OHIE: Blockchain Scaling Made Simple

Haifeng Yu Ivica Nikolić Ruomu Hou Prateek Saxena
Department of Computer Science, National University of Singapore
haifeng@comp.nus.edu.sg inikolic@nus.edu.sg houruomu@comp.nus.edu.sg prateeks@comp.nus.edu.sg

Abstract-Many blockchain consensus protocols have been proposed recently to scale the throughput of a blockchain with available bandwidth. However, these protocols are becoming increasingly complex, making it more and more difficult to produce proofs of their security guarantees. We propose a novel permissionless blockchain protocol OHIE which explicitly aims for simplicity. OHIE composes as many parallel instances of Bitcoin's original (and simple) backbone protocol as needed to achieve excellent throughput. We formally prove the safety and liveness properties of OHIE. We demonstrate its performance with a prototype implementation and large-scale experiments with up to 50,000 nodes. In our experiments, OHIE achieves linear scaling with available bandwidth, providing about 4-10Mbps transaction throughput (under 8-20Mbps per-node available bandwidth configurations) and at least about 20x better decentralization over prior works.

I. Introduction

Blockchain protocols power several open computational platforms, which allow a network of nodes to agree on the state of a distributed ledger periodically. The distributed ledger provides a total order over *transactions*. Nodes connect to each other via an open peer-to-peer (P2P) overlay network, which is *permissionless*: it allows any computer to connect and participate in the computational service without registering its identity with a central authority. A blockchain consensus protocol enables every node to *confirm* a set of transactions periodically, batched into chunks called blocks. The protocol ensures that honest nodes all agree on the same total ordering of the confirmed blocks, and that the set of confirmed blocks grow over time. The earliest such protocol was in Bitcoin [39], and has spurred interest in many blockchain platforms since.

The seminal Bitcoin protocol, published about a decade ago [39], laid some of the key foundations for modern blockchain protocols. But as Bitcoin gained popularity, its low throughput has been cited as glaring concern resulting in high costs per transaction [10]. Currently, Ethereum and Bitcoin process only about 5KB or 10 transactions per second on average, which is less than 0.2% of the average available bandwidth in their respective P2P networks [19]. Many recent research efforts have thus focused on improving the transaction throughput, resulting in a series of beautiful designs for permissionless blockchains [2], [12], [13], [20], [27], [28], [32], [33], [46], [47].

Bitcoin's core consensus protocol—called Nakamoto consensus—still stands out in one critical aspect: it is remarkably *simple*. Nakamoto consensus can be fully described in a few dozens of lines of pseudo-code. Such simplicity makes it extensively amenable to re-parameterization in hundreds

of deployments, and more importantly, a series of formal proofs on its security guarantees have been carefully and independently established in several research works [17], [18], [24], [26], [41].

The importance of keeping constructions *simple* enough to allow such formal proofs and cross validations cannot be over-emphasized. Consensus protocols are notoriously difficult to analyze in the presence of byzantine failures. Formal proofs/analysis are especially important to protocols that are difficult to upgrade once deployed: Upgrades of blockchain protocols after deployment (i.e., "hard forks") cause both philosophical disagreements and financial impact.

Some recent high-throughput blockchain protocols do strive to retain the simplicity of Nakamoto consensus. Unfortunately, many of them do not come with formal end-to-end security proofs. As an example, Conflux [32] is a recent high-throughput blockchain protocol with an elegant design. But it has only provided informal security arguments (Section 3.3 in [32]). Appendix E shows that in our simulation, as the throughput of Conflux increases, the security properties of Conflux deteriorate. Such an undesirable property of Conflux is hard to discover via informal arguments. The Conflux paper itself also presented effective attacks on a prior protocol (Phantom [46]) that comes without proofs.

Our goal. This work aims to develop a *simple* blockchain consensus protocol, which should admit formal end-to-end proofs on safety and liveness, while retaining the high throughput achieved by state-of-the-art blockchain protocols. Specifically, we aim to achieve:

- 1) Near-optimal resilience: Tolerate an adversarial computational power fraction f close to $\frac{1}{2}$, which is near-optimal;
- 2) Throughput approaching a significant fraction of the raw network bandwidth: The raw available network bandwidth in the P2P network constitutes a crude throughput upper bound for all blockchain protocols. We aim to achieve a throughput, in terms of transactions processed per second, that approaches a significant fraction of this raw network bandwidth.²

¹Related observations on Conflux have also been independently made by Bagaria et al. [5] and Fitzi et al. [15].

²Note that the raw available bandwidth is only a rather *crude* upper bound, and hence in practice it is unlikely for the throughput to reach this upper bound. For example, this crude upper bound does not take into account factors such that TCP slow start, probabilistic block generation, probabilistic hot-spots in the P2P overlay network, overheads for determining which blocks to gossip, and so on.

3) Decentralization: Many dynamically selected block proposers should be able to add blocks to the blockchain per second, rather than for example, having one leader or a small committee add blocks over a long period of time. More block proposers make transactions less susceptible to censorship [34], [36], and a DoS attack against a small number of nodes will no longer impact the availability of the entire system [23].

Our approach. This work proposes OHIE,³ a novel blockchain protocol for the permissionless setting. Specifically, OHIE composes many *parallel* instances of the Nakamoto consensus protocol. OHIE first applies a simple mechanism to force the adversary to evenly split its adversarial computational power across all these chains (i.e., Nakamoto consensus instances). Next, OHIE proposes a simple solution to securely arrive at a global order for blocks across all the parallel chains, hence achieving consistency.

The *modularity* of OHIE enables us to prove (under any given constant $f < \frac{1}{2}$) its safety and liveness properties via a *reduction* from those of Nakamoto consensus. Our proof invokes existing theorems on Nakamoto consensus, making the proof modular and streamlined [41]. By running as many (e.g., 1000) chains as the network bandwidth permits, OHIE's throughput scales with available network bandwidth. Finally, the parallel chains in OHIE lead to excellent decentralization, since many miners can simultaneously add new blocks.

Our results. We have implemented a prototype of OHIE, the source code of which is publicly available [1]. We have evaluated it on Amazon EC2 with up to 50,000 nodes, under similar settings as in prior works [20], [32]. Our evaluation first shows that OHIE's throughput scales linearly with available bandwidth, as is the case with state-of-the-art protocols [13], [20], [28], [33], [47]. For example, under configurations with 8-20Mbps per-node bandwidth, OHIE achieves about 4-10Mbps transaction throughput. This translates to close to 1000 to 2500 transactions per second, assuming 500-byte average transaction size as in Bitcoin. Such throughput is about 550% of the throughput of AlgoRand [20] and 150% of the throughput of Conflux [32] under similar available bandwidth. This suggests that while explicitly focusing on simplicity, OHIE retains the high throughput property of modern blockchain designs. Second, regardless of the throughput, the confirmation latency for blocks in OHIE is always below 10 minutes in our experiments, under security parameters comparable to Bitcoin and Ethereum deployments. (The confirmation latencies in Bitcoin and Ethereum are 60 minutes and 3 minutes, respectively.) Finally, our experiments show that the decentralization factor of OHIE is at least about 20x of all prior works.

II. SYSTEM MODEL AND PROBLEM

System model. Our system model and assumptions directly follow several prior works (e.g., [26], [41]). We model hash

functions as random oracles, and assume that some random genesis blocks are available from an initial trusted setup. We consider a permissionless setting, where nodes have no preestablished identities. We use standard proof-of-work (PoW) puzzles, a form of sybil resistance, to limit the adversary by computation power. We assume that the entire network has total n units of computational power, and some reasonable estimation of n is known. Out of this, the adversary controls fn units of computational power, with f being any constant below $\frac{1}{2}$. The adversary can deviate arbitrarily from the prescribed protocol, and hence is byzantine. We assume that some procedure to estimate the total computation power exists a-priori [18]. Standard PoW schemes help ascertain this periodically. For instance, in Bitcoin, the rate of PoW solutions is adjusted (periodically) to be approximately 10 minutes, and the PoW difficulty essentially maps to the estimated total computation power in the network. We can use the same mechanism in our design.

Given a fixed block size (e.g., 20 KB), we assume that honest nodes form a well-connected synchronous P2P overlay network, so that an honest node can broadcast (via gossiping) such a block with a maximum latency of δ to other honest nodes. Our protocol, much like Bitcoin, can tolerate variations in the actual propagation delay. Network partitioning attacks can delay block delivery arbitrarily and can cause honest nodes to lose inter-connectivity [4]. Defences to mitigate these attacks are an important area of research; however, they are outside the scope of the design of the consensus protocol. If the network becomes completely asynchronous, blockchain consensus is considered impossible [41]. In the presence of partitions, the CAP theorem suggests that protocols can either choose liveness or safety, but not both [21], [22]; we choose liveness—the same as Nakamoto consensus. The adversary sees every message as soon as it is sent. The adversary can arbitrarily inject its own messages into the system at any time (this captures the selfish mining attack [14], where newly mined blocks are injected at strategic points of time).

Problem definition. A blockchain protocol should enable any node at any time to output a sequence of total-ordered blocks, which we call the *sequence of confirmed blocks* (or SCB in short). For example, in the Bitcoin protocol, the SCB is simply the blockchain itself after removing the last 6 blocks. *Safety* and *liveness*, in the context of blockchain protocols, correspond to the **consistency** and **quality-growth** properties of the SCB. Informally, these two properties mean that the SCB's on different honest nodes at different time are always consistent with each other, and that the total number of *honest blocks* (i.e., blocks generate by honest nodes) in the SCB grows over time at a healthy rate. We leave the formal definitions of these properties to Section V.

Having **consistency** and **quality-growth** is sufficient to enable a wide range of different applications. For example, **consistency** prevents double-spending in a cryptocurrency—if two transactions spend the same coin, all nodes honor only the

³The word "ohie" comes from the Maori language and means "simple".

first transaction in the total order ⁴. Similarly, any "conflicting" state updates by smart contracts running on the blockchain can be ordered consistently by all nodes, by following the ordering in the SCB. In fact, ensuring a total order is key to support many different consistency properties [6], [29], [30] for applications, including for smart contracts [3].

Finally, if the same transaction is included in multiple blocks in the SCB, then the first occurrence will be processed, while the remaining occurrences will be ignored. To avoid such waste, miners should ideally pick different transactions to include in blocks. For example, among all transactions, a miner can pick those transactions whose hashes are the closest to the hash of the miner's public key. Note that this does not impact safety or liveness at all; it simply reduces the possibility of multiple inclusions (in different blocks) of the same transaction.

III. CONCEPTUAL DESIGN

OHIE composes k (e.g., k=1000) parallel instances of Nakamoto consensus. Intuitively, we also call these k parallel instances as k parallel chains. Each chain has a distinct genesis block, and the chains have ids from 0 to k-1 (which can come from the lexicographic order of all the genesis blocks). Within each instance, we follow the longest-path-rule in Nakamoto consensus.⁵ The miners in OHIE extend the k chains concurrently.

A. Mining in OHIE

Consider any fixed block size (e.g., 20 KB), and the corresponding block propagation delay δ (e.g., 2 seconds). Existing results [26], [41] on Nakamoto consensus show that for any given constant $f < \frac{1}{2}$, there exists some constant c such that if the *block interval* (i.e., average time needed to generate the next block on the chain) is at least $c \cdot \delta$, then Nakamoto consensus will offer some nice security properties. (Theorem 2 later makes this precise.) As an example, for f = 0.43, it suffices [41] for c = 5. Such $c \cdot \delta$, together with n, then maps to a certain *PoW difficulty* p in Nakamoto consensus, which is the probability of mining success for one hash operation. (Theorem 2 gives the precise mapping.) Given p, the hash of a valid block in Nakamoto should have $\log_2 \frac{1}{p}$ leading zeros.

In OHIE, to tolerate the same f as above, the hash of a valid block should have $\log_2 \frac{1}{kp}$ leading zeros, where the value p is chosen to be the same as in the above Nakamoto consensus protocol. Next, the last $\log_2 k$ bits of the hash⁶ of the OHIE block will index to one of the k chains in OHIE, and the block

will be assigned to and will extend from that chain. We have assumed the hash function to be a random oracle, and note that $\log_2 \frac{1}{kp} + \log_2 k = \log_2 \frac{1}{p}$. Hence for any *given* chain in OHIE, the probability of one hash operation (*either* done by honest nodes *or* done by the adversary) generating a block for that chain is exactly p, which is the same as in Nakamoto consensus. Similarly, the block interval for any given chain in OHIE will be the same as in Nakamoto consensus.

The above relation can be formalized: Taking all mechanisms in OHIE (especially the Merkle tree mechanism described next) into account, Lemma 3 later will prove that the behavior of any given chain in OHIE almost follows exactly the same distribution as the behavior of the single chain in Nakamoto consensus. Note that different chains in OHIE are still *correlated*, since a block is assigned to exactly one chain. But we will be able to properly bound the probability of all bad events (whether correlated or not) via a simple union bound.

Finally, since OHIE has k parallel chains, on expectation there will be total k blocks (across all chains) generated every $c \cdot \delta$ time, instead of just one block. Our experiments later will confirm the following simple yet critical property: Propagating many parallel blocks has minimal negative impact on the block propagation delay δ , as compared to propagating a single such block, until we start to saturate the network bandwidth of the system. Hence in OHIE, we use as large a k as possible to effectively utilize all the bandwidth in the system, subject to the condition that δ is minimally impacted.

B. Security of Individual Chains

In Nakamoto consensus, a new block B extends from some existing block A. The PoW computes over B, which contains the hash of A as a field, cryptographically binding the extension of A by B. In OHIE, however, a miner does not know which chain a new block will extend until it finishes the PoW puzzle, the last $\log_2 k$ bits of which then determine the chain extended.

To deal with this, in OHIE, a miner uses a Merkle tree [37] to bind to the last blocks of all the k chains in its local view. Specifically, let A_i be the last block⁸ of chain i, for $0 \le i \le k-1$. The miner computes a Merkle tree using $hash(A_0)$ through $hash(A_{k-1})$ as the tree leaves. The root of the Merkle tree is included in the new block B as an input to the PoW puzzle. After B is mined, the integer i that corresponds to the last $\log_2 k$ bits of hash(B) determines the block A_i from which B extends. When disseminating B in the network, a miner includes $hash(A_i)$ and the Merkle proof of $hash(A_i)$ in the message. (The value of i will be directly obtained from the hash of B.) The Merkle proofs are standard, consisting of $\log_2 k$ off-path hashes [37].

Intuitively, the above design binds each successful PoW to a single existing block on a single chain from which

⁴Transactions spending the same coin as earlier ones in the total order can simply be skipped as invalid by the user.

⁵Nakamoto consensus, strictly speaking, maintains a tree of blocks [39]. The longest-path-rule selects the longest path from the root (i.e., genesis block) to some leaf of the tree, where the leaf is chosen such that the path length is maximized. This path is often referred to as the chain.

 $^{^6\}mathrm{Under}$ our random oracle assumption, any $\log_2 k$ bits of the hash (other that the first $\log_2\frac{1}{kp}$ bits) work. In implementation, one can choose to use any portion (other that the first $\log_2\frac{1}{kp}$ bits) of the hash as appropriate, based on the specific hash function.

⁷Namely, total $\log_2 \frac{1}{p}$ positions in the block's hash must match some predetermined values, respectively.

