On the Proper Order of Markov Chain Model for Daily Precipitation Occurrence in the Contiguous United States

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ABSTRACT

Markov chains are widely used tools for modeling daily precipitation occurrence. Given the assumption that the Markov chain model is the right model for daily precipitation occurrence, the choice of Markov model order was examined on a monthly basis for 831 stations in the contiguous United States using long-term data. The model order was first identified using the Bayesian information criteria (BIC). The maximum-likelihood estimates of the Markov transition probabilities were computed from 100 bootstrapped samples and were then used to generate 50-yr precipitation occurrence series. The distributions of dry- and wet-spell lengths in the resulting series were then compared with observations using a two-sample Kolmogorov–Smirnov (K-S) test. The results suggest that the most parsimonious model, as identified by the BIC, usually (in approximately 68% of the cases) reproduced the wet- and dry-spell length distributions. However, the K-S test often indicated a second-order model when the BIC indicated a first-order model. In a smaller number of cases, the BIC indicated a higher-order model than the K-S test. In both cases, the differences were found to be due to the distribution of wet spells rather than dry spells. It is concluded that models chosen on the basis of the BIC may not adequately reproduce the distributions of wet and dry spells for some locations and times of year.

1. Introduction

The occurrence of precipitation is an important parameter for describing local precipitation climates and modeling their agricultural and hydrological impacts. Markov chain models were first used by Gabriel and Neumann (1962) and are statistical models in which the probability of precipitation on the current day is conditioned on whether or not precipitation occurred on some number of previous days. Although other types of models have been used to produce precipitation occurrence time series (e.g., alternating renewal processes), two-state (i.e., precipitation occurs or does not occur) Markov chain models are most commonly applied, and Roldán and Woolhiser (1982) found that they provided better results than an alternating renewal process for five stations in the United States. For most applications (e.g., Katz 1977; Richardson 1981; Wilks 1992), a first-order (dependence only on the previous day) model has been used, although several studies (Chin 1977; Buishand 1978; Racsko et al. 1991; Guttorp 1995; Katz and Parlange 1998; Wilks 1999; Wan et al. 2005) noted shortcomings in the first-order model in some climates, such as dry spells of inadequate length or frequency and underestimation of variability, and recommended the use of higher models for some seasons and locations. Conversely, Harrison and Waylen (2000) found that a zeroth-order model (i.e., depending only on the overall wet-day probability) was sufficient for some stations and times of year for stations in Costa Rica. These results may arise from geographical differences but also

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suggest that additional investigations of space and time variations in Markov model order are needed.

In addition to the basic climatological relevance of precipitation occurrence, most daily stochastic weather models—for example, "weather generator" (WGEN; Richardson and Wright 1984), "extended version of weather generator" (WXGEN; Wallis and Griffiths 1995), "Long Ashton Research Station weather generator" (LARS-WG; Semenov and Barrow 1997), "weather generator for climate inputs" (CLIGEN; Nicks and Gander 1994), "generation of weather elements for multiple applications" (GEM; Johnson et al. 2000), and "spectral generator" (SPECGEN; Schoof et al. 2005)—simulate temperature and solar radiation conditioned on wet/dry status. In these situations, shortcomings in the simulation of precipitation occurrence can lead to undesired results throughout the model. Further assessment of model order should therefore improve the performance of stochastic weather models and provide guidance for other types of impact studies. The purpose of this paper is to examine the proper order of Markov chain model for daily precipitation occurrence in the contiguous United States using long-term data records from a large number of climatically diverse stations. The use of these relatively long data records will allow robust parameter estimation and a large range of both wet and dry spells for comparison of observed and simulated precipitation occurrence series.

The majority of prior studies that have assessed the order of Markov chain model for precipitation occurrence have used one of two criteria: the Akaike information criterion (AIC; Akaike 1974) or the Bayesian information criterion (BIC; Schwarz 1978) (e.g., Harrison and Waylen 2000; Kottegoda et al. 2004). More recent studies have preferentially used BIC, which has been demonstrated to be asymptotically unbiased (Katz 1981). Here we apply the BIC and additionally evaluate the resulting model order choices by comparing observed and model-generated wet- and dry-spell distributions using a two-sample Kolmogorov–Smirnov (K-S) test. We also apply a bootstrap resampling technique to investigate the sampling variability associated with model order choices based on these criteria.

The remainder of this paper is structured as follows. In section 2, the data and station network used to examine the precipitation occurrence series are described. Section 3 describes the application of Markov chain models and the evaluation criteria used to assess the proper model order. The results of the analysis are presented in section 4. In section 5, the results are summarized and discussed in terms of implications for precipitation occurrence modeling.

