National workshop on Applications of Deep Learning in Computer Vision SSN Engineering College

Introduction to Deep Learning

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Logistic Regression

Linear classification

- $\mathcal{D} = \{(\mathbf{x}_1, y_1), (\mathbf{x}_2, y_2), \dots, (\mathbf{x}_N, y_N)\} \triangleright y \text{ is binary-valued}$ (discrete-valued)
- ► Linear combination of features of x

$$w_0 + w_1 x_1 + \ldots + w_d x_d = w_0 + \sum_{i=1}^d w_i x_i$$

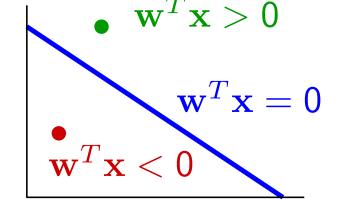
 $w_0 x_0 + w_1 x_1 + \ldots + w_d x_d = \sum_{i=0}^d w_i x_i$, $x_0 = 1$

Vectors in matrix notation

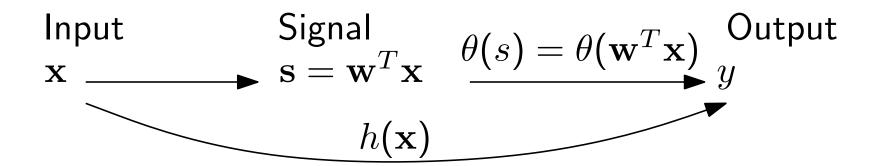
$$\mathbf{x} = egin{bmatrix} x_0 \ x_1 \ dots \ x_d \end{bmatrix} \quad \mathbf{w} = egin{bmatrix} w_0 \ w_1 \ dots \ x_d \end{bmatrix} \quad \mathbf{w} = egin{bmatrix} w_0 \ w_1 \ dots \ x_d \end{bmatrix} \quad \hat{y} = \operatorname{sign}(\mathbf{w}^T \mathbf{x})$$

Linear combination $\sum_{i=0}^{d} w_i x_i = \mathbf{w}^T \mathbf{x} = \mathbf{w}.\mathbf{x}$

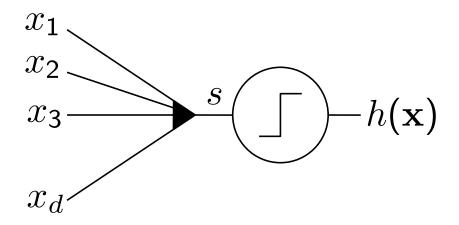
$$\hat{y} = egin{cases} +1 & ext{if } \mathbf{w}^T \mathbf{x} > 0 \ -1 & ext{if } \mathbf{w}^T \mathbf{x} < 0 \end{cases}$$
 $\hat{y} = ext{sign}(\mathbf{w}^T \mathbf{x})$



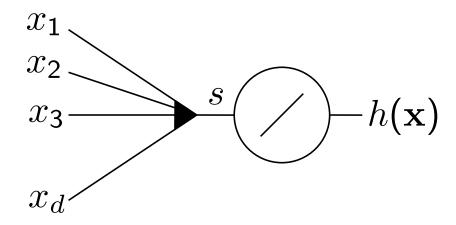
Linear models



Linear classification $h(\mathbf{x}) = \text{sign } (\mathbf{w}^T \mathbf{x})$

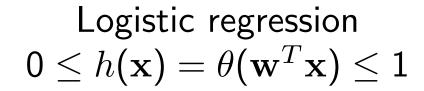


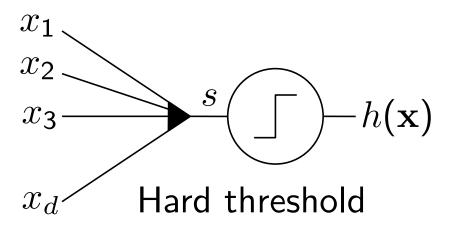
Linear regression $h(\mathbf{x}) = \mathbf{w}^T \mathbf{x}$

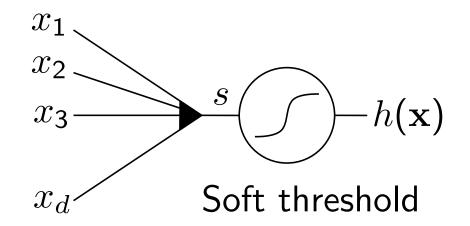


Logistic regression

Linear classification $h(\mathbf{x}) = \text{sign } (\mathbf{w}^T \mathbf{x})$

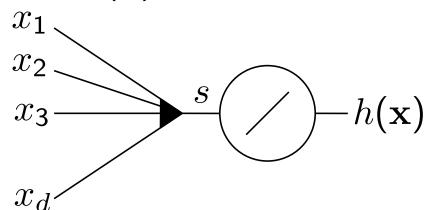






Linear regression

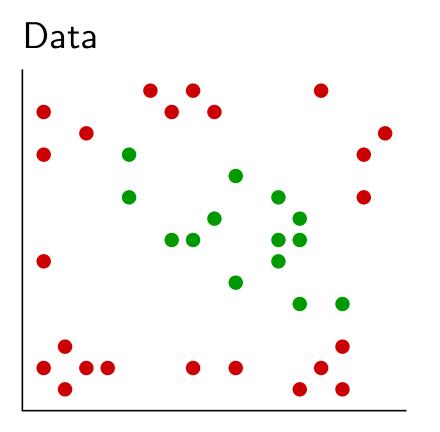
$$h(\mathbf{x}) = \mathbf{w}^T \mathbf{x}$$