 $^{^8}$ Exactly the same as in Nakamoto consensus, the last block here refers to the very last block on the longest path from the genesis block. In particular, this last block is not yet partially-confirmed. (In fact, none of the last T blocks on the path are partially-confirmed.)

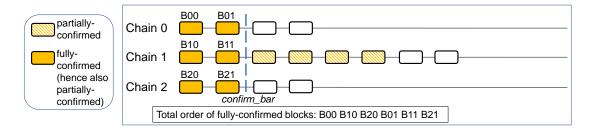


Fig. 1: Illustrating confirm_bar under T=2. Here chain 0, 1, and 2 have 2, 6, and 2 partially-confirmed blocks, respectively. On chain 1, only the first 2 blocks are fully-confirmed.

the new block extends. For further understanding, let us consider the following example scenario. The adversary may intentionally choose A_0 through A_{k-1} all from (say) chain 3, for constructing the Merkle tree. Assume the adversary finds a block B whose hash has $\log_2 \frac{1}{kn}$ leading zeros. If the last $\log_2 k$ bits of B does not equal to 3, then B will not be accepted by any honest node. Otherwise B will be accepted, and B can only extend from A_3 (instead of any other block) on chain 3. The reason is that the honest nodes will need to verify the 4th leaf (which corresponds to chain 3) on the Merkle tree. Only A_3 can pass such verification. Also note that the adversary may intentionally not use the last block on chain 3 as A_3 . This is not a problem, since the security of OHIE ultimately inherits (see Section V) from the security of Nakamoto consensus [41], and the adversary in Nakamoto consensus can already extend from any block (instead of extending from the last block).

C. Ordering Blocks across Chains - A Starting Point

Section V will show that each individual chain in OHIE inherits the proven security properties of Nakamoto consensus [41]. For example, with high probability, all blocks on a chain except the last T blocks (for some parameter T) are confirmed — the ordering of these confirmed blocks on the chain will no longer change in the future. This however does not yet give us a total ordering of all the confirmed blocks across all the k chains in OHIE. Recall from Section II that a node needs to generate an SCB (i.e., a total order of all confirmed blocks) satisfying **consistency** and **quality-growth**. To avoid notational collision, from this point on, we call all blocks on a chain except the last T blocks as partially-confirmed. Once a partially-confirmed block is added to SCB, it becomes fully-confirmed.

One way to design the SCB is to first include the first partially-confirmed block on each of the k chains (there are total k such blocks, and we order them by their chain ids), and then add the second partially-confirmed block on each of the k chains, and so on. This would work well, if every chain has the same number of partially-confirmed blocks.

When the chains do not have the same number of partially-confirmed blocks, we will need to impose a *confirmation bar* (denoted as confirm_bar) that is limited by the chain with the smallest number of partially-confirmed blocks (see Figure 1).

Blocks after confirm_bar cannot be included in the total order yet. This causes a serious problem, since with blocks extending chains at random, some chains can have more blocks than others. In fact in our experiments (results not shown), such imbalance appears to even grow unbounded over time.

D. Ordering Blocks across Chains - Our Approach

Imagine that the longest chain is 8 blocks longer than the shortest chain. Our basic idea to overcome the previous problem is that when the next block on the shortest chain is generated, we simply view it as 8 blocks worth. Figure 2 illustrates this idea. Here each block has two additional fields used for ordering blocks across chains, denoted as a tuple (rank, next_rank). In the total ordering of fully-confirmed blocks, the blocks are ordered by increasing rank values, with tie-breaking based on the *chain ids*. The *chain id* of a block is simply the id of the chain to which the block belongs. For any new block B that extends from some existing block A, we directly set B's rank to be the same as A's next_rank. Putting it another way, A's next_rank specifies (and fixes) the rank of B. A genesis block always has rank of 0 and next_rank of 1.

Determining next_rank. Properly setting the next_rank of a new block B is key to our design. A miner sees all the chains, and can infer the expected rank of the next upcoming block on each chain (before B is added to its chain). For example, at time t_1 in Figure 2, the next_rank of the current last block (not the last partially-confirmed block) on each of the three chains is 1, 5, and 1, respectively. Hence these will be the rank of the upcoming blocks on those chain, respectively. Let x denote the maximum (i.e., 5) among these values, and x corresponds to the "longest" chain (in terms of rank) among the k chains. Regardless of which chain the new block B ends up belonging to, we want B to help that chain to increase its rank to catch up with the "longest" chain. Hence the node generating B should directly set B's next rank to be x, or any value larger than 9 x. (To prevent adversarial manipulation, a careful implementation requiring an additional trailing

 $^{^9}$ Using a value larger than x will cause B's chain to exceed the length of the currently "longest" chain (in terms of rank). This is not a problem since the other chains will catch up with B's chain once a new honest block is added to each of those chains. We do not need the chains to have exactly the same length all the time.

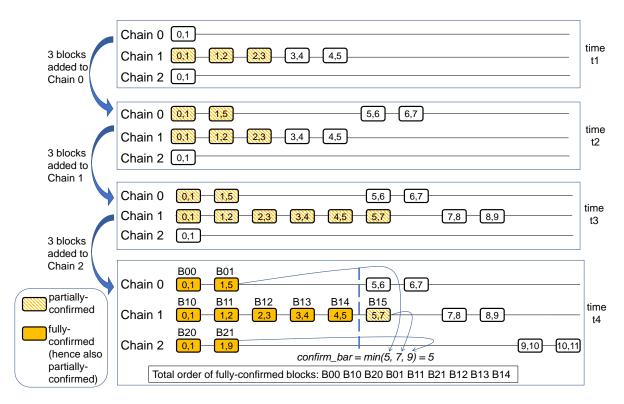


Fig. 2: Each block has a tuple (rank, next_rank). Our example here assumes that once a block is generated, it is seen by all nodes immediately. Note that OHIE does not need such an assumption. We use T=2 in this figure.

field is needed—see Section IV.) Finally, we always ensure that B's next_rank is at least one larger than B's rank, regardless of x. This guarantees that the rank values of blocks on one chain are always increasing.

In the example in Figure 2, from time t_1 to t_2 , there are 3 new blocks (with tuples (1,5), (5,6), and (6,7), respectively) added to chain 0. For the first new block added (i.e., the block with tuple (1,5)), the value of x is 5, and hence the block's next_rank is set to be 5. For the second new block (i.e., the block with tuple (5,6)), the value of x is still 5, while the rank of this block is already 5. Hence we set the block's next_rank to 6.

Determining the total order. We can now establish a total order among the blocks in the following way. Consider any given honest node at any given time and its local view of all the chains. Let y_i be the next_rank of the last partially-confirmed block on chain i in this view. For example, at time t_4 in Figure 2, we have $y_0 = 5$, $y_1 = 7$ and $y_2 = 9$. Note that the position of a partially-confirmed block on its respective chain will not change anymore, and hence all these y_i 's are "stable". Let confirm_bar $\leftarrow \min_{i=1}^k y_i$. Then, the next partially-confirmed block on any chain must have a rank no smaller than confirm_bar. This means that the node must have seen all partially-confirmed blocks whose rank is smaller than confirm_bar. Thus, it is safe (see Lemma 4) to deem all partially-confirmed blocks whose rank is smaller than confirm_bar as fully-confirmed, and include them in

SCB. Finally, all the fully-confirmed blocks will be ordered by increasing rank values, with tie-breaking favoring smaller chain ids. As an example, in Figure 2, at time t_4 , we have confirm_bar being 5. Hence, the 9 partially-confirmed blocks whose rank is below 5 become fully-confirmed.

Summary. By properly setting the rank values of all the blocks, we ensure that the chains remain balanced in terms of the rank's of their respective last blocks. This is regardless of how imbalanced the chains are in terms of the total number of blocks, hence avoiding the earlier imbalance problem in Section III-C.

IV. IMPLEMENTATION DETAILS

We call the Nakamoto consensus protocol (p,λ,T) -Nakamoto, where the hash of a valid block needs to have $\log_2\frac{1}{p}$ leading zeros, λ is the security parameter (i.e., the length of the hash output), and T is the number of blocks that we remove from the end of the chain in order to obtain partially-confirmed blocks. We call the OHIE protocol (k,p,λ,T) -OHIE, where λ and T are the same as above, and where k is the number of chains in OHIE. In (k,p,λ,T) -OHIE, the hash of a valid block should have $\log_2\frac{1}{kp}$ leading zeros.

For simplicity, the value of k is fixed in our current design of OHIE—adjusting k on-the-fly could potentially be possible via a view change mechanism, but we consider it as future work. The value of T in OHIE can be readily adjusted on-the-fly by individual nodes, without needing any coordination. Because the security of OHIE inherits from the security of Nakamoto

```
1: d \leftarrow \log_2(\frac{1}{kn});
 2: V_i \leftarrow \{(\text{genesis block of chain } i, \text{ the attachment } i\}
     for genesis block of chain i), for 0 \le i \le
 3: M \leftarrow \text{Merkle} tree of the hashes of the k
     genesis blocks;
 4: trailing ← hash of genesis block of chain 0;
 6: OHIE() {
      repeat forever {
 7:
           ReceiveState();
 8:
           Mining();
10:
           SendState();
11:
12: }
13:
14: Mining() {
15:
      B.transactions \leftarrow get_transactions();
       B.\mathtt{root} \leftarrow \mathtt{root} of Merkle tree M;
16:
17:
      B.\mathtt{trailing} \leftarrow \mathtt{trailing};
18:
       B.\mathtt{nonce} \leftarrow \mathtt{new} \ \mathtt{nonce}();
       B.\mathtt{hash} \leftarrow hash(B);
19:
       if (\widehat{B}.\text{hash has }d\text{ leading zeroes}) {
20:
           i \leftarrow \text{last } \log_2 k \text{ bits of } \widehat{B}.\text{hash};
21:
           \widehat{B}.leaf \leftarrow leaf i of M;
22:
           \widehat{B}.leaf_proof \leftarrow M.MerkleProof(i);
23:
           ProcessBlock(B, \widehat{B});
24:
25:
26: }
27:
28: SendState() {
29:
        send V_i (0 \le i \le k-1) to other nodes;
        // In implementation, only need to send those
     blocks not sent before.
31: }
32:
33: ReceiveState() {
        foreach (B, \widehat{B}) \in received state do
            ProcessBlock(B, \widehat{B});
36: // A block B_1 should be processed before B_2,
     if \widehat{B_2}.leaf or \widehat{B_2}.trailing points to B_1.
37: }
```

```
38: ProcessBlock(B, \widehat{B}) {
      // do some verifications
       i \leftarrow \text{last } \log_2 k \text{ bits of } \widehat{B}.\text{hash};
       verify that \widehat{B}.hash has d leading zeroes;
       verify that hash(B) = \widehat{B}.hash;
       verify that \widehat{B}.leaf is leaf i in the Merkle tree, based on B.root and
      B.\mathtt{leaf\_proof};
       verify that \widehat{B}.\mathtt{leaf} = \widehat{A}.\mathtt{hash} for some block (A, \widehat{A}) \in V_i;
       verify that B.\mathtt{trailing} = \widehat{C}.\mathtt{hash} for some block (C,\widehat{C}) \in \bigcup_{j=0}^{k-1} V_j;
45:
       if (any of the above 5 verifications fail) then return;
46:
47:
       // compute rank and next rank values
48:
        \widehat{B}.\mathtt{rank} \leftarrow \widehat{A}.\mathtt{next\_rank};
49:
        \widehat{B}.next_rank \leftarrow \widehat{C}.next_rank;
50:
       if (\widehat{B}.\mathtt{next\_rank} \leq \widehat{B}.\mathtt{rank}) then \widehat{B}.\mathtt{next\_rank} \leftarrow \widehat{B}.\mathtt{rank} + 1;
51:
52:
       // update local data structures
53:
        V_i \leftarrow V_i \cup \{(B, \widehat{B})\};
54:
55:
       update trailing;
56:
       update Merkle tree M;
57: }
58:
59: OutputSCB() {
       // determine partially-confirmed blocks and confirm_bar
60:
       for (i = 0; i < k; i++) {
            W_i \leftarrow \text{get\_longest\_path}(V_i);
62:
            \mathtt{partial}_i \leftarrow \mathtt{blocks} in W_i except the last T blocks;
63:
            (B_i, B_i) \leftarrow \text{the last block in partial}_i;
64:
            y_i \leftarrow \widehat{B_i}.\mathtt{next\_rank};
65:
66:
       \text{all\_partial} \leftarrow \cup_{i=0}^{k-1} \text{partial}_i;
67:
       confirm_bar \leftarrow \min_{i=0}^{k-1} y_i;
68:
69:
       // determine fully-confirmed blocks and SCB
70:
71:
       foreach (B, \widehat{B}) \in \text{all partial } \{
72:
            if (\widehat{B}.\mathtt{rank} < \mathtt{confirm\_bar}) then L \leftarrow L \cup \{(B,\widehat{B})\};
73:
74:
       sort blocks in L by rank, tie-breaking favoring smaller chain id;
75:
76:
       return L;
77: }
```

Fig. 3: Pseudo-code of the (k, p, λ, T) -OHIE protocol.

consensus, using different T values in OHIE has a similar effect as in Nakamoto consensus. For example, a user can use a larger T for a higher security level. To simplify discussion, however, we consider some fixed T value.

Overview. Figure 3 gives the pseudo-code of OHIE, as run by each node. In the main loop (Line 8 to 10) of OHIE, a node receives messages from others, makes one attempt for solving the PoW (i.e., makes one query to the random oracle or hash function), and then sends out messages. The messages contain OHIE blocks. Such a main loop is exactly the same as in Nakamoto consensus [26], [41]. Note that the block generation rate is significantly lower than the rate of queries to the random oracle. Hence, most of the time, the main loop will not have any new messages to receive/send. In an actual implementation, such sending/receiving of messages can be

done in a separate thread.

The function OutputSCB() can be invoked whenever needed. It produces the current SCB, by exactly following the description in Section III-D.

Key data structures. Each OHIE node maintains sets V_0 through V_{k-1} . V_i initially contains the genesis block for chain i. During the execution, V_i is a tree of blocks, containing all those blocks with a path to that genesis block. We use W_i to denote the longest path (from the genesis block to some leaf) on this tree. All blocks on W_i , except the last T blocks, are partially-confirmed.

The Merkle tree M is constructed by using the hash of the very last block on W_i ($0 \le i \le k-1$) as the k leaves. Each node further maintains a trailing variable, which is the hash of the *trailing block*. The *trailing block* is the block with the

largest next_rank value, among all blocks in $\cup_{i=0}^{k-1} V_i$. (Note that the trailing block may or may not be in $\cup_{i=0}^{k-1} W_i$.) If there are multiple such blocks, we let the trailing block be the one with the smallest chain id. Both M and trailing should be properly updated whenever the node receives new blocks.

Block generation/verification. A block B in Ohie consists of some transactions, a fresh nonce, the Merkle root, and a B.trailing field. All these fields are fed into the hash function, during mining. Section III-B explains why we include the Merkle root. The following explains the B.trailing field.

As explained in Section III-D, regardless of which chain the new block B ends up belonging to, we want B to help that chain to increase its rank to catch up with the "longest chain" (in terms of rank). To do so, all we need is to set B's next rank to be large enough. A naive design is to let the creator of B directly set B's next_rank, based on its local V_i 's. Such a design enables the adversary to pick the maximum possible value¹⁰ for that field. Doing so exhausts the possible values for next rank, since the next rank of blocks on a given chain needs to keep increasing.

