# Circular Convolution
## Using DFT – IDFT / frequency Domain

→ (Matrix Method) –

<u>Note:</u> Length of two signals should be same. If not, use zero padding

Ex: ①

$x(n) = \{1, 2, 3, 4\} \rightarrow l_1 = 4$

$h(n) = \{4, 5, 6\} \rightarrow l_2 = 3$

$\because l_1$ & $l_2$ are not same

$\therefore h(n) = \{4, 5, 6, \underline{0}\}$

⇒ steps to do: $x(n) \circledast h(n)$

i) DFT of $x(n) \rightarrow X(K)$

ii) DFT of $h(n) \rightarrow H(K)$

iii) $X(K) \cdot H(K) = Y(K)$

iv) $\text{IDFT}\left(Y(K)\right) = y(n)$.

(1) $x(n) = \{2, 3, 4, 5\}$ & $h(n) = \{5, 2, 3, 4\}$

Step 1:

$$X(K) = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 2 \\ 3 \\ 4 \\ 5 \end{bmatrix}$$

$$= \begin{bmatrix} 2+3+4+5 \\ 2-3j-4+5j \\ 2-3+4-5 \\ 2+3j-4-5j \end{bmatrix} = \begin{bmatrix} 14 \\ -2+2j \\ -2 \\ -2-2j \end{bmatrix}$$

Step 2:

$$H(K) = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 5 \\ 2 \\ 3 \\ 4 \end{bmatrix}$$

$$= \begin{bmatrix} 5+2+3+4 \\ 5-2j-3+4j \\ 5-2+3-4 \\ 5+2j-3-4j \end{bmatrix} = \begin{bmatrix} 14 \\ 2+2j \\ 2 \\ 2-2j \end{bmatrix}$$

Step 3: $X(K) \cdot H(K) = Y(K)$.

$= \{14, -2+2j, -2, -2-2j\} \cdot \{14, 2+2j, 2, 2-2j\}$

$\therefore Y(K) = \{196, -8, -4, -8\}$

Step 4: IDFT of $Y(K) \rightarrow y(n)$.

$$y(n) = \frac{1}{4}\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & j & -1 & -j \\ 1 & -1 & 1 & -1 \\ 1 & -j & -1 & j \end{bmatrix}\begin{bmatrix} 196 \\ -8 \\ -4 \\ -8 \end{bmatrix}$$

$$= \frac{1}{4}\begin{bmatrix} 196 - 8 - 4 - 8 \\ 196 - \cancel{8j} + 4 + \cancel{8j} \\ 196 + 8 - 4 + 8 \\ 196 + \cancel{8j} + 4 - \cancel{8j} \end{bmatrix}$$

$$= \frac{1}{4}\begin{bmatrix} 176 \\ 200 \\ 208 \\ 200 \end{bmatrix} = \begin{bmatrix} 44 \\ 50 \\ 52 \\ 50 \end{bmatrix}$$

$$\therefore \boxed{y(n) = \{44, 50, 52, 50\}}$$