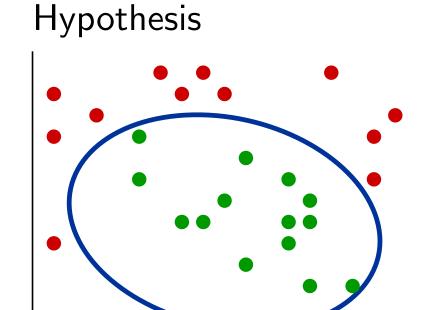


Logistic regression — overview

- $h(\mathbf{x}) = \frac{1}{1 + e^{-\mathbf{w}^T \mathbf{x}}} = P(y|\mathbf{x}), \text{ hypothesis set}$
- $e(h(\mathbf{x}_i), y) = \ln(1 + e^{-y_i \mathbf{w}^T \mathbf{x}_i})$, cross-entropy error
- $E(\mathbf{w}) = \frac{1}{N} \sum_{i=1}^{N} \ln(1 + e^{-y_i \mathbf{w}^T \mathbf{x}_i})$, in-sample error or likelihood
- ightharpoonup Minimize in-sample error $E(\mathbf{w})$
- ► Gradient $\nabla_{\mathbf{w}} E(\mathbf{w})$, in which direction of \mathbf{w} error decreases most rapidly and how much
- ► Gradient descent: $\mathbf{w}_{(t+1)} \leftarrow \mathbf{w}_{(t)} + \eta \nabla_{\mathbf{w}} E(\mathbf{w})$

Linear is limited

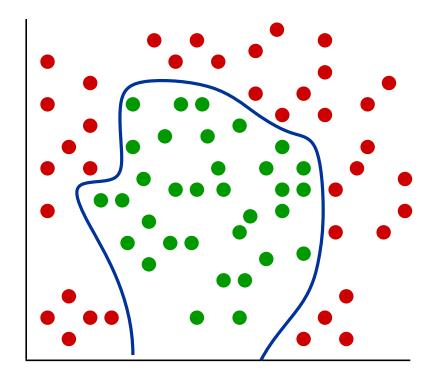




- Credit limit is affected by 'years in residence'
- But not in a linear way!

Idea

- Generalization of perceptron.
- Can model complex (non-linear) target function.
- ightharpoonup Fits the data by minimizing in-sample error E
- Flexible, but risk of overfitting.

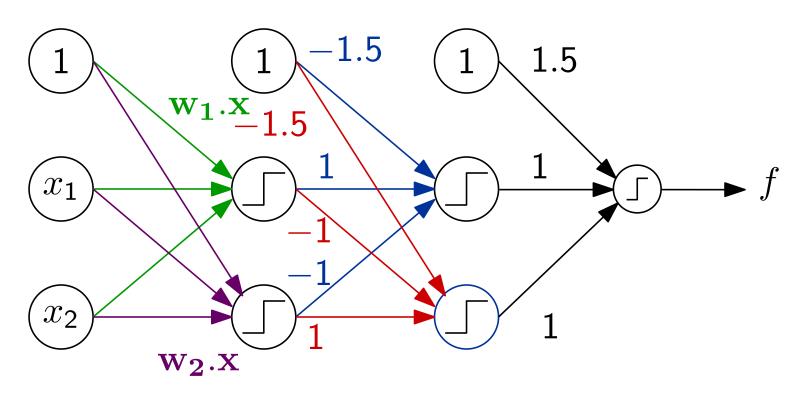


Neural Networks

Idea

- Generalization of perceptron.
- Can model complex (non-linear) target function.
- \blacktriangleright Fits the data by minimizing in-sample error E
- ► Flexible, but risk of overfitting.

Multi-layer perceptron (MLP) of XNOR



3-layer feed-forward

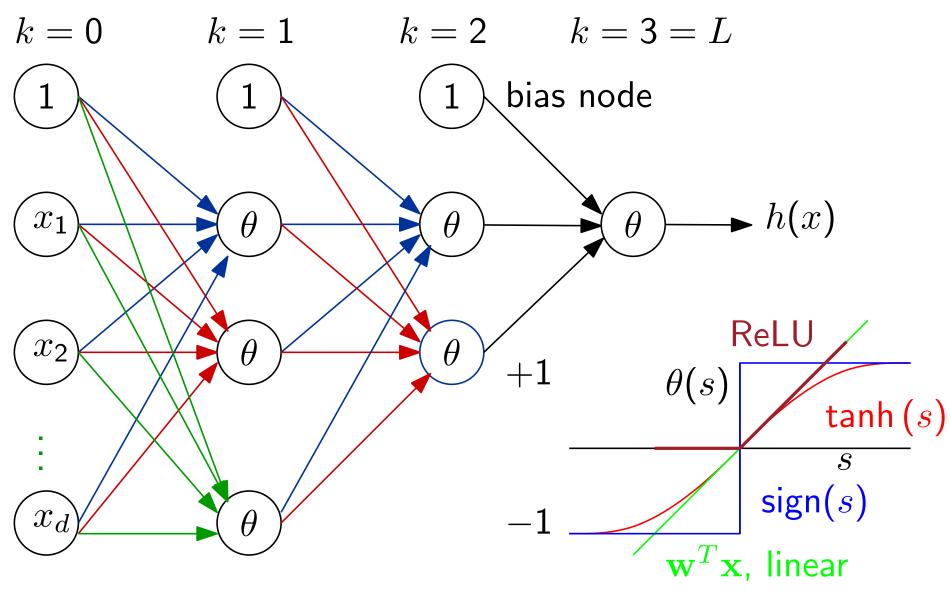
$$h_1(\mathbf{x}) = \mathbf{w}_1^T \mathbf{x}$$
 $h_1(\mathbf{x}).\overline{h_2(\mathbf{x})}$ $\underline{h_1(\mathbf{x})}.\overline{h_2(\mathbf{x})}$ + $h_2(\mathbf{x}) = \mathbf{w}_2^T \mathbf{x}$ $\overline{h_1(\mathbf{x})}.h_2(\mathbf{x})$ $h_1(\mathbf{x}).h_2(\mathbf{x})$

Between input and output, multiple hidden layers.

