Computational Fluid Dynamics

Midterm-Endterm Project

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1. Formulation of the problem:

Find numerical solution of two-dimensional compressible Navier-Stokes equation:

1. Continuity equation

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) = 0$$

2. Momentum equation

$$\begin{cases} \dfrac{\partial \rho u}{\partial t} + \dfrac{\partial}{\partial x}(\rho u^2 + P + \tau_{xx}) + \dfrac{\partial}{\partial y}\left(\rho uv - \tau_{yx}\right) = 0 \\ \dfrac{\partial \rho v}{\partial t} + \dfrac{\partial}{\partial x}\left(\rho uv - \tau_{xy}\right) + \dfrac{\partial}{\partial y}\left(\rho v^2 + P + \tau_{yy}\right) = 0 \end{cases}$$

3. Energy equation

$$\frac{\partial}{\partial t}(E_t) + \frac{\partial}{\partial x}\left((E_t + P)u + q_x - u\tau_{xx} - v\tau_{xy}\right) + \frac{\partial}{\partial y}\left((E_t + P)v + q_y - u\tau_{yx} - v\tau_{yy}\right) = 0$$

4. Ideal gas law

$$P = \rho R T$$

Where,

$$E_t = \rho\left(e + \frac{|V|^2}{2}\right) \quad V = \sqrt{u^2 + v^2} \quad e = C_v\, T$$

$$\tau_{xx} = \lambda(\nabla V) + 2\mu\frac{\partial u}{\partial y}$$

$$\tau_{yy} = \lambda(\nabla V) + 2\mu\frac{\partial v}{\partial y}$$

$$\tau_{xy} = \mu\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)$$

$$\tau_{yx} = \mu\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)$$

$$\lambda = -\frac{2}{3}\mu \text{ - Second viscosity}$$

$$q_x = -k\frac{\partial T}{\partial x}$$

$$q_y = -k\frac{\partial T}{\partial y}$$

$$k = \frac{\mu * \gamma * C_v}{Pr}$$

$$C_v = \frac{R}{\gamma - 1} - specific\ heat\ at\ constant\ volume$$

with the following boundary conditions:

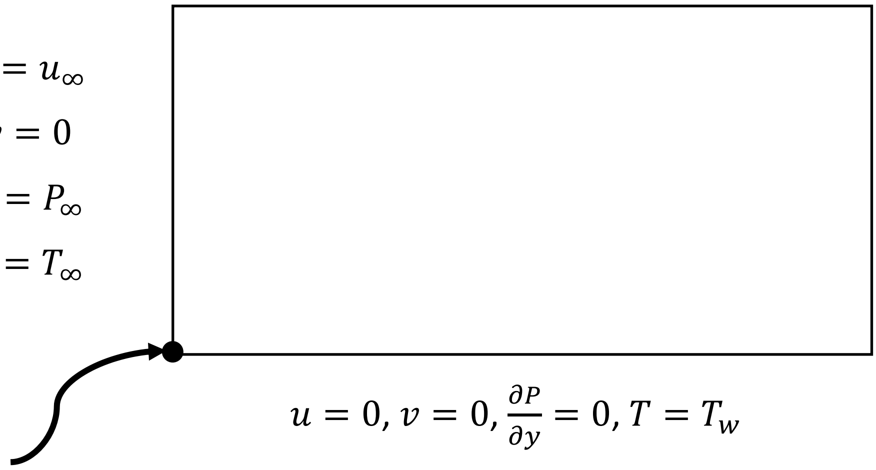
$$u = u_\infty, v = 0, P = P_\infty, T = T_\infty$$

$u = u_\infty$

$v = 0$

$P = P_\infty$

$T = T_\infty$



$$\frac{\partial u}{\partial x} = 0$$

$$\frac{\partial v}{\partial x} = 0$$

$$\frac{\partial P}{\partial x} = 0$$

$$\frac{\partial T}{\partial x} = 0$$

$$u = 0, v = 0, \frac{\partial P}{\partial y} = 0, T = T_w$$

$$u = 0, v = 0, P = P_\infty, T = T_w$$

And initial values:

$M = 4.0 - Mach\ number$

$L_D = 0.00001\ m - (on\ axis\ x)$

$L_H = 5L_D\ m - \ (on\ axis\ y)$

$a = 340.28\ m/s - freestrean\ speed\ of\ sound$

$\rho_\infty = 101325.0\ N/m^2 - freestrean\ pressure$

$T_\infty = 288.16\ K - freestrean\ Tempreture$

$\frac{T_w}{T_\infty} = 1$

$\gamma = 1.4$

$\mu = 1.7894 * 10^{-5} \frac{kg}{m} * s - \ Dynamic\ Viscosity$

$T_o = 288.16\ K - Initial\ Tempreture$

$R = 287\ J/(kg * K) - Gas\ constant\ for\ perfect\ gas$

Pr = 0.71 - Prandtl number for perfect gas

Necessary formulas:

$$M = \frac{u}{a}$$

$$\mu = \mu_0 \left(\frac{T}{T_0}\right)^{\frac{3}{2}} * \frac{T_0 + 110}{T + 110} - \text{Sutherland's Law}$$

2. Analytical Solution:

-

3. Numerical Solution:

Continuity equation

$$\frac{\partial \rho}{\partial t}+\frac{\partial}{\partial x}(\rho u)+\frac{\partial}{\partial y}(\rho v)=0$$

Momentum equation

$$\begin{cases}\dfrac{\partial \rho u}{\partial t}+\dfrac{\partial}{\partial x}(\rho u^2+P+\tau_{xx})+\dfrac{\partial}{\partial y}(\rho uv-\ \tau_{yx})=0\\ \dfrac{\partial \rho v}{\partial t}+\dfrac{\partial}{\partial x}(\rho uv-\tau_{xy})+\dfrac{\partial}{\partial y}(\rho v^2+P+\tau_{yy})=0\end{cases}$$

Energy equation

$$\frac{\partial}{\partial t}(E_t)+\frac{\partial}{\partial x}\left((E_t+P)u+q_x-u\tau_{xx}-v\tau_{xy}\right)+\quad \frac{\partial}{\partial y}\left((E_t+P)v+q_y-u\tau_{yx}-v\tau_{yy}\right)=0$$

Ideal gas law

$$P=\rho RT$$

First, for convenience we need to combine the equations into a compact vector form:

$$\frac{\partial U}{\partial t}+\frac{\partial E}{\partial x}+\frac{\partial F}{\partial y}=0$$

Where:

$$U=\begin{pmatrix}U_1\\U_2\\U_3\\U_5\end{pmatrix}=\begin{pmatrix}\rho\\ \rho u\\ \rho v\\ E_t\end{pmatrix}$$

$$E=\begin{pmatrix}E_1\\E_2\\E_3\\E_5\end{pmatrix}=\begin{pmatrix}\rho u\\ \rho u^2+P-\tau_{xx}\\ \rho uv-\tau_{xy}\\ (E_t+P)u+q_x-u\tau_{xx}-v\tau_{xy}\end{pmatrix}$$

$$F=\begin{pmatrix}F_1\\F_2\\F_3\\F_5\end{pmatrix}=\begin{pmatrix}\rho v\\ \rho uv-\ \tau_{yx}\\ \rho v^2+P+\tau_{yy}\\ (E_t+P)v+q_y-u\tau_{yx}-v\tau_{yy}\end{pmatrix}$$

The first row of the vector corresponds to the continuity equation. Likewise, the second, third rows are the momentum equations, while the fourth row is the energy equation.

Now, we are moving from a continuous medium to a discrete one to use the numerical method:

$$\frac{U_{i,j}^{n+1}-U_{i,j}^{n}}{\Delta t}+\frac{E_{i+1,j}^{n}-E_{i,j}^{n}}{\Delta x}+\frac{F_{i,j+1}^{n}-F_{i,j}^{n}}{\Delta y}=0$$

$$U_{i,j}^{n+1}=U_{i,j}^{n}+\Delta t\left(-\frac{E_{i+1,j}^{n}-E_{i,j}^{n}}{\Delta x}-\frac{F_{i,j+1}^{n}-F_{i,j}^{n}}{\Delta y}\right)$$

For solving the compressible two-dimensional Navier-Stokes equation we are going to use the MacCormack Explicit Method.

