多變量分析: 作業一

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4.1

(a)

$$f(x_1, x_2) = \frac{1}{1.2\sqrt{2}\pi} e^{-\frac{{x_1}^2+1.6x_1x_2+2{x_2}^2-(4.8\sqrt{2}+2)x_1-(1.6\sqrt{2}+12)x_2+4.8\sqrt{2}+19}{1.44}}$$

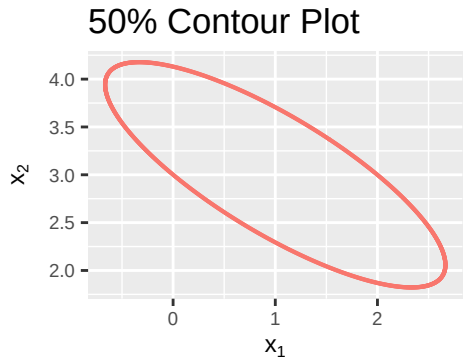
(b)

$$\frac{{x_1}^2+1.6x_1x_2+2{x_2}^2-(4.8\sqrt{2}+2)x_1-(1.6\sqrt{2}+12)x_2+4.8\sqrt{2}+19}{0.36}$$

(c) Contour Plot

Plot $Distance^2 < \chi_2^2(.50) = 1.39$:

$f(\mathbf{x}) = \frac{1}{2\pi det(\Sigma)^{(1/2)}} e^{-\chi_2^2(.50)/2}$



4.6

Ans: (a)(c)(d)

- (a), (c): $\because\ Cov(X_1, X_2) = Cov(X_2, X_3) = 0$
- (d): $\because\ both\ X_1\ and\ X_3\ are\ independent\ to\ X_2$

4.19

(a)

$By\ lecture\ 4\ in\ class,\ it\ follows\ \chi_6^2$

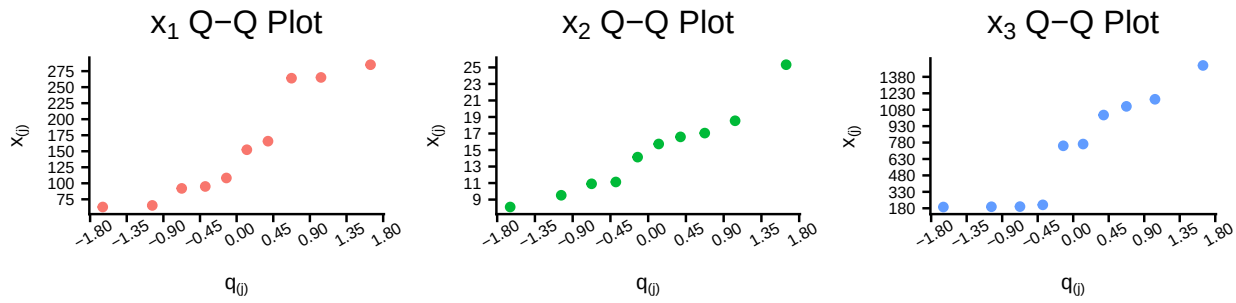
(b)

$By\ CLT,\ \overline{X} \xrightarrow{d} N_6(\mu, \frac{1}{20}\Sigma),\ and\ \sqrt{n}(\overline{X} - \mu) \xrightarrow{d} N_6(0, \Sigma)$

(c)

$By\ textbook,\ 19S\ follows\ Wishart\ distribution\ with\ 19\ d.f.$

4.24 (a)



The distribution of x_1 seems to deviate from a normal distribution a lot, since the curve of the Q-Q plot isn't straight and looks much more like an S-shaped curve.

The distribution of x_2 is much closer to normal. If the most upper-right point is removed, the curve from the Q-Q plot would resemble a straight line.

4.26

(a): Equations

$$\mathbf{S} = \begin{pmatrix} s_{11} & s_{12} \\ s_{12} & s_{22} \end{pmatrix} = \begin{pmatrix} 10.62 & -17.71 \\ -17.71 & 30.85 \end{pmatrix}, \bar{\mathbf{x}} = \begin{pmatrix} 5.20 \\ 12.48 \end{pmatrix}$$

$$distance^2 = (\mathbf{x}_j - \bar{\mathbf{x}})^T \mathbf{S}^{-1} (\mathbf{x}_j - \bar{\mathbf{x}})$$
$$= (0.7538828748926251\,(x_{j2} - 12.841) + 1.25693419723757\,(x_{j1} - 5.2))\,(x_{j2} - 12.841)$$
$$+ (x_{j1} - 5.2)\,(1.25693419723757\,(x_{j2} - 12.841) + 2.189804123676667\,(x_{j1} - 5.2))$$

(b)

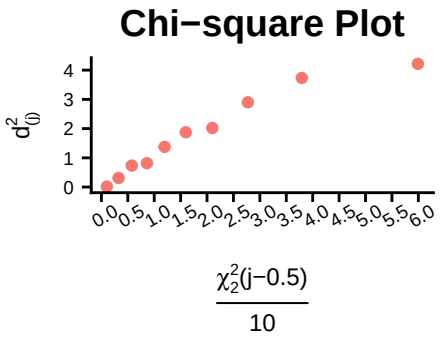
- $distance^2 = (1.88, 2.02, 2.9, 0.74, 0.31, 0.02, 3.73, 0.82, 1.38, 4.22)$

Observations **within estimated 50% probability contour** is equivalent to observations with $distance^2 < \chi_2^2(.50) = 1.39$.

Exactly **50%** of observations fall within the estimated 50% probability contour, i.e. 50% of observations have $distance^2 < \chi_2^2(.50) = 1.39$.

(c): Chi-square plot

- **Ordered distance²**: 0.02, 0.31, 0.74, 0.82, 1.38, 1.88, 2.02, 2.9, 3.73, 4.22



Plotting Source Code

R Code: https://liao961120.github.io/notes/HW.html