# 多變量分析 HW3

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## Q1: 9.12

$$\mathbf{S_n} = \frac{23}{24}\mathbf{S} = \begin{pmatrix} 0.01061 & 0.00768 & 0.00782 \\ 0.00768 & 0.00615 & 0.00575 \\ 0.00782 & 0.00575 & 0.00649 \end{pmatrix}$$

### (a) Specific Variances

$\mathbf{S_n} \approx \hat{\mathbf{L}}\hat{\mathbf{L}}^T + \hat{\mathbf{\Psi}}$, where $diag(\mathbf{S_n}) = diag(\hat{\mathbf{L}}\hat{\mathbf{L}}^T) + diag(\hat{\mathbf{\Psi}})$
Hence, $\hat{\mathbf{\Psi}} = diag(\hat{\mathbf{\Psi}}) = diag(\mathbf{S_n} - \hat{\mathbf{L}}\hat{\mathbf{L}}^T)$

$$\hat{\mathbf{\Psi}} = \begin{pmatrix} 0.000166 & 0.000000 & 0.000000 \\ 0.000000 & 0.000495 & 0.000000 \\ 0.000000 & 0.000000 & 0.000639 \end{pmatrix}$$

### (b) Communalities

$\sigma_{ii} = \ell_{i1}^2 + \ell_{i2}^2 + \ ... \ + \ell_{im}^2 + \psi_i$
$h_i^2 = \ell_{i1}^2 + \ell_{i2}^2 + \ ... \ + \ell_{im}^2 = 0.022$

### (c) Proportion of variance explained by the factor

$\frac{s_{11}+s_{22}+ \ ... \ +s_{pp}}{h_i^2} = 0.9441$

### (d) Residual Matrix

$$\mathbf{S_n} - \hat{\mathbf{L}}\hat{\mathbf{L}}^T - \hat{\mathbf{\Psi}} = \begin{pmatrix} 0.000000 & -0.000166 & -0.000164 \\ -0.000495 & 0.000000 & -0.000493 \\ -0.000637 & -0.000637 & 0.000000 \end{pmatrix}$$

## Q2: 9.32

#### (a) S PC

| | RC1 | RC2 |
|---|---|---|
| SalePr | ** 563.912 ** | ** 264.834 ** |
| YrHgt | 0.233 | 1.227 |
| FtFrBody | -30.645 | 87.494 |
| PrctFFB | -1.316 | 1.935 |
| Frame | 0.181 | 0.658 |
| BkFat | 0.03 | -0.006 |
| SaleHt | 0.142 | 1.536 |

#### (b) S ML

```
              ML1        ML2
SalePr   563.89339 264.871486
YrHgt      0.23324   1.226832
```

```
FtFrBody -30.64816  87.487307
PrctFFB   -1.31595   1.935205
Frame      0.18131   0.657605
BkFat      0.02998  -0.005524
SaleHt     0.14229   1.535597
```

**(c) R PC**

```
             RC1      RC2
SalePr    0.5766  0.70502
YrHgt     0.9437 -0.11889
FtFrBody  0.7271 -0.35380
PrctFFB   0.5410 -0.69161
Frame     0.9385 -0.03266
BkFat    -0.2015  0.76513
SaleHt    0.9121 -0.12960
```

**(d) R ML**

```
              ML1     ML2
SalePr    0.63298 -0.1934
YrHgt     0.84706  0.4986
FtFrBody  0.34237  0.6982
PrctFFB   0.07614  0.9255
Frame     0.84784  0.4424
BkFat    -0.06745 -0.5144
SaleHt    0.70427  0.5173
```

**(e) Compare**

**(f) scatter plots of factor2 vs factor1 in (a) and (c)**

## Q3

## no principle component method