Feed-forward MLP

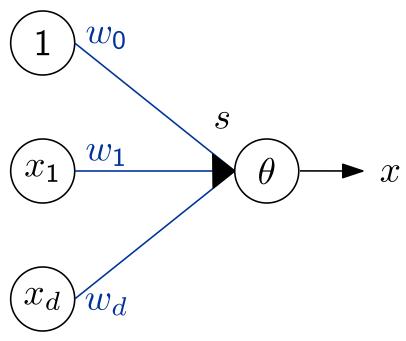
- ightharpoonup Each layer feed-forward to the next layer only. Layer i to layer i+1 only.
- No feed-backward, no jump forward to other layers.
- Perceptron has just input and output.
- Multi-layer perceptron
 - \blacktriangleright input layer with d_{in} nodes,
 - ightharpoonup output layer with d_{out} node,
 - ▶ 2 hidden layers with 3 nodes (hidden units) each.
- Architecture of MLP: number of hidden layers and number of hidden units in each layer.
- A 3-layer MLP with suitably many hidden units can model any target function — can fit any data.
- May overfit! May not generalize.

Neural network



input x hidden layers $1 \le k < L$ output layer L

Activation function



- Desirable that activation functions are differentiable, and
- ► Expressed in terms of the function itself.

- ► Tanh: output $\theta(s)$ $\theta(s) = \tanh(s) = \frac{e^s e^{-s}}{e^s + e^{-s}}$
- $ightharpoonup \operatorname{ReLU}$: output $\theta(s) = \max(s,0)$
- Sigmoid: output $\theta(s) = \frac{1}{1 + e^{-s}}$

Output-input for one layer

$$(\mathbf{s^{(k)}})^{\mathbf{T}} = (\mathbf{x^{(k-1)}})^{\mathbf{T}}.\mathbf{W^{(k)}}$$

 $\mathbf{x^{(k)}} = \theta(\mathbf{s^{(k)}})$

Forward propagation idea

$$\mathbf{x}^{(0)} \xrightarrow{W^{(1)}} \mathbf{s}^{(1)} \xrightarrow{\theta} \mathbf{x}^{(1)} \xrightarrow{W^{(2)}} \mathbf{y}$$

$$\begin{bmatrix} x_{0}^{(0)} & s_{0}^{(1)} & s_{0}^{(1)} & s_{0}^{(1)} & s_{0}^{(1)} & s_{0}^{(2)} & s$$

Training error

► Training error

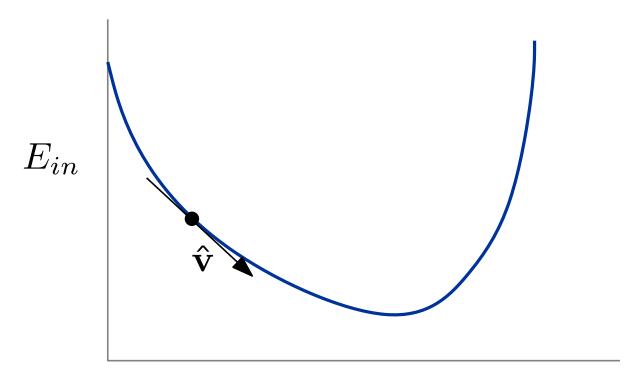
$$E(\mathbf{w}) = \frac{1}{N} \sum_{i=1}^{N} e_i$$

$$= \frac{1}{N} \sum_{i=1}^{N} (\hat{y}_i - y_i)^2$$

▶ To minimize $E(\mathbf{w})$, solve $\nabla E(\mathbf{w}) = \mathbf{0}$ for \mathbf{w} .

Iterative method: gradient descent

- General method for non-linear optimization
- ightharpoonup Start at $\mathbf{w}_{(0)}$, descend the surface along the steepest slope
- ▶ Let $\hat{\mathbf{v}}$ (unit vector) be the direction of steepest slope.
- ightharpoonup Take fixed-size steps along $\hat{\mathbf{v}}$: $\mathbf{w}_{(t+1)} = \mathbf{w}_{(t)} + \eta \hat{\mathbf{v}}$



Stochastic Gradient Descent (SGD)

- 1. Initialize $\mathbf{w}_{(0)}$ 2. for $t=1,2,3,\ldots$ until \mathbf{w} converges do
 3. for $i=1\ldots N$ do
 4. Compute gradient $\nabla E(\mathbf{w})$ 5. Update weight: $\mathbf{w}(t+1) \leftarrow \mathbf{w}(t) \eta \nabla E(\mathbf{w}(t))$
- 6. Return w

- ► For updating w once, it considers only one training example, chosen at random.
- Converges faster, but may oscillate.

Backpropagation idea

Sensitivity of layer
$$k$$
, $\delta^{(k)} = \frac{\partial e}{\partial s^{(k)}}$

lackbox Partial derivatives in layer k-1 in terms of partial derivatives of layer k

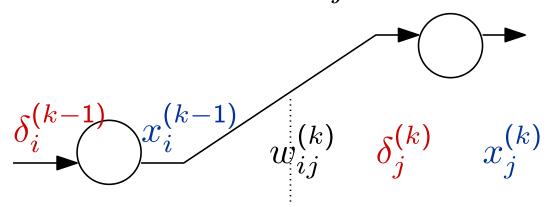
$$\frac{\partial e(\mathbf{w})}{\partial W^{(k)}} = x^{(k-1)} \times (\delta^{(k)})^T$$

$$\delta^{(k)} = \theta'(s^{(k)}) \otimes [W^{(k+1)} \times \delta^{(k+1)}]_1^{D_k}$$

$$\delta^{(L)} = 2(\hat{\mathbf{y}} - \mathbf{y})\theta'(\hat{\mathbf{y}})$$

Backpropagation algorithm

- 1: Initialize all weights w_{ij}^k at random
- 2: for $t = 1, 2 \dots$ do
- 4: Forward: Compute all $x_j^{(k)}$
- 5: Backward: Compute all $\delta_i^{(k)}$
- 6: Update weights: $w_{ij}^{(k)} \leftarrow w_{ij}^{(k)} \eta x_i^{(k-1)} \delta_j^{(k)}$
- 7: Return the final weights w_{ij}^k



