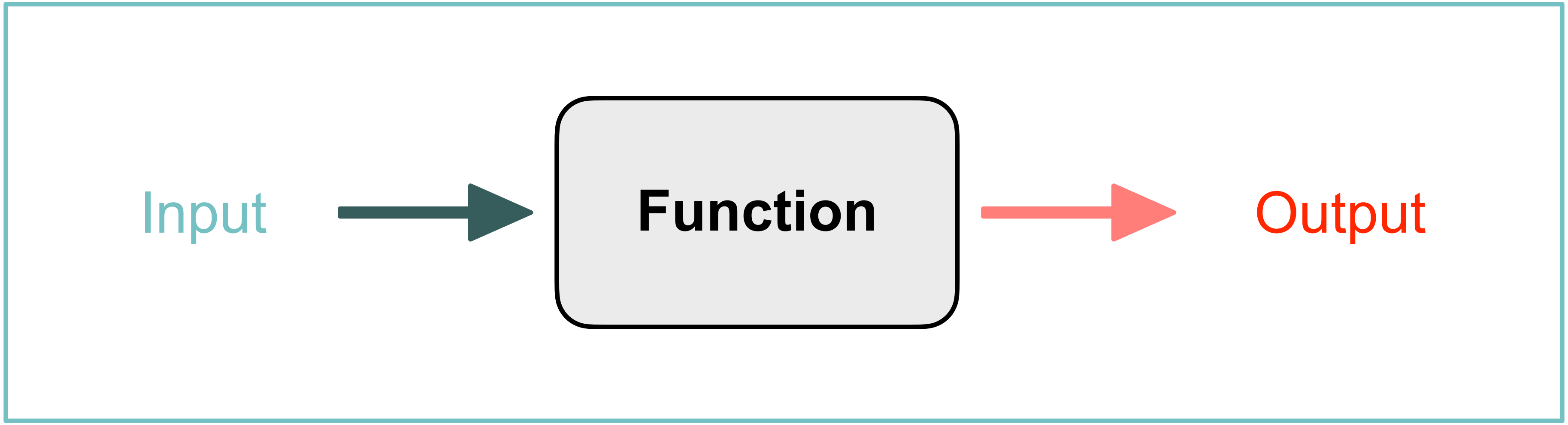
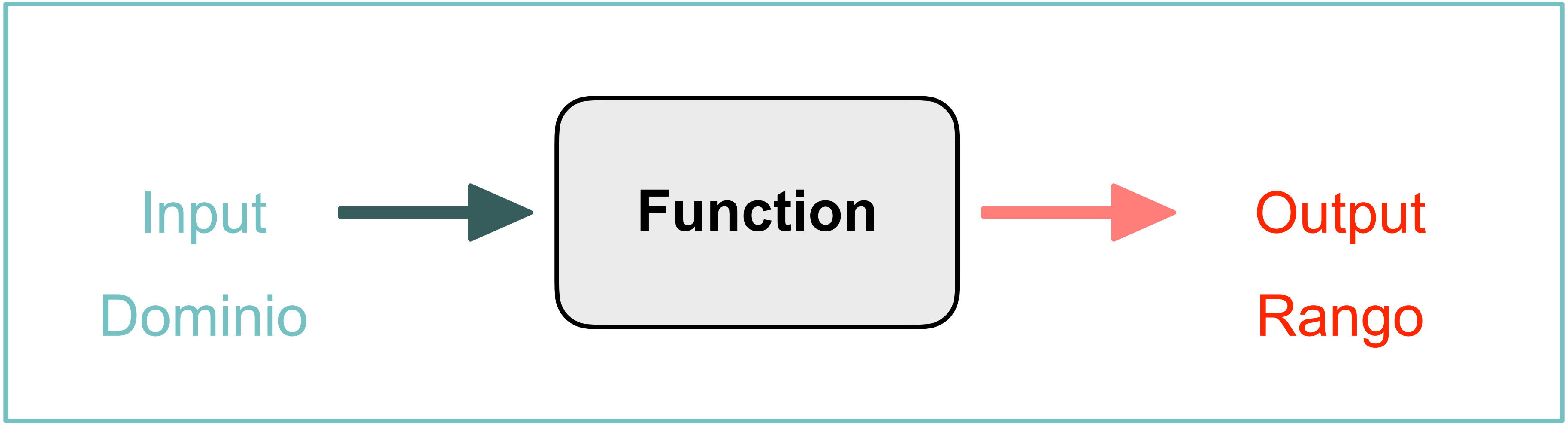
Function Program



Calculus



Calculus

Input

Dominio

Function

Output

Rango

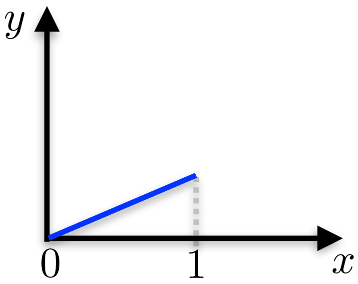
$x \in \mathbb{R}^+ \ (a \in \mathbb{R}^+)$

