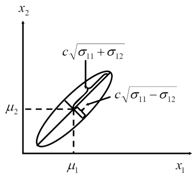
Contents

0.0.1 Multivariate Normal Distribution

$N_p(\boldsymbol{\mu}, \boldsymbol{\Sigma}) = \frac{1}{(2\pi)^{p/2}|\boldsymbol{\Sigma}|^{1/2}}\, e^{\frac{-(\boldsymbol{X}-\boldsymbol{\mu})^T\boldsymbol{\Sigma}^{-1}(\boldsymbol{X}-\boldsymbol{\mu})}{2}}$

0.0.2 Contour of Distance



$c^2 = (\boldsymbol{X}-\boldsymbol{\mu})^T_{1\times p}\boldsymbol{\Sigma}^{-1}(\boldsymbol{X}-\boldsymbol{\mu})_{p\times 1}$
$= \frac{1}{\lambda_1}y_1^2 + \frac{1}{\lambda_2}y_2^2 + \cdots + \frac{1}{\lambda_p}y_p^2$
($p = 2$ in this case) $\lambda_1 = \sigma_{11} + \sigma_{12}$, $\lambda_2 = \sigma_{11} - \sigma_{12}$ are the eigenvalues of $\boldsymbol{\Sigma} = \begin{pmatrix}\sigma_{11} & \sigma_{12}\\ \sigma_{21} & \sigma_{22}\end{pmatrix}$

0.0.3 Linear Combinations of Variables

Let $\boldsymbol{Z} = \boldsymbol{CX}$, then
$\boldsymbol{\mu_z} = E(\boldsymbol{Z}) = E(\boldsymbol{CX}) = \boldsymbol{C\mu_x}$,
$\boldsymbol{\Sigma_z} = Cov(\boldsymbol{Z}) = Cov(\boldsymbol{CX}) = \boldsymbol{C\Sigma_x C^T}$

0.0.4 Assessing Normality

1. **Univariate**: Q-Q plot
2. **Bivariate**: Scatter plot
3. **Multivarite**: Chi-square plot

0.0.4.1 Q-Q Plot

Plot $q_{(j)}$(Z score, or standard normal quantile) against $x_{(j)}$.
$P[Z \le q_{(j)}] = p_{(j)} = \frac{j-0.5}{n}$, where $x_{(1)} \le x_{(2)} \le \cdots \le x_{(j)} \le \cdots \le x_{(n)}$.

0.0.4.2 Chi-square Plot

Plot $d^2_{(j)}$ against $q_{c,\,p}(\frac{j-0.5}{n})$, where p is the degrees of freedom of the chi-square quantile and is also the dimension of the multivariate normal N_p.
$d^2_{(j)} = (\boldsymbol{x}_j - \bar{\boldsymbol{x}})^T\boldsymbol{S}^{-1}(\boldsymbol{x}_j - \bar{\boldsymbol{x}})$

0.0.5 Correlation between principle component Y_i and variable X_k

Parameter
$\rho_{Y_i,\,X_k} = \frac{e_{ik}\sqrt{\lambda_i}}{\sqrt{\sigma_{kk}}}$

Estimator

$r_{\hat{Y}_i,\,\hat{X}_k} = \frac{\hat{e}_{ik}\sqrt{\hat{\lambda}_i}}{\sqrt{s_{kk}}}$

$, i, k = 1, 2, \cdots, p$

$$
\begin{aligned}
y &= (a-x)(b+x)(c-x)(d+x) \\
&= [ab + (a-b)x + x^2][cb + (c-d)x + x^2] \\
&= 0
\end{aligned} \tag{1}
$$