Issues in Back-propagation algorithm

- ► It has been shown that finding weights to minimize error is NP-complete [Blum and Rivest, 1989]
- Not guaranteed to converge
- Over-fitting to the training samples
- Error minimization process can get trapped in a local minima
- Extremely slow convergence
- Selection of initial weights may be critical
- Selection of optimal parameters
- Extremely difficult to explain the model and its predictions

Deep Neural Networks

Large datasets

- ► Today, we have very large datasets available especially for images, videos, speech, and text.
- ► However, traditional feature engineering and classifiers based on given-representations have saturated — "shallow" networks have relatively small model parameters to capture information content of large data sets.
- More than two hidden layers.
- Very deep neural networks: up to hundreds of layers.

Feature Engineering

- Major focus of machine learning has been selecting and extracting appropriate features
- Images: color, texture, edges, connected components, shapes, moments, SIFT, SURF, HOG, Wavelets-based, etc.
- ► Text: n-grams, character n-grams, POS tags, Named Entities, tags from shallow parsing, word sense disambiguation, etc.
- ► Speech: Mel Cepstral coefficients (MFCC), energy, zero crossing rates, pitch, timbre, etc.
- Which features are suitable for a given task? Leads to feature engineering.
- Which classifier is better? Ensemble of classifiers?
- ▶ Is a machine learning algorithm really "intelligent"?

Challenges with deep neural networks

- Vanishing gradients
- Extremely slow convergence
- Lack of generalization
- Lack of large data number of model parameters increases with depth
- Lack of labeled data supervised learning requires examples with corresponding targets
- Computational inefficiency

Vanishing gradient in backpropagation

- Network parameters are updated proportional to the partial derivative of the cost function w.r.t. the current parameters in each iteration of training.
- \blacktriangleright tanh has gradients in the range (0, 1)
- ightharpoonup Chain rule multiplies n of these small numbers to compute gradients of the leftmost layers in an n-layer network.
- ightharpoonup Gradient decreases exponentially with n.
- Effectively preventing some parameters from changing their values.
- ► Earlier layers train very slowly, if at all.
- May completely stop the neural network from further training.
- Several improvements combined together: ReLU, LSTM, skip connections used in residual neural networks.

New deep learning ideas

- New activation functions
- Regularization methods
- Initialization methods
- Data augmentation
- Optimization techniques
- ► GPU-based and distributed algorithms

Activation Function: Rectifier

Rectifier is a simple activation function defined as

$$f(x) = \max(0, x)$$

- Neuron using rectifier is popularly known as ReLU (Rectified Linear Unit)
- One variation of this is "softplus" function

$$f(x) = \ln(1 + ex)$$

$$f'(x) = \frac{e^x}{1 + e^x} = \frac{1}{1 + e^{-x}}$$

- ReLU converges faster than networks using traditional 'tanh' [Krizhevsky et al., 2012]
- Variations of ReLU

Activation Function: Softmax

- Squashes a k-dimension vector of real values to k-dimension vector of real values in the range (0,1) that add up to 1
- ightharpoonup Converts a k-dimension vector to probability distribution over k different possible outcomes
- Useful in multi-class classification
- ► For example, softmax of [1,2,3,4,1,2,3] is [0.024, 0.064, 0.175, 0.475, 0.024, 0.064, 0.175]

Regularization methods: Dropout

- Set the output of a hidden neuron to 0 with a probability of 0.5 (effectively "drop" those neurons out).
- Dropped out neurons do not take part in forward pass and back-propagation.
- Hence, different architecture is sampled every time, but these networks share weights.
- ► Reduces the complex co-adaption of neurons and reduces the model parameters to be learned.
- However, dropout almost doubles the number of iterations required.

Data augmentation

- We may not have enough data for learning a large number of model parameters (especially labeled data)
- Augment the existing data with some "label-preserving" operations
 - Image translations and horizontal reflections. Given, 256x256 images, extract random 224x224 patches and their horizontal reflections – object labels do not change!
 - ► Alter the intensities of RGB channels one idea may be to perform PCA to find principal components and add a magnitude of this to the intensities.

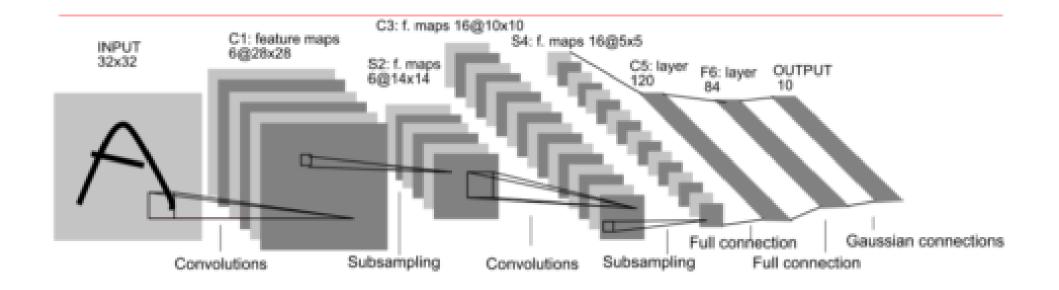
Stochastic Gradient Descent

- ► Stochastic Gradient Descent (SGD) incrementally updates the weights for each training example.
- This may result in wide swings the weights (which may be good sometimes!)
- To overcome this problem, learning rate is slowly decreased as the learning progresses.
- ▶ In other words, fluctuations are permitted initially, but controlled with the progress of the iterations simulated annealing idea.
- ▶ It is possible to take best of both worlds and perform mini-batch updates – gradients are calculated for a batch of n training examples.
- Mini-Batch updates provide better exploitation of GPU and distributed computing

Challenges in Mini-Batch SGD

- How to choose appropriate learning rate? A high value may lead to undesirable fluctuations.
- ► What may a better schedule for reducing the learning rate? Extremely slow schedule may lead us to a better minima, but then convergence will also be extremely slow.
- Should all the parameters have the same learning rate?

