Unit - III

Non-Parametric Tests.

$+$ 1^{st}

$-$ 2^{nd}

$0 \leftarrow 20$ $40 \rightarrow 50$

20 40

$25 \leftrightarrow 35$

μ, σ t, z, χ^2, F

1. No parameters are involved.
2. Distribution free test
3. Assumption req^d is that the parent pop^{ln} follows continuous dist^{n}.
4. It uses sample data such as the signs of measurements, order relationships or category frequencies.
5. Stressing or compressing the scale does not alter them (results).

As a result, null distⁿ of the non-parametric test statistic can be determined without parameters of the parent poplⁿ distⁿ.

+ −

Ranks

$\mu, \sigma, \bar{x}, s$

CONTINUOUS.

If the parent poplⁿ distⁿ is not normal, the inferences (conclusions) drawn from these test based on parametric test may be seriously effected. These effects may be more serious if the sample size is small.

$\bar{x}_1, \bar{x}_2$

Difference bet^{n} Parametric and Non-Parametric Tests-

Parametric	Non-Parametric
1. Information about popl^{n} is completely known.	1. Inf. about popl^{n} is unknown.
2. Specific assumptions are made regarding the popl^{n}	2. No assumptions are made regarding the popl^{n}.
3. Null hypothesis is made on parameters of the parent popl^{n}. $\left.\begin{array}{l} H_0: \mu_1 = \mu_2 \\ H_0: \sigma_1 = \sigma_2 \end{array}\right\}$ Parameters of popl^{n}	3. The null hypothesis is free from parameters.

Parametric	Non-parametric
(vi) Tests are applicable only for variable	Tests are applicable for both variable and attributes.
(vii) No parametric test exist for Nominal scale data	Non-parametric test do exist for nominal or ordinal scale data
(viii) Parametric Test is powerful, if it exist	It is not so powerful like parametric test.
(ix) Test available for testing the interaction in analysis of variance model	No non-parametric test is available for testing the interaction in ANOVA model.

(1) **Sign Test :**

To test the null hypothesis that the median of a distⁿ is equal to some value. It can be used

(a) in place of one sample t-test

(b) " " " paired " "

(c) for ordered categorical data where a numerical scale is inappropriate but where it is possible to rank the obs.

The obs. in a sample of size n are $x_1, x_2, \ldots x_n$. The null hypothesis is that the popl^{n} median is equal to some value M_0, i.e. $H_0: M = M_0$ against $H_1: M \neq M_0$.

H_1 may be one tailed or two-tailed, i.e. $H_1 : \underline{M > M_0}$ or $H_1 : \underline{M < M_0}$

one tailed

or H_1 $M \neq M_0$

↓

2 tailed

Suppose that $S+$ of the obs. are greater than M_0 and $S-$ are smaller than M_0. Values of x which are exactly equal to M_0 are ignored; the sum of $S+$ and $S-$ may therefore be less than n. We may denote this sum by n_0

$M_0 : 7$

$n = 6$

$n = 5$

$2 - 5$ ✓

$5 - 2$ ✓

$7 = 0 \rightarrow$ ignore

$4 - 3$ ✓

$6 - 1$ ✓

$1 - 6$ ✓

Under the null hypothesis, we would expect half of x's to be above the median and half below. Therefore, under the null hypothesis both $S+$ and $S-$ follow a binomial dist^{n} with

$p = \frac{1}{2}$ and $n = n_0$

Test procedure:

Ex: Following data represents marks scored by students. (Marks out of 60).

54	32	41	22	31	46	43	44	39	35	21	52	21
55	23	48	28	27	51	36	27	40	38	35	48	

Past experience shows that 50% of students scored marks 45 or above. Use sign test to decide whether this group is inferior to the previous group.

Sol: Here, $H_0 : M = 45$ and $H_1 : M < 45$

Wilcoxon Signed Rank Test:

Distribution free test → Non-parametric Test.

used to test the null hypo that the median of a distⁿ is equal to some value. It can be used (a) in place of one sample t-test (b) in place of paired t-test or (c) for ordered categorical data where a numerical scale is inappropriate but where it is possible to rank the obs.