MacCormack Method

The basic algorithm is predictor-corrector method.

1. Predictor step

$$\bar{U}_{i,j}^{n+1} = U_{i,j}^{n} + \Delta t\left(-\frac{E_{i+1,j}^{n} - E_{i,j}^{n}}{\Delta x} - \frac{F_{i,j+1}^{n} - F_{i,j}^{n}}{\Delta y}\right)$$

2. Corrector step

$$U_{i,j}^{n+1} = \frac{1}{2}\left(U_{i,j}^{n} + \bar{U}_{i,j}^{n+1} - \frac{\Delta t}{\Delta x}\left(\bar{E}_{i,j}^{n+1} - \bar{E}_{i-1,j}^{n+1}\right) - \frac{\Delta t}{\Delta y}\left(\bar{F}_{i,j}^{n+1} - \bar{F}_{i,j-1}^{n+1}\right)\right)$$

4. Code:

Initializing data:

```
import numpy as np
import matplotlib.pyplot as plt
m = 4  # Mach's number
L_d = 0.00001
L_h = L_d
a = 340.28  # speed of sound
P_free = 101325  # freestream pressure
T_free = 288.16  # freestream temperature
T_w = 288.16  # since T_w/T_free = 1
gamma = 1.4
mu_0 = 1.7894 * 10 ** (-5)  # viscousity
T_0 = 288.16 # initial temperature
R = 287 # Gas const
Pr = 0.71  # Prandtl number for perfect gas

u_free = m * a
c_v = R / (gamma - 1)
ro_free = P_free / (R * T_free)
dx = L_d / 25
dy = L_h / 25
x_list = np.arange(0, L_d + dx, dx)
y_list = np.arange(0, L_h + dy, dy)
dt = 1.5*10 ** (-10)
n = 0
u = [np.ones((len(y_list), len(x_list))) * u_free]
v = [np.zeros((len(y_list), len(x_list)))]
P = [np.ones((len(y_list), len(x_list))) * P_free]
T = [np.ones((len(y_list), len(x_list))) * T_free]
ro = [np.ones((len(y_list), len(x_list))) * ro_free]
mu = [np.zeros((len(y_list), len(x_list)))]
k = [np.zeros((len(y_list), len(x_list)))]
```

Boundary function and applying it:

```
def boundary(u, v, P, T, ro):
    # left boundary condition
    for j in range(len(y_list)):
        u[j][0] = u_free
        v[j][0] = 0
        P[j][0] = P_free
        T[j][0] = T_free
        ro[j][0] = ro_free
    # upper boundary condition
    for i in range(len(x_list)):
        u[-1][i] = u_free
        v[-1][i] = 0
        P[-1][i] = P_free
        T[-1][i] = T_free
        ro[-1][i] = ro_free
    # lower boundary condition
    for i in range(len(x_list)):
        u[0][i] = 0
        v[0][i] = 0
        P[0][i] = 2 * P[1][i] - P[2][i]
        T[0][i] = T_w
        ro[0][i] = P[0][i] / (R * T[0][i])
    # right boundary condition
    for j in range(len(y_list)):
        u[j][-1] = 2 * u[j][-2] - u[j][-3]
        v[j][-1] = 2 * v[j][-2] - v[j][-3]
        P[j][-1] = 2 * P[j][-2] - P[j][-3]
        T[j][-1] = 2 * T[j][-2] - T[j][-3]
        ro[j][-1] = P[j][-1] / (R * T[j][-1])
    # boundary for first point
    u[0][0] = 0
    v[0][0] = 0
    P[0][0] = P_free
    T[0][0] = T_w
    ro[0][0] = ro_free
    return (u, v, P, T, ro)

u[n], v[n], P[n], T[n], ro[n] = boundary(u[n], v[n], P[n], T[n], ro[n])
```