LeNet5



- ► Trained using 32x32 pixel size images from MNIST (at most 20x20 characters centered in 28*28)
- Cx are convolutional layers (C1, C3, C5)
- Sx are sumsampling (pooling) layers (S1, S4)
- ► One fully connected hidden layer (F6) with 84 neurons and an output layer with 10 neurons

Convolutional Neural Networks

High dimensional input

- ightharpoonup Add one layer k with D_k neurons.
- ▶ Add $(D_{k-11} + 1) \times D_k$) parameters $W^{(k)}$ and bias.
- Optimization become intractable.
- Images input is very high-dimensional.
- ► Each pixel of an image is a feature. If image is 100 by 100 pixels, then there are 10,000 features.

Convolutional neural networks

- Reduces the number of parameters in a deep neural network with many units without losing too much in the quality of the model.
- ► CNNs have found applications in image and text processing.

Learn locally

- ► In images, pixels that are close to one another usually represent the same type of information: sky, water, leaves, fur, bricks.
- ► Exception from the rule are the edges: the parts of an image where two different objects "touch" one another.
- ▶ Train the network to recognize regions of the same information as well as the edges \Rightarrow Can predict the object represented in the image.
- Split the image into square patches using a moving window approach.
- ► Learn multiple smaller regression models at once.
- ► Train small regression model for a square patch as input learns to detect a specific kind of pattern in the input patch.
- One to detect the sky; another one for the grass, the third one for edges of a building.

Convolution

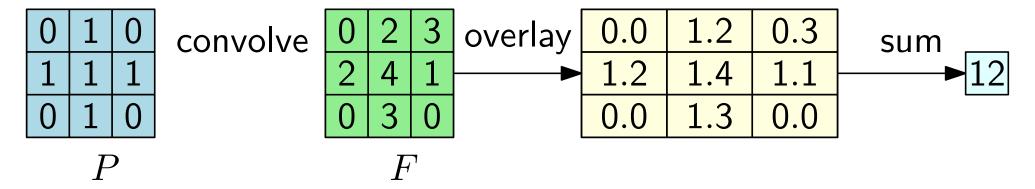
- ▶ Input image is made of black and white pixels.
- ightharpoonup A patch is a 3 imes 3.
- ► Learn a regression model to detect a cross pattern in patches.

$$\mathbf{P} = egin{bmatrix} 0 & 1 & 0 \ 1 & 1 & 1 \ 0 & 1 & 0 \end{bmatrix}$$

Learn a 3×3 parameter matrix \mathbf{F} (filter): parameters corresponding to 1's will be positive and corresponding to 0's will be close to 0.

$$\mathbf{F} = \begin{bmatrix} 0 & 2 & 3 \\ 2 & 4 & 1 \\ 0 & 3 & 0 \end{bmatrix}$$

Convolution between two matrices



 ${f P}$ convolution ${f F}={f 12}$

- ightharpoonup Higher the convolution value, the more similar ${f F}$ is to ${f P}$.
- ► A bias added before applying activation function.

$$\mathbf{P} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \end{bmatrix}, \mathbf{F} = \begin{bmatrix} 0 & 2 & 3 \\ 2 & 4 & 1 \\ 0 & 3 & 0 \end{bmatrix}$$

 ${f P}$ convolution ${f F}={f 9}$

► Each filter slides (convolves) across the input image, left to right, top to bottom, and convolution is computed at each slide.

A filter convolves across an image

Cor	<u>ı</u> v 1	•			Conv 4		
1 0	0	1	-1 2	4	1 0 0 1	-1 2	4 -1 7
1 0	1	0	4 -2		1 0 1 0	4 -2	2
1 1	0	0	Filter		1 1 0 0		
0 1	0	1	$\lfloor 1 \rfloor$		0 1 0 1	1	
Cor	1v 2	<u>-</u>	Bias		Conv 5		
1 0	0	1	-1 2	4 -1	1 0 0 1	-1 2	4 -1 7
1 0	1	0	4 -2		1 0 1 0	4 -2	2 7
1 1	0	0			1 1 0 0		
0 1	0	1	1		0 1 0 1	1	
Cor	1v 3	3			Conv 6		
1 0	0	1	-1 2	4 -1 7	1 0 0 1	-1 2	4 -1 7
1 0	1	0	4 -2		1 0 1 0	4 -2	2 7 6
1 1	0	0			1 1 0 0		
0 1	0	1	1		0 1 0 1	1	

Convolution layer

- One convolution layer made of multiple filters.
- ► Filter matrices and bias are parameters learned through gradient descent.
- ReLU activationfunction in convolution layer.
- Output layer activation function?
- In convolution layer k, each filter outputs one matrix, D_k filters output D_k matrices.
- ▶ If layer k+1 is convolutional, it treats the D_k matrices as D_k image matrices, volume of depth D_k .
- Convolution of a patch of a volume is the sum of convolutions of the corresponding patches of individual matrices of the volume.
- Input image is a volume of 3 channels: R, G, B.

