

Circular ConvolutionUsing DFT - IDFT / Frequency Domain→ (Matrix Method) -

Note: Length of two signals should be same.
If not, use Zero padding

Ex:

$$\textcircled{1} \quad x(n) = \{1, 2, 3, 4\} \rightarrow l_1 = 4$$

$$h(n) = \{4, 5, 6\} \rightarrow l_2 = 3$$

$\therefore l_1$ & l_2 are not same.

$$\therefore h(n) = \{4, 5, 6, 0\}$$

⇒ Steps to do: $x(n) \circledast h(n)$

i) DFT of $x(n) \rightarrow X(K)$

ii) DFT of $h(n) \rightarrow H(K)$

iii) $X(K) \cdot H(K) = Y(K)$

iv) IDFT ($Y(K)$) = $y(n)$

① $x(n) = \{2, 3, 4, 5\}$ & $h(n) = \{5, 2, 3, 4\}$

Step 1:

$$X(K) = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 2 \\ 3 \\ 4 \\ 5 \end{bmatrix}$$

$$= \begin{bmatrix} 2 + 3 + 4 + 5 \\ 2 - 3j - 4j + 5j \\ 2 - 3 + 4 - 5 \\ 2 + 3j - 4 - 5j \end{bmatrix} = \begin{bmatrix} 14 \\ -2 + 2j \\ -2 \\ -2 - 2j \end{bmatrix}$$

Step 2:

$$H(K) = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 5 \\ 2 \\ 3 \\ 4 \end{bmatrix}$$

$$= \begin{bmatrix} 5 + 2 + 3 + 4 \\ 5 - 2j - 3 + 4j \\ 5 - 2 + 3 - 4 \\ 5 + 2j - 3 - 4j \end{bmatrix} = \begin{bmatrix} 14 \\ 2 + 2j \\ 2 \\ 2 - 2j \end{bmatrix}$$

Step 3: $X(K) \cdot H(K) = Y(K)$

$$= \{14, -2 + 2j, -2, -2 - 2j\} \cdot \{14, 2 + 2j, 2, 2 - 2j\}$$

$$\therefore Y(K) = \{196, -8, -4, -8\}$$

Step 4: IDFT of $Y(K) \rightarrow y(n)$.

$y(n) =$	1	1	1	1		19.6
	4	1	j	-1	$-j$	-8
		1	-1	1	-1	-4
		1	$-j$	-1	j	-8

$=$	1	19.6 - 8 - 4 - 8
	4	$19.6 - 8j + 4 + 8j$
		$19.6 + 8j - 4 + 8$
		$19.6 + 8j + 4 - 8j$

$=$	1	17.6	$=$	44
	4	20.0		50
		20.8		52
		20.0		50

$$\therefore y(n) = \{44, 50, 52, 50\}$$