Functions for calculating thermal conductivity and viscosity:

```
def change_mu(T):
    global m_0, T_0
    return mu_0 * ((T / T_0) ** (3 / 2)) * ((T_0 + 110) / (T + 110))  # Sutherland's law for perfect gas


def change_k(mu, gamma, c_v, Pr):
    return (mu * (gamma * c_v)) / Pr

for j in range(len(y_list)):
    for i in range(len(x_list)):
        mu[n][j][i] = change_mu(T[n][j][i])

for j in range(len(y_list)):
    for i in range(len(x_list)):
        k[n][j][i] = change_k(mu[n][j][i], gamma, c_v, Pr)
```

Function for τ_{xx}:

```
def TAU_XX(u, v, mu, case):
    dx_list = np.zeros((len(y_list), len(x_list)))
    dy_list = np.zeros((len(y_list), len(x_list)))

    if case == 'predict_e':
        for j in range(len(y_list)):
            for i in range(len(x_list)):
                if i == 0:
                    dx_list[j][i] = (u[j][i + 1] - u[j][i]) / dx
                else:
                    dx_list[j][i] = (u[j][i - 1] - u[j][i]) / dx

    else:
        for j in range(len(y_list)):
            for i in range(len(x_list)):
                if i == len(x_list) - 1:
                    dx_list[j][i] = (u[j][i] - u[j][i - 1]) / dx
                else:
                    dx_list[j][i] = (u[j][i + 1] - u[j][i]) / dx

    for j in range(len(y_list)):
        for i in range(len(x_list)):
            if j == 0:
                dy_list[j][i] = (v[j + 1][i] - v[j][i]) / dy
            elif j == len(y_list) - 1:
                dy_list[j][i] = (v[j][i] - v[j - 1][i]) / dy
            else:
                dy_list[j][i] = (v[j + 1][i] - v[j - 1][i]) / (2 * dy)

    return -(2 / 3) * mu * (dx_list + dy_list) + 2 * mu * (dx_list)
```

Note: For more stability in the equations, we use a different approximation scheme (forward, backward, central) depending on the situation.

Function for τ_{yy}:

```
def TAU_YY(u, v, mu, case):
    dx_list = np.zeros((len(y_list), len(x_list)))
    dy_list = np.zeros((len(y_list), len(x_list)))

    if case == 'predict_f':
        for j in range(len(y_list)):
            for i in range(len(x_list)):
                if j == 0:
                    dy_list[j][i] = (v[j + 1][i] - v[j][i]) / dy
                dy_list[j][i] = (v[j][i] - v[j - 1][i]) / dy
    else:
        for j in range(len(y_list)):
            for i in range(len(x_list)):
                if j == len(y_list) - 1:
                    dy_list[j][i] = (v[j][i] - v[j - 1][i]) / dy
                else:
                    dy_list[j][i] = (v[j + 1][i] - v[j][i]) / dy

    for j in range(len(y_list)):
        for i in range(len(x_list)):
            if i == 0:
                dx_list[j][i] = (u[j][i + 1] - u[j][i]) / dx
            elif i == len(x_list) - 1:
                dx_list[j][i] = (u[j][i] - u[j][i - 1]) / dx
            else:
                dx_list[j][i] = (u[j][i + 1] - u[j][i - 1]) / (2 * dx)

    return -(2 / 3) * mu * (dx_list + dy_list) + 2 * mu * (dy_list)
```