2. Data

The data used in this study are daily precipitation measurements from a subset of 831 stations from the National Weather Service (NWS) Cooperative Observing Program network (see http://www.nws.noaa.gov/climate and Fig. 1). As shown, the stations analyzed provide good spatial coverage of the contiguous United States and thus represent a range of geographical and climatological variability. Each station included has at least 70 yr of daily data with less than 5% missing data. Many of the records have lengths exceeding 100 yr. Based on the work of Dobi-Wantuch et al. (2000), we use the lowest measurable precipitation value to differentiate wet days and dry days. For the data used here, that equates to 0.254 mm.

3. Method

Markov chain models are commonly used tools for simulating time series of discrete random variables. Such models can be described by two properties: the number of different values that the variable can have (known as the "state") and the number of previous values used to determine the state-to-state transition probabilities (known as the "order"). In the case of precipitation occurrence, the variable X_t is binary, having only two states: occurrence ($X_t = 1$) or nonoccurrence ($X_t = 0$) of precipitation. Thus, the simplest Markov model for precipitation occurrence is the two-state, first-order model, which is described by four transition probabilities:

$$p_{00} = \Pr\{X_t = 0 | X_{t-1} = 0\}, \tag{1a}$$

$$p_{01} = \Pr\{X_t = 1 | X_{t-1} = 0\}, \tag{1b}$$

$$p_{10} = \Pr\{X_t = 0 | X_{t-1} = 1\}, \quad \text{and} \tag{1c}$$

$$p_{11} = \Pr\{X_t = 1 | X_{t-1} = 1\}. \tag{1d}$$

Note that $p_{00} + p_{01} = 1$ and $p_{10} + p_{11} = 1$. The model is therefore fully defined by two transition probabilities: p_{01} (the probability that precipitation will occur tomorrow if precipitation did not occur today) and p_{11} (the probability that precipitation will occur tomorrow if precipitation occurred today). These probabilities can easily be computed from observed precipitation occurrence time series. Their maximum-likelihood estimates, $\hat{p}_{01}$ and $\hat{p}_{11}$, are given by

$$\hat{p}_{01} = \frac{n_{01}}{n_{00} + n_{01}} \quad \text{and} \tag{2a}$$

$$\hat{p}_{11} = \frac{n_{11}}{n_{10} + n_{11}}, \tag{2b}$$

FIG. 1. Map of the contiguous United States showing the locations of the 831 cooperative weather stations used in this study.

where n_{01} is the historical count of wet days that followed dry days, n_{00} is the historical count of dry days that followed dry days, and so on. The first-order model described above can be easily generalized to higher orders, although the number of parameters increases exponentially, being 2^k for a kth-order model (Wilks 1999).

To determine which model order is most appropriate for a particular station, several evaluation criteria exist. For example, the AIC (Akaike, 1974) and the BIC (Schwarz, 1978) both aim to identify the best model with the fewest number of parameters and are based on the log-likelihood functions of the transition probabilities. Because of the demonstrable advantages of the BIC and in light of the large sample sizes in this study, the BIC is used as the primary evaluation criterion. It is given by

$$\mathrm{BIC}(m) = -2L_m + s^m[\ln(n)], \tag{3a}$$

where L_m is the log-likelihood for a model of order m, s is the number of states, and n is the sample size. For the two-state precipitation occurrence process described above, this simplifies to

$$\mathrm{BIC}(m) = -2L_m + 2^m[\ln(n)]. \tag{3b}$$

The log-likelihoods for two-state Markov chain models of orders 1, 2, and 3 are given by Wilks (2006) as

$$L_1 = \sum_{i=0}^{1}\sum_{j=1}^{1} n_{ij}\ln(\hat{p}_{ij}), \tag{4a}$$

$$L_2 = \sum_{h=0}^{1}\sum_{i=0}^{1}\sum_{j=0}^{1} n_{hij}\ln(\hat{p}_{hij}), \quad \text{and} \tag{4b}$$

$$L_3 = \sum_{g=0}^{1}\sum_{h=0}^{1}\sum_{i=0}^{1}\sum_{j=0}^{1} n_{ghij}\ln(\hat{p}_{ghij}). \tag{4c}$$

In this study, the BIC is used to determine the proper order of Markov chain model for each of the 831 locations described in section 2. The models are individually fit to each calendar month to examine seasonal variations.

Previous studies (e.g., Harrison and Waylen 2000) have cautioned against choosing a precipitation occurrence model based on the BIC alone. Instead, statistical tests can be performed on data generated using Markov models to assess their agreement with observations. For example, Lowry and Guthrie (1968) assessed Markov model order using a chi-square test. Because the chi-square test requires binning of the data for categories with low counts, the nonparametric two-sample K-S test (Wilks 2006) is used here. The maximum-likelihood estimates for the Markov chain parameters are used to generate a 50-yr precipitation occurrence series for each station. The distributions of the generated wet- and dry-spell lengths from each station and each order

