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Problem 1. Blackbody Radiators

Stefan estimated that the power per unit area radiated from the surface of the sun was 43.5 times greater than that of a metal bar heated to 1950 deg C. What is the temperature of the sun?

Temperature of Metal = $T_m = 1950 \text{ deg.C} = 2223 \text{ K}$
 Power of Metal by Stefan-Boltzmann Law is :

$$P = \sigma T_m^4$$

Power of Sun = 43.5 times Power of Metal. Hence

$$\sigma T_s^4 = 43.5 \sigma T_m^4$$

$$T_s = (43.5 T_m^4)^{0.25} = 5709.023 \text{ K}$$

Based on this temperature, what wavelength λ of light does the sun emit most intensely, in nm? What frequency of light, in s^{-1} ? What color does this correspond to?

Wien's Constant $W = 2897768 \text{ nm} \cdot \text{K}$
 By Wien's Displacement Law, $\lambda_{\text{max}} T_s = W$

$$\lambda_{max} = W/T_s = 507.577nm$$

The frequency ν is given by:

$$\nu = \frac{c}{\lambda_{max}} = 5.906 \times 10^{14} s^{-1}$$

This corresponds to green light.

What is the ultraviolet catastrophe, and what did Planck have to assume to circumvent it?

Answer given in Python Notebook

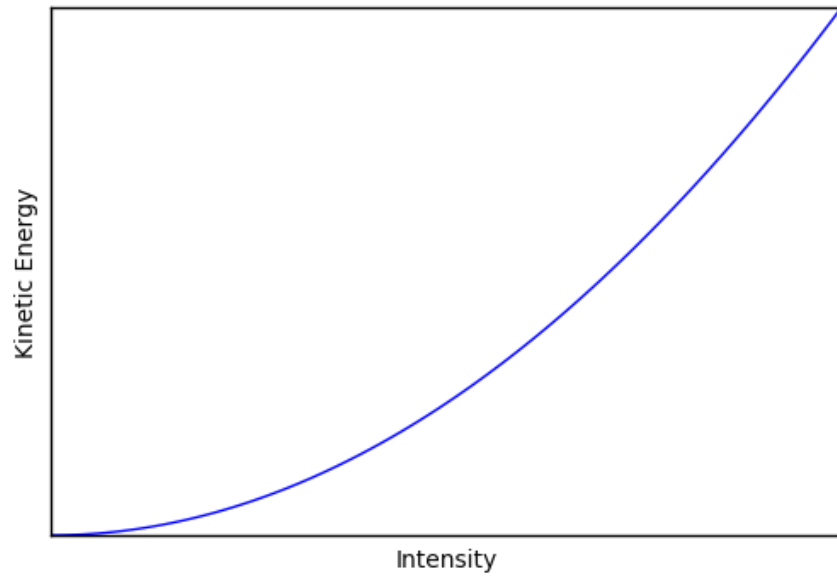
Problem 2. Photoelectric effect

You set up an experiment in which you shine light of varying intensity and constant frequency at a metal surface and measure the maximum kinetic energy of the emitted electrons. As an accomplished student of classical physics, you know that the energy contained in a wave is proportional to the square of its intensity. Based on this knowledge, sketch how you *expect* the kinetic energy of the electrons to vary in the experiment. Briefly justify your answer.