Functions for τ_{xy}:

```
def TAU_XY(u, v, mu, case):
    dx_list = np.zeros((len(y_list), len(x_list)))
    dy_list = np.zeros((len(y_list), len(x_list)))

    if case == 'predict_e' or case == 'correct_e':

        for j in range(len(y_list)):
            for i in range(len(x_list)):
                if j == 0:
                    dy_list[j][i] = (u[j + 1][i] - u[j][i]) / (dy)
                elif j == len(y_list) - 1:
                    dy_list[j][i] = (u[j][i] - u[j - 1][i]) / (dy)
                else:
                    dy_list[j][i] = (u[j + 1][i] - u[j - 1][i]) / (2 * dy)

        if case == 'predict_e':
            for j in range(len(y_list)):
                for i in range(len(x_list)):
                    if i == 0:
                        dx_list[j][i] = (v[j][i + 1] - v[j][i]) / dx
                    else:
                        dx_list[j][i] = (v[j][i] - v[j][i - 1]) / dx
        else:
            for j in range(len(y_list)):
                for i in range(len(x_list)):
                    if i == len(x_list) - 1:
                        dx_list[j][i] = (v[j][i] - v[j][i - 1]) / dx
                    else:
                        dx_list[j][i] = (v[j][i + 1] - v[j][i]) / dx

    elif case == 'predict_f' or case == 'correct_f':
        for j in range(len(y_list)):
            for i in range(len(x_list)):
                if i == 0:
                    dx_list[j][i] = (v[j][i + 1] - v[j][i]) / (dx)
                elif i == len(x_list) - 1:
                    dx_list[j][i] = (v[j][i - 1] - v[j][i]) / (dx)
                else:
                    dx_list[j][i] = (v[j][i + 1] - v[j][i - 1]) / (2 * dx)

        if case == 'predict_f':
            for j in range(len(y_list)):
                for i in range(len(x_list)):
                    if j == 0:
                        dy_list[j][i] = (u[j + 1][i] - u[j][i]) / (dy)
                    else:
                        dy_list[j][i] = (u[j][i] - u[j - 1][i]) / (dy)
        else:
            for j in range(len(y_list)):
                for i in range(len(x_list)):
                    if j == len(y_list) - 1:
                        dy_list[j][i] = (u[j][i] - u[j - 1][i]) / (dy)
                    else:
                        dy_list[j][i] = (u[j + 1][i] - u[j][i]) / (dy)

    return mu * (dx_list + dy_list)
```

Functions for q_x, q_y:

```
def Q_X(k, T, case):
    dT_list = np.zeros((len(y_list), len(x_list)))
    if case == 'predict_e':
        for j in range(len(y_list)):
            for i in range(len(x_list)):
                if i == 0:
                    dT_list[j][i] = (dT_list[j][i + 1] - dT_list[j][i]) / dx
                else:
                    dT_list[j][i] = (dT_list[j][i] - dT_list[j][i - 1]) / dx
    else:
        for j in range(len(y_list)):
            for i in range(len(x_list)):
                if i == len(x_list) - 1:
                    dT_list[j][i] = (dT_list[j][i] - dT_list[j][i - 1]) / dx
                else:
                    dT_list[j][i] = (dT_list[j][i + 1] - dT_list[j][i]) / dx

    return -k * dT_list


def Q_Y(k, T, case):
    dT_list = np.zeros((len(y_list), len(x_list)))
    if case == 'predict_f':
        for j in range(len(y_list)):
            for i in range(len(x_list)):
                if j == 0:
                    dT_list[j][i] = (dT_list[j + 1][i] - dT_list[j][i]) / dy
                else:
                    dT_list[j][i] = (dT_list[j][i] - dT_list[j - 1][i]) / dy
    else:
        for j in range(len(y_list)):
            for i in range(len(x_list)):
                if j == len(y_list) - 1:
                    dT_list[j][i] = (dT_list[j][i] - dT_list[j - 1][i]) / dy
                else:
                    dT_list[j][i] = (dT_list[j + 1][i] - dT_list[j][i]) / dy

    return -k * dT_list
```

```
U1_ = np.zeros((len(y_list), len(x_list)))
U2_ = np.zeros((len(y_list), len(x_list)))
U3_ = np.zeros((len(y_list), len(x_list)))
U5_ = np.zeros((len(y_list), len(x_list)))
ro_ = np.zeros((len(y_list), len(x_list)))
u_ = np.zeros((len(y_list), len(x_list)))
v_ = np.zeros((len(y_list), len(x_list)))
T_ = np.zeros((len(y_list), len(x_list)))
e_ = np.zeros((len(y_list), len(x_list)))
P_ = np.zeros((len(y_list), len(x_list)))
```

Implementation of MacCormark Algorithm:

Predictor:

```
for _ in range(50):
    tau_xx = TAU_XX(u[n], v[n], mu[n], 'predict_e')
    tau_yy = TAU_YY(u[n], v[n], mu[n], 'predict_e')
    tau_xy_e = TAU_XY(u[n], v[n], mu[n], 'predict_e')
    tau_xy_f = TAU_XY(u[n], v[n], mu[n], 'predict_f')

    q_x = Q_X(k[n], T[n], 'predict_e')
    q_y = Q_Y(k[n], T[n], 'predict_f')

    U1 = ro[n]
    U2 = ro[n] * u[n]
    U3 = ro[n] * v[n]
    U5 = ro[n] * (c_v * T[n] + (u[n] ** 2 + v[n] ** 2) / 2)

    E1 = ro[n] * u[n]
    E2 = ro[n] * (u[n] ** 2) + P[n] - tau_xx
    E3 = ro[n] * u[n] * v[n] - tau_xy_e
    E5 = (ro[n] * (c_v * T[n] + (u[n] ** 2 + v[n] ** 2) / 2) + P[n]) * u[n] - u[n] * tau_xx - v[n] * tau_xy_e + q_x

    F1 = ro[n] * v[n]
    F2 = ro[n] * u[n] * v[n] - tau_xy_f
    F3 = ro[n] * (v[n] ** 2) + P[n] - tau_yy
    F5 = (ro[n] * (c_v * T[n] + (u[n] ** 2 + v[n] ** 2) / 2) + P[n]) * v[n] - u[n] * tau_xy_f - v[n] * tau_yy + q_y

    for j in range(len(y_list) - 1):
        for i in range(len(x_list) - 1):
            U1_[j][i] = U1[j][i] - (dt / dx) * (E1[j][i + 1] - E1[j][i]) - (dt / dy) * (F1[j + 1][i] - F1[j][i])
            U2_[j][i] = U2[j][i] - (dt / dx) * (E2[j][i + 1] - E2[j][i]) - (dt / dy) * (F2[j + 1][i] - F2[j][i])
            U3_[j][i] = U3[j][i] - (dt / dx) * (E3[j][i + 1] - E3[j][i]) - (dt / dy) * (F3[j + 1][i] - F3[j][i])
            U5_[j][i] = U5[j][i] - (dt / dx) * (E5[j][i + 1] - E5[j][i]) - (dt / dy) * (F5[j + 1][i] - F5[j][i])

    for j in range(1, len(y_list) - 1):
        for i in range(1, len(x_list) - 1):
            ro_[j][i] = U1_[j][i]
            u_[j][i] = U2_[j][i] / U1_[j][i]
            v_[j][i] = U3_[j][i] / U1_[j][i]
            e_[j][i] = ((U5_[j][i] / U1_[j][i]) - ((u_[j][i] ** 2 + v_[j][i] ** 2) / 2))
            T_[j][i] = e_[j][i] / c_v
            P_[j][i] = ro_[j][i] * R * T_[j][i]

    u_, v_, P_, T_, ro_ = boundary(u_, v_, P_, T_, ro_)

    mu_ = np.zeros((len(y_list), len(x_list)))
    for j in range(len(y_list)):
        for i in range(len(x_list)):
            mu_[j][i] = change_mu(T_[j][i])

    k_ = np.zeros((len(y_list), len(x_list)))
    for j in range(len(y_list)):
        for i in range(len(x_list)):
            k_[j][i] = change_k(mu_[j][i], gamma, c_v, Pr)
```