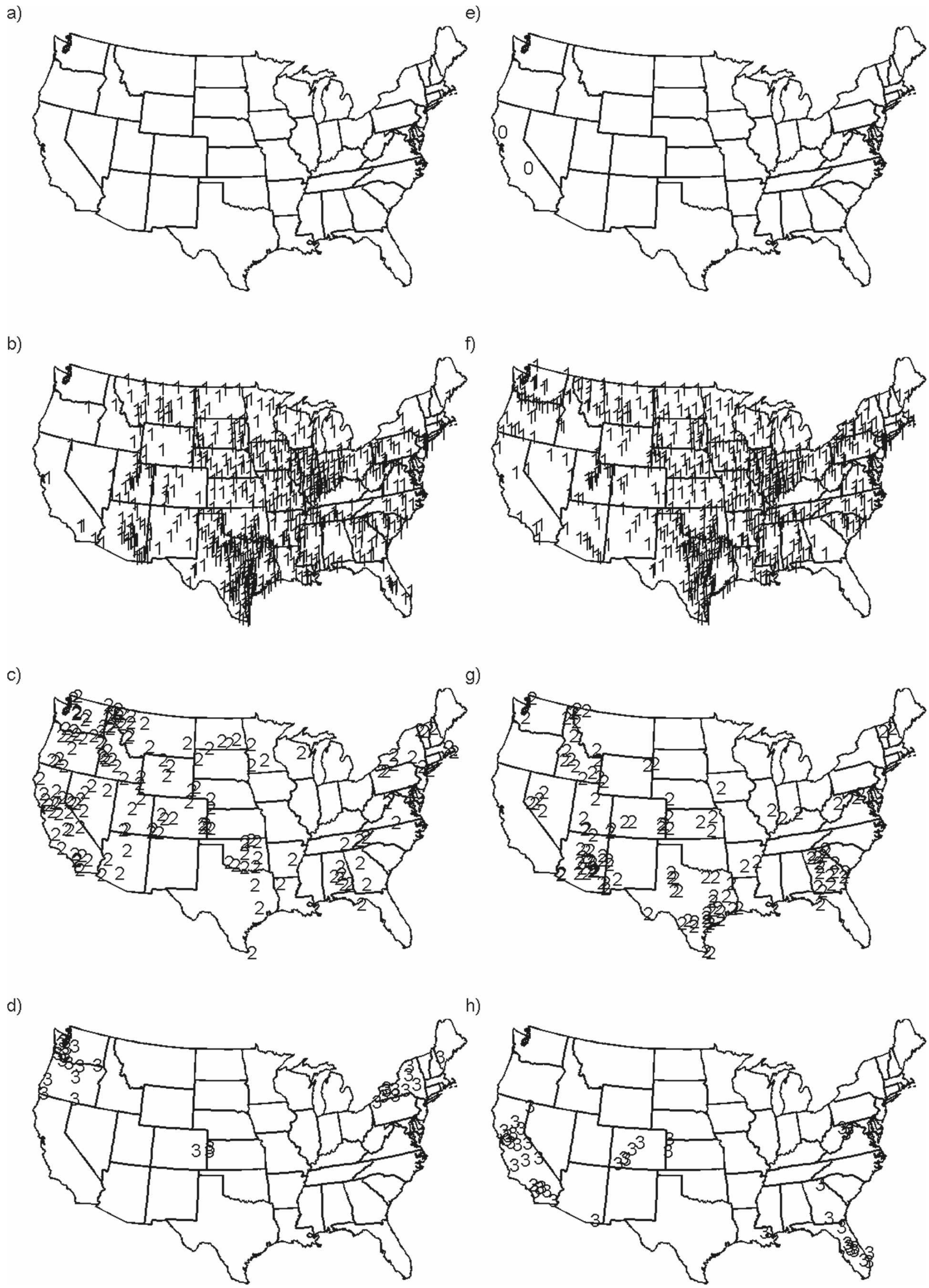

FIG. 2. Markov model-order choices based on the BIC for (a)–(d) January and (e)–(h) July. The number at each station location corresponds to the chosen model order. Individual panels are used for each order to aid visualization of results.

TABLE 1. Summary of monthly Markov model-order choices based on the BIC and K-S test. The table entries are the number of stations (out of 831) at which zeroth-, first-, second- and third-order models are identified. The values given in parentheses indicate the average number (out of 100) of the bootstrap samples that indicate the specified model order.