$b \in \mathbb{R}^+ - \{0\}$

$(a < b)$

$$f(x) = \frac{x}{b}$$

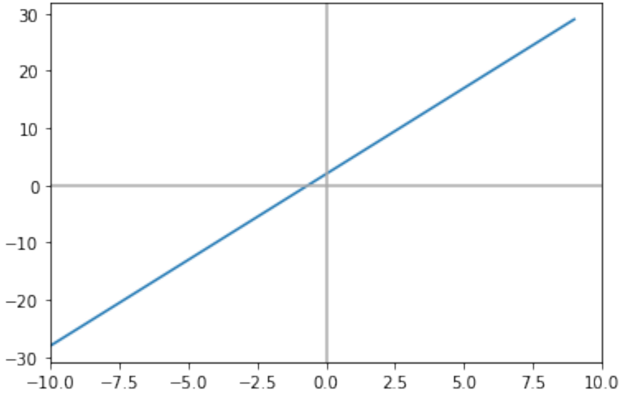
$$0 \leq f(x) \leq 1$$



Functions

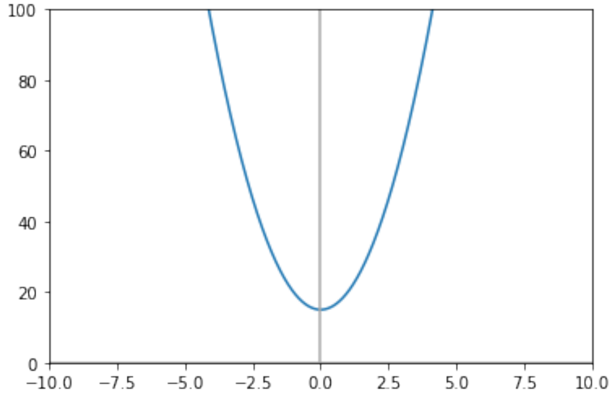
We can define a function as a relation between variables. This relationship associates each element of a set (**input / domain**) to a single element of another set (**output / range**)

$$f(x) = 3x + 2$$



linear

$$f(x) = 5x^2 + 15$$

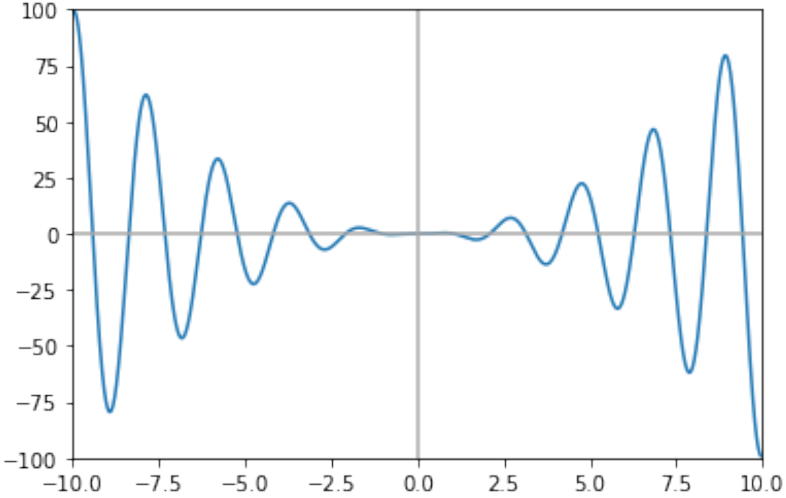


nonlinear

Functions

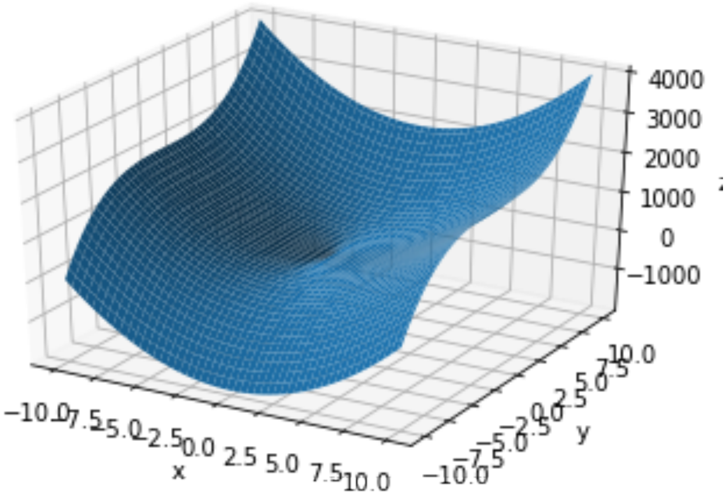
More examples:

$$f(x) = x^2 sin(3x)$$



nonlinear

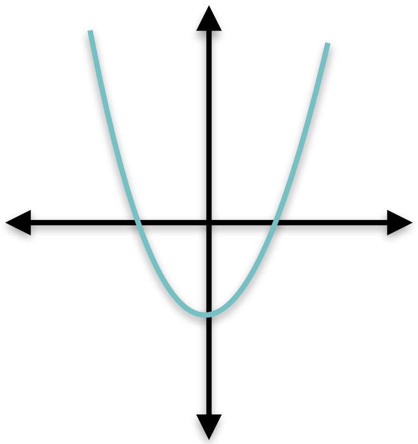
$$f(x, y) = 2y^3 + 20x^2 + 8$$



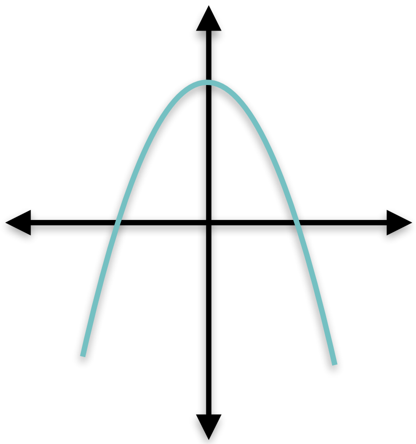
nonlinear

Functions

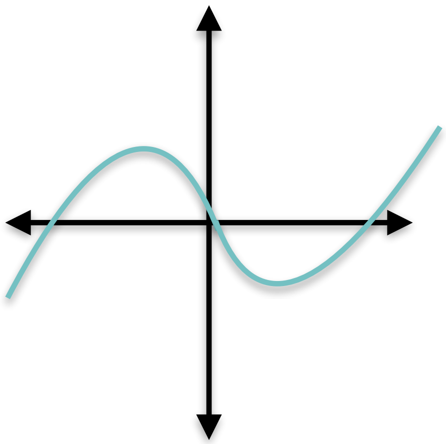
More examples:



Convex Function



Concave Function



non-Convex Function

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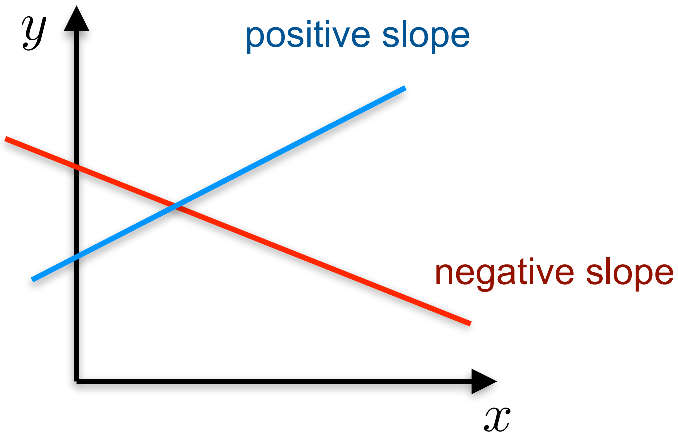
Slopes

The slope of a function tells how much the variable y changes in comparison to the variable x.