Corrector:

```
    ro = np.append(ro, [np.zeros((len(y_list), len(x_list)))], axis=0)
    u = np.append(u, [np.zeros((len(y_list), len(x_list)))], axis=0)
    v = np.append(v, [np.zeros((len(y_list), len(x_list)))], axis=0)
    P = np.append(P, [np.zeros((len(y_list), len(x_list)))], axis=0)
    T = np.append(T, [np.zeros((len(y_list), len(x_list)))], axis=0)

    mu = np.append(mu, [np.zeros((len(y_list), len(x_list)))], axis=0)
    k = np.append(k, [np.zeros((len(y_list), len(x_list)))], axis=0)
    e = np.zeros((len(y_list), len(x_list)))

    tau_xx = TAU_XX(u_, v_, mu_, 'correct_e')
    tau_yy = TAU_YY(u_, v_, mu_, 'correct_e')
    tau_xy_e = TAU_XY(u_, v_, mu_, 'correct_e')
    tau_xy_f = TAU_XY(u_, v_, mu_, 'correct_f')

    q_x = Q_X(k_, T_, 'correct_e')
    q_y = Q_Y(k_, T_, 'correct_f')

    E1_ = ro_ * u_
    E2_ = ro_ * (u_ ** 2) + P_ - tau_xx
    E3_ = ro_ * u_ * v_ - tau_xy_e
    E5_ = (ro_ * (c_v * T_ + (u_ ** 2 + v_ ** 2) / 2) + P_) * u_ - u_ * tau_xx - v_ * tau_xy_e + q_x

    F1_ = ro_ * v_
    F2_ = ro_ * u_ * v_ - tau_xy_f
    F3_ = ro_ * (v_ ** 2) + P_ - tau_yy
    F5_ = (ro_ * (c_v * T_ + (u_ ** 2 + v_ ** 2) / 2) + P_) * v_ - u_ * tau_xy_f - v_ * tau_yy + q_y
```

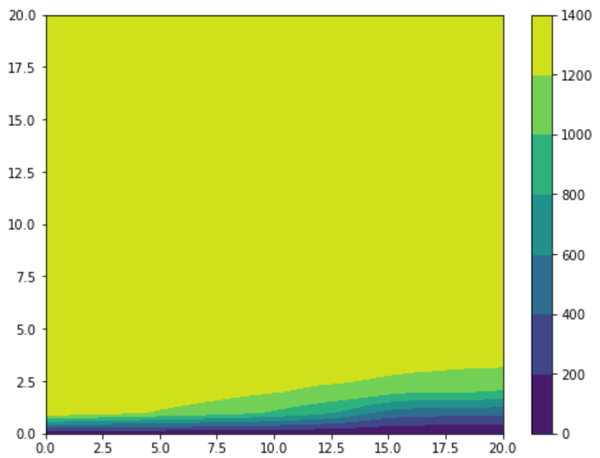
```
    for j in range(len(y_list) - 1):
        for i in range(len(x_list) - 1):
            U1[j][i] = 0.5 * (U1[j][i] + U1_[j][i] - (dt / dx) * (E1_[j][i] - E1_[j][i - 1]) - (dt / dy) * (
                    F1_[j][i] - F1_[j - 1][i]))
            U2[j][i] = 0.5 * (U2[j][i] + U2_[j][i] - (dt / dx) * (E2_[j][i] - E2_[j][i - 1]) - (dt / dy) * (
                    F2_[j][i] - F2_[j - 1][i]))
            U3[j][i] = 0.5 * (U3[j][i] + U3_[j][i] - (dt / dx) * (E3_[j][i] - E3_[j][i - 1]) - (dt / dy) * (
                    F3_[j][i] - F3_[j - 1][i]))
            U5[j][i] = 0.5 * (U5[j][i] + U5_[j][i] - (dt / dx) * (E5_[j][i] - E5_[j][i - 1]) - (dt / dy) * (
                    F5_[j][i] - F5_[j - 1][i]))

    for j in range(1, len(y_list) - 1):
        for i in range(1, len(x_list) - 1):
            ro[n + 1][j][i] = U1[j][i]
            u[n + 1][j][i] = U2[j][i] / U1[j][i]
            v[n + 1][j][i] = U3[j][i] / U1[j][i]
            e[j][i] = (U5[j][i] / U1[j][i]) - ((u[n + 1][j][i] ** 2 + v[n + 1][j][i] ** 2) / 2)
            T[n + 1][j][i] = e[j][i] / c_v
            P[n + 1][j][i] = ro[n + 1][j][i] * R * T[n + 1][j][i]

    u[n + 1], v[n + 1], P[n + 1], T[n + 1], ro[n + 1] = boundary(u[n + 1], v[n + 1], P[n + 1], T[n + 1], ro[n + 1])

    for j in range(len(y_list)):
        for i in range(len(x_list)):
            mu[n + 1][j][i] = change_mu(T[n + 1][j][i])

    for j in range(len(y_list)):
        for i in range(len(x_list)):
            k[n + 1][j][i] = change_k(mu[n + 1][j][i], gamma, c_v, Pr)
    n += 1
```