This is why in OHIE, each node maintains a trailing variable (i.e., the hash of the trailing block¹¹). The miner sets B.trailing \leftarrow trailing, and B.trailing (as part of B) is fed into the hash function. When a node u receives a new block B, the node will verify whether it has already seen some block C whose hash equals B.trailing. If so, u sets next rank of B to be the same as the next_rank of C. Otherwise Bwill not be accepted. Doing so prevents the earlier attack.

Note that the adversary can still lie, and claim some arbitrary block as its trailing block. Theorem 1 in Section V, however, will prove that this does *not* cause any problem. Intuitively, by doing so, the adversary is simply refusing to help a chain to grow its rank to catch up. But the next honest block on the chain will enable the chain to grow its rank properly, and to immediately catch up.

Block attachment. Some information about a block is not available until after the block is successfully mined, and we include such information in an attachment for the block. The attachment for a block is always stored/disseminated together with the block. We always use B to denote an attachment for a block B. (Note that \widehat{B} is not fed into the hash function when we compute the hash of B.) Specifically, let i be the last $\log_2 k$ bits in B's hash. B will include leaf i of the Merkle tree, and the corresponding Merkle proof. For convenience, Balso contains the hash of B, in its \widehat{B} .hash field.

An attachment \widehat{B} further contains a rank value and a next rank value, for the block B. (The block B itself actually does not have a rank or next_rank field.) When a node receives a new block B and its attachment B, the node independently computes the proper values for \widehat{B} .rank and

 \widehat{B} .next rank based on its local information, and use those values (Line 49 to 51 in Figure 3). Lemma 5 will prove that except with an exponentially small probability (i.e., excluding hash collisions and so on), for any block B, all honest nodes will assign exactly the same value to \widehat{B} .rank (\widehat{B} .next_rank). Finally, a genesis block B always has \widehat{B} .rank = 0 and $B.\mathtt{next} \mathtt{rank} = 1.$

V. SECURITY GUARANTEES OF OHIE

Our analysis results presented in this section hold under all possible strategies of the adversary.

A. Overview of Guarantees

Formal framework. Our formal framework directly follows several prior works (e.g., [26], [41]). All executions we consider are of polynomial length with respect to the security parameter λ . We model hash functions as random oracles. The execution of the system comprises a sequence of ticks, where a tick is the amount of time needed to do a single proof-ofwork query to the random oracle by an honest node. Hence in each tick, each honest node does one such query, while the adversary does up to fn such queries. We allow these fnqueries to be done sequentially, which only makes our results stronger. Define Δ (e.g., 2×10^{12}) to be δ (e.g., 2 seconds) divided by the duration of a tick (e.g., 10^{-12} second) — hence a message sent by an honest node will be received by all other honest nodes within Δ ticks. A block is an honest block if it is generated by some honest node, otherwise it is a malicious block. For two sequences S_1 and S_2 , S_2 is a prefix of S_1 iff S_1 is the concatenation of S_2 and some sequence S_3 . Here S_3 may be empty — hence S_1 is also a prefix of itself. Finally, recall from Section IV the definitions of (p, λ, T) -Nakamoto and (k, p, λ, T) -OHIE.

Main theorem on OHIE. We will eventually prove the fol-

Theorem 1. Consider any given constant $f < \frac{1}{2}$. Then there exists some positive constant c such that for $\tilde{a}ll \ p \leq \frac{1}{c\Delta n}$ and all $k \geq 1$, the (k, p, λ, T) -OHIE protocol satisfies all the following properties, with probability at least 1-k. $exp(-\Omega(\lambda)) - k \cdot exp(-\Omega(T))$:

- (growth) On any honest node, the length of each of the
- k chains increases by at least T blocks every $\frac{2T}{pn}$ ticks. (quality) On any honest node and at any time, every Tconsecutive blocks on any of the k chains contain at least $\frac{1-2f}{1-f}T$ honest blocks.
- (consistency) Consider the SCB S_1 on any node u_1 at any time t_1 , and the SCB S_2 on any node u_2 at any time t_2 . Then either S_1 is a prefix of S_2 or S_2 is a prefix of S_1 . Furthermore, if $(u_1 = u_2 \text{ and } t_1 < t_2)$ or $(u_1 \neq u_2)$ and $t_1 + \Delta < t_2$), then S_1 is a prefix of S_2 .
- (quality-growth) For all integer $\gamma \geq 1$, the following property holds after the very first $\frac{2T}{pn}$ ticks of the execution: On any honest node, in every $(\gamma+2)\cdot\frac{2T}{pn}+2\Delta$ ticks,

 $^{^{10}\}mathrm{next_rank}$ must have a finite domain in actual implementation.

¹¹Our trailing block is the block with the largest next_rank in $\cup_{i=0}^{k-1} V_i$. One could further require the trailing block to be in $\cup_{i=0}^{k-1} W_i$. The security guarantees of OHIE (in Section V) and our proofs also hold for such an interaction decimal trailing block is the security guarantees. alternative design.

¹²Here u_1 (t_1) may or may not equal u_2 (t_2).

at least $\gamma \cdot k \cdot \frac{1-2f}{1-f}T$ honest blocks are newly added to

Values of c, λ , and T. The value of c in Theorem 1 will be exactly the same as the c in Theorem 2 next. If we want the properties in Theorem 1 to hold with probability $1 - \epsilon$, then both λ and T should be $\Theta(\log \frac{1}{\epsilon} + \log k)$. The value of ϵ needed by a real application (e.g., a cryptocurrency system) is typically orders of magnitude smaller than $\frac{1}{k}$ — hence λ and T are usually just $\Theta(\log \frac{1}{\epsilon})$.

Four properties. The growth and quality in Theorem 1 are about the individual component chains in OHIE. For consistency, considering individual chains obviously is not sufficient. Hence, Theorem 1 proves consistency¹³ for the SCB, which is the final total order of the fully-confirmed blocks. Theorem 1 also proves the quality-growth of SCB, showing that SCB will incorporate more honest blocks at a certain rate. Ultimately, **consistency** corresponds to the *safety* of OHIE, while quality-growth captures the liveness of OHIE.

B. Existing Result as a Building Block

Our proof later will invoke the following theorem from [41] on Nakamoto consensus.

Theorem 2. (Adapted from Corollary 3 in [41].) Consider any given constant $f<\frac{1}{2}$. Then there exists some positive constant c such that for all $p\leq\frac{1}{c\Delta n}$, the (p,λ,T) -Nakamoto protocol satisfies all the following properties, with probability at least $1 - exp(-\Omega(\lambda)) - exp(-\Omega(T))$:

- (growth) On any honest node, the length of the chain
- increases by at least T blocks every $\frac{2T}{pn}$ ticks.

 (quality) On any honest node and at any time, every T consecutive blocks on the chain contain at least $\frac{1-2f}{1-f}T$ honest blocks.
- (consistency) Let S_1 (S_2) be the sequence of blocks on the chain on any node u_1 (u_2) at any time t_1 (t_2), excluding the last T blocks on the chain. Then either S_1 is a prefix of S_2 or S_2 is a prefix of S_1 .

Combing the **growth** and **quality** properties immediately leads to quality-growth for (p, λ, T) -Nakamoto:

• (quality-growth) Let S be the sequence of blocks on the chain on any given honest node, excluding the last Tblocks. Then after the very first $\frac{2T}{pn}$ ticks of the execution, in every $\frac{2T}{pn}$ ticks, at least $\frac{1-2f}{1-f}T$ honest blocks are newly added to S.

C. Proof for Theorem 1

Overview of proof for Theorem 1. Our first step is to obtain a reduction from (p, λ, T) -Nakamoto to (k, p, λ, T) -OHIE. Consider any given adversary A for (k, p, λ, T) -OHIE, and any given chain i $(0 \le i \le k-1)$ in the execution of (k, p, λ, T) -OHIE against A. Our reduction step will show that there exists some adversary \mathcal{A}' for (p, λ, T) -Nakamoto with the following property: Except some exponentially small probability and after proper mapping from blocks in (k, p, λ, T) -OHIE to blocks in (p, λ, T) -Nakamoto, the behavior of chain i in the execution of (k, p, λ, T) -OHIE against \mathcal{A} follows exactly the same distribution as the behavior of the (single) chain in the execution of (p, λ, T) -Nakamoto against \mathcal{A}' . Such a reduction implies that existing properties on (p, λ, T) -Nakamoto directly carry over to each individual chain in (k, p, λ, T) -OHIE.

Our second step is to show that conditioned upon all the existing properties (in Theorem 2) on (p, λ, T) -Nakamoto holding for each individual chain in (k, p, λ, T) -OHIE, the SCB generated in OHIE must satisfy the properties in Theorem 1, except with some exponentially small probability. The reasoning in this step will center around the rank and next_rank values of the blocks.

Formal concepts needed for reduction. Consider any (blackbox and potentially randomized) adversary \mathcal{A} for OHIE. Define EXEC(OHIE, k, p, λ , T, A) to be the random variable denoting the joint states of all the honest nodes and the adversary throughout (i.e., at every tick) the entire execution resulted from running (k, p, λ, T) -OHIE against A. Define random variable Chainview_i(EXEC(OHIE, k, p, λ , T, \mathcal{A})) to be the joint state of chain i on every honest node throughout this execution. Recall that in OHIE, a node maintains k sets of blocks, V_0 through V_{k-1} , and chain i corresponds to the longest path in V_i . One could imagine that for each (u, t) pair, Chainview_i() contains a component describing chain i on the honest node u at tick t.

We similarly define EXEC(Nakamoto, p, λ , T, \mathcal{A}') for the execution resulted from running (p, λ, T) -Nakamoto against adversary \mathcal{A}' . Also, we similarly define random variable Chainview(EXEC(Nakamoto, p, λ , T, \mathcal{A}')) to be the joint state of the (single) chain on every honest node throughout this execution.

For two random variables X and Y over some finite domain Z, define their variation distance to be ||X - Y|| = $0.5\sum_{z\in Z}|\Pr[X=z]-\Pr[Y=z]|$. If X and Y are both parameterized by λ , we say that $X(\lambda)$ is strongly statistically close to $Y(\lambda)$ iff $||X - Y|| = exp(-\Omega(\lambda))$.

Our reduction lemma. The following is our reduction lemma:

Lemma 3. Consider any given i where $0 \le i \le k-1$, and any given adversary A for (k, p, λ, T) -OHIE. There exists some adversary \mathcal{A}'_i for (p, λ, T) -Nakamoto such that the following two random variables are strongly statistically close:

- Chainview_i(EXEC(OHIE, k, p, λ , T, \mathcal{A}))
- $\sigma_i^{\tau}(\text{Chainview}(\text{EXEC}(Nakamoto, p, \lambda, T, \mathcal{A}_i'))$

Here τ is the randomness in EXEC(Nakamoto, p, λ , T, \mathcal{A}'_i). The random variable $\sigma_i^{\tau}(\mathtt{Chainview}())$ is the same as Chainview(), except that we replace each block B' in Chainview() with another block $\sigma_i^{\tau}(B')$. The mapping $\sigma_i^{\tau}(A')$ is some one-to-one mapping from each block B' among all the blocks on all the nodes in (p, λ, T) -Nakamoto to some block $\sigma_i^{\tau}(B')$ on all the nodes in (k, p, λ, T) -OHIE. The mapping $\sigma_i^{\tau}()$ guarantees that i) B' is an honest block iff $\sigma_i^{\tau}(B')$ is an

¹³Different prior works [17], [41] define consistency slightly differently. Our definition here is either equivalent or stronger than those in the prior works.

honest block, and ii) B' extends from A' iff $\sigma_i^{\tau}(B')$ extends from $\sigma_i^{\tau}(A')$.

The crux of proving this lemma is to construct the adversary \mathcal{A}'_i . In our proof, \mathcal{A}'_i simulates a certain execution of OHIE against A. More precisely, A'_i simulates all the OHIE nodes, as well as the adversary A in a black-box fashion. The adversary \mathcal{A}'_i simultaneously interacts with the (real) nodes running (p, λ, T) -Nakamoto, while ensuring that the simulated OHIE execution and the real Nakamoto execution are properly "coupled". We defer the complete proof of this lemma to Appendix B through D. To be fully rigorous, the complete proof needs additional formalism and also needs to fully specify the (p, λ, T) -Nakamoto protocol by pseudo-code.

From individual chains to SCB. The following lemma (proof in Appendix A) establishes the connection from the individual chains in OHIE to the SCB in OHIE:

Lemma 4. If the three properties in Theorem 2 hold for each of the k chains in (k, p, λ, T) -OHIE, then with probability at least $1 - exp(-\Omega(\lambda))$, the SCB in OHIE satisfies the **consistency** and **quality-growth** properties in Theorem 1.

Final proof. Using all the lemmas, we can now prove Theorem 1:

Proof. (for Theorem 1) We set the constant c in Theorem 1 to be the same as the c in Theorem 2. For any given i where $0 \le i$ $i \le k-1$, Lemma 3 and Theorem 2 tell us that for chain i in (k, p, λ, T) -OHIE, with probability at least $1 - exp(-\Omega(\lambda))$ $exp(-\Omega(T)) - exp(-\Omega(\lambda))$, the three properties in Theorem 2 hold for that chain. Hence with probability at least 1-k. $exp(-\Omega(\lambda)) - k \cdot exp(-\Omega(T))$, the properties in Theorem 2 hold for all k chains in (k, p, λ, T) -OHIE. The **growth** and quality properties in Theorem 1 then directly follow. Applying Lemma 4 further leads to the consistency and quality-growth properties in Theorem 1.

D. Discussion and Comparison

Plug-in alternative results. Our analysis invokes the results in [41]. The analysis in [41] is just one of the many works [17], [18], [24], [26], [41] that analyze Nakamoto-style protocols. A highlight of our proof on OHIE is that it invokes the existing guarantees on (p, λ, T) -Nakamoto as black-box. Hence alternative results on (p, λ, T) -Nakamoto directly translates to alternative results on OHIE.

Specifically, Theorem 2 (adopted from [41]) has the following three quantitative measures:

- The value of c, where ¹/_{cΔn} is the upper limit on p.
 The value of x = ^{2T}/_{pn} ticks (i.e., the growth rate), which is the time needed for the chain length to grow by T blocks.
- The value of $y = \frac{1-2f}{1-f}T$ (i.e., the *quality rate*), which is the number of honest blocks among every T consecutive blocks on the chain.

Other analyses [17], [18], [24], [26] have obtained alternative results on x and y values for (p, λ, T) -Nakamoto. They have also derived various sufficient conditions¹⁴ for consistency in (p, λ, T) -Nakamoto, which all ultimately translate to requirement on the value of c.

We can directly plug in alternative c, x, and y values from alternative analyses on (p, λ, T) -Nakamoto to obtain alternative results on OHIE. If we do so, then the value of c in Theorem 1 will be exactly the same as the given c. The **consistency** property in Theorem 1 will remain unchanged. The remaining properties in Theorem 1 will become:

- (growth) The length of each of the k chains on each honest node will increase by at least T blocks every xticks.
- (quality) On any honest node and at any time, every Tconsecutive blocks on any of the k chains must contain at least y honest blocks.
- (quality-growth) For all integer $\gamma > 1$, after the very first x ticks of the execution, on any honest node in every $(\gamma + 2) \cdot x + 2\Delta$ ticks, at least $\gamma \cdot k \cdot y$ honest blocks are newly added to SCB.