	Zeroth order		First order		Second order		Third order	
Month	BIC	K-S	BIC	K-S	BIC	K-S	BIC	K-S
Jan	0	52 (73.7)	645 (91.4)	487 (84.8)	158 (74.4)	209 (86.6)	28 (82.1)	4 (61.5)
Feb	0	47 (72.3)	703 (93.0)	531 (87.3)	108 (75.7)	192 (87.2)	20 (76.5)	12 (67.2)
Mar	0	25 (67.5)	680 (90.1)	561 (86.5)	132 (74.1)	195 (88.3)	19 (69.8)	7 (63.6)
Apr	0	15 (76.4)	725 (89.0)	610 (85.7)	104 (74.7)	144 (90.6)	2 (65.0)	18 (59.1)
May	0	26 (83.5)	748 (93.1)	601 (85.1)	81 (77.0)	179 (88.0)	2 (56.5)	7 (61.9)
Jun	0	41 (80.8)	654 (87.0)	525 (82.4)	171 (75.2)	215 (86.1)	6 (71.5)	10 (59.9)
Jul	2 (75.5)	105 (79.0)	659 (89.7)	468 (85.0)	128 (75.9)	172 (85.5)	42 (81.4)	28 (63.0)
Aug	0	104 (77.8)	689 (91.5)	519 (83.6)	132 (75.0)	142 (84.9)	10 (78.8)	12 (60.9)
Sep	0	12 (71.6)	711 (88.6)	633 (86.7)	118 (71.6)	156 (84.0)	2 (63.0)	16 (61.7)
Oct	0	4 (70.5)	738 (91.6)	697 (89.6)	92 (73.5)	91 (87.7)	1 (38.0)	2 (52.0)
Nov	0	12 (70.5)	698 (90.4)	586 (87.6)	96 (74.7)	136 (86.5)	37 (84.8)	15 (62.5)
Dec	0	26 (72.4)	643 (90.8)	507 (85.2)	150 (78.9)	188 (87.2)	38 (86.3)	19 (62.7)

of Markov chain are tested against observations under the null hypothesis that the observed and generated spell lengths are drawn from the same underlying process. Rejection of this hypothesis for either wet or dry spells is interpreted as evidence that the Markov order used is not adequate. The test is performed here with $\alpha = 0.05$.

To understand better the effects of sampling variability and the degree of overlap between recommendations of different model order, we use a bootstrap resampling technique (Efron 1982). The bootstrap was implemented by randomly choosing (with replacement) data from the historical record and estimating the Markov parameters for models of order 0–3, computing the value of the BIC, generating a 50-yr precipitation occurrence series, and evaluating the wet- and dry-spell distributions against the observations using the K-S test. This process was repeated 100 times using randomly chosen data of length equal to the historical record.

4. Results

a. Model-order choices based on BIC

Application of the method described in section 3 results in 9972 total models (831 stations × 12 months) for each of the 100 bootstrapped samples. The model order is chosen as the order that minimizes the BIC for the majority of the 100 bootstrapped samples. Based on these criteria, a first-order model is chosen for the overwhelming majority (83.2%) of station-months. This result agrees with previous studies that have assumed first-order Markov dependence [e.g., Richardson and Wright (1984)]. The second-order model is chosen for 14.7% of cases. The third- and zeroth-order models are chosen far less frequently (2.1% and 0.02%, respectively). The overall model-order choice exhibits considerable spatial variability (Fig. 2) as well as important seasonal variations (Table 1). Seasonal variations in model choice also occur at individual stations. Although the BIC overwhelmingly supports use of the first-order model, closer inspection of the results shows that the second-order model is recommended for at least a single month at 580 (69.8%) stations examined and for at least two months at 352 (42.4%) stations. For a smaller number of stations, the higher-order Markov chains are chosen according to the BIC for more than two months. Deviations from first-order Markov behavior are principally observed in three cases:

1) The first case includes seasonally varying locations (Table 1) for which second-order Markov chains are recommended. The BIC leads to second-order models for many locations in the western United States throughout the year. Second-order models are also recommended for eastern stations during the winter, with a progression into the interior United States during the spring months. During June, second-order models are recommended at many locations throughout the contiguous United States.
2) The second case is observed during the winter months at stations in the Pacific Northwest and New England, where the BIC-based models are third order. In these regions, multiday precipitation events occur during winter in association with arriving cyclones (Pacific Northwest), lake-effect snow (western New England), or East Coast cyclones (eastern New England).
3) The third case occurs during the midsummer months for stations in California and Florida, where the BIC-based models are third order. At these loca-