5. Results:

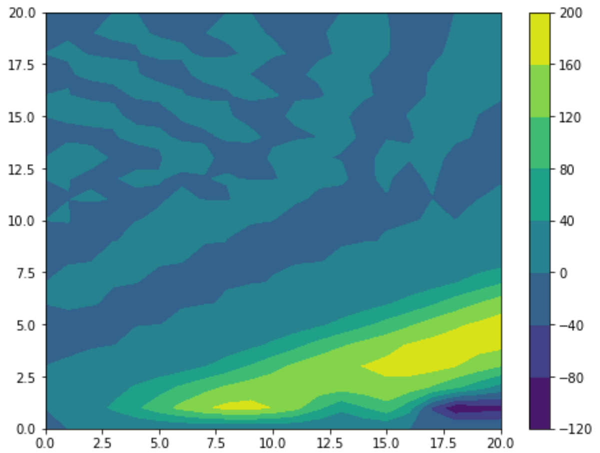
Code for plotting graph:

```
fig, ax = plt.subplots()
cp = ax.contourf(u[n-1])
fig.set_figwidth(8)
fig.set_figheight(6)
fig.colorbar(cp)
plt.show()
```

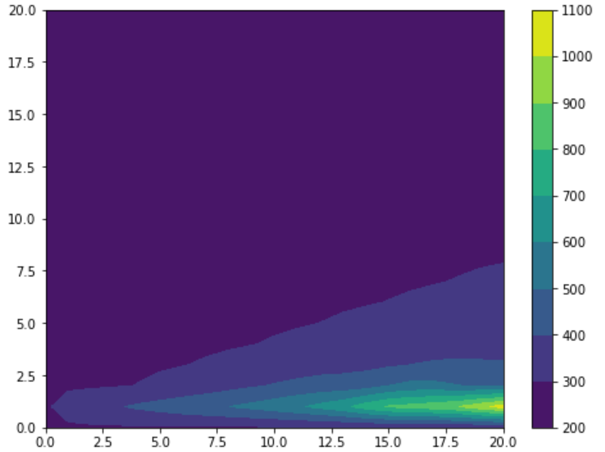
$\underline{Contour\ for\ u}$:



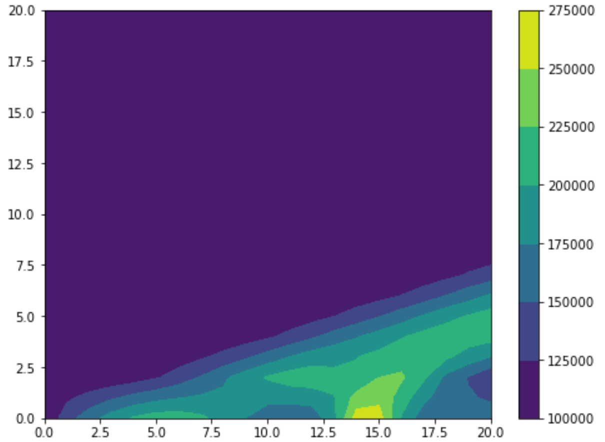
$\underline{Contour\ for\ v}$:



$Contour\ for\ T$:



$Contour\ for\ P$:



6. Conclusion:

We solved the two-dimensional compressible Navier-Stokes equation with the MacCormark Method which consists of two steps. Forward difference operators are used in the predictor step and backward difference operators are used in the corrector step. This method is convenient for solving but has shortcomings. The main drawback is that this is an explicit method, and we are limited by the stability condition. We will not be able to achieve the desired result without choosing the appropriate parameters. Having picked up the parameters, we have modeled a supersonic flow around a flat plate. From the results, we see the boundary layer. Due to the presence of a viscous boundary layer, the plate has a curvature. In addition, we can notice that the dissipation of energy in the boundary layer caused temperatures on the walls.