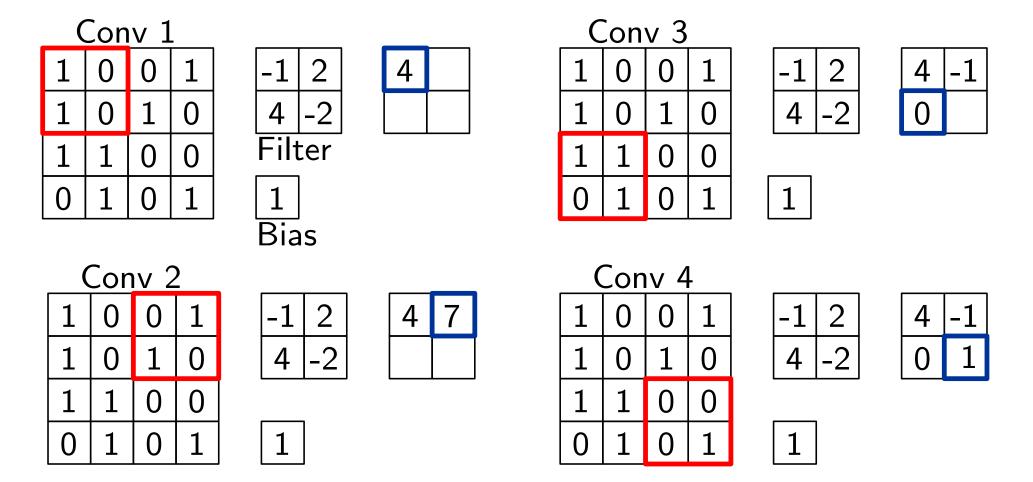
Convolution of a volume

3 1 -2 4 4 1 0 5 2 2 1 0 1 -2 -1 2	2 -1 3 -2 1 -1 1 2 -3 1 0 1 2 -1 -1 0 -1 2 -3 2 1 0 4 3 1 0 2 -5 1 -3 1 -2	
- <u>2</u> 5	 	

$$(-2.3 + 3.1 + 5.4 + -1.1) + (-2.2 + 3. -1 + 5. -3 + -1.1) +$$

 $(-2.1 + 3. -1 + 5.2 + -1. -1) + (-2)$

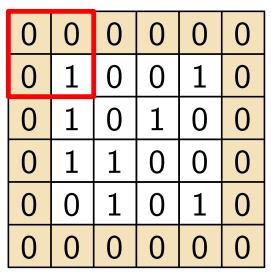
Stride



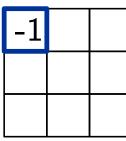
- ► Step size of the sliding window.
- ► Larger the slide, the smaller the output matrix.

Padding

Conv 1



-1	2
4	-2
Filt	er



1

Bias

- Border around the image how many cells wide?
 - ► The larger the padding, the larger the output matrix.
 - ► Helpful with larger filters allows to better "scan" the boundaries of the image.

Conv	2
\mathbf{C}	_

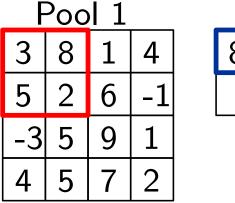
0	0	0	0	0	0
0	1	0	0	1	0
0	1	0	1	0	0
0	1	1	0	0	0
0	0	1	0	1	0
0	0	0	0	0	0

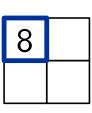
-1	2
4	-2

-1	0	

1

Pooling with filter size 2 and stride 2





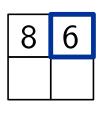
	<u> </u>	<u> </u>	
3	8	1	4
5	2	6	-1
-3	5	9	1
4	5	7	2

 $D_{\alpha} = 1.2$

8	6
5	

	1 001 2		
3	8	1	4
5	2	6	-1
-3	5	9	1
4	5	7	2

Pool 2



	Pod	ol 4	•
3	80	1	4
5	2	6	-1
-3	5	9	1
4	5	7	2

8	6
5	9

- Pooling applies a fixed operator, max or average no parameters to learn. Max pooling popular.
- Hyperparameters: filter size, stride. Usually, pooling layer follows a convolution layer.
- Reduces the number of parameters, speeds up training.

Recurrent Neural Networks

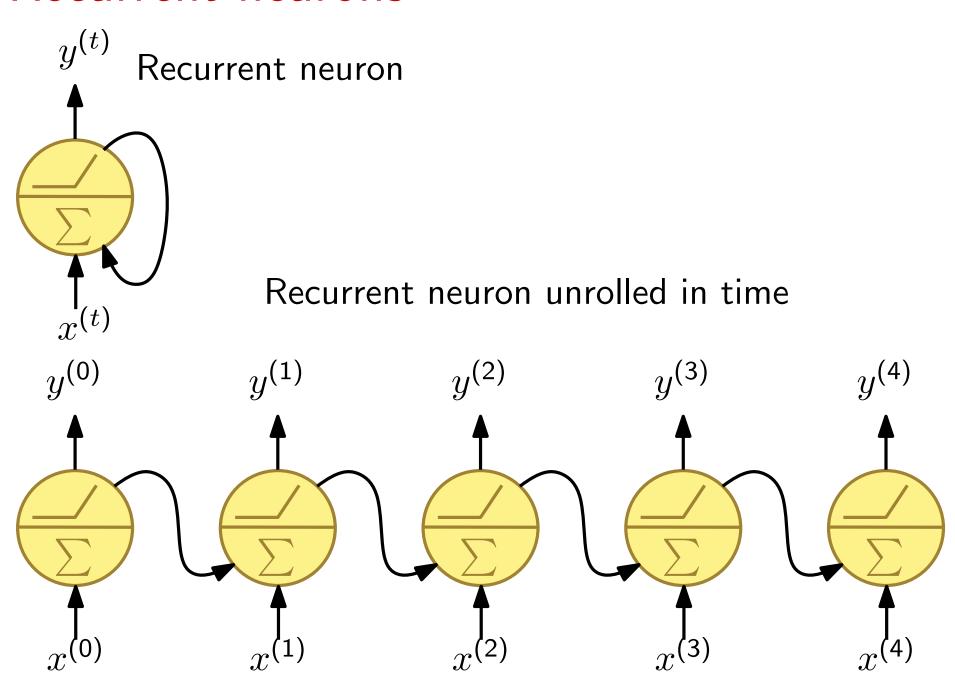
Recurrent Neural Networks (RNN)

- Used to label, classify, or generate sequences.
- ► A sequence is a matrix: each row is a feature vector and the order of rows matters.
- ▶ To label a sequence is to predict a class for each feature vector in a sequence.
- ► To classify a sequence is to predict a class for the entire sequence.
- ► To generate a sequence is to output another sequence (of a possibly different length) somehow relevant to the input sequence.