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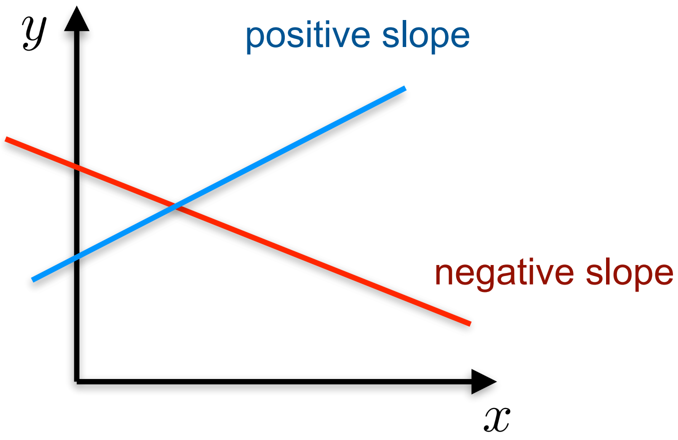

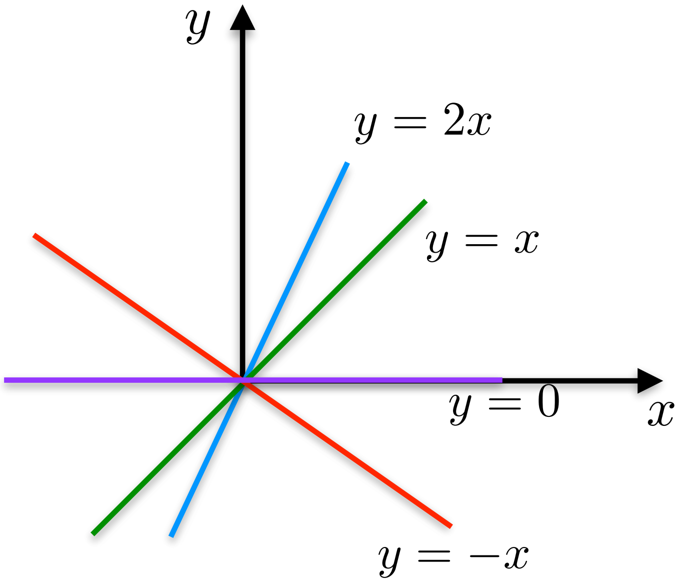

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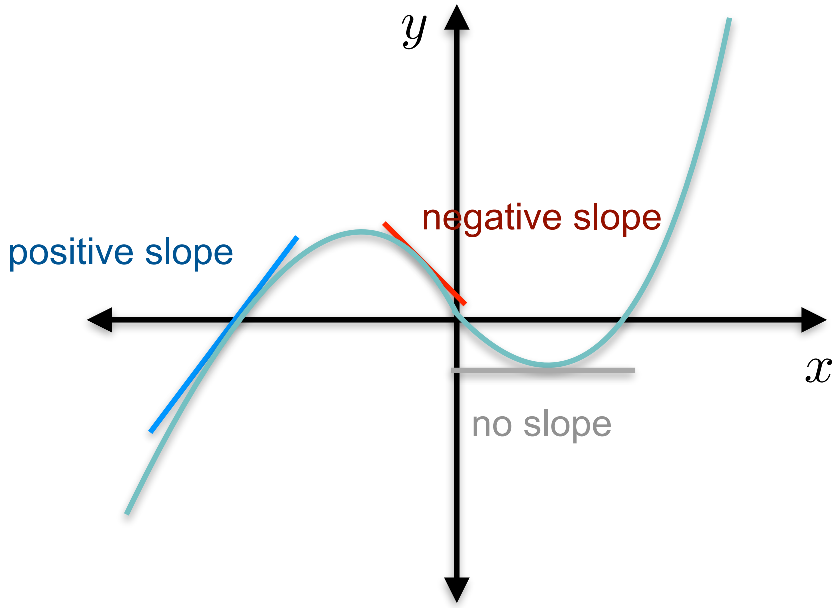
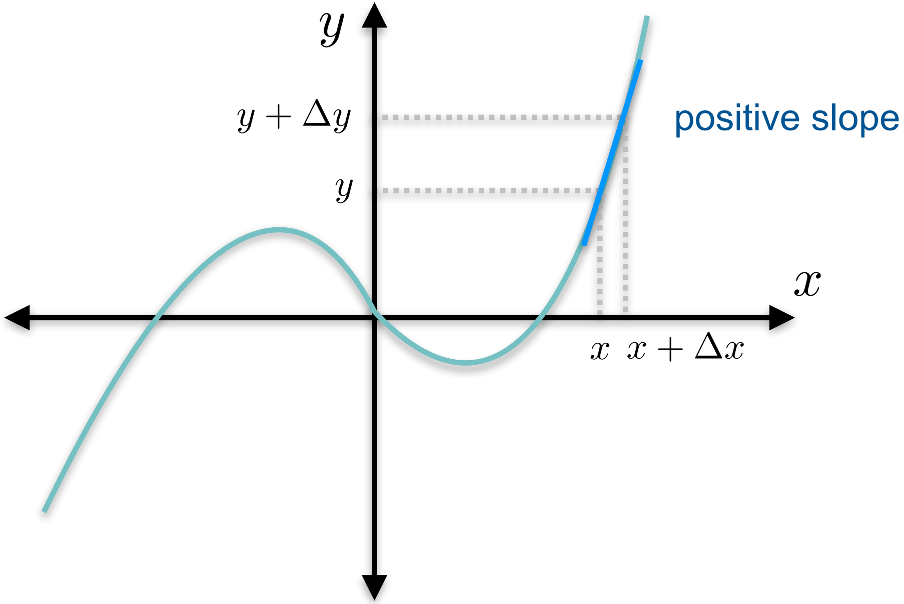
Non linear equations:



Derivatives

Mathematical definition: The derivative at a point of any function is the rate of change of this function at this point. When Δx approaches 0, the rate of change corresponds to the slope of the tangent at this point



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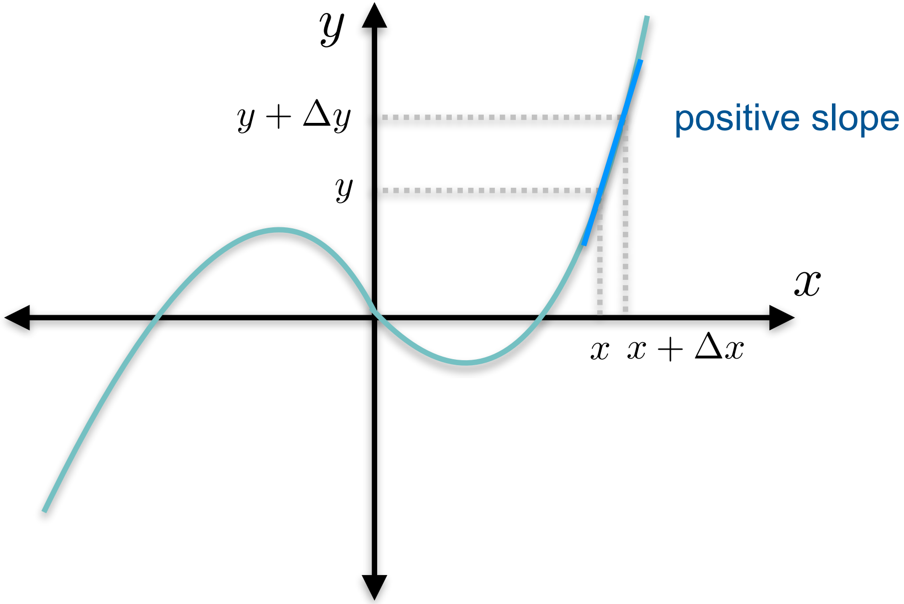

$$slope = \frac{\Delta y}{\Delta x}$$

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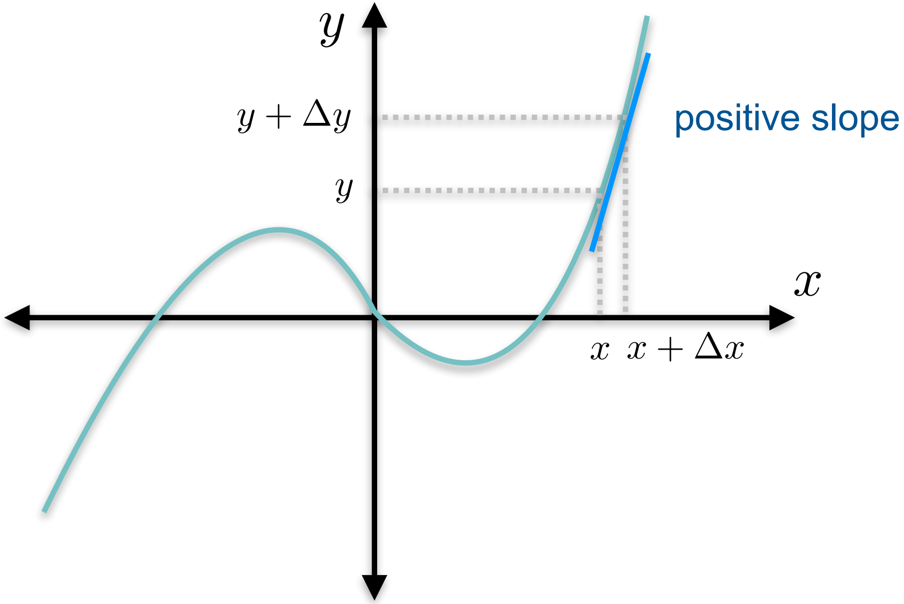

$$slope = \frac{\Delta y}{\Delta x}$$

$$\boxed{f'(x) = \frac{df(x)}{dx} = \frac{f(x + \Delta x) - f(x)}{\Delta x}}$$

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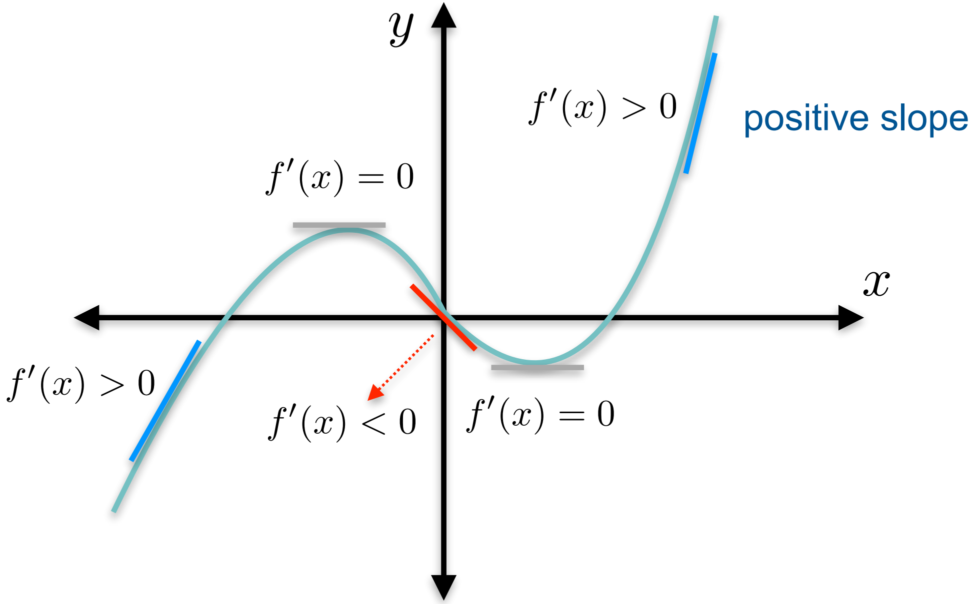

