Kayvan_Shah_Assignment3_solution

February 16, 2023

1 Assignment 3: Bias & Variance (60 points)

1.1 Due Thursday, Feburary 16, 2023 at 6PM

Turning the .ipynb notebook, and a viewable version of the notebook, such as html or pdf.

This assignment aims to replicate the bias/variances figures 4.5 and 4.6 in section 4.7 of Alpaydin, deepening the understanding of bias and variance.

1.2 Question one (20 points)

Replicate figures 4.5 in section 4.7 of Alpyadin 4th edition

[1]:
```
import random
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
```

The ground thruth function for the regression is f(x) = 3 cos(2x)

Generate 100 sample datasets with f(x) + Gaussian white noise (N(0,1)). Each dataset will have 20 points randomly selected x from [0,5] with corresponding target points.

[2]:
```
# Ground truth target function
def f(x):
    return 3 * np.cos(1.3 * x)

# seed
np.random.seed(62)
# x
x = np.random.uniform(0.0, 5.0, [100, 20])
x = np.sort(x)

# Ground truth targets
g = f(x)
# Add white noise
noisy = np.random.normal(0, 1, [100, 20])
# y
y = g + noisy
```

```python
# use linspace(0,5,100) as test set to plot the images
x_test = np.linspace(0,5,100)
```

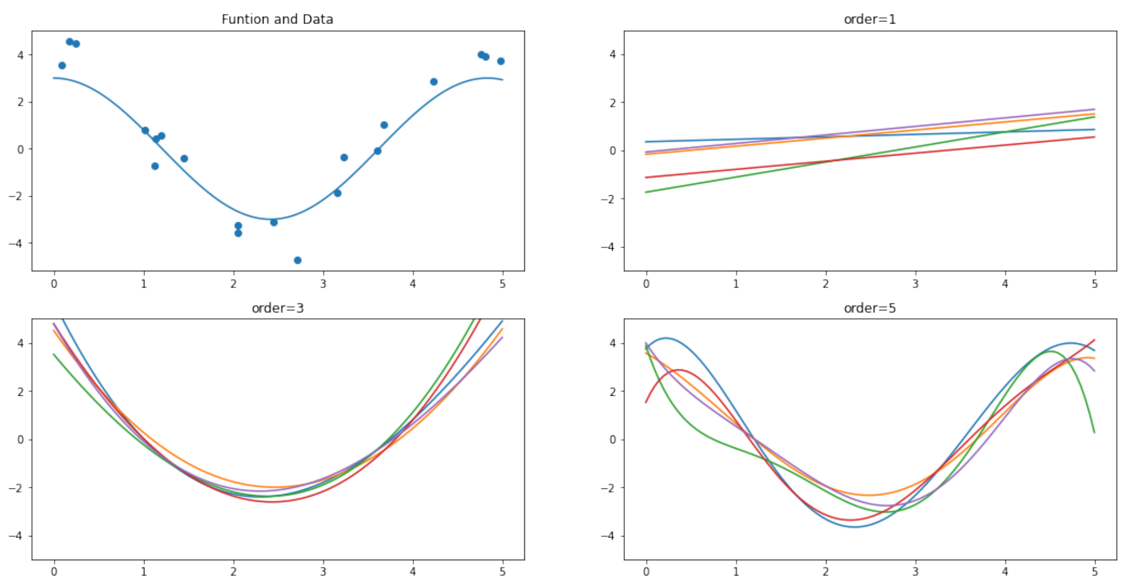
TODO: Use the First 5 datasets to generate 4 plots. - Figure one: Function f(x) = 3 cos(2x) and one noisy dataset sampled from the function, namely "Function, and data". - Figure two: Generate five polynomial fits of degree ONE based on the first five datasets and name this figure with "Order 1" - Figure three: Generate five polynomial fits of degree THREE based on the first five datasets and name this figure with "Order 3" - Figure four: Generate five polynomial fits of degree FIVE based on the first five datasets and name this figure with "Order 5" - For figures two, three, and four, please add a dotted line as an average line for the five fits.

Please use x_test to plot all the model functions, not just the ground truth function. This will make all the higher polynomial models look smoother.