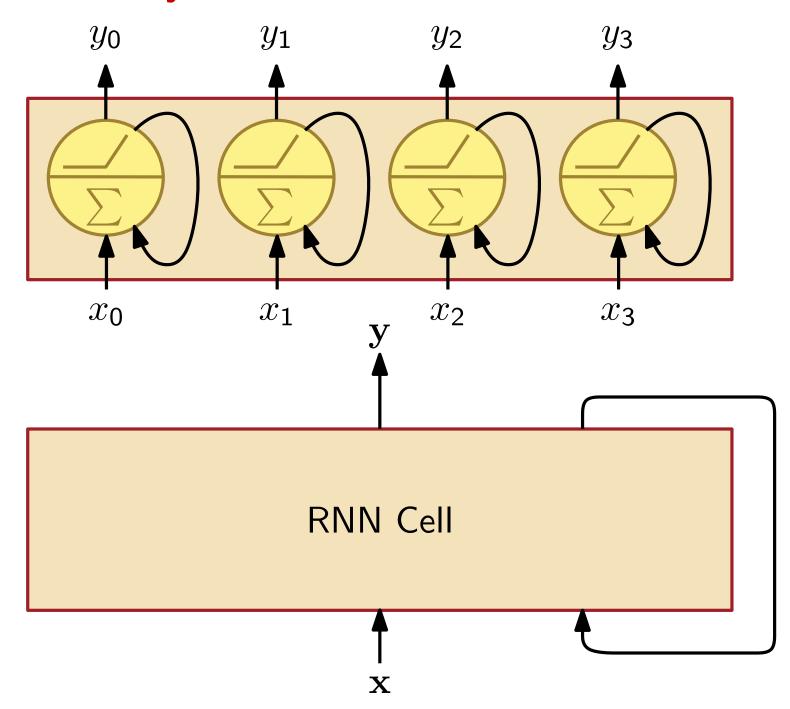
Applications

- ► Text processing
- Speech processing

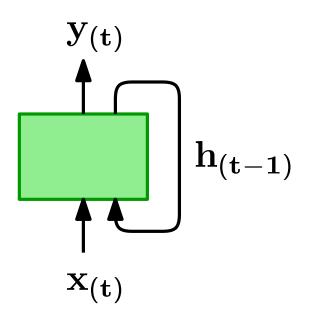
Recurrent neurons



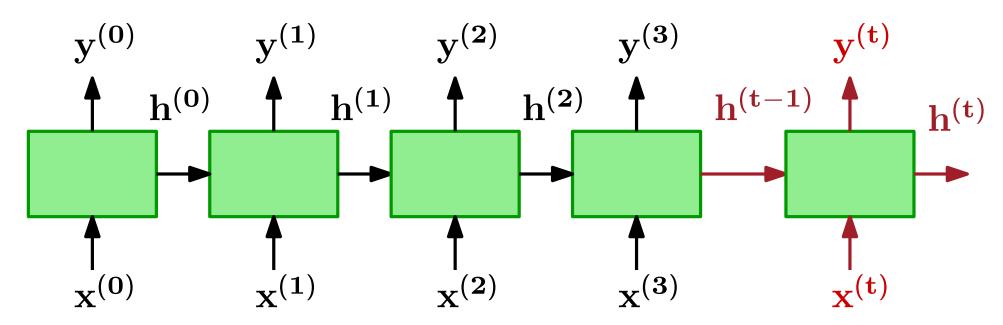
Recurrent layer



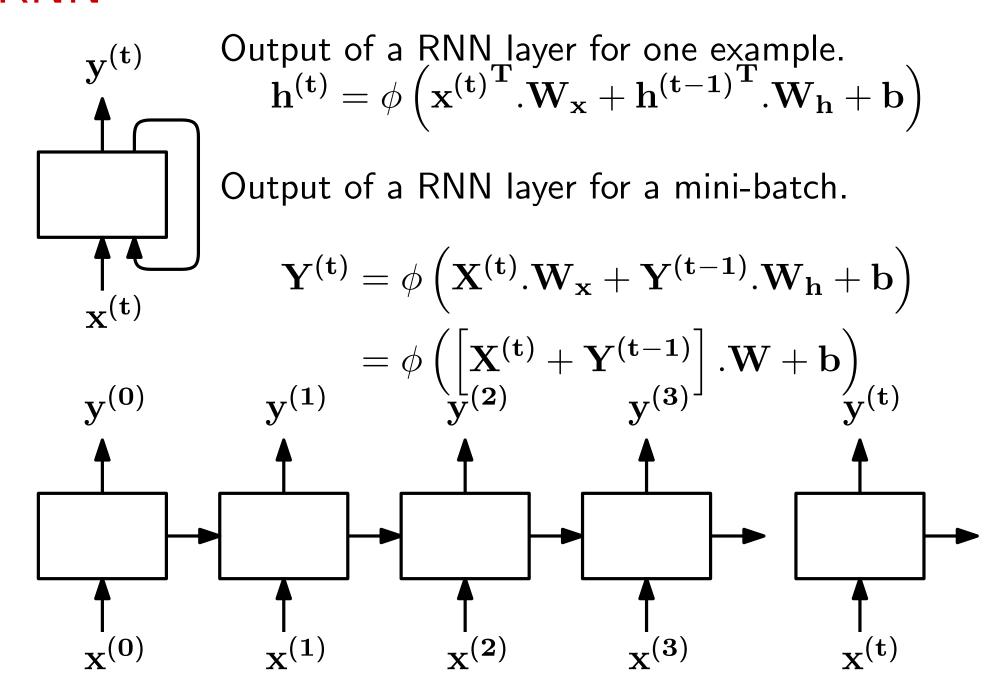
Memory cell



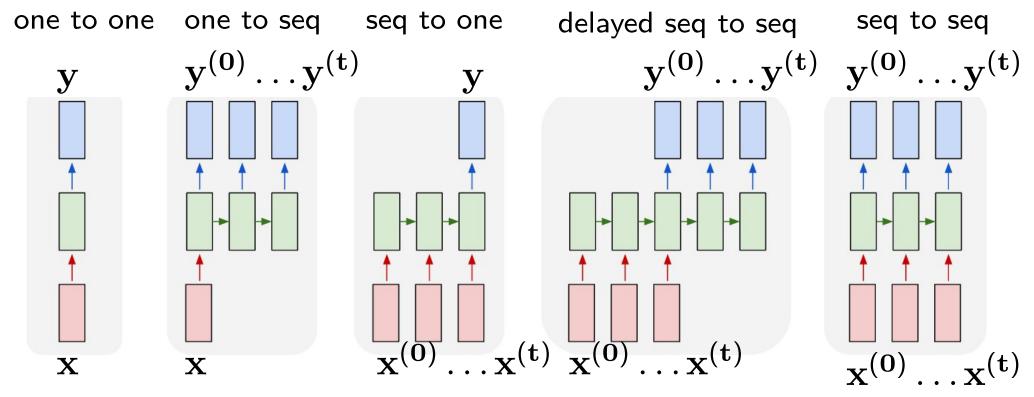
$$\mathbf{h^{(t)}} = f(\mathbf{h^{(t-1)}}, \mathbf{x^{(t)}})$$
$$\mathbf{y^{(t)}} = g(\mathbf{h^{(t-1)}}, \mathbf{x^{(t)}})$$



RNN



Input Output sequences



- one to one: Image classification
- one to seq : Image captioning
- seq to one: Text classification
- delayed seq to seq: Translation
- seq to seq: Time series prediction

Difficulties

- Propagation through time is used to compute the parameters.
- Because of the sequential nature of the input, backpropagation has to "unfold" the network over time.
- ► The longer the input sequence, the deeper is the unfolded network.
- ightharpoonup Even if our RNN has just one or two recurrent layers, both tanh and softmax suffer from the vanishing gradient problem.
- ► Long-term dependencies
- ► The feature vectors from the beginning of the sequence tend to be "forgotten", because the state of each unit becomes significantly affected by the feature vectors read more recently.
- ► In text or speech processing, the cause-effect link between distant words in a long sentence can be lost.

Gated RNN

- ► Long short-term memory (LSTM) networks.
- Networks based on the Gated Recurrent Unit (GRU).
- Units make decisions about what information to store, and when to allow reads, writes, and erasures.
- ► Those decisions are learned from data and implemented through the concept of gates.
- ► There are several architectures of gated units.
- Minimal gated GRU and is composed of a memory cell, and a forget gate.
- ► Can store information in their units for future use.
- ▶ Reading, writing, and erasure of information stored in each unit is controlled by activation functions – values in the range (0, 1).

Tools

Python



- Very high-level data structures.
- Clean syntax.
- Eco-system (comes with batteries attached!)
- De Facto programming language of ML.
- A large collection of ML packages.
- Python 2.7, Python 3

