$$\frac{\partial C}{\partial b_j^l} = \delta_j^l$$ Rate of change of the cost with respect to any bias in the network:

Take the left hand side,

$$\frac{\partial C}{\partial b_j^l} = \frac{\partial C}{\partial z_j^l}\frac{\partial z_j^l}{\partial b_j^l} = \frac{\partial C}{\partial z_j^l}\frac{\partial}{\partial b_j^l}\sum_k w_{jk}^l a_k^{l-1} + b_j^l$$

$$= \frac{\partial C}{\partial z_j^l}\cdot(0+1)$$

$$\frac{\partial C}{\partial b_j^l} = \delta_j^l$$ (From equation 29)

Rate of change of the cost with respect to any weight in the network:

$$\frac{\partial C}{\partial w^{l}_{jk}} = a^{l-1}_{k} \delta^{l}_{j}$$

<u>Proof:</u>

Take the left hand side

$$\frac{\partial C}{\partial w^{l}_{jk}} = \frac{\partial C}{\partial z^{l}_{j}} \frac{\partial z^{l}_{j}}{\partial w^{l}_{jk}}$$

$$= \delta^{l}_{j} \cdot \frac{\partial}{\partial w^{l}_{jk}} \sum_{k} w^{l}_{jk} a^{l-1}_{k} + b^{l}_{j}$$

$$\frac{\partial C}{\partial w^{l}_{jk}} = \delta^{l}_{j} \cdot a^{l-1}_{k}$$