Comparing with prior results. With the above discussion, we can now easily compare Theorem 1 with any of the prior results [17], [18], [24], [26], [41]. Consider the result from any such prior analyses, with certain resulting c, x, and yvalues. (This also implies that in that particular result, the rate of quality-growth is y new honest blocks every x ticks.) Now let us plug in such c, x, and y values into Theorem 1. Then OHIE will provide exactly the same quantitative guarantees as that prior result, in terms of the value of c, the growth rate, and the quality rate. The only difference will be regarding the quality-growth of SCB. Under the given x and y, for all integer $\gamma \geq 1$, in OHIE $\gamma \cdot k \cdot y$ honest blocks are newly added to the SCB every $(\gamma+2) \cdot x + 2\Delta$ ticks. Since 2Δ is dominated by $(\gamma+2) \cdot x$, the average rate of honest blocks being added to the SCB is about $k \cdot y$ honest blocks every x ticks, in the long term. Such a rate is k times of the rate of quality-growth in the corresponding prior result. This also intuitively explains why OHIE increases throughput by k times.¹⁵

Confirmation latency. Finally, Theorem 1 also indirectly gives OHIE's guarantee on transaction confirmation latency (also called wait-time in [17], [41]): Assume that we want the properties in Theorem 1 to hold with $1 - \epsilon$ probability, and let us invoke the theorem with $p = \Theta(\frac{1}{c\Delta n})$. By **qualitygrowth**, at least one honest block is newly added to SCB within $\Theta(T\Delta) = \Theta((\log \frac{1}{2} + \log k)\Delta)$ ticks. Now if a transaction is injected into all honest nodes continuously for $\Theta((\log \frac{1}{\epsilon} + \log k)\Delta)$ ticks, this transaction must be included in the SCB after those ticks. Hence, the confirmation latency is simply $\Theta((\log \frac{1}{\epsilon} + \log k)\Delta)$ ticks. As explained earlier, this usually becomes $\Theta(\Delta \log \frac{1}{\epsilon})$ in practical settings. Such a confirmation latency is the same as in [17], [41], and is

¹⁴For example, the sufficient condition derived in [41] is that $\alpha(1-2(\Delta+1)\alpha) \geq (1+\eta)\beta$ for some positive constant η .

¹⁵Of course, k cannot be unbounded. In practice, increasing k beyond a certain point will cause a non-trivial increase in Δ , when the system starts to saturate the network bandwidth. Our experiments later will quantify this.

fundamental in all Nakamoto-style protocols: Each new block takes $\Theta(\Delta)$ ticks, and $\Theta(\log \frac{1}{\epsilon})$ new blocks are needed for the "confidence" to reach $1 - \epsilon$.

VI. EXPERIMENTAL EVALUATION

Methodology. We have implemented a prototype of OHIE in C++, with total around 4,700 lines of code. We use Amazon's EC2 virtual machines (or EC2 instances) for evaluation. We rent m4.2xlarge instances in 14 cities around the globe¹⁶, with each instance having 8 cores and 1Gbps bandwidth. The one-way latency between two random EC2 instances is about 90-140ms, which is consistent with AlgoRand's experiments [20] and with measurements in Bitcoin and Ethereum [19]. To avoid excessive monetary expenses on EC2, we run two sets of experiments. Our *macro experiments* run 12,000 to 50,000 OHIE nodes on up to 1,000 EC2 instances to evaluate the end performance of OHIE, while our *micro experiments* run 1,000 OHIE nodes on 20 EC2 instances to determine OHIE internal parameters.

In all experiments, the OHIE nodes form a P2P overlay by each node connecting to 8 randomly selected peers, and pernode bandwidth is up to 20Mbps, since we run 50 nodes per EC2 instance. This setup is the same as in the experiments of AlgoRand [20] and Conflux [32]. All results reported are averaged over 5 runs, each lasting until measurements stabilize in that run.

A. Choosing Block Size and Block Interval in OHIE

We first use micro experiments to measure the *block propagation delay (BPD)* for a single block (with no parallel propagations) to reach 99% of the nodes. We consider different bandwidth configurations where the per-node bandwidth ranges from 8Mbps to 20Mbps. We observe that the BPD for 20 KB blocks is about 1.7-1.9 seconds, across all our bandwidth configurations. Similar BPD values are observed for block sizes ranging from 10 KB to 64 KB.

We further observe that such BPD does not significantly increase as the network size increases: Even in our macro experiments with 50,000 nodes, the BPD values are still only about 3.2 seconds for 10-20 KB blocks. This is consistent with theoretical expectations about random graphs, ¹⁷ and the fact that BPD is proportional to the average number of hops between two nodes.

Smaller blocks enable better decentralization, but also incur more protocol overheads. Taking all factors into account, we choose a block size of 20 KB for OHIE. We then set p to correspond to a block interval of about 10 seconds on *each* chain. Based on Section V and results in [41], a block interval of $5 \times BPD$ is sufficient to tolerate f = 0.43.

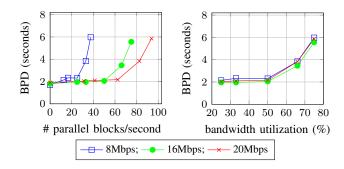


Fig. 4: BDP under different per-node bandwidth configurations (8-20Mbps). [**left**]: BPD vs. number of blocks propagated in parallel per second. The BPD for a single block (non-parallel case) is plotted as x-axis value of 0. [**right**]: BPD vs. fraction of raw bandwidth utilized by parallel block propagation.

B. Efficient Parallel Propagation of Blocks

At the network level, a key difference between OHIE and other high-throughput protocols (e.g., Algorand [20]) is that with its large number of parallel chains, OHIE propagates a large number of small blocks in parallel. In contrast, protocols such as Algorand usually propagate a single large block (e.g., of 1 MB).

Now for OHIE, the critical empirical question is whether propagating many parallel blocks will have significant negative impact on BPD, as compared to propagating a single such block. We use micro experiments to answer this critical question. Figure 4(left) plots the BPD for 20 KB blocks (results for block sizes ranging from 10-64KB are similar and not shown), as a function of number of parallel block propagations. For example, "40 parallel blocks/sec" means that we inject 40 new blocks (of size 20 KB each) into the network per second. The figure shows that under 20Mbps raw bandwidth, even 60 parallel blocks per second will not cause any substantial increase in BPD.

Figure 4(right) presents the same results from a different perspective — it plots how BPD changes as the fraction of raw bandwidth used by the parallel block propagation increases. It shows that, consistently under all our bandwidth configurations, parallel block propagation can effectively utilize a rather significant fraction (about 50%) of the raw bandwidth, without significant negative impact on BPD. This simple yet important finding lays the empirical foundation for parallel chain designs. In particular, we can hope OHIE to eventually achieve a throughput approaching a significant fraction of the raw network bandwidth, by using a sufficient number of parallel chains.

C. End-to-end Performance of OHIE

We finally use macro experiments to evaluate the end-to-end performance of OHIE, in terms of its throughput, decentralization factor, and confirmation latency. By default, all our results will be from running 12,000 nodes on 1000 EC2 instances, with different per-node bandwidth configurations (8-20Mbps). We have also experimented with 50,000 nodes on

¹⁶This is the maximal number of cities with such instances.

¹⁷In random Erdos-Renyi graphs, roughly speaking, the average number of hops between two nodes increases only logarithmically with the number of nodes.

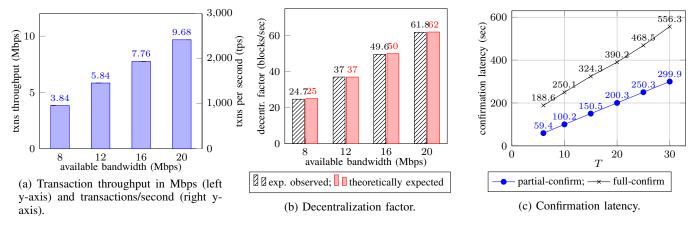


Fig. 5: OHIE's throughput, decentralization factor, and confirmation latency.

1000 EC2 instance, with 20 Mbps per-node bandwidth. Those results are within 1% of the results under the corresponding experiment with 12,000 node, and we do not report those separately.

We always use 20 KB blocks, with block interval being 10 seconds on each chain, as determined in Section VI-A. We choose k according to the available bandwidth, such that $k \times \text{block size/block interval} \approx 0.5 \times \text{available bandwidth}.$ Specifically, for 8Mbps, 12Mbps, 16Mbps, and 20Mbps pernode available bandwidth, we use k = 250, 370, 500, and 620, respectively. Since these k values are not powers of 2, we do not use the last $\log_2 k$ bits of the block hash to decide which chain a block belongs to. Instead, let x be the last 48 bits of the block hash, and we assign the block to chain i where $i = x \mod k$. Assuming the hash function is a random oracle, doing so will assign each block to a uniformly random chain, except some negligible probability. 18 Finally, to be able to run multiple nodes on each EC2 instance, we do not have the nodes solve PoW puzzles. Instead, each node produces ("mines") a new block after some exponentially distributed time that corresponds to the mining difficulty.

Throughput. Figure 5a shows that the throughput of OHIE indeed scales up roughly linearly with the available bandwidth. In fact, the throughput of OHIE always reaches a rather significant fraction (about 50%) of the raw available network bandwidth of the system. Under 20Mbps available bandwidth, the throughput of OHIE is about 9.68Mbps, or about 2,420 transactions per second (assuming 500-byte average transaction size as in Bitcoin). As a quick comparison, OHIE achieves about 550% of the throughput of AlgoRand [20] under similar available bandwidth. Compared to Conflux under 20Mbps available bandwidth [32], the throughput of OHIE is about 150% of the throughput of Conflux. This suggests that while explicitly focusing on simplicity, OHIE still retains the high throughput property of modern blockchain designs.

Decentralization factor. Another advantage of OHIE is its *decentralization factor*, in terms of the number of distinct

confirmed blocks per second. Figure 5b shows that the decentralization factor of OHIE increases linearly with the available bandwidth. This is expected since the number of parallel blocks (or chains) increases with the available bandwidth. In particular, OHIE achieves a decentralization factor of about 61.8 under 20Mbps available bandwidth. This is at least about 20x higher than those reported in experiments on previous permissionless protocols, among which Omniledger [28] reported the best decentralization factor of about 3.1 (i.e., 25 blocks in 8.1 seconds).

Confirmation latency. Let T be the number of blocks that we remove from the end of the chain in order to obtain partially-confirmed blocks in each individual chain in OHIE. For example, Bitcoin and Ethereum use T=6 and T=10 to 15, respectively. For comparable security, our analysis in Section V suggests using a T that is $\Theta(\log k)$ larger. Given that our k is no larger than 2^{14} , we use T=20 to 30 in OHIE. Note that T has impact only on confirmation latency, and has no impact on the throughput or decentralization factor of OHIE.

Figure 5c plots the average time for a block to become partially-confirmed and fully-confirmed on all nodes, under 20Mbps bandwidth configuration. It shows that a block takes about 1-5 minutes to become partially-confirmed, and then about another 2-4 minutes to become fully-confirmed. As a reference point, the confirmation latencies in Bitcoin and Ethereum are presently 60 and 3 minutes, respectively.

For a conservative T=30, we have done further experiments confirming (not plotted in Figure 5c) that the partial and full confirmation latencies remain stable under various bandwidth configurations from 8Mbps to 20Mbps. Putting it another way, the latency does not deteriorate as the throughput of OHIE increases.

VII. RELATED WORK

OHIE uses PoW under the same permissionless model as Nakamoto consensus. There have been works in alternative models, such as using Proof-of-Stake [7], [8], [20], [25] or assuming a permissioned setting [11], [16], [38]. PoW-based

 $^{^{18} \}mbox{This}$ is not exactly uniformly random since the range of x is not an exact multiple of k. But the impact is negligible.

permissionless blockchain protocols have largely followed two paradigms: The first based on extensions to Nakamoto consensus, and the second based on utilizing classical byzantine agreement (BA). We discuss these two categories one by one.

Nakamoto consensus. Existing deployments of Nakamoto consensus (e.g., Bitcoin) and its variant the GHOST protocol [45] (e.g., Ethereum) achieve only about 5KByte/s throughput. Bitcoin-NG is a variant of Nakamoto consensus, where many micro-blocks are proposed by the same proposer, after the proposer is chosen by a key block [13]. This helps to significantly improve throughput without comprising security, but at the same time, still has limited decentralization: A single block proposer is responsible for generating all (micro) blocks for an extended period of time (i.e., 10 minutes), inviting censorship attacks and DoS attacks. In comparison, OHIE achieves superior decentralization — in our experiments, in each second, up to about 60 different proposers propose blocks concurrently.

Scaling Nakamoto consensus. Phantom [46] and Conflux [32] have attempted to scale Nakamoto consensus, by having blocks reference more than one previous blocks. Section I has already discussed these two works.

Chainweb [35], [43] is another attempt to scale Nakamoto consensus, by maintaining k parallel chains. Unlike OHIE, Chainweb does not have any mechanism to prevent the adversary from focusing entirely on one of the k chains. Chainweb's original analysis [35], [43] only considers a few specific attack strategies [41], instead of all possible adversaries. Fitzi et al. [15] have shown that an adversary focusing on a specific chain can cause the confirmation latency in Chainweb to increase quadratically in k. In contrast, OHIE does not suffer from such a problem. Furthermore, different from OHIE where the rank values balance all the chains, Chainweb requires synchronous growth of all chains, needing to periodically stall fast-growing chains. Kiffer et al. [26] have provided a further analysis of Chainweb, which does not support the highthroughput claim by Chainweb. In their analysis, a natural form of Chainweb is "bounded by the same throughput as Nakamoto protocol for the same consistency guarantee" [26].

Concurrent with and independent of this work, there are two online non-refereed technical reports [5], [15] proposing a similar approach to composing multiple parallel chains. While their high-level designs share similarities with ours, in their designs, one of the k parallel chains is designated as a special chain, and blocks on the other chains are related to blocks on that special chain [5], [15]. In OHIE, all k chains are equal and symmetric. More importantly, these two concurrent and independent works [5], [15] do not have any implementation details or experimental evaluation, while we have a prototype implementation as well as large-scale evaluation on Amazon EC2. In particular, our large-scale experiments confirm that propagating many parallel blocks does *not* negatively impact block propagation delay, which lays the empirical foundation for parallel chain designs.

Finally, Spectre [44] confirms blocks without guaranteeing a

total order, while the inclusive protocol [31] includes as many non-conflicting transactions as possible. These works provide weaker consistency notions than OHIE, which guarantees a total order. These weaker notions may suffice for cryptocurrency payments, but for smart contracts, total ordering is key in resolving state conflicts [29], [30] and in building towards higher abstractions of consistency [3], [12], [28].

Blockchain protocols relying on byzantine agreement. Some PoW-based permissionless blockchain protocols [2], [12], [27], [28], [33], [42], [47] build upon classical byzantine agreement (BA) protocols. BA protocols require a committee of pre-agreed identities among which the BA protocol can run. Such a committee can either be established using Nakamoto consensus itself or using previous rounds of BA.