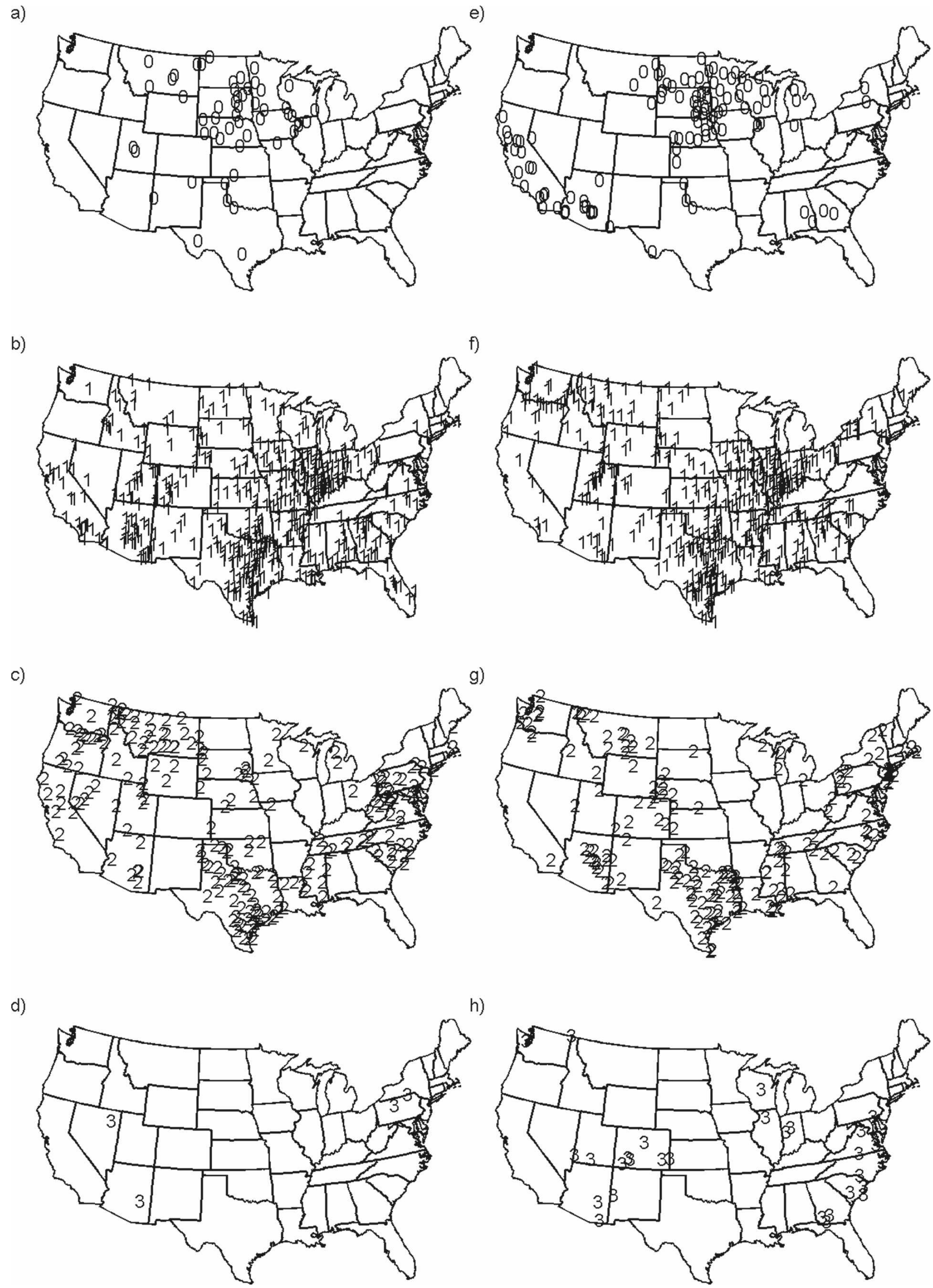

FIG. 3. As in Fig. 2, but for the K-S test.

tions, conditions are either persistently dry with rare precipitation events (California) or persistently wet with daily sea-breeze-induced precipitation events (Florida).

The BIC-based results presented above are based on a simple majority of the results of the 100 samples produced by the bootstrap resampling procedure. It is therefore important to consider the consistency of the

TABLE 2. Comparison of monthly Markov model-order choices based on the BIC and K-S tests. The table entries indicate the number of stations (out of 831) at which the model order chosen according to BIC and K-S tests are equal and the number at which the K-S test suggests using a model of greater or lesser order than the BIC. For example, there are five cases in which the K-S-based model is two orders less than the BIC-based model during January.

	K-S-based order minus BIC-based order						
Month	−3	−2	−1	0	1	2	3
Jan	0	5	117	481	146	3	0
Feb	0	4	92	530	149	7	0
Mar	0	3	103	522	158	2	0
Apr	1	1	77	571	124	13	0
May	0	6	77	559	164	7	0
Jun	1	9	125	487	162	7	0
Jul	8	17	149	444	137	18	0
Aug	1	23	143	478	126	6	0
Sep	0	0	76	611	123	7	0
Oct	0	2	53	670	67	2	0
Nov	0	1	43	585	112	8	0
Dec	0	5	101	470	155	9	0

BIC-based model-order recommendations among these samples to gauge the importance of choosing a higher-order model. These percentages, averaged over all stations, are shown in Table 1 (in parentheses) and indicate that the first-order BIC-based recommendations are highly consistent among the bootstrapped samples. For example, in February, 703 stations use first-order models and, at those 703 stations, the first-order model is recommended on average for 93% (93/100) of the bootstrapped samples. The second-order BIC-based recommendations are slightly less consistent but generally remain near 75%. The third-order BIC recommendations are also generally consistent, with approximately 70%–80% of the bootstrapped samples exhibiting agreement during the winter and summer. During the transition seasons, when the BIC-based order is rarely greater than 2, the percentages are much lower. These results suggest that the BIC-based order estimates are generally consistent among the bootstrapped samples and do not arise from sampling variations.