The intensity I varies with kinetic energy, K as

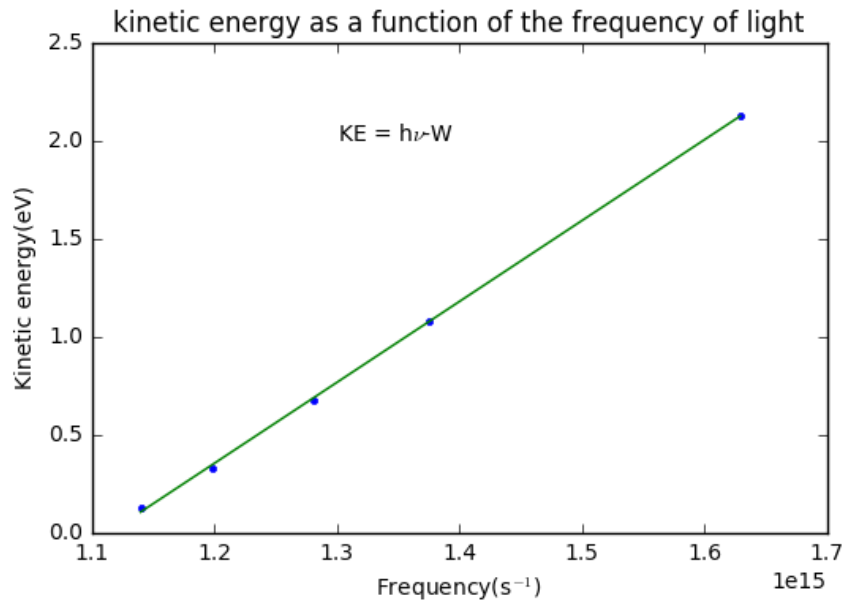
$$I = K^2$$

The plot should look like:



Not finding a result that you like, you set up another experiment in which you vary the frequency of light at constant intensity. Not finding a result that you like, you set up another experiment in which you vary the frequency of light at constant intensity.

From the given table, by fitting the data, you get:



The negative intercept gives the Work Function.

The slope gives the value of Planck's constant.

$W = 4.598 \text{ eV}$, Planck's constant = $4.127 \times 10^{-15} \text{ eV} \cdot \text{s}$

What is the metal? Hint : It is a coinage metal.

The metal can be copper or silver

(For a detailed explanation, see the Python Notebook)

Problem 3. Diffraction

The spacing between atoms in a Ag crystal is approximately 2.9 \AA , a distance that can be measured by scattering photons of a comparable wavelength off the crystal. What is the energy (in eV) of a photon of wavelength 2.9 \AA ? What part of the electromagnetic spectrum does this correspond to?

Given $\lambda = 2.9 \text{ \AA}$

$$E = \frac{hc}{\lambda} = 4275.86 \text{ eV}$$

This wavelength corresponds to X-rays.

Suppose you have a device that produces these photons at a power of $1 \mu\text{ W}$. How many photons/s does this correspond to?

Given Power, $P = 1 \mu\text{ W} = 6.2415 \times 10^{12} \text{ eV/s}$

$$n = P/E = 1.4597 \times 10^9 \text{ photons/s}$$

The Ag spacing can also be measured by scattering *electrons* off a crystal. To what speed (in m/s) would an electron need to be accelerated to have the necessary de Broglie wavelength? What fraction of the speed of light is this?

Mass of electron, $m_e = 9.109 \times 10^{-31} \text{ kg}$

Momentum of photon, $p = h/\lambda = 2.284 \times 10^{-24} \text{ kg m/s}$

Speed of photon, $v = p/m_e = 2.508 \times 10^6 \text{ m/s}$

It is 0.00837 of the speed of light.

Problem 4. The Bohr Atom

Calculate the energies of an electron in the $n = 1$ and $n = 2$ orbits, in eV.

Given $n_1 = 1$, $n_2 = 2$

By Bohr's formula,

$$E_1 = -13.606/(n_1)^2 = -13.606 \text{ eV}$$

$$E_2 = -13.606/(n_2)^2 = -3.401 \text{ eV}$$

Would light need to be absorbed or emitted to cause an electron to jump from the $n = 1$ to the $n = 2$ orbit? What wavelength of light does this correspond to?

$$\Delta E = E_2 - E_1 = 10.205 \text{ eV}$$

$$\lambda = hc/\Delta E = 121.515 \text{ nm}$$

Light would need to be absorbed for the electron to jump

What is the circumference of the $n = 2$ orbit? What is the de Broglie wavelength of an electron in the $n = 2$ orbit? How do these compare?

Bohr constant for Radius, $a_0 = 0.529 \text{ \AA}$

Radius for $n(n=2)$ orbital = $a_0 \times n^2$

Circumference of $n = 2$ orbit is given by:

$$C_2 = 2 \pi * a_0 * n^2 = 1.329 * 10^{-9} m$$

From the course outline, the electron momentum is:

$$p_n = e^2 m_e 2\pi / (4\pi\epsilon_0 h n) = 9.963 * 10^{-25} kgm/s$$

The electron wavelength is given by:

$$\lambda = h/p_n = 6.6501 * 10^{-10} m$$

By comparing both values, it is clear that $C_2 = 2 * \lambda$