Different from OHIE and other Nakamoto-style protocols, which can tolerate any constant $f < \frac{1}{2}$, BA-based blockchain protocols [2], [20], [27], [33], [42], [47] all require $f < \frac{1}{3}$. A key bottleneck in BA-based designs, as acknowledged in prior works, is that of establishing (or replenishing) committees with 200-2000 identities each. This large committee size is necessary to ensure the requisite resilience f. The latency of replenishing committees can be between in tens of seconds [2], [20], minutes [33], [47], or hours [28]. After committees are established and when there are no attacks, BA protocols tend to have smaller confirmation latencies [20], [27], [47] than Nakamoto-style protocols. Finally, we point out that classical BA protocols, such as PBFT [9], can be considerably more complex to implement and verify, as compared to Nakamoto consensus.

Sharding designs. A subset of the BA-based blockchain protocols employ sharding [28], [33], [47], where many parallel shards process blocks in parallel. This helps to improve decentralization and throughput, at the cost of additional complexity. But due to the overheads associated with each shard, these protocols [28], [33], [47] typically use a small number of shards (no more than 25). The best decentralization was achieved in [28], with about 25 blocks proposed every 8 seconds. In comparison, OHIE does not involve pre-assigning nodes to its k chains, and can easily use as large a k as needed. The decentralization factor of OHIE in our experiments (i.e., about 600 blocks every 10 seconds) is about 20x higher than the sharding designs.

VIII. CONCLUSION

We present OHIE, a permissionless proof-of-work blockchain protocol that composes parallel instances of Nakamoto consensus securely. OHIE has a simple implementation and a modular safety and liveness proof. It achieves linear scaling with available bandwidth and at least about 20x better decentralization over prior works.

ACKNOWLEDGMENTS

We thank Hung Dang, Seth Gilbert, Aquinas Hobor, Ilya Sergey, Shruti Tople, and Muoi Tran for their helpful feedbacks on this work. We thank Sourav Das for help on formatting the figures in this paper. This work is partly supported by

the sponsors of the Crystal Center at National University of Singapore. All opinions and findings presented in this work are those of the authors only.

REFERENCES

- [1] Ohie blockchain scaling. https://github.com/ivicanikolicsg/OHIE, 2019.
- [2] ABRAHAM, I., MALKHI, D., NAYAK, K., REN, L., AND SPIEGELMAN, A. Solida: A blockchain protocol based on reconfigurable byzantine consensus. In *International Conference on Principles of Distributed Systems (OPODIS)* (2017).
- [3] AL-BASSAM, M., SONNINO, A., BANO, S., HRYCYSZYN, D., AND DANEZIS, G. Chainspace: A sharded smart contracts platform. In Network and Distributed System Security Symposium (NDSS) (2018).
- [4] APOSTOLAKI, M., ZOHAR, A., AND VANBEVER, L. Hijacking bitcoin: Routing attacks on cryptocurrencies. In *IEEE Symposium on Security and Privacy* (2017).
- [5] BAGARIA, V., KANNAN, S., TSE, D., FANTI, G., AND VISWANATH, P. Deconstructing the blockchain to approach physical limits. arXiv preprint arXiv:1810.08092v2 (2018).
- [6] BAILIS, P., AND GHODSI, A. Eventual consistency today: limitations, extensions, and beyond. Commun. ACM (2013).
- [7] BENTOV, I., PASS, R., AND SHI, E. Snow white: Provably secure proofs of stake. IACR Cryptology ePrint Archive (2016).
- [8] BONNEAU, J., MILLER, A., CLARK, J., NARAYANAN, A., KROLL, J. A., AND FELTEN, E. W. Sok: Research perspectives and challenges for bitcoin and cryptocurrencies. In *IEEE Symposium on Security and Privacy* (2015).
- [9] CASTRO, M., AND LISKOV, B. Practical byzantine fault tolerance. In Symposium on Operating Systems Design and Implementation (OSDI) (1999).
- [10] CROMAN, K., DECKER, C., EYAL, I., GENCER, A. E., JUELS, A., KOSBA, A. E., MILLER, A., SAXENA, P., SHI, E., SIRER, E. G., SONG, D., AND WATTENHOFER, R. On scaling decentralized blockchains - (A position paper). In Financial Cryptography and Data Security - FC International Workshops (BITCOIN) (2016).
- [11] DANEZIS, G., AND MEIKLEJOHN, S. Centrally banked cryptocurrencies. In Network and Distributed System Security Symposium (NDSS) (2016)
- [12] DECKER, C., SEIDEL, J., AND WATTENHOFER, R. Bitcoin meets strong consistency. In *International Conference on Distributed Computing and Networking (ICDCN)* (2016).
- [13] EYAL, I., GENCER, A. E., SIRER, E. G., AND VAN RENESSE, R. Bitcoin-NG: A Scalable Blockchain Protocol. In NSDI (2016).
- [14] EYAL, I., AND SIRER, E. G. Majority is not enough: Bitcoin mining is vulnerable. *Communications of the ACM 61*, 7 (2018), 95–102.
- [15] FITZI, M., GAZI, P., KIAYIAS, A., AND RUSSELL, A. Parallel chains: Improving throughput and latency of blockchain protocols via parallel composition. Cryptology ePrint Archive, report 2018/1119, version 20181130:165620, Nov. 2018. https://eprint.iacr.org/2018/1119.
- [16] GARAY, J., AND KIAYIAS, A. Sok: A consensus taxonomy in the blockchain era, 2018. https://eprint.iacr.org/2018/754.pdf.
- [17] GARAY, J., KIAYIAS, A., AND LEONARDOS, N. The bitcoin backbone protocol: Analysis and applications. In EUROCRYPT (2015).
- [18] GARAY, J. A., KIAYIAS, A., AND LEONARDOS, N. The bitcoin backbone protocol with chains of variable difficulty. In CRYPTO (2017).
- [19] GENCER, A. E., BASU, S., EYAL, I., VAN RENESSE, R., AND SIRER, E. G. Decentralization in bitcoin and ethereum networks. In *Financial Cryptography and Data Security (FC)* (2018).
- [20] GILAD, Y., HEMO, R., MICALI, S., VLACHOS, G., AND ZELDOVICH, N. Algorand: Scaling byzantine agreements for cryptocurrencies. In Symposium on Operating Systems Principles (2017).
- [21] GILBERT, S., AND LYNCH, N. Brewer's conjecture and the feasibility of consistent, available, partition-tolerant web services. SIGACT News 33, 2 (2002), 51–59.
- [22] GILBERT, S., AND LYNCH, N. A. Perspectives on the CAP theorem. IEEE Computer 45, 2 (2012), 30–36.
- [23] HEILMAN, E., KENDLER, A., ZOHAR, A., AND GOLDBERG, S. Eclipse attacks on bitcoin's peer-to-peer network. In *USENIX Security Sympo*sium (2015).
- [24] KIAYIAS, A., AND PANAGIOTAKOS, G. Speed-security tradeoffs in blockchain protocols. *IACR Cryptology ePrint Archive* 2015 (2015), 1019.

- [25] KIAYIAS, A., RUSSELL, A., DAVID, B., AND OLIYNYKOV, R. Ouroboros: A provably secure proof-of-stake blockchain protocol. In CRYPTO (2017).
- [26] KIFFER, L., RAJARAMAN, R., AND ABHI SHELAT. A Better Method to Analyze Blockchain Consistency. In CCS (2018).
- [27] KOKORIS-KOGIAS, E., JOVANOVIC, P., GAILLY, N., KHOFFI, I., GASSER, L., AND FORD, B. Enhancing bitcoin security and performance with strong consistency via collective signing. In *USENIX Security Symposium* (2016).
- [28] KOKORIS-KOGIAS, E., JOVANOVIC, P., GASSER, L., GAILLY, N., AND FORD, B. OmniLedger: A Secure, Scale-Out, Decentralized Ledger via Sharding. In *IEEE Symposium on Security and Privacy* (2018).
- [29] KUNG, H. T., AND PAPADIMITRIOU, C. H. An optimality theory of concurrency control for databases. In SIGMOD (1979).
- [30] LAMPORT, L. How to make a multiprocessor computer that correctly executes multiprocess programs. *IEEE Trans. Computers* 28, 9 (1979), 690–691.
- [31] LEWENBERG, Y., SOMPOLINSKY, Y., AND ZOHAR, A. Inclusive block chain protocols. In *International Conference on Financial Cryptography* and Data Security (2015).
- [32] LI, C., LI, P., ZHOU, D., XU, W., LONG, F., AND YAO, A. Scaling nakamoto consensus to thousands of transactions per second, 2018. Arxiv preprint, arXiv:1805.03870v4.
- [33] LUU, L., NARAYANAN, V., ZHENG, C., BAWEJA, K., GILBERT, S., AND SAXENA, P. A secure sharding protocol for open blockchains. In CCS (2016).
- [34] LUU, L., VELNER, Y., TEUTSCH, J., AND SAXENA, P. Smartpool: Practical decentralized pooled mining. In USENIX Security Symposium (2017).
- [35] MARTINO, W., QUAINTANCE, M., AND POPEJOY, S. Chainweb: A Proof-of-Work Parallel-Chain Architecture for Massive Throughput (DRAFT v15), 2018. https://kadena.io/download/178/.
- [36] MCCORRY, P., HICKS, A., AND MEIKLEJOHN, S. Smart contracts for bribing miners. IACR Cryptology ePrint Archive (2018).
- [37] MERKLE, R. C. A digital signature based on a conventional encryption function. In CRYPTO (1987).
- [38] MILLER, A., XIA, Y., CROMAN, K., SHI, E., AND SONG, D. The honey badger of BFT protocols. In CCS (2016).
- [39] NAKAMOTO, S. Bitcoin: A peer-to-peer electronic cash system, 2008. https://bitcoin.org/bitcoin.pdf.
- [40] NATOLI, C., AND GRAMOLI, V. The balance attack against proof-ofwork blockchains: The r3 testbed as an example, 2016. Arxiv preprint, arXiv:1612.09426.
- [41] PASS, R., SEEMAN, L., AND SHELAT, A. Analysis of the blockchain protocol in asynchronous networks. In EUROCRYPT (2017).
- [42] PASS, R., AND SHI, E. Hybrid consensus: Efficient consensus in the permissionless model. In *International Symposium on Distributed Computing* (2017).
- [43] QUAINTANCE, M., AND MARTINO, W. Chainweb Protocol Security Calculations (WORK IN PROGRESS DRAFT v7), 2018. https://kadena.io/download/187/.
- [44] SOMPOLINSKY, Y., LEWENBERG, Y., AND ZOHAR, A. Spectre: A fast and scalable cryptocurrency protocol. *IACR Cryptology ePrint Archive* (2016).
- [45] SOMPOLINSKY, Y., AND ZOHAR, A. Secure high-rate transaction processing in bitcoin. In *International Conference on Financial Cryptography and Data Security* (2015).
- [46] SOMPOLINSKY, Y., AND ZOHAR, A. Phantom: A scalable blockdag protocol, 2018. https://eprint.iacr.org/2018/104/20180330:121321.
- [47] ZAMANI, M., MOVAHEDI, M., AND RAYKOVA, M. RapidChain: Scaling Blockchain via Full Sharding. In CCS (2018).

APPENDIX

A. Proof for Lemma 4

Lemma 4 directly follows from Lemma 6 and Lemma 7, which we will state and prove next. To prove Lemma 6 and Lemma 7, we will need Lemma 5. Lemma 5 shows that except some exponentially small probability, given a block B and regardless of whether B is an honest block or malicious

block, the value of \widehat{B} (and in particular, the value of \widehat{B} .rank / \widehat{B} .next_rank) must be the same on all honest nodes. This avoids the need of reasoning about potentially different \widehat{B} .rank (\widehat{B} .next_rank) values on different honest nodes. In the following, we will state and prove Lemma 5 through 7, one by one.

Lemma 5. Consider the execution of (k, p, λ, T) -OHIE against any given adversary A. With probability at least $1 - exp(-\Omega(\lambda))$, there will never be two honest nodes u_1 and u_2 adding $(B_1, \widehat{B_1})$ and $(B_2, \widehat{B_2})$ to their local set of blocks, respectively, such that $B_1 = B_2$ and $\widehat{B_1} \neq \widehat{B_2}$.

Proof. Define bad1 to be the event that the execution (of (k, p, λ, T) -OHIE against \mathcal{A}) contains some invocations $hash(x_1)$ and $hash(x_2)$ such that $x_1 \neq x_2$ and $hash(x_1) = hash(x_2)$. Given that we model the hash function as a random oracle, and given that the length of the execution is $poly(\lambda)$, we have $Pr[bad1] = exp(-\Omega(\lambda))$.

Next define bad2 to be the event that the execution (of (k, p, λ, T) -OHIE against \mathcal{A}) contains some invocation of hash() that belongs to one of the following two categories:

- For some B, some honest node invokes hash(B) at Line 42 of Figure 3 and the verification at that line succeeds (i.e., hash(B) indeed equals B.hash), despite that hash(B) has never been previous invoked (by either some honest node or the adversary) in the execution.
- Some honest node invokes hash() at Line 43 of Figure 3 and the Merkle verification at that line succeeds, despite the following: Let $hash(x_1)$, $hash(x_2)$, ..., $hash(x_{\log_2 k})$ be the $\log_2 k$ hash invocations done by the honest node during the successful Merkle verification. There exists some x_i $(1 \le i \le \log_2 k)$ such that $hash(x_i)$ has never been previously invoked (by either some honest node or the adversary) in the execution.

Again given that we model the hash function as a random oracle, and given that the length of the execution is $poly(\lambda)$, one can easily verify that $\Pr[\mathtt{bad2}] = exp(-\Omega(\lambda))$.

A simple union bound then shows that with probability at least $1-exp(-\Omega(\lambda))$, neither bad1 nor bad2 happens. It now suffices to prove that conditioned upon neither of the bad events happening, if $B_1=B_2$, then we must have $\widehat{B_1}=\widehat{B_2}$. An attachment consists of five fields, namely, hash, leaf, leaf_proof, rank, and next_rank. We will reason about these one by one.

Since $B_1=B_2$, and since $(B_1,\widehat{B_1})$ and $(B_2,\widehat{B_2})$ have been accepted by u_1 and u_2 respectively, we trivially have $\widehat{B_1}$.hash $=\widehat{B_2}$.hash. We next prove $\widehat{B_1}$.leaf $=\widehat{B_2}$.leaf and $\widehat{B_1}$.leaf_proof $=\widehat{B_2}$.leaf_proof. Since u_1 has verified the Merkle proof before accepting $(B_1,\widehat{B_1}),u_1$ must have invoked hash(x) for some x of length 2λ , with the return value of such invocation being B_1 .root. Since bad1 and bad2 do not happen, such an x must be unique. Let $x_0|x_1\leftarrow x$, where x_0 and x_1 are both of length λ . Following this process for $\log_2 k$ steps (we are effectively tracing down the Merkle tree), we will find a unique x_\perp value, which corresponds to leave i of

the Merkle tree, with i being the last $\log_2 k$ bits of $\widehat{B_1}$.hash. Since bad1 and bad2 do not happen, in order for $(B_1,\widehat{B_1})$ to pass the verification by u_1 , we must have $\widehat{B_1}$.leaf $= x_{\perp}$. By the same argument and since B_1 .root $= B_2$.root, we must also have $\widehat{B_2}$.leaf $= x_{\perp}$. Hence we conclude $\widehat{B_1}$.leaf $= \widehat{B_2}$.leaf. A similar argument shows that $\widehat{B_1}$.leaf_proof $= \widehat{B_2}$.leaf_proof as well.

