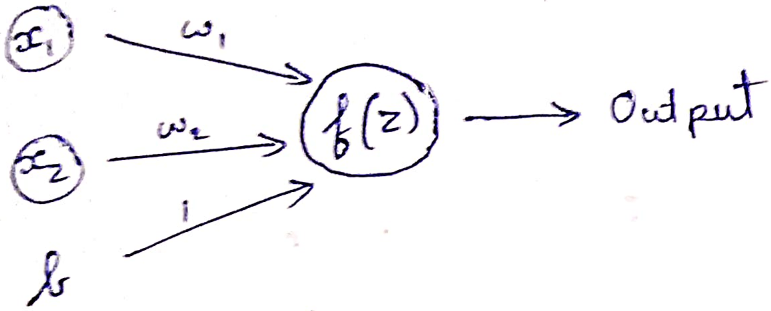
Part I : Perceptron

$$z = w.x + b$$



$$f(z) = \begin{cases} 1, & z \geq 0 \\ 0, & z < 0 \end{cases}$$

When we multiply weights and bias with positive constant $c > 0$

$$z' = c(w.x + b)$$
$$= c*w.x + b*c$$

The decision boundary,

$$f(z') = \begin{cases} 1, & c(wx + b) \geq 0 \\ 0, & c(wx + b) < 0 \end{cases}$$

After dividing both sides by c

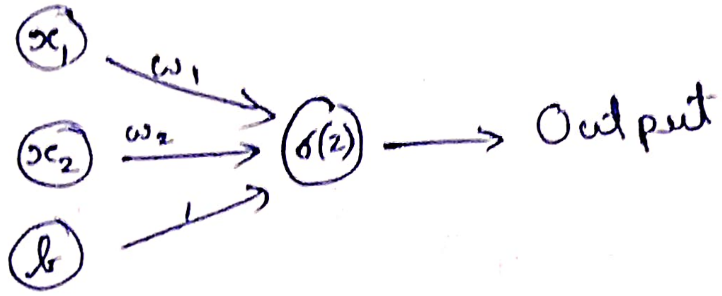
if $c > 0$, ~~the~~

$$f(z') = \begin{cases} 1, & wx + b \geq 0 \\ 0, & wx + b < 0 \end{cases}$$

if $c < 0$

$$f(z') = \begin{cases} 1, & wx + b \leq 0 \\ 0, & wx + b > 0 \end{cases}$$

Part II: Sigmoid neurons



$$z = wx + b \qquad \sigma(z) = \frac{1}{1+e^{-z}}$$

Multiplying weights and bias by $c > 0$

$$z' = cw.x + cb$$

$$\sigma(z') = \frac{1}{1 + e^{-(cw.x + cb)}}$$

$$\sigma(z') = \frac{1}{1+e^{-c(w.x+b)}} = \frac{1}{1+e^{-cz}}$$

Case 1: $w.x + b > 0$

$$\cancel{\lim_{c\to\infty} \sigma(z') = \frac{1}{1+}}$$

$$\lim_{c\to\infty} \sigma(z') = \lim_{c\to\infty} \frac{1}{1+e^{-cz}}$$

$$= \frac{1}{1+e^{-\infty}}$$

$$= \frac{1}{1+0}$$

$$\lim_{c \to \infty} \sigma(z') = 1$$

Case 2 : $wx + b < 0$

$$\lim_{c \to \infty} \sigma(z') = \lim_{c \to \infty} \frac{1}{1+e^{-cz}}$$

$$= \lim_{c \to \infty} \frac{1}{1+e^{cz}} \quad \text{(Since } z < 0\text{)}$$

$$= \cancel{\lim} \frac{1}{1+e^{\infty}}$$

$$\lim_{c \to \infty} \sigma(z') = 0$$

The obtained result is same as

$$f(z) = \begin{cases} 1, & z \geq 0 \\ 0, & z < 0 \end{cases} \quad \text{in perceptron}$$

except when $z = 0$

Case 3 : $wx + b = 0$

$$\lim_{c \to \infty} \sigma(z') = \lim_{c \to \infty} \frac{1}{1+e^{-cz}} = \frac{1}{1+e^{0}} = \frac{1}{1+1}$$

$$= 0.5$$