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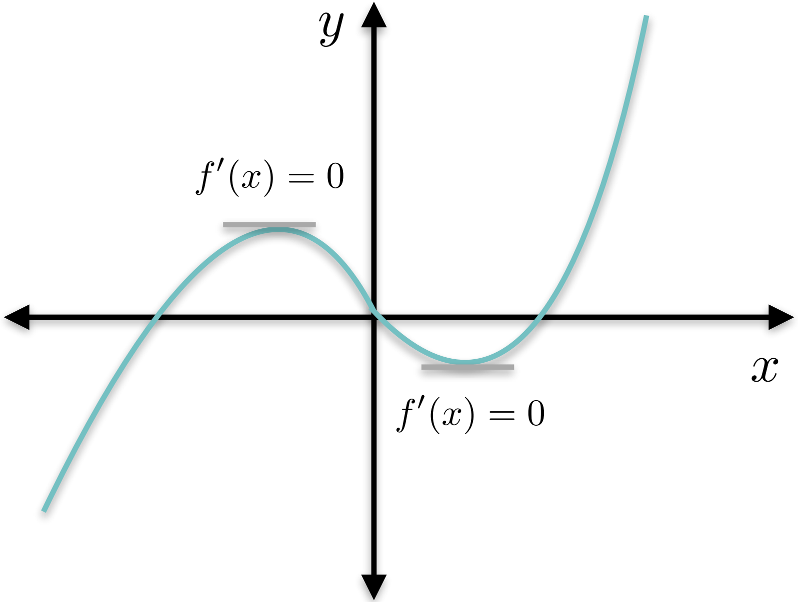
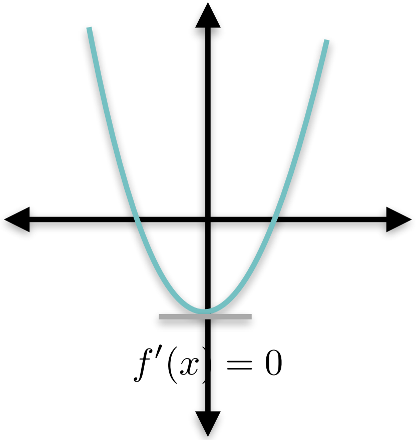
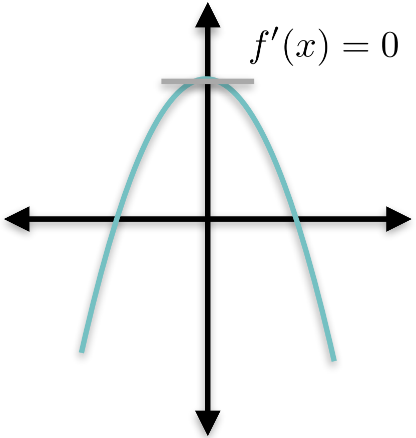
Derivatives Applications

- **Optimization problems:** Search for the maximums or minimums of a function.





Convex



Concave

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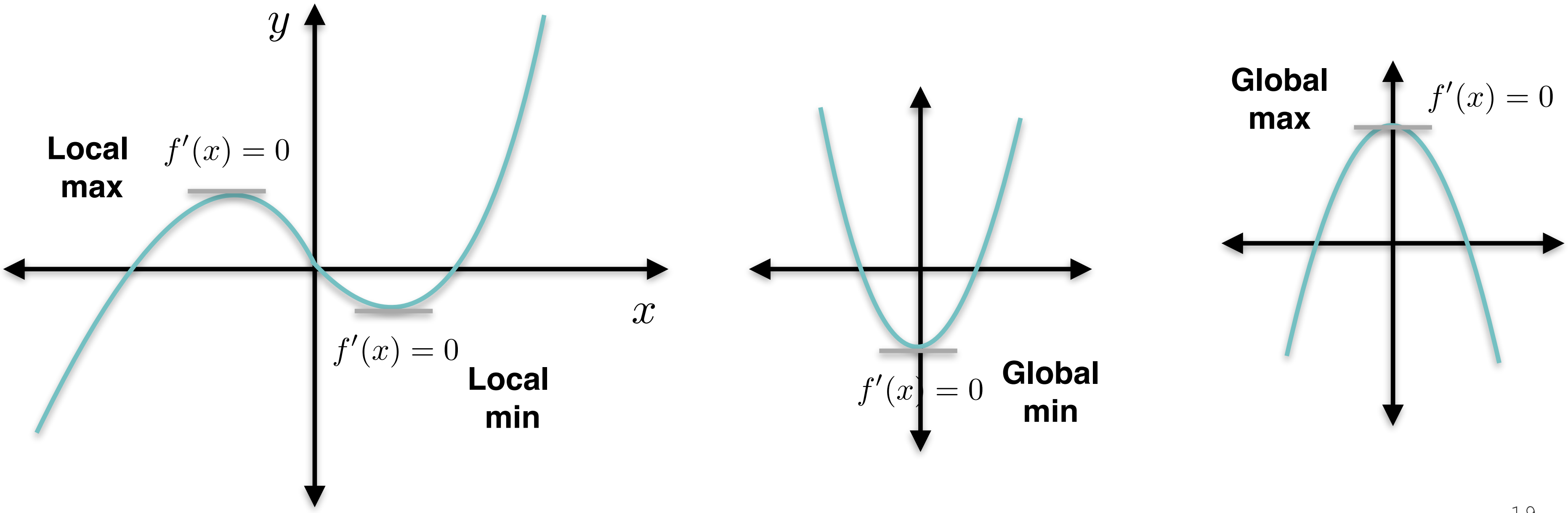

