HW Assignment-3

Question 1

Probability distribution of a Gaussian distribution

$$\mathrm{P(x|}\mu,\mathrm{V)} = \frac{1}{\sqrt{2\pi V}} e^{-\frac{(x-\mu)^2}{2V}}$$

The probability values of the Gaussian Distribution over X

$$\mathrm{P(X|}\mu,\mathrm{V)} = \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi V}} e^{-\frac{(x_i-\mu)^2}{2V}}$$

Taking the log-likelihood (£)

$$£ = \log \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi V}} e^{-\frac{(x_i-\mu)^2}{2V}} = \sum_{i=1}^{N} \log\left(\frac{1}{\sqrt{2\pi V}} e^{-\frac{(x_i-\mu)^2}{2V}}\right) = \sum_{i=1}^{N}\left(\log\frac{1}{\sqrt{2\pi V}} + \log e^{-\frac{(x_i-\mu)^2}{2V}}\right)$$

$$£ = \sum\left(\log 1 - \log\sqrt{2\pi V} + \log e\,.-\frac{(x_i-\mu)^2}{2V}\right) = \sum\left(\frac{1}{2}.\log 2\pi V - \frac{(x_i-\mu)^2}{2V}\right)$$

$$£ = -\frac{N}{2}.\log 2\pi V - \frac{1}{2V}\sum (x_i-\mu)^2$$

To find the MLE of log-likelihood we maximize the function by taking partial derivative of μ and V and set it to zero.

MLE of μ

$$\frac{\partial £}{\partial \mu} = 0 = -\frac{1}{2V}\sum \frac{\partial}{\partial \mu}(x_i-\mu)^2 = -\frac{1}{2V}\sum 2(x_i-\mu).-1$$

$$\sum_{i=1}^{N} x_i - \sum_{i=1}^{N} \mu = \sum_{i=1}^{N} x_i - N\mu = 0$$

$$\sum_{i=1}^{N} x_i = N\mu$$

$$\boldsymbol{\mu = \frac{1}{N}\sum_{i=1}^{N} x_i}$$

MLE of V

$$\frac{\partial £}{\partial V} = 0 = -\frac{N}{2}.\frac{1}{2\pi V}.2\pi + \frac{1}{2V^2}\sum (x_i-\mu)^2 = -\frac{N}{2V} + \frac{1}{2V^2}\sum (x_i-\mu)^2$$

$$\frac{N}{2V} = \frac{1}{2V^2}\sum (x_i-\mu)^2$$

$$\boldsymbol{V = \frac{1}{N}\sum (x_i-\mu)^2}$$

Question 2

Method 1

Use Kit Test\Disease	Yes	No	
Positive	90	792	882
Negative	10	9108	9118
	100	9900	10000

$$P(disease \mid +ve) = \frac{90}{882} = 0.102 = 10.2\%$$

Method 2

$$P(disease) = 0.01$$

$$P(+ve \mid disease) = 0.9$$

$$P(+ve \mid no\ disease) = 0.08$$

$$P(disease \mid +ve) = \frac{P(+ve \mid disease) * P(disease)}{P(+ve)} = \frac{0.9 * 0.01}{0.0882} = 0.102 = 10.2\%$$

$$P(+ve) = P(+ve \mid disease) * P(disease) + P(+ve \mid no\ disease) * P(no\ disease)$$

$$P(+ve) = 0.9 * 0.01 + 0.08 * 0.99 = 0.0882$$