In[730]:=
```
(*Both Objective Functions take two vectors as input and outputs
  some measure of their difference – basically, they are distances.*)

(*Shape Objective Function*)
ANG[v_, w_] := Total[v * w] / (Total[v * v] * Total[w * w]) ^ (1 / 2);
SOF[v_, w_] := (ANG[v, w] – 1) ^ 2;
(*Normalized Least Squares Objective Function*)
NLS[v_, w_] := Total[(v / (Total[v * v]) ^ (1 / 2) – w / (Total[w * w]) ^ (1 / 2)) ^ 2];
```

In[740]:=
```
Clear[a, b, k];
a = {a1, a2};
b = k {b1, b2};
(*We can ensure that scaling does not impact anything at all. In fact,
k is simplified away.*)
Rules = {k > 0};
Collapse = Refine @@ Append[{Simplify@#}, Rules] &;
SReduce = Expand@Simplify@Expand@# &;
S1 = Collapse@SOF[a, b]
N1 = SReduce@Collapse@NLS[a, b];
N1 = 2 * (Simplify@(N1 / 2 – 1) + 1)
(*Both functions are rotationally invariant.*)
```

Out[744]=
$$\left(-1+\frac{a1\,b1+a2\,b2}{\sqrt{\left(a1^2+a2^2\right)\left(b1^2+b2^2\right)}}\right)^2$$

Out[746]=
$$2\left(1+\frac{-a1\,b1-a2\,b2}{\sqrt{a1^2+a2^2}\,\sqrt{b1^2+b2^2}}\right)$$

In[739]:=
```
(*We should be able to see the asymptotic behavior of these two objective
 functions by comparing the distance of a fixed vector to an arbitrary vector.*)
Module[{siz = 200, dis = 0.1,
  vec = {5, 0}
 },
 P3 = Plot3D[# /. {b1 → vec[[1]], b2 → vec[[2]]},
    {a1, -dis, dis}, {a2, -dis, dis}, ImageSize → siz, PlotTheme -> "Web"] &;
 C3 = ContourPlot[# /. {b1 → vec[[1]], b2 → vec[[2]]}, {a1, -dis, dis},
    {a2, -dis, dis}, ImageSize → siz] &;
 Target = {S1, N1, S1 / N1};
 Quiet@Grid[
   {{"Shape", "Least Squares", "SOF/NLS"},
    P3 /@ Target, C3 /@ Target},
   Frame → All]
]
(*VEC here is the arbitrarily picked example vector.
  The objective function surface merely rotates as VEC changes angle.
  The surface does not change as VEC's magnitude changes.*)
```

Out[739]=

Shape	Least Squares	SOF/NLS