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Minimization or Maximization of Functions

Find $x \in [-2, 2]$ to maximize or minimize $f(x) = x^2 - 4x + 3$

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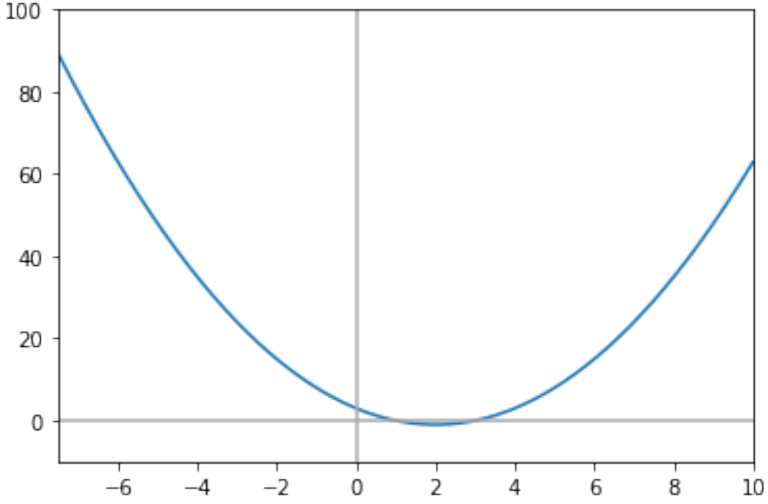

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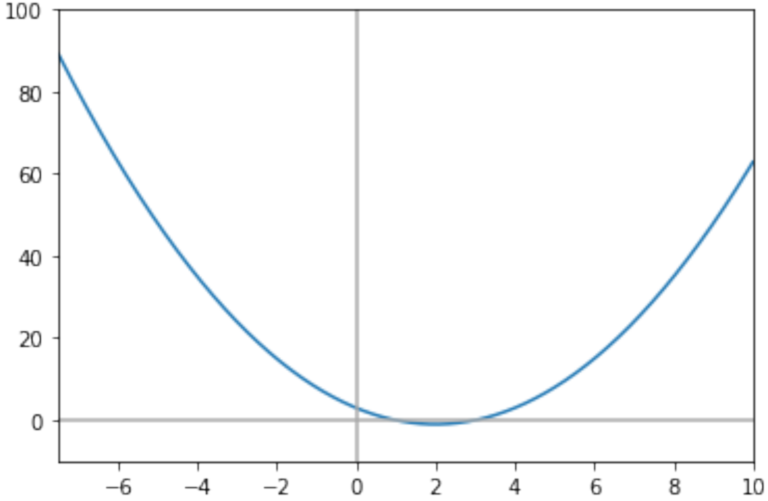

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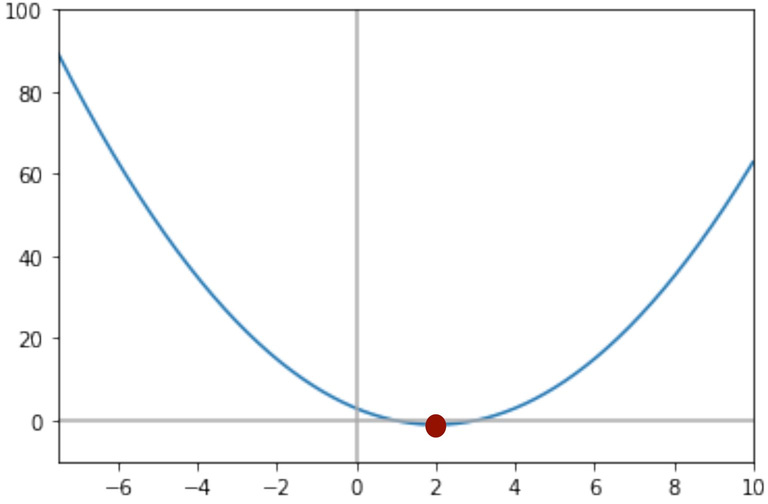
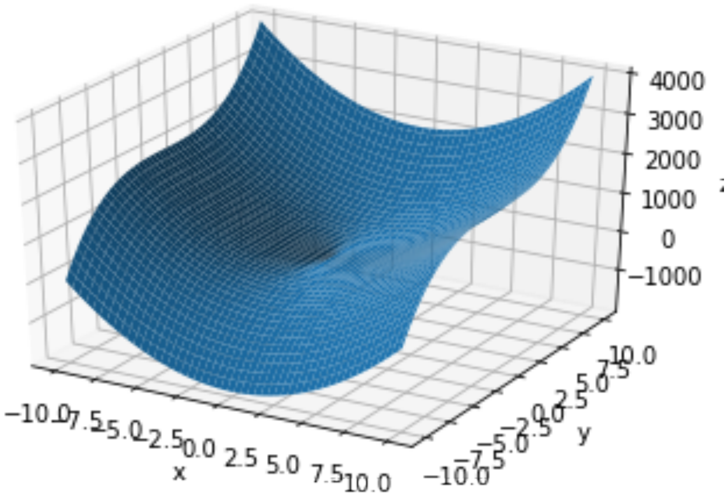
$$\boxed{x = \frac{4}{2} = 2}$$



Gradient

Definition: It is define as the vector compose by the derivatives of a function. This vector points in the direction of greatest increase of a function.

$$\nabla f(\vec{x}) = \begin{bmatrix} \frac{\partial f}{\partial x_1} \\ \vdots \\ \frac{\partial f}{\partial x_n} \end{bmatrix} \begin{array}{l} \longrightarrow \text{partial derivate in } x_1 \\ \\ \longrightarrow \text{partial derivate in } x_n \end{array}$$