Note: If the number of observations/pairs is such that $\frac{n(n+1)}{2}$ is large enough, i.e. $\left(\frac{n(n+1)}{2} > 20\right)$ a normal approximation can be used with

$\mu = \frac{n(n+1)}{4}$ and $\sigma^2 = \frac{n(n+1)(2n+1)}{24}$

$n > 20$

$\frac{n(n+1)}{2} > 20$

$$Z = \frac{W - \mu}{\sigma} \sim N(\mu, \sigma^2)$$

Ex: Let $n = 14$, $W+ =$ Sum of ranks with +ve sign $= 64.5$

$W- =$ " " " , -ve sign $= 40.5$

$W = \min(W+, W-) = \min(64.5, 40.5) = 40.5$

$d = 21$ with $\alpha = 0.05$

cal < tab

$W > d$, we reject H_0.

Ex: $W+ = 71$, $W- = 7$, $n = 12$. Use Wilcoxon's method to find if there is any diff. in the drugs given to 12 patients with median 1.65 hours with 2 tied ranks.

Sol:- No: of pairs $= \frac{n(n+1)}{2} = \frac{\cancel{12}^{6} \times 13}{\cancel{2}} = 78 > 20$

So we have to use Normal approximation.

$\mu = \frac{n(n+1)}{4} = \frac{\cancel{12}^{3}(13)}{\cancel{4}} = 39$, $\sigma^2 = \frac{n(n+1)(2n+1)}{24} = \frac{\cancel{12}(13)(25)}{\cancel{24}\,2}$

$= 162.5$

Since there are 2 tied ranks, $t = 2$.

$\therefore \frac{t^3 - t}{48} = \frac{2^3 - 2}{48} = 0.125$

$\therefore$ Variance $\sigma^2 = 162.5 - 0.125 = 162.375$

$\sigma = \sqrt{162.375} = 12.7426$

$\therefore z = \dfrac{x - \mu}{\sigma} = \dfrac{71 - 39}{12.7426} = \dfrac{32}{12.7426}$ 6

$= 2.51$

Cal $z = 2.51$

tab z at 5 % l.o.s $= 1.645$

$\therefore$ Cal $z >$ tab z, we reject H_0.

Run Test :

Used for examining whether or not a set of observations constitutes a random sample from an infinite popl^{n}.

Test for randomness is of major importance because the assumption of randomness underlies statistical inference.

Departure from randomness can take many forms.

H_0 : Sample values come from a random sequence

H_1 : " " " " " non- " "

	H_1	H_0
	1	2
	2	7
	3	9
2	4	1
4		13
6		11
~~7~~ 8		8

Each observation is denoted by '+' sign if it is more than the previous number and denoted by '–' sign if it is less than the previous number. Total no: of runs up (+) and runs down (–) is counted. Too few runs indicate that the sequence is not random (less persistency) and too many runs also indicate that the sequence is not random (zigzag).

Critical value: Critical value for the test is obtained from the table for a given value of n at a desired level of significance (α). Let this value be 'r_c'.

Decision Rule: If r_c (lower) $\leq r_c \leq r_c$ (upper), accept H_0. Otherwise reject H_0.

Tied values: If an observation is equal to its preceding observation denote it by zero. While counting the # of runs ignore it and reduce the value of n accordingly.

$n=9$
$n=7$

~~31~~ 25	+	25	−
12	−	36	+
12	0	17	−
12	0	17	0
21	+	12	−
27	+	18	+
30	+	18	0
12	−	37	+

r_c(upper) = 3
r_c(lower) = 2

$n=6$

$n=8$

Large Sample Sizes: When sample size > 25, the critical value r_c can be obtained using a normal dist^n approximation.

Test statistic is Z

$$Z = \frac{r - \mu}{\sigma} \quad \text{where } \mu = \frac{2n-1}{3}, \; \sigma = \sqrt{\frac{16n-29}{90}}$$

By comparing cal Z with tab Z we can make decision about H_0.

Decision criterion: if $|Z| \geq Z_{\alpha/2}$, we reject H_0.