Next we move on to the rank and next_rank fields. Let A_1 be any block on u_1 such that $\widehat{A_1}$.hash $= x_\perp$, where x_\perp is obtained as above. Similarly define A_2 . Since bad1 and bad2 do not happen, we must have $A_1 = A_2$. Next, let C_1 be any block on u_1 such that $\widehat{C_1}$.hash $= B_1$.trailing, and let C_2 be any block on u_2 such that $\widehat{C_2}$.hash $= B_2$.trailing $= B_1$.trailing. Similarly, we must also have $C_1 = C_2$.

Consider the DAG \mathcal{G}_1 consisting of all the blocks on u_1 as vertices, where for each block B_1 , there is an edge to B_1 from the corresponding A_1 , and another edge to B_1 from the corresponding C_1 . We do a topological sort of all the vertices in \mathcal{G}_1 , and assume that B_1 is the jth block in the topological sort. (Note that j is based on B_1 and \mathcal{G}_1 , and has nothing to do with B_2 .) We will do an induction on j to show that for all $B_1 = B_2$, we have $\widehat{B_1}$.rank = $\widehat{B_2}$.rank and $\widehat{B_1}$.next_rank = $\widehat{B_2}$.next_rank.

The induction basis for j from 1 to k trivially holds (these are the k genesis blocks). Now assume that the previous claim holds for all $j < j_1$, and we prove the claim for $j = j_1$. Since B_1 is the j_1 -th block in the topological sort, the position of A_1 and C_1 in the topological sort must be before j_1 . Furthermore, we have shown earlier that $A_1 = A_2$ and $C_1 = C_2$. We can thus invoke the inductive hypothesis on A_1 and C_1 , which shows that $\widehat{A_1}.\mathtt{next_rank} = \widehat{A_2}.\mathtt{next_rank}$ and $\widehat{C_1}.\mathtt{next_rank} = \widehat{C_2}.\mathtt{next_rank}$. By Line 49 in Figure 3, $\widehat{B_1}.\mathtt{rank}$ is set to be the same as $\widehat{A_1}.\mathtt{next_rank}$, while $\widehat{B_2}.\mathtt{rank}$ is set to be the same as $\widehat{A_2}.\mathtt{next_rank}$. Hence we have $\widehat{B_1}.\mathtt{rank} = \widehat{B_2}.\mathtt{rank}$. A similar argument shows $\widehat{B_1}.\mathtt{next_rank} = \widehat{B_2}.\mathtt{next_rank}$.

Lemma 6. If the three properties in Theorem 2 hold for each of the k chains in (k, p, λ, T) -OHIE, then with probability at least $1 - exp(-\Omega(\lambda))$, (k, p, λ, T) -OHIE satisfies the **consistency** property in Theorem 1.

Proof. Lemma 5 shows that with probability at least $1-\exp(-\Omega(\lambda))$, there will never be two honest nodes adding (B_1,\widehat{B}_1) and (B_2,\widehat{B}_2) to their respective local set of blocks, such that $B_1=B_2$ and $\widehat{B}_1\neq\widehat{B}_2$. This means that for each block B accepted by an honest node, its rank and next_rank on this honest node will be the same as the corresponding values on all other honest nodes. Hence we can directly refer to the rank and next_rank of a block B, and no longer need to consider the values of \widehat{B} .rank and \widehat{B} .next_rank on individual nodes. All our following discussion will be conditioned on this.

The following restates the **consistency** property in Theorem 1, which we need to prove:

• (consistency) Consider the SCB S_1 on any node u_1 at

any time t_1 , and the SCB S_2 on any node u_2 at any time t_2 .¹⁹ Then either S_1 is a prefix of S_2 or S_2 is a prefix of S_1 . Furthermore, if $(u_1 = u_2 \text{ and } t_1 < t_2)$ or $(u_1 \neq u_2 \text{ and } t_1 + \Delta < t_2)$, then S_1 is a prefix of S_2 .

Let the view of node u_1 at time t_1 be Ψ_1 , and the view of node u_2 at time t_2 be Ψ_2 . Let x_1 and x_2 be the confirm_bar in Ψ_1 and Ψ_2 , respectively. Without loss of generality, assume $x_1 \leq x_2$. Let S_1 and S_2 be the SCB in Ψ_1 and Ψ_2 , respectively. The next will prove that S_1 is a prefix of S_2 . In the proof, we will sometimes use set operations over S_1 and S_2 . For example, $S_1 \cap S_2$ refers to the set of common blocks in S_1 and S_2 .

Let $F_1(i)$ be the sequence of partially-confirmed blocks on chain i in Ψ_1 . Let $G_1(i)$ be the prefix of $F_1(i)$ such that $G_1(i)$ contains all blocks in $F_1(i)$ whose rank is smaller than x_1 . Similarly define $F_2(i)$ and $G_2(i)$, where $G_2(i)$ contains those blocks in $F_2(i)$ whose rank is smaller than x_2 . We first prove the following two claims:

- For all i where $0 \le i \le k-1$, $G_1(i)$ is a prefix of $G_2(i)$. To prove this claim, note that by the **consistency** property in Theorem 2, either $F_1(i)$ is a prefix of $F_2(i)$ or $F_2(i)$ is a prefix of $F_1(i)$. If $F_1(i)$ is a prefix of $F_2(i)$, then together with the fact that $x_1 \le x_2$, it is obvious that $G_1(i)$ is a prefix of $G_2(i)$. If $F_2(i)$ is a prefix of $F_1(i)$, let $F_2(i)$ is a prefix of the last block in $F_2(i)$. This also means that the next_rank of that block is at least $F_2(i)$. By our design of confirm_bar, we know that $F_2(i)$ is a prefix of $F_2(i)$ must also be a prefix of $F_2(i)$.
- For all i where $0 \le i \le k-1$ and all block $B \in G_2(i) \setminus$ $G_1(i)$, B's rank must be no smaller than x_1 . We prove this claim via a contradiction and assume that B's rank is smaller than x_1 . Together with the fact that B is in $G_2(i) \setminus G_1(i)$, we know that B is in $F_2(i)$ but not in $F_1(i)$. Hence $F_2(i)$ cannot be a prefix of $F_1(i)$. Then by the **consistency** property in Theorem 2, $F_1(i)$ must be a prefix of $F_2(i)$. Let x_4 be the rank of the last block D in $F_1(i)$. Since $F_1(i)$ is a prefix of $F_2(i)$, both D and B must be in $F_2(i)$, and D must be before B in $F_2(i)$. Since the blocks in $F_2(i)$ must have increasing rank values, we know that D's rank must be smaller than B's. Hence we have $x_4 = D$'s rank < B's rank $\le x_1 - 1$, or more concisely, $x_4 < x_1 - 1$. On the other hand, since D's rank is x_4 , its next_rank must be at least x_4+1 . By our design of confirm_bar in Ψ_1 , we know that $x_1 \leq x_4 + 1$ and hence $x_4 \ge x_1 - 1$. This yields a contradiction.

Now we can use the above two claims to prove that S_1 is a prefix of S_2 . S_1 consists of all the blocks in $G_1(0)$ through $G_1(k-1)$, while S_2 consists of all the blocks in $G_2(0)$ through $G_2(k-1)$. Since $G_1(i)$ is a prefix of $G_2(i)$ for all i, we know that $S_1 \subseteq S_2$. For all blocks in S_1 (which is the same as $S_2 \cap S_1$), the sequence S_1 orders them in exactly the same way as the sequence S_2 . For all block $B \in S_2 \setminus S_1$, by the second claim above, we know that B's rank must be no smaller than

 x_1 . Hence in the sequence S_2 , all blocks in $S_2 \setminus S_1$ must be ordered after all the blocks in $S_2 \cap S_1$ (whose rank must be smaller than x_1). This completes our proof that S_1 is a prefix of S_2 .

Finally, if $u_1 = u_2$ and $t_1 < t_2$, then since confirm_bar on a node never decreases over time, we must have $x_1 \le x_2$. Similarly, if $u_1 \ne u_2$ and $t_1 + \Delta < t_2$, then by time t_2 , node u_2 must have seen all blocks seen by u_1 at time t_1 . By the **consistency** property in Theorem 2, all partially-confirmed blocks on u_1 at time t_1 must also be partially-confirmed on u_2 at time t_2 . Thus we must also have $x_1 \le x_2$. Putting everything together, if $(u_1 = u_2 \text{ and } t_1 < t_2)$ or $(u_1 \ne u_2 \text{ and } t_1 + \Delta < t_2)$, then S_1 must be a prefix of S_2 .

Lemma 7. If the three properties in Theorem 2 hold for each of the k chains in (k, p, λ, T) -OHIE, then with probability at least $1 - exp(-\Omega(\lambda))$, (k, p, λ, T) -OHIE satisfies the quality-growth property in Theorem 1.

Proof. Same as the reasoning in the beginning of the proof for Lemma 6, we first invoke Lemma 5 to show that with probability at least $1 - exp(-\Omega(\lambda))$, there will never be two honest nodes adding $(B_1, \widehat{B_1})$ and $(B_2, \widehat{B_2})$ to their respective local set of blocks, such that $B_1 = B_2$ and $\widehat{B_1} \neq \widehat{B_2}$. All our following discussion will be conditioned on this, and we will be able to directly refer to the rank and next_rank of a block B.

The following restates the **quality-growth** property in Theorem 1, which we need to prove:

• (quality-growth) For all integer $\gamma \geq 1$, the following property holds after the very first $\frac{2T}{pn}$ ticks of the execution: On any honest node, in every $(\gamma+2)\cdot\frac{2T}{pn}+2\Delta$ ticks, at least $\gamma\cdot k\cdot\frac{1-2f}{1-f}T$ honest blocks are newly added to SCB.

Consider any given honest node u, and any given time t_0 (in terms of ticks from the beginning of the execution), where t_0 is after the very first $\frac{2T}{pn}$ ticks of the execution. By the **growth** property in Theorem 2, at time t_0 , the length of every chain in (k, p, λ, T) -OHIE must be at least T. By the **growth** property and the **quality** property in Theorem 2, we know that from time t_0 to time $t_1 = t_0 + \gamma \cdot \frac{2T}{pn}$ on node u, every chain in (k, p, λ, T) -OHIE has at least $\gamma \cdot \frac{1-2f}{1-f}T$ honest blocks becoming newly partially-confirmed. Let α_i denote the set of such newly partially-confirmed honest blocks on chain i (for $0 \le i \le k-1$). We have $|\alpha_i| \ge \gamma \cdot \frac{1-2f}{1-f}T$ for all i, and $|\bigcup_{i=0}^{k-1} \alpha_i| \ge \gamma \cdot k \cdot \frac{1-2f}{1-f}T$. To prove the lemma, it suffices to show that by time $t_4 = t_0 + (\gamma + 2) \cdot \frac{2T}{pn} + 2\Delta$ on node u, all blocks in α_i will have become fully-confirmed for all $0 \le i \le k-1$. Without loss of generality, we only need to prove for α_0 .

Let x be the largest next_rank among all the blocks in α_0 . By definition of such x, the rank value of every block in α_0 must be smaller than x. Let y be the length of chain 0 on node u at time t_1 . By time $t_2 = t_1 + \Delta$, all honest nodes will have received all blocks in α_0 . Furthermore, the length of chain 0 on all honest nodes at time t_2 must be at least y. Together with

¹⁹Here u_1 (t_1) may or may not equal u_2 (t_2).

the **consistency** property in Theorem 2, we know that all the blocks in α_0 must be partially-confirmed on all honest nodes by time t_2 . Thus starting from t_2 , whenever an honest node (including node u) mines a block, by the design of OHIE, the next_rank of the new honest block will be at least x. We will invoke this important property later.

Next let us come back to the honest node u, and consider any one of the chains. From time $t_3=t_2+\Delta$ to time $t_4=t_3+\frac{4T}{pn}$ on node u, by the **growth** property in Theorem 2, the length of this chain must have increased by at least 2T blocks. The first T blocks among all these blocks must have been partially-confirmed on node u at time t_4 . By the **quality** property in Theorem 2, these first T blocks must contain at least $\frac{1-2f}{1-f}T$ honest blocks. For $T\geq \frac{1-f}{1-2f}$, these first T blocks must contain at least one honest block B that is partially-confirmed. Since B is first seen by u no earlier than t_3 , we know that this honest block B must have been generated (either by u or by some other honest node) no earlier than t_2 . By our earlier argument, the next_rank of B must be at least x.

Finally, note that we actually have one such B on every chain on node u at time t_4 . This means that on node u at time t_4 , the confirm_bar is at least x. We earlier showed that for all blocks in α_0 , their rank values must all be smaller than x. Hence such a confirm_bar enables all blocks in α_0 to become fully-confirmed on node u at time t_4 . Observe that $t_4 = t_0 + (\gamma + 2) \cdot \frac{2T}{pn} + 2\Delta$, and we are done.

B. Additional Formalism for the Proof of Lemma 3

This section introduces some additional modelling and formalism, to prepare for the proof of Lemma 3 in Appendix D.

PoW based on a random oracle. In OHIE, for performing and verifying PoW, all the nodes have access to a common random orale H1 (i.e., the hash function, which is viewed as a random oracle). We need to properly model PoW based on such a random oracle, and we use exactly the same approach as in Pass et al.'s analysis on Nakamoto consensus [41]. Specifically, let the function f() be the random function corresponding to H1. The random oracle H1 has two interfaces: For mining a block, an honest node invokes H1.compute(B) to compute f(B) (i.e., Line 19 of Figure 3 will become " \widehat{B} .hash \leftarrow H1.compute(B)"). Such mining is successful iff f(B) has a certain number of leading zeroes. For verifying a received block, an honest node invokes H1.verify(B, y) to check whether f(B) equals y (i.e., Line 42 of Figure 3 will become "verify that $\mathtt{H1.verify}(B, B.\mathtt{hash}) = \mathtt{true}$ "). The adversary may invoke $\operatorname{H1.compute}(B)$ and $\operatorname{H1.verify}(B,y)$ in arbitrary ways — for example, the adversary may choose not to invoke H1.verify(B, y), but instead invoke H1.compute(B)to verify whether f(B) = y. In each tick, an honest node can invoke H1.compute() only once, and the adversary can invoke H1.compute() at most fn times. On the other hand, H1.verify() invocations are "free" and can be invoked any number of times (except that the total number of such invocations is still bounded by length of the entire execution). As explained in [41], separating these two interfaces of H1 helps to capture the assumption that doing a PoW incurs non-trivial cost, while verifying a PoW does not.

Related to the above, and also following the same approach in Pass et al. [41], in OHIE each block B is sent together with its corresponding f(B) value. Specifically, the attachment \widehat{B} contains a field \widehat{B} .hash to store f(B). This is needed so that the receiver can verify B by invoking the "free" interface H1.verify $(B,\widehat{B}.\text{hash})$, instead of invoking H1.compute(B) to recompute f(B).