Anaconda



https://anaconda.org/

- Most popular Python data science platform.
- ► Leads open source projects like Anaconda, NumPy and SciPy that form the foundation of modern data science.

NumPy



http://www.numpy.org/

Fundamental package for scientific computing with Python.

- A powerful N-dimensional array object.
- Sophisticated functions.
- ightharpoonup Tools for integrating C/C++ and Fortran code.
- Useful linear algebra, Fourier transform, and random number capabilities.

scikit-learn



- http://scikit-learn.org/stable/
- Classification: SVM, Nearest neighbors, Random forests.
- ► Regression: SVR, Ridge regression, Lasso.
- Clustering: k-means, Spectral clustering.
- ▶ PCA, Feature selection, Non-negative matrix factorization.
- Model selection: Grid search, Cross validation, Metrics.
- Preprocessing: Feature extraction.

TensorFlow



- Open source software library for numerical computation using data flow graphs.
- Nodes in the graph represent mathematical operations, while the graph edges represent the multidimensional data arrays (tensors) communicated between them.
- Deploy computation to one or more CPUs or GPUs in a desktop, server, or mobile device with a single API.
- Originally developed by Google Brain Team for machine learning and deep neural networks research.
- General enough to be applicable in a wide variety of other domains as well.

Keras



- ► High-level neural networks API, written in Python and capable of running on top of TensorFlow, CNTK, or Theano.
- Allows for easy and fast prototyping.
- Supports both convolutional networks and recurrent networks, as well as combinations of the two.
- ► Runs seamlessly on CPU and GPU.

PyTorch



- Open source machine learning framework based on the Torch library.
- Applications such as computer vision and natural language processing.
- Developed by Facebook's Al Research lab (FAIR).
- ▶ Although the Python interface is more polished and the primary focus of development, PyTorch also has a C++ interface.

References

- ► Ian Goodfellow, Yoshua Bengio, Aaron Courville, "Deep Learning", 2016
- ► Eli Stevens, Luca Antiga, Thomas Viehmann, "Deep Learning with PyTorch", 2020