Ex: Given $n = 11$, $r_c(\text{upper}) = 8$, $r_c(\text{lower}) = 3$, $r = 7$.
What will be the decision made on H_0?

Solⁿ: Since $r_c(\text{lower}) \leq r \leq r_c(\text{upper})$

we accept H_0.

Ex: $n = 40$, $\alpha = 0.05$, runs $= 26 = r$. Give your decision on H_0.

Solⁿ $n > 25$, we use Z test.

$$Z = \frac{r - \mu}{\sigma}, \quad \mu = \frac{2n+1}{3} = \frac{2 \times 40 + 1}{3} = \frac{81}{3} = \underline{27}$$

$$\sigma = \sqrt{\frac{16n - 29}{90}} = \sqrt{\frac{16 \times 40 - 29}{90}} = \sqrt{\frac{620 - 29}{90}} = \sqrt{\frac{591}{90}}$$

$$= \sqrt{6.5667}$$

$$= 2.5626$$

$$\therefore Z = \frac{40 - 27}{2.5626} = \frac{13}{2.5626} = \underline{\underline{5.07}}$$

$\text{tab } Z_{0.025} = 1.96$. $\therefore$ cal $Z >$ tab Z, we reject H_0.

Wald - Wolfowitz Run Test :

Two sample run test $\rightarrow$ used to examine whether two random sample come from popl$^{\underline{n}}$ having same dist$^{\underline{n}}$. This test can detect differences in averages or spread or any other important aspect bet$^{\underline{n}}$ the two popl$^{\underline{n}}$s. This test is efficient when each sample size is moderately large (≥ 10)

H_0 : Two samples come from the popl$^{\underline{n}}$s having the same dist$^{\underline{n}}$

H_1 : " " " " " " " different dist$^{\underline{n}}$

<u>Test statistic</u>: Let r denote the no: of runs. To obtain r, list the (n+m) observations from two samples in order of magnitude

X	Y	X	Y
12	5		45
18	7	~~36~~	~~29~~
19	11	~~41~~	~~18~~
35	18	~~12~~	~~7~~
36	29	~~18~~	~~11~~
41	34	~~35~~	~~5~~
46	45	~~19~~	~~34~~
		46	

<u>Test statistic</u>: Let r denote the no: of runs. To obtain r, list the (n+m) observations from two samples in order of magnitude. Denote observations from one sample by x's and the other by Y. Count the no: of runs.

<u>Critical value</u>: Difference in location results in few runs and diffence in spread also result in few no: of runs. Consequently, the region for this test is always one-sided. The critical value to decide whether or not the no: of runs is few is obtained from the table. The table value gives critical value

'r_c' for n (size of sample 1) and m (size of sample 2) at 5% level of significance.

<u>Decision Rule:</u> If $r \leq r_c$, reject H_0

Tie: In case x and y observations have same value, place the obs. x(y) first if the run of x(y) obs in continuing.

Large Sample Size: For sample size larger than 20, we can use Z statistic as given below

$(n+m) > 20$

$n \to x$ $\quad y \to m$

$$Z = \frac{r - \mu}{\sigma} \sim N(0,1)$$

where $\mu = 1 + \frac{2nm}{n+m}$, $\sigma = \sqrt{\frac{2nm(2nm-n-m)}{(n+m)^2(n+m-1)}}$

$\} 20$

$n+m$
$= 25+5$
$= 25 > 20$

Ex: Given: $n = 21$, $m = 28$, r = no: of runs = 33, give your decision on H_0.

Sol^{n}: Since $n = 21$, $m = 28$ $\Rightarrow n + m = 21 + 28 = 49$ observations.

$\Rightarrow n + m > 20$

So we will use Z test.

