To Prove:

The choice of Δv which minimizes $\nabla C \cdot \Delta v$ is $\Delta v = -\eta \nabla C$, where $\eta = \frac{\varepsilon}{\|\nabla C\|}$

Proof:

Cauchy-Schwarz inequality: $|u \cdot v| \le \|u\| \cdot \|v\|$

In our case: $|\nabla C \cdot \Delta v| \le \|\nabla C\| \cdot \|\Delta v\|$

$$|\nabla C \cdot \Delta v| \le \|\nabla C\| \cdot \varepsilon \qquad (\because \|\Delta v\| = \varepsilon)$$

This means,

$$-\|\nabla C\| \cdot \varepsilon \le \nabla C \cdot \Delta v \le \|\nabla C\| \cdot \varepsilon$$

Minimum possible value of $\nabla C \cdot \Delta v$ is $-\|\nabla C\| \cdot \varepsilon$

Cauchy-Schwarz inequality becomes equality when both vectors are parallel.

In $\boxed{\nabla C \cdot \Delta V = -\|\nabla C\| \cdot \varepsilon}$, Δv and ∇C points in opp direction

Therefore,

$\Delta V = -\eta \nabla C$, where η is a positive constant

~~∅~~ we know that, $\varepsilon = \|\Delta V\|$

$$= \|-\eta \nabla C\|$$

$$\varepsilon = \eta \|\nabla C\|$$

$$\eta = \frac{\varepsilon}{\|\nabla C\|}$$