b. *Model choice based on spell-length distributions*

To assess the applicability of the BIC-based models, the transition probabilities [Eq. (1)] for Markov chains of order 0, 1, 2, and 3 were used to stochastically simulate 50-yr series for each station and month. This generation was accomplished by generating a uniform [0, 1] random number and comparing it with the appropriate transition probability (based on the previous day's wet/dry status). If the random number was less than the transition probability, a wet day was simulated. Otherwise, a dry day was simulated. The distributions of wet and dry spells from the observed and generated series were then compared using a two-sample K-S test. If the null hypothesis that the observed and generated data were drawn from the same underlying process could not be rejected for either the wet- or dry-spell distributions, then the order being evaluated accurately represented the distribution of wet and dry periods and was considered acceptable for the station and month under consideration. The final model order chosen was the lowest model for which at least 50 of the 100 generated series resulted in an inability to reject the null hypotheses of the K-S test for both wet- and dry-spell-length distributions. For a small number of stations (approaching 10% during winter months), these criteria were not met, suggesting that model order higher than 3 may be necessary to consistently produce series that mimic the observed distributions of wet and dry spells.

Although the results based on spell-length distributions are in general agreement with those derived from consideration of the BIC, there are several major differences (Table 1; Fig. 3). Model-order recommendations from the K-S approach are also predominantly first-order (71.8% of station-months), but a larger number of second-order (21.6%) and zeroth-order models (5.0%) are recommended, whereas fewer third-order models (1.6%) are recommended. The zeroth-order model, which is seldom identified by the BIC, is found to adequately reproduce wet- and dry-spell-length distributions for multiple stations in the northern plains during the winter and summer months and at stations in the dry climates of the Southwest during spring and summer (Table 1, Fig. 3). The spell-length criteria results in second-order models for many locations during winter (Pacific Northwest, Central Texas, mid-Atlantic; see Fig. 3) with the greatest prevalence in the lee of the Rocky Mountains during the transition seasons and summer months (Fig. 3). The greater prevalence of second-order models found here is consistent with the results of Hayhoe (2000), who found an improvement in wet and dry sequences at three Canadian stations when using a second-order Markov model rather than a first-order model.

Also included in Table 1 are the number of bootstrapped samples (out of 100) for which the chosen model order was consistently identified. As noted above, the model order was chosen based on a simple majority of the bootstrapped samples. The numbers reported in Table 1 are much higher than 50 and are generally consistent with those from the BIC-based analysis. The consistency among bootstrapped samples

