INVESTIGATION OF RESOLVING SETS AND METRIC DIMENSION

KYLE RYAN
ADVISORS: TREVOR MCGUIRE, WARREN SHREVE

ABSTRACT. For any graph G it is possible to describe each of its vertices uniquely with respect to an ordered subset of vertices of G called a resolving set. In this project, we investigate properties of associated with minimal resolving sets, known as bases and conditions under which changing G affects its bases.

1. INTRODUCTION

For this investigation we will begin with several basic defintions that might not be part of a typical introduction to graph theory.

Definition 1.1 (Representation of vertex(with respect to W)). :
For an ordered subset $W = (w_1, w_2, ..., w_i, w_{i+1}, ...)$ of vertices in G, the representation of a vertex $v \in G$ with respect to W denoted $rep(v/W)$ is:
$rep(v/W) = (d(v, w_1), d(v, w_2), ..., d(v, w_i), d(v, w_{i+1}), ...)$

Definition 1.2 (Resolving Set). :
An ordered subset $W \subset V(G)$ is called a resolving set of G if
$\forall v_1, v_2 \in V(G)$, $rep(v_1/W) \neq rep(v_2/W)$.
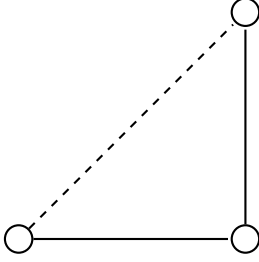
Definition 1.3 (Basis of a graph). :
A resolving set W of G is called a *basis of G*.

Definition 1.4 (Metric Dimension). :
The metric dimension of a graph G is the order of any basis for G.

Finding the metric dimension or any basis for a graph is currently considered to be an NP-Hard problem. Metric dimension is known for some subsets of graphs which will be discussed later, but in general it is very difficult or simply time consuming to determine. The intent of this paper is to investigate whether there is a set of graphs for which having a known metric dimension allows us to determine metric dimension after adding or removing an edge. Furthermore we will attempt to investigate what properties of these graphs make it possible to determine metric dimension for them after adding or removing an edge.

Example 1.5. The path graph P_3 has a metric dimension of 1. Simply calling W the set containing one leaf node of P_3 is sufficient to construct a basis for P_3. By adding any edge to P_3, P_3 becomes K_3 which has a metric dimension of 2.

Dogan Comez.

2. ENCODABILITY OF GRAPHS WITH RESPECT TO THEIR BASES

Definition 2.1 (Representation set of a graph). :
The representation set of a graph with respect to a basis W is the set $H = \{r(v/W) \mid v \in G\}$.

Definition 2.2 (Unique Encoding). :
If we can construct a graph G from its representation set H by connecting vertices in G if their representations differ by no more than 1 in each place then we say that G is uniquely encoded by H. Furthermore we say that G is uniquely encodeable with respect to its basis W or G is enccodable under W.
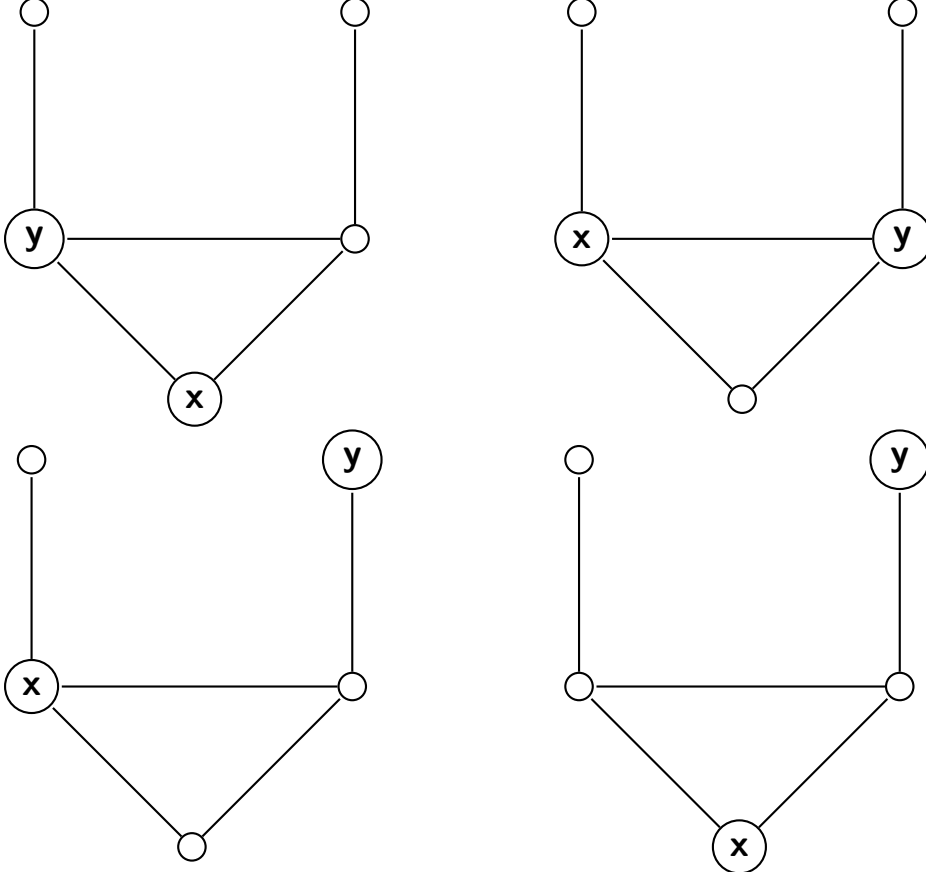
Lemma 2.3. G *is encodable under* W *if and only if* $\forall$ *nonadjacent* v_1, $v_2 \in G$: $\exists\ w \in W\ s.t.\ |d(v_1, w) - d(v_2, w)| > 1$

Proof. Suppose that we construct a graph G and basis W $\square$

Example 2.4. Observe the following

3. Upper and Lower Bounds on Metric Dimension

Lemma 3.1. *Let G be a graph with a bridge (u,v) s.t. $G - (u,v)$ has components H_1, H_2 such that H_1 has at least one degree > 2 vertex. Then for any basis W of G, $W \nsubseteq H_2$.*

Proof. $\square$

Lemma 3.2. *Let G be a graph with a cut vertex v and basis W. If $deg(v) > 2$ or if for two components H_1, H_2 of $G - v$, H_1 has a vertex of degree > 2, $W \nsubseteq H_2$.*

Proof. $\square$

Lemma 3.3. *Let G be any graph. Let H be the graph obtained by edge contractions on G. If only the number of degree 2 vertices is changed,*

Proof. $\square$

[**?**]

References

Department of Mathematics 2750, North Dakota State University, PO BOX 6050, Fargo, ND 58108-6050, USA

E-mail address: kyleryanmn@gmail.com, trevor.mcguire@ndsu.edu, warren.shreve@ndsu.edu