Random oracle used in Merkle tree. In OHIE, the nodes also use a hash function to construct and verify the Merkle tree. It will be convenient to model this hash function as a separate random oracle H2 that is independent of H1.20 Since a single random oracle can be easily used to implement two independent random oracles, doing so does not make any difference in implementation. The oracle H2 also has two interfaces, H2.compute(x) and H2.verify(x,y). An honest node invokes H2.compute(x) for constructing the Merkle tree (Line 3 and Line 56 of Figure 3), and invokes H2.verify(x, y)for verifying the Merkle tree (Line 43 of Figure 3). The adversary may invoke these two interfaces in arbitrary ways — for example, it may choose to invoke H2.compute(x) for verifying the Merkle proof. All invocations of H2.compute(x)and H2.verify(x,y) are "free". Separating these two interfaces helps to make our discussion less cumbersome — for example, we can refer to "the invocation of the random oracle H2 at Line 43 of Figure 3" simply as "H2.verify()".

Pseudo-code for Nakamoto consensus. To make our reduction in Lemma 3 rigorous, we specify the Nakamoto consensus protocol in Figure 6 — this specification is the same as in Pass et al.'s work [41]. Several aspects of the specification are worth some further clarification.

First, as explained above, we use H1'.compute() and H1'.verify() to model random-oracle-based PoW (Line 16 and Line 22 in Figure 6). To avoid notation collision, we use H1' to refer to the random oracle in Nakamoto consensus.

Second, exactly the same as in Pass et al.'s work [41], in Figure 6, each block B' in Nakamoto consensus is sent together with its hash value, which is stored in the attachment $\widehat{B'}$. (Hence each block B' in Nakamoto consensus now also has an attachment.) As explained earlier, doing so serves to enable the receiver to verify a block by invoking the "free" interface $\mathrm{H1'.verify}()$.

Finally, usually in Nakamoto consensus, a valid block's hash needs to have $\log_2\frac{1}{p}$ leading zeroes. To simplify discussion later, we instead require a valid block's hash to have $\log_2(\frac{1}{kp})$ leading zeroes and $\log_2 k$ trailing zeroes. Since we model hash functions as random oracles and since $\log_2(\frac{1}{kp}) + \log_2 k =$

²⁰Note that the leaves on our Merkle tree are hashes of blocks. The hashes of the blocks are obtained using H1, while the non-leave nodes on the Merkle tree are computed using H2.

```
1: V' \leftarrow \{(\text{genesis block, genesis block's attachment})\};
 2: last' \leftarrow hash of the genesis block;
 3:
 4: Nakamoto() {
 5:
      repeat forever {
             ReceiveState();
 6:
 7:
             Mining();
 8:
             SendState();
 9:
      }
10: }
11:
12: Mining() {
      B'.transactions' \leftarrow get_transactions(); B'.prev' \leftarrow last'; B'.nonce' \leftarrow new_nonce();
13:
15:
      \widehat{B'}.hash' \leftarrow H1'.compute(B');
      if (\widehat{B'}.\mathtt{hash'} has \log_2(\frac{1}{kp}) leading zeroes and \log_2 k
     trailing zeroes) then ProcessBlock(B', \widehat{B'});
18:
19:
20: ProcessBlock(B', \widehat{B'}) {
     verify that \widehat{B}'.hash' has \log_2(\frac{1}{kn}) leading zeroes and
     \log_2 k trailing zeroes;
      verify that \mathtt{H1'}.\mathtt{verify}(B',\widehat{B'}.\mathtt{hash'}) = \mathtt{true};
      verify that B'.prev' = \widehat{A'}.hash' for some block
     (A', A') \in V';
24:
       if (any of above 3 verifications fails) then return;
      V' \leftarrow V' \cup \{(B', \widehat{B'})\};
26:
      last' \leftarrow hash of the last block on the longest path in V';
27:
28: }
29:
30: SendState() {
        send V' to other nodes;
31:
       // In implementation, only need to send those blocks not
33:
34:
35: ReceiveState() {
        foreach (B', \widehat{B'}) \in \text{received state do}
36:
            ProcessBlock(B', B');
37:
38: // Note that a block B'_1 should be processed before B'_2 if
     B_2'.prev' points to B_1'.
39: }
```

Fig. 6: Pseudo-code of Nakamoto consensus.

 $\log_2 \frac{1}{p}$, such a change does not make any material difference for the protocol.

Modelling adversarial message delay. The adversary can manipulate the message delay in arbitrary ways, as long as the delay does not exceed Δ . The adversary also sees every message as soon as it is sent. To model this, we follow exactly the same approach as in [41]: We imagine that honest nodes do not communicate with each other directly. Instead, an honest node always directly sends its message to the adversary (with no delay). The adversary then must deliver this message to all other honest nodes within Δ ticks, without modifying the content of the message.

C. Overview of Proof for Lemma 3

Since the proof for Lemma 3 in Appendix D is long, we first give an overview here. Recall from Appendix B that OHIE uses two random oracles H1 and H2, and they are invoked via interfaces H1.compute(), H1.verify(), H2.compute(), and H2.verify().

The first conceptual step in the proof of Lemma 3 is to clean up various low-probability bad events such as collisions, so that the execution (of OHIE against the given adversary \mathcal{A}) becomes easier to reason about. Specifically, related to H1, there are two bad events:

- The execution contains some invocation $\mathtt{H1.verify}(x,y)$ such that $\mathtt{H1.compute}(x)$ has not been previously invoked and yet $\mathtt{H1.verify}(x,y)$ returns true.
- The execution contains some invocations $\texttt{H1.compute}(x_1)$ and $\texttt{H1.compute}(x_2)$ such that $x_1 \neq x_2$ and $\texttt{H1.compute}(x_1) = \texttt{H1.compute}(x_2)$.

The first step in the proof of Lemma 3 modifies the semantics of $\mathtt{H1.verify}(x,y)$ and $\mathtt{H1.compute}(x)$, so that the above two bad events will never happen. The proof will further modify the semantics of $\mathtt{H2.verify}(x,y)$ and $\mathtt{H2.compute}(x)$ in a similar way. We will prove that all these modifications only change the distribution of the execution by some exponentially small amount.

The next conceptual step is the central part of the proof. Without loss of generality, let us consider i=0 in Lemma 3, and we simply use \mathcal{A}' to denote \mathcal{A}'_0 . For any given adversary \mathcal{A} for (k,p,λ,T) -OHIE, we need to construct another adversary \mathcal{A}' for (p,λ,T) -Nakamoto such that chain 0 in the execution of (k,p,λ,T) -OHIE against \mathcal{A} behaves almost exactly the same as the chain in the execution of (p,λ,T) -Nakamoto against \mathcal{A}' .

To facilitate understanding, all notations for the execution of (p,λ,T) -Nakamoto against \mathcal{A}' will come with the superscript "'", while notations for the execution of (k,p,λ,T) -OHIE will not. Let u' denote any honest node in (p,λ,T) -Nakamoto. We construct \mathcal{A}' in the following way: \mathcal{A}' is composed of a $\mathit{middlebox}$ and the adversary \mathcal{A} —putting it another way, \mathcal{A}' will invoke \mathcal{A} as a black-box. The honest nodes in (p,λ,T) -Nakamoto only interact with the middlebox. The middlebox internally simulates an execution of (k,p,λ,T) -OHIE running against \mathcal{A} , by simulating all the OHIE honest nodes and by invoking \mathcal{A} . We will use u to denote any (simulated) honest node in this simulated execution.

The design of the middlebox is the key. In each tick, to simulate the honest node u (which runs OHIE), the middlebox observes the outgoing message from the corresponding honest node u^{\prime} (which runs Nakamoto):

• If u' sends out a new block B', it means that u' successfully mined B' in this tick. The middlebox then simulates the mining by u. When u invokes $\mathrm{H1.compute}()$, the middlebox directly returns $\widehat{B'}$.hash as the result. Since $\widehat{B'}$.hash is already known to have $\log_2(\frac{1}{kp})$ leading zeroes and $\log_2 k$ trailing zeroes, this will cause u to

successfully mine a block B, and B will eventually be assigned to chain 0. The middle box records the mapping from B' to B in a table, and records the mapping from B to the query result (i.e., $\widehat{B'}$.hash) in another table.

• If u' does not send out any message, it means that u' has failed to mine a block in this tick. The middle-box then simulates the mining by u. When u invokes H1.compute(), the middlebox returns a uniformly random value y, conditioned upon that y does not correspond to successful mining in Nakamoto. (In other words, y should either not have $\log_2(\frac{1}{kp})$ leading zeroes or not have $\log_2 k$ trailing zeroes.) Finally, the middlebox does proper bookkeeping as earlier.

The middle-box also needs to interact with the simulated adversary \mathcal{A} :

- The middle box feeds all messages sent by the simulated honest nodes into A.
- If A want to deliver a message to a simulated honest node u, the middlebox feeds that message into u, and observes whether u add any block B to its chain 0. If yes, the middlebox converts B (which is an OHIE block) to B' (which is a Nakamoto block), and delivers B' to the corresponding u'.
- If \mathcal{A} invokes $\operatorname{H1.compute}(B)$ (i.e., the PoW query), the middlebox converts B (which is an OHIE block) to B' (which is a Nakamoto block). The middlebox then "tunnels" this request to the random oracle $\operatorname{H1}'$ in the Nakamoto execution, by invoking $\operatorname{H1}'.\operatorname{compute}(B')$ and then relaying the response to \mathcal{A} . (This invocation of $\operatorname{H1}'.\operatorname{compute}()$ will be "charged" against \mathcal{A}' , of which the middlebox is a part.) Finally, the middlebox does proper bookkeeping.
- All invocations of H1.verify(), H2.compute(), and H2.verify() by A are answered by the middlebox looking up its internal tables.

Finally, one of the tables maintained by the middlebox will directly give us the mapping σ_0^{τ} as needed by Lemma 3.

D. Proof for Lemma 3

Proof. (for Lemma 3) Without loss of generality, we prove this lemma for i=0. To simplify notation, we use \mathcal{A}' to denote \mathcal{A}'_0 in the lemma. Let H1 and H2 be the two random oracles used in EXEC(OHIE, k, p, λ , T, \mathcal{A}), and H1' be the (single) random oracle used in EXEC(Nakamoto, p, λ , T, \mathcal{A}'). We will design and reason about several hybrid executions. For convenience, define Hybrid1 to be EXEC(OHIE, k, p, λ , T, \mathcal{A}).

Designing Hybrid2. We define Hybrid2 to be the same as Hybrid1, except that in Hybrid2 we change the semantics of all invocations to H1.compute(), H1.verify(), H2.compute(), and H2.verify(). To do so, we maintain three tables T_1 , T_2 , and T_3 . The three tables contain entries in the form of (B, hash), (x, y), and (B, B'), respectively. Let the k genesis blocks of OHIE be $(B_0, \widehat{B_0})$, $(B_1, \widehat{B_1})$, ..., $(B_{k-1}, \widehat{B_{k-1}})$, and let the genesis block of Nakamoto be $(B', \widehat{B'})$. Initially, T_1

contains entries $(B_i, \widehat{B_i}.hash)$ for each $i \in [0, k-1]$, T_2 is empty, and T_3 contains the entry (B_0, B') .

- In Hybrid2, when H2.verify(x,y) is invoked, we return true if $(x,y) \in T_2$, and false otherwise. We define bad1 to be the event that the execution Hybrid1 contains some invocation H2.verify(x,y) such that H2.compute(x) has not been previously invoked and yet H2.verify(x,y) returns true. Given that H2 is a random oracle, and given that the length of the execution Hybrid1 is $poly(\lambda)$, we have $Pr[bad1] = exp(-\Omega(\lambda))$.
- In Hybrid2, when H2.compute(x) is invoked, if there exists some y such that (x, y) ∈ T₂, we directly return y. (By our design, such a y must be unique.) Otherwise we choose a uniformly random binary string y of λ length. We next check whether T₂ contains any entry in the form of (x₂, y) for any x₂. If yes, we re-choose a uniformly random y, until there is no collision. We next insert (x, y) into T₂, and returns y to the invoking party.
 - We define bad2 to be the event that the execution Hybrid1 contains some invocations H2.compute (x_1) and H2.compute (x_2) such that $x_1 \neq x_2$ and H2.compute (x_1) = H2.compute (x_2) . By similar reasoning as above, we have $\Pr[\text{bad2}] = exp(-\Omega(\lambda))$.
- In Hybrid2, when H1.verify(B,y) is invoked, we return true if $(B,y) \in T_1$, and false otherwise. We define bad3 to be the event that the execution Hybrid1 contains some invocation H1.verify(B,y) such that H1.compute(B) has not been previously invoked and yet H1.verify(B,y) returns true. By similar reasoning as above, we have $\Pr[\text{bad3}] = exp(-\Omega(\lambda))$.
- In Hybrid2, when H1.compute(B) is invoked, if there exists some y such that $(B,y) \in T_1$, we directly return y. Otherwise we check whether there is some B' such that $(B,B') \in T_3$. If not, we construct B' based on B (as described next), and add (B,B') into T_3 . We then invoke H1'.compute(B'), and let the return value be y. We add (B,y) to T_1 , and return y as the answer to H1.compute(B).

To construct B'based on B, we first B'.transactions' \leftarrow B.transactions, and set B'.nonce' to be a uniformly randomly chosen nonce. Next we check whether T_2 contains any entry in the form of $(x_0|x_1, B.root)$ for some x_0 and x_1 , each of length λ . (Given that T_2 contains no collisions, x_0 and x_1 must be unique, if they do exist.) If yes, we continue to look for an entry in the form of $(x_{00}|x_{01},x_0)$, for some x_{00} and x_{01} each of length λ . We repeat such a step for $\log_2 k$ times, to obtain $x_0, x_{00}, x_{000}, \ldots$ Let x_{\perp} be the last value in the previous sequence of $\log_2 k$ values, and we set B'.prev $\leftarrow x_{\perp}$. If any of the above $\log_2 k$ steps cannot proceed due to T_2 not containing the needed entry, we set B'.prev $\leftarrow B$.root.

We define bad4 to be the event that in the execution of Hybrid2, the table T_3 contains two entries (B_1, B'_1) and (B_2, B'_2) such that $B_1 = B_2$ or $B'_1 = B'_2$. Since each

B' comes with a new nonce and since the length of the execution is $poly(\lambda)$, we have $Pr[bad4] = exp(-\Omega(\lambda))$.

Execution Hybrid2 is close to Hybrid1. Conditioned upon none of the four bad events (bad1 through bad4) happening, one can verify that the joint distribution of the return values of all the invocations of the four interfaces (i.e., H1.compute(), H1.verify(), H2.compute(), and H2.verify()) is the same in Hybrid1 and Hybrid2. Since by a union bound, the probability of none of the four bad events happening is at least $1-exp(-\Omega(\lambda))$, we conclude that Hybrid1 and Hybrid2 are strongly statistically close.

Designing Hybrid3. We define Hybrid3 to be the same as Hybrid2, except that i) the honest nodes in Hybrid3 run (p, λ, T) -Nakamoto, rather than the protocol in Hybrid2, and ii) the adversary \mathcal{A}' in Hybrid3 is composed of a special middlebox and the adversary A in Hybrid2. The middle-box converts i) incoming messages to A' so that they can be fed into A, and ii) outgoing messages from A so that they can be sent by \mathcal{A}' (to the honest nodes running Nakamoto consensus). Furthermore, all the honest nodes in Hybrid2 will be simulated by the middlebox in Hybrid3. Consider any given honest node u' in Hybrid3, which runs (p, λ, T) -Nakamoto and maintains a single chain. The middlebox internally simulates a counterpart of u', and let us call this simulated counterpart as node u. This (simulated) node u runs the protocol in Hybrid2, and hence maintains k parallel chains. We will next explain how the middlebox exactly works. To avoid notation collision, we will use u' to denote any honest node in Hybrid3, and uto denote any honest node in Hybrid2 and any (simulated) honest node in Hybrid3 as simulated by the middlebox.