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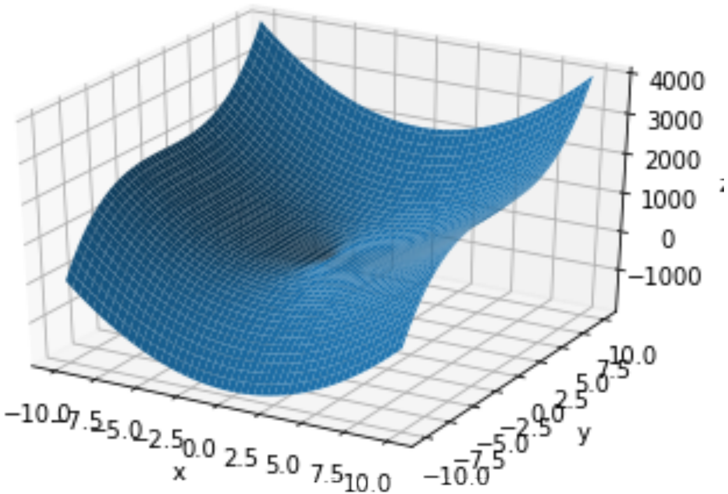

$$f(x, y) = 20x^2 + 2y^3 + 8$$

$$\nabla f(x, y) = \left[\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right] = [40x, 6y^2]$$

Gradient Descent

It is an optimization algorithm used to minimize a function by iteratively moving in the direction of steepest descent as defined by the negative of the gradient.

It is widely used on almost every ML algorithm.

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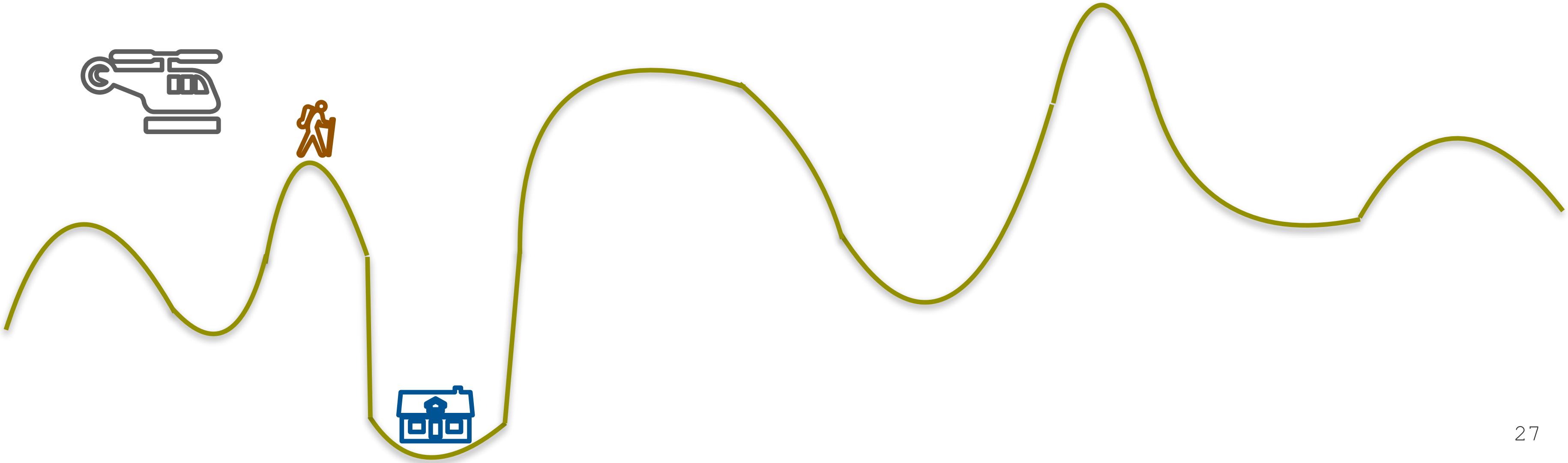
Main Idea



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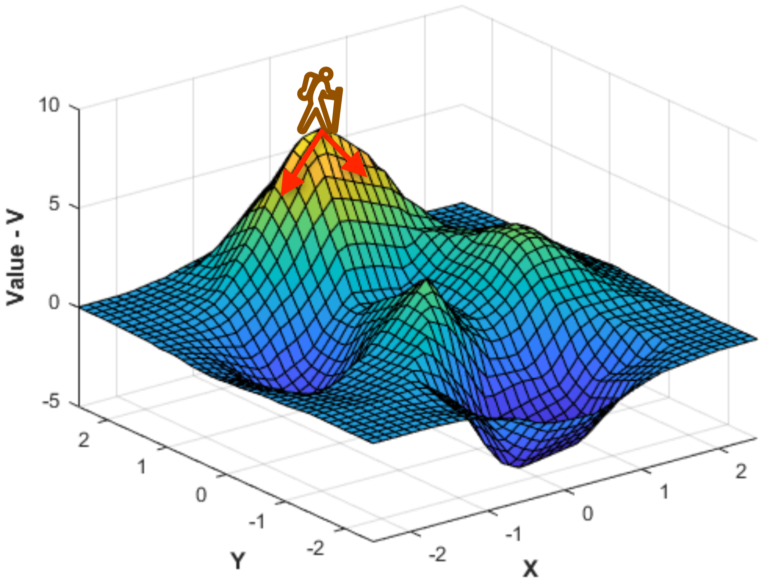
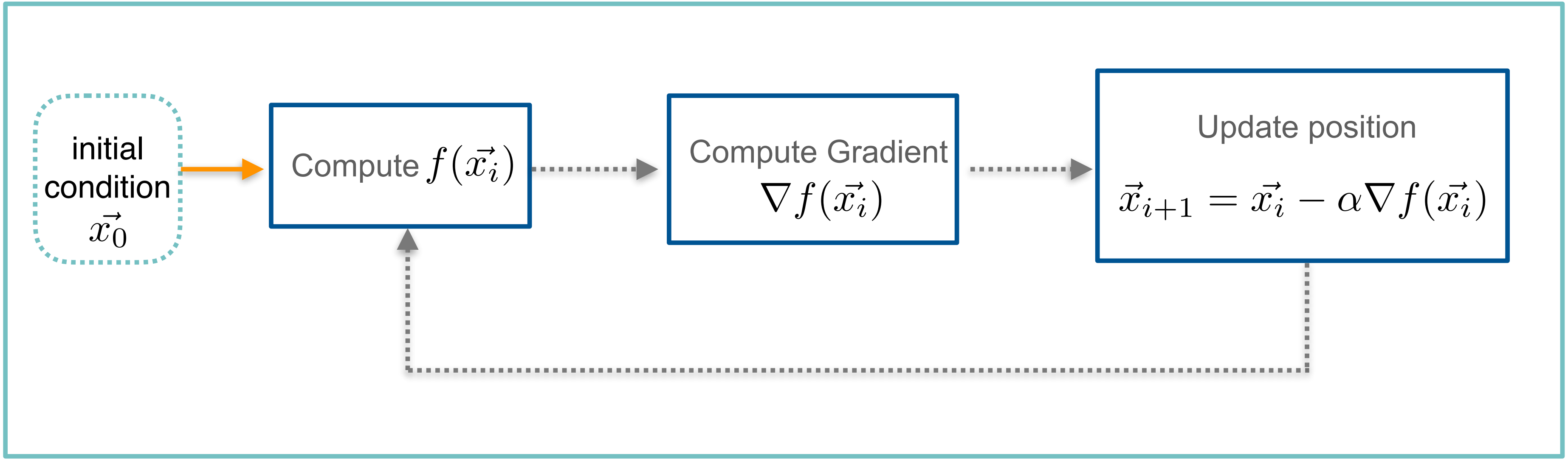