Hint: You can use the Sklearn's PolynomialFeatures and LinearRegression. - https://scikit-learn.org/stable/modules/linear_model.html#polynomial-regression-extending-linear-models-with-basis-functions

[3]:
```python
# model
from sklearn.preprocessing import PolynomialFeatures
from sklearn.linear_model import LinearRegression
from sklearn.pipeline import Pipeline

def linear_model_predict(X, Y, order):
    # fit one polynomial model of degree `order`
    ## Insert your code BEGIN
    model = Pipeline(
        [
            ('poly', PolynomialFeatures(degree=order)),
            ('linear', LinearRegression(fit_intercept=False))
        ]
    )
    model = model.fit(X.reshape(-1,1), Y)
    ## Insert your code END
    return model

def plot_figure(x, y, x_test, order):
    # plot five curves corresponding to the polynomial of degree `order`
    # plot the average of these five curves
    ## Insert your code BEGIN
    for i in range(5):
        model = linear_model_predict(x[i], y[i], order)
        plt.plot(x_test, model.predict(x_test.reshape(-1,1)))
        plt.title(f"order={order}")
    ## Insert your code END
```

```python
# show the plots
fig, axs = plt.subplots(2, 2, figsize=(18, 9))


# figure one
plt.subplot(2, 2, 1)
## Insert your code BEGIN
plt.scatter(x[0],y[0])
plt.plot(x_test, f(x_test))
plt.title("Funtion and Data")
## Insert your code END

# figure two
plt.subplot(2, 2, 2)
plt.ylim(-5, 5)
plot_figure(x, y, x_test, order=1)

# figure three
plt.subplot(2, 2, 3)
plt.ylim(-5, 5)
plot_figure(x, y, x_test, order=3)

# figure four
plt.subplot(2, 2, 4)
plt.ylim(-5, 5)
plot_figure(x, y, x_test, order=5)
```



1.3 Question 2 (40 points)

TODO: Generate Figure 4.6 from Alpaydin 4th Edition

The x-axis is the order of polynomial model, from 1 to 5. the y-axis is the error. The plot should contain three curves: total error, bias error and variance error.

Use all 100 dataset to compute the total error, bias error and variance error functions by using total error equation (4.36): $Ex[(E[r|x] - g(x))^2|x] = (E[r|x]) - E_X(g(x))^2 + E_X[(g(x) - E_X[g(x)])^2]$

Evaluate each of the three error functions with 10 equally spaced values starting from 0 and ending at 5, i.e. np.linspace(0, 5, 10)

TODO: For each of the five polynomial models, print the average predictions, $E_X[g(x)]$**, at np.linspace(0, 5, 10)**

Hint: Average prediction at point x means computing the average value of the predictions of 100 models generated by 100 datasets. The point x should range from np.linspace(0, 5, 10)

TODO: Generate and print a DataFrame with 5 rows, one for each order and 4 columns. The 4 columns are: * Order * Bias error * Variance error * Total error

Hint: Average prediction at point x means computing the average value of the predictions of 100 models generated by 100 datasets. The point x should range from np.linspace(0, 5, 10)

Hint: For bias error $(E[r|x]) - E_X(g(x))^2$, $E[r|x] = f(x)$ **and** $E_X[g(x)]$ **is the average over 100 models from the 100 datasets. Then, you can approximate bias error by average over x in np.linspace(0, 5, 10) of** $(E[r|x] - E_X[g(x)])2$**.**

Hint: For For variance error, you need to have a nested loops (for each dataset and for x in np.linespace(0, 5, 10)) to get the average variance error.

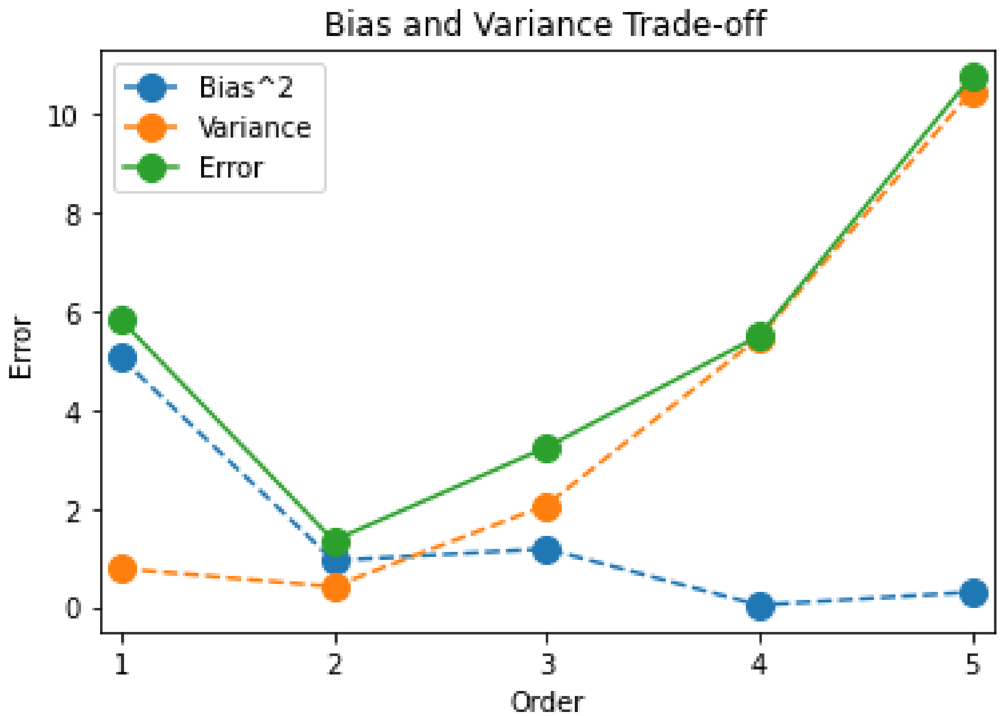
Hint: The total error is the sum of bias error and variance error.