Under H_0, $r \sim N[\mu, \sigma^2]$ where

$$\mu = 1 + \frac{2nm}{n+m} = 1 + \frac{2 \times 21 \times 28}{21 + 28} = 1 + \frac{1176}{49} = 1 + 24 = 25$$

$$\sigma^2 = \frac{2nm(2nm - n - m)}{(n+m)^2(n+m-1)} = \frac{2 \times 21 \times 28(2 \times 21 \times 28 - 21 - 28)}{(21+28)^2(21+28-1)} = 11.5$$

$$Z = \frac{33 - 25}{\sqrt{11.5}} = \frac{8}{3.3912} = \underline{\underline{2.36}}$$

$Z_{0.05} = 1.96$

$Z_{cal} > Z_{0.05}$

we reject H_0 at 5% l.o.s.

Kruskal - Wallis Test : (1952)

Used when the assumptions of ANOVA are not met. They both asses for significant differences on a continuous dependent variable by a grouping independent variable (with three or more groups). In the ANOVA, we assume that the dist^{n} of each group is normally distributed and there is approximately equal variance on the scores of each group. However, in Kruskal-Wallis Test, we do not have any of these assumptions. Like all non-parametric tests, this test is also not as powerful as ANOVA.

(2) The cases of each group are independent.
(3) The measurement scale should be least ordinal.

Procedure:

(1) Arrange the data in ascending order.
(2) Assign rank to them in ascending order. In case of a repeated value, or a tie, assign ranks to them by averaging their ranks.
(3) Then sum up the different ranks, eg: $R_1, R_2 \ldots R_i$ for each of the different groups.

(iv) To calculate the value, apply the following formula:

$$* \quad H = \left[\frac{12}{N(N+1)} \sum_{i=1}^{c} \frac{R_i^2}{n_i}\right] - 3(N+1) \quad \text{where } N = \sum_{i=1}^{c} n_i \qquad (4)$$

n_i = no: of observations in the i^{th} group.

<u>Correction for ties:</u>

$$* \quad CF = 1 - \frac{\sum t_i(t_i^2 - 1)}{N^3 - N}$$

where t_i = no: of tied values within group i.

$$H_c = \frac{H}{CF}$$

where H is calculated by using (4)

Ex: Given: $N = 21$, $c = 3$, $R_1 = 131$, $N_1 = 8$; $R_2 = 58$, $N_2 = 7$; $R_3 = 42$, $N_3 = 6$. Decide whether to accept or reject the null hypo H_0 where H_0: there is no significant difference among the three groups.

Sol^n: Here, $N = 21$, $c = 3$.

$$\therefore H = \left[\frac{12}{N(N+1)} \sum \frac{R_i^2}{N_i}\right] - 3(N+1)$$

$$= \frac{\cancel{12}\,6}{21 \times \cancel{22}\,11}\left[\frac{(131)^2}{8} + \frac{(58)^2}{7} + \frac{(42)^2}{6}\right] - 3 \times 22$$

$$= \frac{6}{21 \times 11}\left[2145.125 + 480.571 + 294\right] - 66$$

$$= \frac{6 \times 2919.696}{231} - 66 = \frac{17518.176}{231} - 66 = 75.836 - 66$$

$$= 9.84$$

In ANOVA, we were interested in testing for k popl^n means; if we reject the null hypothesis, then it implies that all the k popl^n means are not equal. But we cannot say that all are significantly different pair wise. There is possibility that out of k popl^n means, two may not show significant difference. Hence, it is essential to pinpoint which pairs differ. This post-hoc test will identify the pairs of means (from at least three) differ. There are no: of post hoc tests.

(1) Fisher's least significant difference (LSD)

(2) Duncan's multiple range test (DMRT)

(1) Fisher's Least Significant Difference (LSD):

Fisher $\to$ 1935

most commonly used test after ANOVA.

$$LSD = t_{\underset{\sim}{N-K},\alpha/2}^{\nu}\sqrt{MSE\left(\frac{1}{n_i}+\frac{1}{n_j}\right)}$$

If $n_i = n_j$, for all $i = 1, 2 \ldots K$, then

$$LSD = t_{N-K,\alpha/2}\sqrt{\frac{2MSE}{n}}$$

Where MSE = mean error sum of squares.

ν = degrees of freedom corresponding to error SS

= $N-K$

n_i = no: of obs. corresponding to treatment A.

n_j = " " " " " B

α = L.O.S.