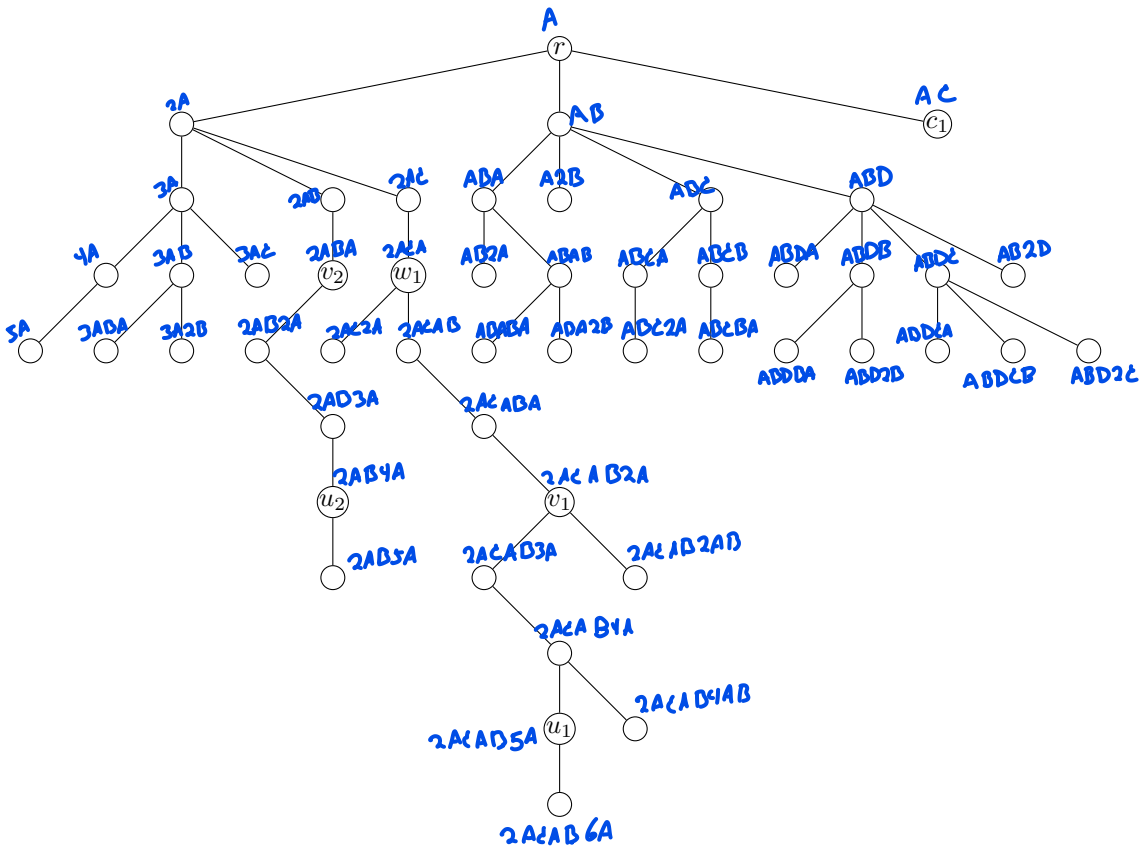

Figure 1: A tree with $\kappa^{(2)}_{nc,\leq 3}(T) = 3$ and $\kappa^{(3)}_{nc,\leq 2}(T) = 4$

$i \to 2$ *(3)* $\Lambda_2 = 8 \not\leq 0$; *compsize(r)* $= 50 - 11 = 39 \not\leq k = 2$.

(4) $\Lambda_2 = 8 \not\leq l = 3$; *compsize(r)* $= 39 \neq k = 2$.

(5) Vertices such that $l(u) = \Lambda_2 = 8$ *in* T_2 *do not exist.*
Set $F_3 = F_2 = \{v_1\}$; $T_3 = T_2 = T - T(w_1)$; $\Lambda_3 = \Lambda_2 - 1 = 7$;
$i \to 3$.

$i \to 3$ *(3)* $\Lambda_3 = 7 \not\leq 0$; *compsize(r)* $= 39 \not\leq k = 2$.

(4) $\Lambda_3 = 7 \not\leq l = 3$; *compsize(r)* $= 39 \neq k = 2$.

(5) Only the lowest vertex in T_3 *has* $l(u) = \Lambda_3 = 7$. *compsize(u)* $\not\geq k = 2$.
Set $F_4 = F_3 = \{v_1\}$; $T_4 = T_3 = T - T(w_1)$; $\Lambda_4 = \Lambda_3 - 1 = 6$;
$i \to 4$.