In each tick, if node u' broadcasts a new block $(B', \widehat{B'})$, then:

- The middle-box simulates the execution of Mining() by
 u. When u invokes H1.compute(B), the middle-box will
 return B'.hash as the answer to H1.compute(B).
- The middle-box adds $(B, \widehat{B}.\text{hash})$ into the table T_1 , where $(B, \widehat{B}.\text{hash})$ was constructed by u during Mining().
- The middle-box adds (B, B') into the table T_3 .
- The middle-box feeds the message sent by \boldsymbol{u} (if any) into $\boldsymbol{\Lambda}$

In each tick, if node u' does not broadcast a new block, then:

- The middle-box simulates the execution of Mining() by u. When u invokes H1.compute(B), the middle-box returns y as the answer. Here y is chosen as a uniformly random binary string of length λ . But if y has $\log_2(\frac{1}{kp})$ leading zeroes and $\log_2 k$ trailing zeroes, the middle-box will rechoose y uniformly randomly, until y does not have such a property.
- The middle-box adds $(B, \widehat{B}.\text{hash})$ into the table T_1 , where $(B, \widehat{B}.\text{hash})$ was constructed by u during Mining().
- The middlebox next constructs B' based on B, in the same process as when $\mathtt{H1.compute}(B)$ is invoked in

- Hybrid2. The middlebox adds (B, B') into the table T_3 .
- The middle-box feeds the message sent by u (if any) into \mathcal{A} .

Recall that in Hybrid3, \mathcal{A} is being used as a component by \mathcal{A}' . In Hybrid3, all invocations of H1.compute(), H1.verify(), H2.compute(), and H2.verify() by \mathcal{A} are processed in the same way in the Hybrid2. In each tick, if \mathcal{A} (as part of \mathcal{A}') intends to deliver a message to node u, then for each tuple (B,\widehat{B}) in the message:

- The middlebox simulates the execution of $\operatorname{ProcessBlock}(B,\widehat{B})$ by node u.
- If u adds (B,\widehat{B}) to V_0 , the middlebox will find B' such that $(B,B')\in T_3$. The middlebox sets $\widehat{B'}$.hash \leftarrow \widehat{B} .hash, and forwards/delivers the tuple $(B',\widehat{B'})$ to node u'. Note that for this step, there must exists some B' such that $(B,B')\in T_3$. The reason is that B already passed the verification by u, and part of that verification process invoked H1.verify $(B,\widehat{B}.\text{hash})$ with a return value of true. Hence we have $(B,\widehat{B}.\text{hash})\in T_1$. One can easily verify that whenever the execution Hybrid3 adds $(B,\widehat{B}.\text{hash})$ into the table T_1 , some entry (B,B') must be added into the table T_3 .

Properties of Hybrid3. Define bad5 to be the event where in Hybrid3, the table T_3 contains two entries (B_1, B_1') and (B_2, B_2') such that either $B_1 = B_2$ or $B_1' = B_2'$. In Hybrid3, a new entry (B, B') is added to T_3 only when some honest node u invokes H1.compute() or \mathcal{A} invokes H1.compute(). When u invokes H1.compute(), both the nonce in B and the nonce in B' are uniformly random. When \mathcal{A} invokes H1.compute(), a new entry (B, B') is added to T_3 only if T_3 does not already contain an entry for B. In such a case, B' in the new entry (B, B') always has a random nonce. Putting both cases together and since the length of the execution is $poly(\lambda)$, we have $Pr[bad5] = exp(-\Omega(\lambda))$.

It is easy to see that Hybrid3 is just an execution of the (p,λ,T) -Nakamoto protocol against the adversary \mathcal{A}' (which is composed of \mathcal{A} and the middlebox). Furthermore, one can (tediously) verify that conditioned upon neither bad4 nor bad5 happening, the view of \mathcal{A} (i.e., the randomness in \mathcal{A} , all the incoming messages to \mathcal{A} , and all the return values for invocations of H1 and H2 by \mathcal{A}) in Hybrid2 follows the same distribution as the view of \mathcal{A} in Hybrid3. Hence conditioned upon neither bad4 nor bad5 happening, the behavior of \mathcal{A} in Hybrid2 follows the same distribution as the behavior of \mathcal{A} in Hybrid3. In turn, conditioned upon neither bad4 nor bad5 happening, the state of all the honest nodes u in Hybrid2 is identically distributed as the state of all the (simulated) honest nodes u in Hybrid3.

Finally, conditioned upon the event bad5 not happening, the table T_3 in Hybrid3 gives a one-to-one mapping σ_0^τ from each block B' in all the V' on all the nodes u' in Hybrid3 to some block $\sigma_0^\tau(B')$ in all the V_i on all the (simulated) nodes u in Hybrid3. Furthermore, σ_0^τ guarantees that B' is an honest block iff $\sigma_0^\tau(B')$ is an honest block, and also guarantees that B'.prev = $\sigma_0^\tau(B')$.leaf. One can verify that in Hybrid3, a

node u' adds a block B' to its V' iff the (simulated) node u adds the block $\sigma_0^{\tau}(B')$ to V_0 .

The following summarizes what we have by now, conditioned upon neither bad4 nor bad5 happening:

- Hybrid1 = EXEC(OHIE, k, p, λ , T, \mathcal{A}).
- Hybrid1 and Hybrid2 are strongly statistically close.
 (This holds even conditioned upon bad4 not happening.)
- The joint state of all the honest nodes u in Hybrid2 is identically distributed as the joint state of all the simulated honest nodes u in Hybrid3.
- An honest node u' in Hybrid3 adds B' to its V' iff the corresponding simulated honest node u in Hybrid3 adds $\sigma_0^{\tau}(B')$ to its V_0 , and we must further have B'.prev = $\sigma_0^{\tau}(B')$.leaf.
- B' is an honest block iff $\sigma_0^{\tau}(B')$ is an honest block.
- Hybrid3 = EXEC(Nakamoto, p, λ , T, \mathcal{A}').

Combining all the above then completes the proof for the lemma. \Box

E. Security Properties of Conflux

This section presents concrete results showing that in our simulation, as the throughput of Conflux [32] increases, the security properties of Conflux deteriorate. Our simulation results are based on the balance attack, which is related to the one in [5], [40]. We do note, however, that our simulation results are obtained only under some example parameters from the Conflux paper [32]. These results may change under other parameters.

We do not claim novelty or research contribution in this section, since the balance attack on Conflux has been folklore and is not the focus of this paper. Instead, our goal is simply to show, concretely, how the security properties of Conflux deteriorate in our simulation. Related observations on Conflux have also been independently made by Bagaria et al. [5] and Fitzi et al. [15].

Possible fix. A simple and easy fix to strengthen Conflux's security properties against the balance attack is for Conflux to operate closer to the parameter range of the GHOST protocol [45], with relatively "low" throughput. (Such relatively "low" throughput will still be much higher than the throughput of the GHOST protocol.) In fact, our simulation, under the "low" throughput setting, already confirms the effectiveness of this fix.

Another good solution for improving Conflux's security properties against the balance attack is for Conflux to explicitly use multiple chains and to explicitly force the adversary to split its computational power across these multiple chains, as in [15]. Other fixes may also exist, which is however beyond the scope of this paper.

Review of Conflux. Conflux is based on the GHOST protocol [45], while GHOST is an improvement over the Bitcoin protocol. Imagine for now that the Bitcoin protocol disseminates all blocks that have ever been mined, regardless of

whether they are on the longest path. Let Ψ be the set of blocks that any given node have seen by any given point of time. Then all blocks in Ψ will conceptually form a direct tree, where block A has a directed edge to block B if A extends from B. With a slight abuse of notation, we also use Ψ to refer to this tree. In the Bitcoin protocol, to produce the total ordering of confirmed blocks, a node will follow the longest-path rule. Namely, the node will output the longest path (after truncating a certain number of blocks at the end) from the root to some leaf of the tree, where the leaf is chosen such that the path length is maximized.

In GHOST, rather than choosing the longest path, a node instead chooses the path in the following way. The node will start from the root A of Ψ , and then find A's child B such that B's subtree has the most blocks, as compared to the subtrees for other children of A. (Namely, B's subtree is the "heaviest".) The path now contains AB. Next, the node finds B's child C such C's subtree is the heaviest, among the subtrees for all children of B. The path now contains ABC. The process continues until we reach a leaf.

Conflux builds upon GHOST by introducing a mechanism for including all blocks in the tree in the final total ordering of blocks, rather than just the blocks on the chosen path. With such a mechanism, Conflux can now obtain much higher throughput by increasing the block generation rate. For example, in one setting, Conflux generates one 4MB block every 5 seconds, despite that the time needed to propagate a 4MB block to all nodes is around 100 seconds. (In GHOST, such high block generation rate is never helpful, since most blocks would not be on the chosen path and would be wasted.)

Setting for the balance attack. For simplicity, we will consider the following setting. We assume that Conflux proceeds in rounds, where each round corresponds to the time needed to propagate a block to all nodes. In each round, an honest node will try to mine a new block, and if successfully, will propagate the new block to all other nodes. Any blocks propagated will be received by all nodes at the end of the current round. In each round, the adversary will also mine new blocks, based on what it has seen up to the previous round. Once the honest nodes send out their newly mined blocks in a given round, we assume that adversary can observe those blocks, and then potentially send out malicious blocks in response to those. We also assume that the adversary can choose to send a malicious block to some honest nodes but not others. Since an honest node always forwards all blocks it receives, those other honest nodes will still receive those malicious blocks one round later. Conflux uses a deterministic tie-breaking rule for choosing which blocks to extend from, when there are multiple candidate blocks. We follow such a design. (Randomized tiebreaking rules will actually make the attack more effective, since randomized rules will result in honest nodes splitting their computation power.) Finally, Conflux mentions a stale block detection heuristic, which we do not consider since with the large threshold used in Conflux, it is unlikely to affect our results.

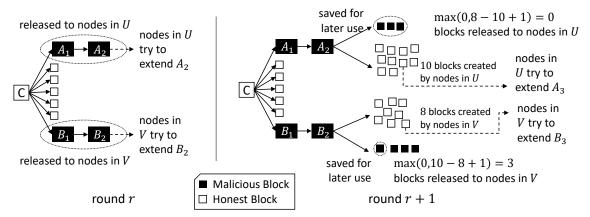


Fig. 7: Adversary's attack strategy in the balance attack.

Intuition behind the balance attack. Before detailing the balance attack, we first provide some intuitions. The balance attack aims to maintain two forks that keep growing. Each fork will have some weight, which is roughly the number of blocks in that fork. Let U be any set consisting of half of all the honest nodes, and let set V contain the remaining honest nodes. Imagine that in round r, nodes in U and nodes in V are working on the two forks, respectively. The adversary wants such property to continue to hold in round r+1. Let x_A and x_B be the number of honest blocks generated by nodes in Uand in V, respectively. The adversary will send y_A malicious blocks (extending the first fork) to nodes in U (but not nodes in V), such that $x_A + y_A > x_B$. Similarly, it sends y_B malicious blocks (extending the second fork) to nodes in V, such that $x_B + y_B > x_A$. Doing so will keep nodes in U and nodes in V continuing working on each of the two forks, respectively, in round r+1. If the adversary do not have enough malicious blocks to follow the above step, the attack stops.

This attack is effective in Conflux but not in GHOST because with Conflux's design, high throughput is achieved by generating many block per round. For example, in one setting, Conflux generates one 4MB block every 5 seconds, while the time needed to propagate a 4MB block is around 100 seconds. This gives about 20 blocks per round. The GHOST protocol never intends to work under such parameters. Because of this particular way of achieving high throughput, in Conflux, the expectations of x_A , x_B , y_A , and y_B will all be much larger than those in GHOST. (The ratio of the expectations of the four quantities remain the same in Conflux as in GHOST.) In particular in GHOST, the expected number of malicious blocks generated by the adversary in one round is below 1, which means that it is often 0. This would render the balance attack ineffective.

Attack strategy. Start from any given round r, the following gives the detailed steps of the balance attack (Figure 7):

1) In round r, let block C be the block that all the nodes are trying to extend from. The adversary generates blocks A_1 extending from C, A_2 extending from A_1 , B_1 extending from C, and B_2 extending from B_1 .

- 2) At the end of round r, the adversary disseminates A_1 and A_2 to all nodes in U, and B_1 and B_2 to all nodes in V.
- 3) In round r+1, all the honest nodes in U (V) will try to extend from A2 (B2). Let xA and xB be the number of (honest) blocks generated by nodes in U and V, respectively. At the same time, the adversary will mine as many malicious blocks as it can, with odd-numbered blocks extending from A2, and even-numbered blocks extending from B2.
- 4) At the end of round r+1, the adversary disseminates total $\max(0,x_B-x_A+1)$ odd-numbered blocks to nodes in U. This will cause nodes in U to see more blocks extending from A_2 than blocks extending from B_2 . Hence in the next round, nodes in U will all extend from some child block A_3 of A_2 . Similarly, the adversary disseminates total $\max(0,x_A-x_B+1)$ even-numbered blocks to nodes in V, so that all nodes in V will later extend from some child block B_3 of B_2 . If the adversary does not have enough blocks to disseminate in this step, the attack stops.
- 5) In round r+2, all the honest nodes in U(V) will try to extend from $A_3(B_3)$. In round r+2 and later, the adversary's strategy is similar as the above two steps, except the following. In any given round r', the adversary may not use up all the blocks it generates. Those blocks can be "saved" for later use. For example, when the adversary needs to disseminate $\max(0, x_B x_A + 1)$ malicious blocks to nodes in U, those malicious blocks do not have to extend from the last block in A_1 's branch they can extend from any block in A_1 's branch.

Results. With the above balance attack, we use simulation to determine how long a fork the adversary can maintain. This would then correspond to the number T of blocks we need to have after a block, in order for the block to get confirmed. As a reference point, Ethereum (which is also based on the GHOST protocol [45]) uses T=15. We consider the top 0.01% of the fork length — this would match the 0.01% "risk tolerance" used in Conflux's evaluation [32]. We vary

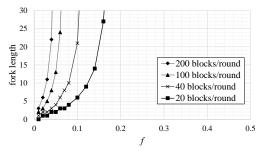


Fig. 8: Fork length in Conflux when under the balance attack.

the expected number of blocks generated per round from 20 to 200. Out of these blocks, on expectation f fraction are malicious blocks. In one of its evaluation settings, Conflux generates one $4\mathrm{MB}$ block every 5 seconds, which would roughly correspond to 20 blocks per round.

Figure 8 presents our simulation results. It shows that with a target fork length of no more than 15, in our simulation Conflux can only tolerate f less than 0.2 when the number of blocks per round is 20. This f deteriorates as the throughput of Conflux (i.e., number of blocks per round) increases. For example, with a target fork length of no more than 15 and with 200 blocks per round, in our simulation Conflux can only tolerate f less than 0.1.