[4]:
```python
## Insert your code BEGIN
# Define any variables or methods that you need here

## Insert your code END

def bias_error(avg_pred, x_eval):
    # For each polynomial order, computes its bias error
    # returns a list of length 5
    five_bias = []
    ## Insert your code BEGIN
    for order in avg_pred:
        diff = np.square(np.array(order) - f(x_eval)).mean()
        five_bias.append(diff)
    ## Insert your code END
    return five_bias
```

```python
def variance_error(avg_pred, models_evals):
    # For each polynomial order, computes its variance error
    # returns a list of length 5
    five_variance = []
    ## Insert your code BEGIN
    five_variance = np.var(models_evals, axis=1).mean(1)
    ## Insert your code END
    return five_variance


# Fit 5 * 100 models, i.e. fit 100 models for each degree in range(1, 6).
# The shape of models_list is (5, 100)
models_list = []
## Insert your code BEGIN
for order in range(1,6):
    model_order = []
    for i in range(100):
        model = linear_model_predict(x[i], y[i], order)
        model_order.append(model)
    models_list.append(model_order)
print("Shape of `models_list` = ", np.shape(models_list))
## Insert your code END


# create evaluation x data
x_eval = np.linspace(0, 5, 10)

# Evaluate each of the 5 * 100 models on `x_eval`
# The shape of models_evals_list is (5,100,10) which is 5 degree with 100␣
↪models and each model predict the 10 x evaluation
models_evals_list = []
## Insert your code BEGIN
for order in models_list:
    order_eval = []
    for model in order:
        preds = model.predict(x_eval.reshape(-1,1))
        order_eval.append(preds)
    models_evals_list.append(order_eval)
print("Shape of `models_evals_list` = ", np.shape(models_evals_list))
## Insert your code END


# For each degree compute the average predictiona at `x_eval`
# The shape `ave_preds_list` is (5,10)
avg_preds_list = []
## Insert your code BEGIN
avg_preds_list = np.array(models_evals_list).mean(1)
```

```python
print("Shape of `avg_preds_list` = ", np.shape(avg_preds_list))
## Insert your code END


bias_lst = bias_error(avg_preds_list, x_eval)

variance_lst = variance_error(avg_preds_list, models_evals_list)

total_error = [x + y for x, y in zip(bias_lst, variance_lst)]

# show the plot
x_points = [1,2,3,4,5]

plt.plot(x_points, bias_lst, linestyle='dashed',label = "Bias^2", marker='o',␣
 ↪markersize=10)
plt.plot(x_points, variance_lst, linestyle='dashed', label = "Variance",␣
 ↪marker='o', markersize=10)
plt.plot(x_points, total_error, linestyle='solid', label = "Error", marker='o',␣
 ↪markersize=10)
plt.legend()
plt.xlim(0.9, 5.1)
plt.xticks(np.linspace(1, 5, 5))
plt.xlabel("Order")
plt.ylabel("Error")
plt.title("Bias and Variance Trade-off")
# Display graph
plt.show()
```

```
Shape of `models_list` =  (5, 100, 2)
Shape of `models_evals_list` =  (5, 100, 10)
Shape of `avg_preds_list` =  (5, 10)
```



[5]:
```
# Error DataFrame
pd.set_option("display.precision", 3)
error_df = pd.DataFrame({
    'Order': range(1,6),
    'Bias Error': bias_lst,
    'Variance Error': variance_lst,
    'Total Error': total_error
})
error_df
```

[5]:

	Order	Bias Error	Variance Error	Total Error
0	1	5.058	0.771	5.829
1	2	0.935	0.407	1.343
2	3	1.173	2.060	3.233
3	4	0.033	5.462	5.495
4	5	0.296	10.464	10.760

[6]:
```
# Average predictions
pd.set_option("display.precision", 3)
pd.DataFrame(avg_preds_list)
```

[6]:

	0	1	2	3	4	5	6	7	8	9
0	-0.326	-0.263	-0.199	-0.136	-0.073	-0.009	0.054	0.118	0.181	0.244

1	4.459	1.772	-0.225	-1.533	-2.150	-2.078	-1.315	0.137	2.279	5.111
2	5.177	2.027	-0.236	-1.655	-2.271	-2.126	-1.261	0.280	2.457	5.227
3	2.983	2.550	0.504	-1.631	-2.857	-2.706	-1.240	0.951	2.748	2.502
4	1.324	2.121	0.442	-1.638	-2.860	-2.690	-1.219	0.924	2.673	2.612