Gradient Descent Algorithm

Procedure



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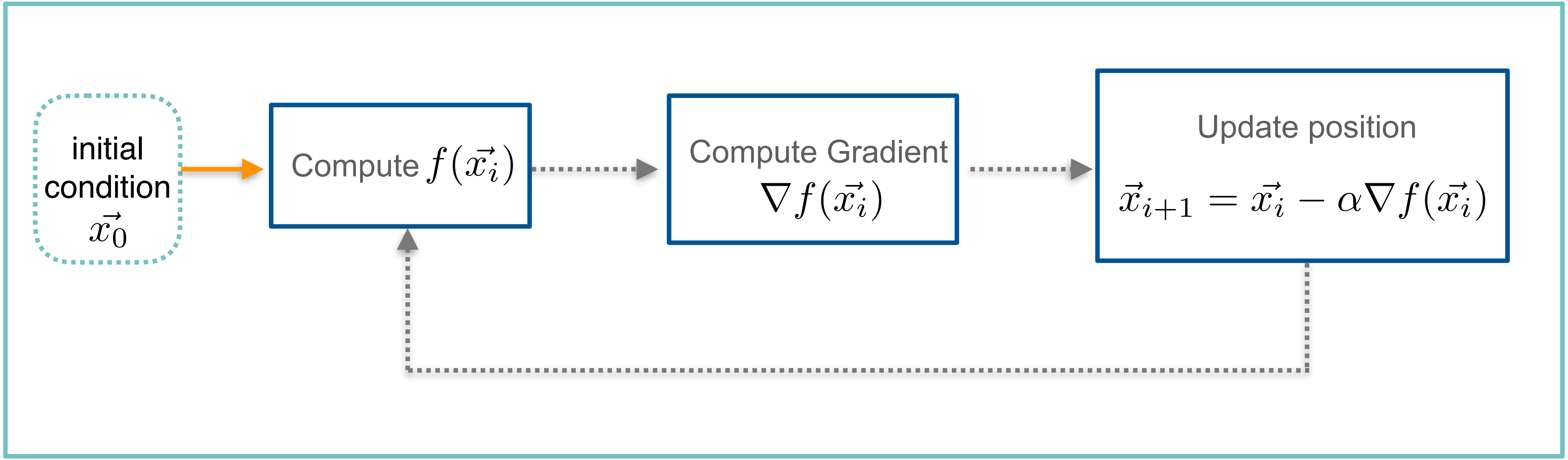

Learning rate α
(Hyperparameter)

Gradient Descent Algorithm

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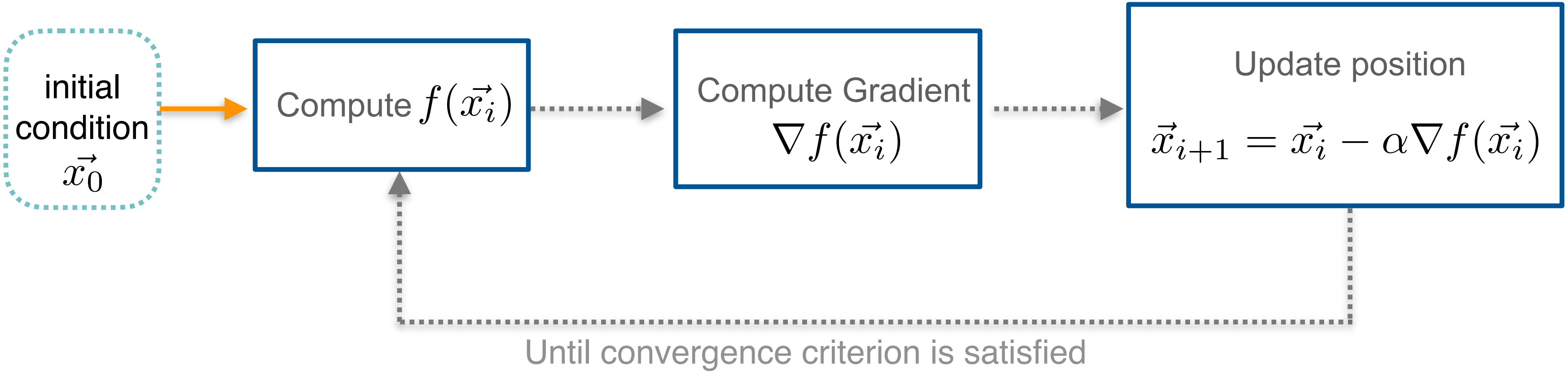

Example: 

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