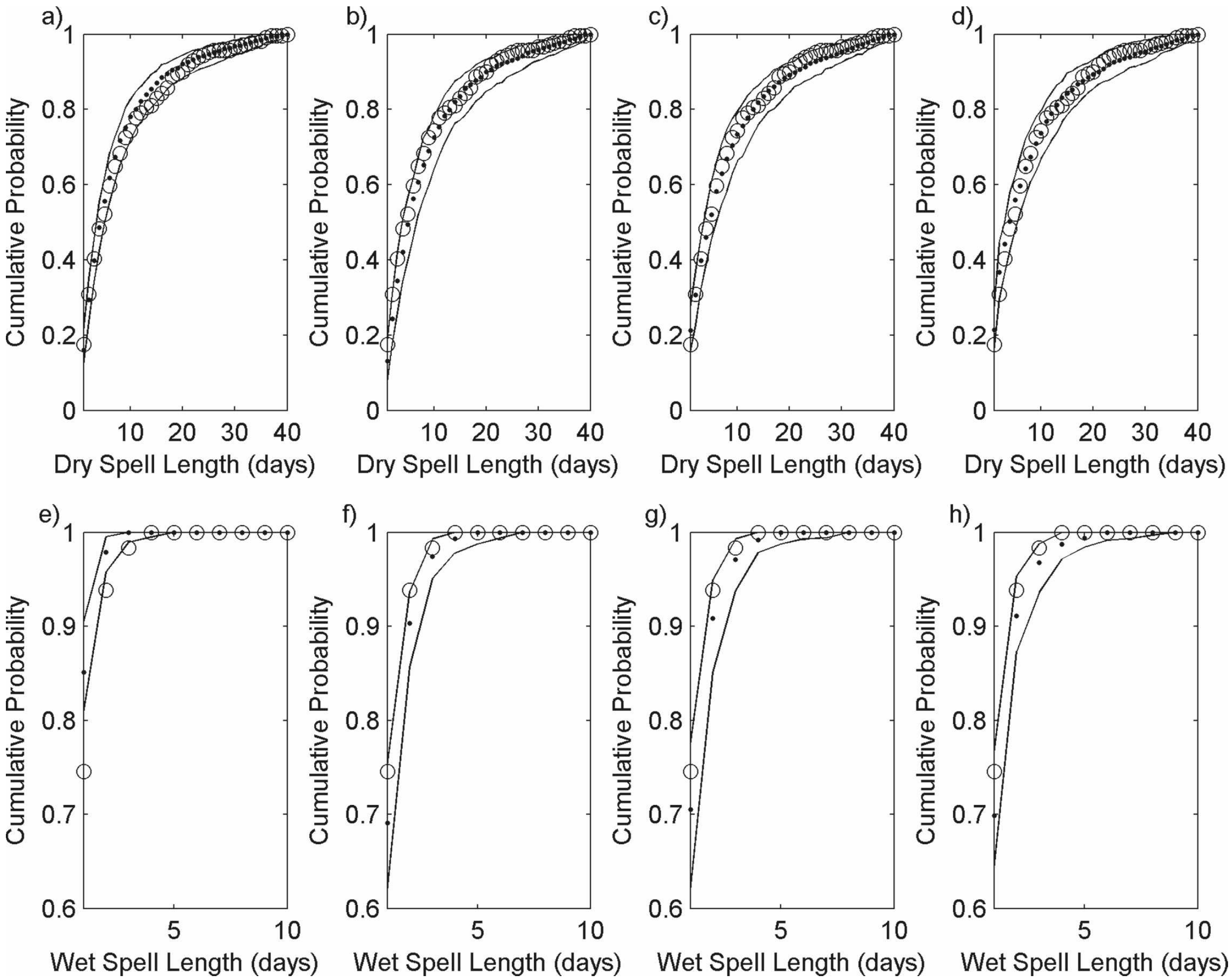

FIG. 4. Cumulative mass functions for (a)–(d) dry and (e)–(h) wet spells for July at Sacaton for observed data and Markov chains of order 0–3 (from left to right). The circles show the observed cumulative probabilities. The dots indicate the median of the modeled cumulative probabilities, and the solid lines indicate the 5th and 95th ordered cumulative probabilities based on bootstrap resampling.

suggests that the model-order identification is not due to sampling variations.

c. *Comparison of model-order choices based on BIC and spell-length distributions*

The model-order recommendations based on the BIC and K-S test are compared in Table 2, which indicates that in the majority of cases, the models identified using BIC considerations correspond to those that reproduce spell-length distributions that are consistent with those in the observed record. In some cases, however, there are considerable differences, including 11 station-months for which the BIC criterion results in third-order models while the spell-length criterion yields zeroth-order models. In a larger number of cases, the model orders differ by one or two orders (Table 2). These cases correspond to both higher and lower spell-based model-order recommendations relative to the BIC. Station-specific results for two stations are shown in Figs. 4 and 5. These results suggest that shortcomings in the first-order model are more closely related to wet-spell distributions than to dry-spell distributions.

At Sacaton, Arizona (Fig. 4), the BIC indicates a second-order model, whereas the spell-length method indicates a zeroth-order model. As shown, all of the models (order 0–3) provide an accurate depiction of the dry-spell distribution. Wet spells at this location are predominantly short-duration events and are relatively rare when compared with dry spells. There are no observed wet spells of longer than 4 days. The differences in the observed and simulated wet-spell distributions in Fig. 4 are not significant for the zeroth-order Markov chain model, even though the observed spell lengths

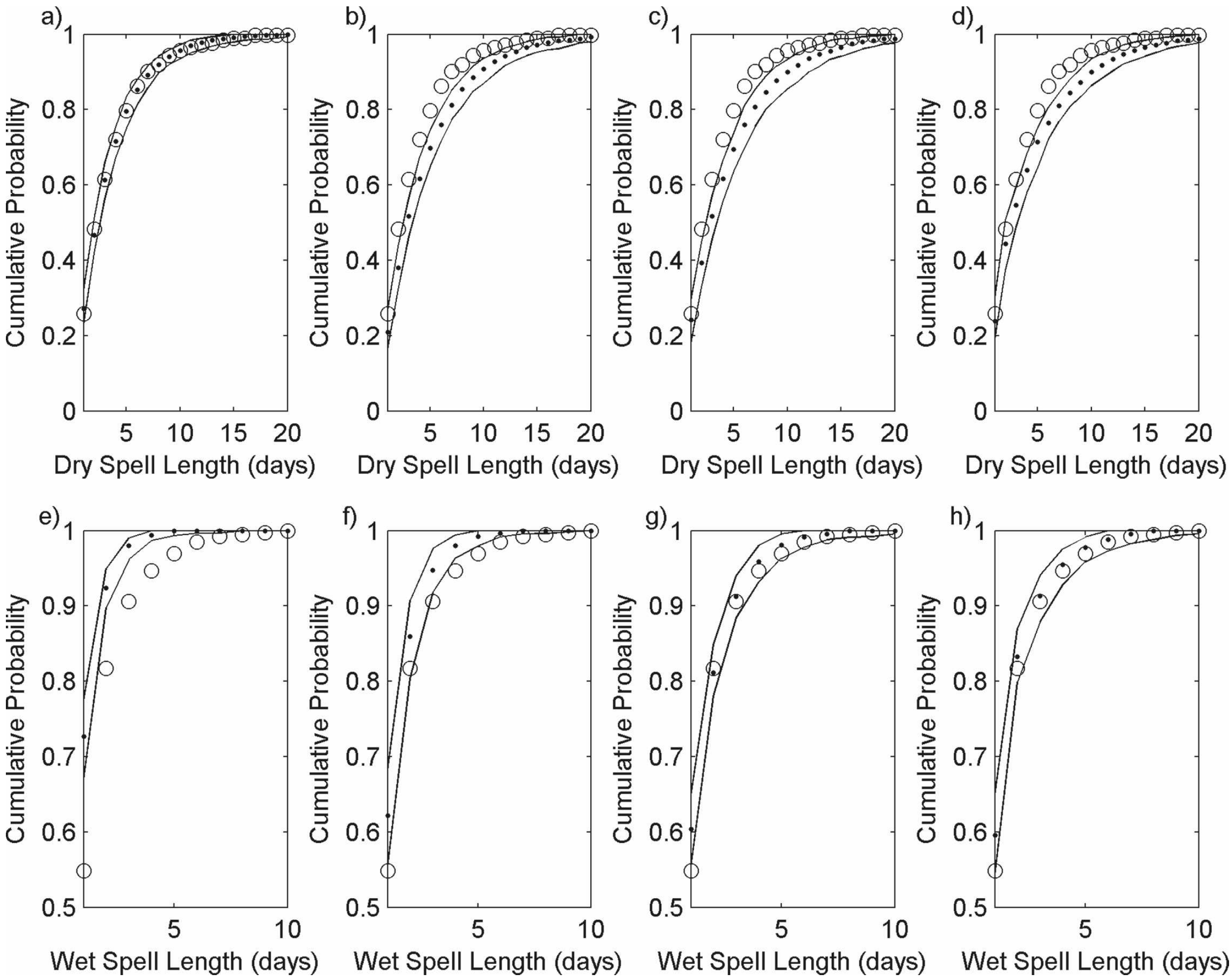

FIG. 5. As in Fig. 4, but at Hopewell.

are outside of the confidence interval resulting from the bootstrap resampling. Similar behavior is observed at most western stations, at which the BIC-based order is often much higher than the K-S-based order.

At Hopewell, Virginia (Fig. 5), the opposite relationship between the BIC- and spell-length-recommended models is found. In this case, the BIC indicates a first-order model, whereas the K-S test indicates a third-order model. As shown in Fig. 5, the zeroth-order Markov chain model adequately reproduces the dry-spell lengths, whereas higher-order models exhibit larger deviations from the observations. The higher-order models, however, result in better simulation of wet spells. At third order, the wet spells are well simulated and there has been some improvement achieved over the second-order model with respect to short dry spells. This example from Hopewell suggests that hybrid-order models might be appropriate for some months. As an alternative, Wan et al. (2005) have suggested retaining first-order Markov dependence and adding a separate model to simulate the annual number of wet days. This approach allowed better representation of the daily precipitation series but did not require as many parameters as higher-order Markov chains.

5. Summary

In this paper, we have examined the proper model order for Markov chain representation of the monthly precipitation occurrence process at 831 stations in the contiguous United States. Previous studies have used the BIC to determine model order. For the stations and data analyzed here, the BIC overwhelmingly indicates a first-order model, although other orders are recommended at some stations for select times of year.

To assess the performance of the Markov chain models, 50-yr series were generated using the maximum-likelihood estimates of the transition probabilities for

Markov chains of order 0–3. The distributions of wet and dry spells were then compared with those in the observed time series to assess the level of agreement between observed and generated precipitation occurrence series. For the majority of the stations and months examined (68.4%), the model order recommended is consistent with the model that reproduces the spell-length distributions. The BIC indicates a higher-order model than the spell-length approach for 13.3% of the station-months, whereas the BIC indicates a lower-order model for 18.3% of the station-months. In most cases, these differences are found to be related to the distribution of wet-spell lengths.

The BIC aims to identify the model that contains the most information but has the fewest parameters. Thus, while there is no reason to expect, a priori, that the BIC- and K-S-based criteria would result in the same model-order recommendation, it is noteworthy that choosing model order based on the BIC criterion alone may result in deficiencies in the lengths of dry and wet spells in the series generated by the Markov model. Practitioners should quantify the nature of differences in Markov chain models of different order and determine whether the magnitude of differences is large enough to warrant a model of different order. Further work will investigate the use of hybrid-order models to address separately the reproduction of wet and dry spells by models of different order.

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