Week – 12 (Classwork Tasks)

The Linear Momentum Principle

1.

Show that, if a system moves from one state of rest to another over a certain time interval, then the average of the total external force over this time interval must be zero.

An hourglass of mass M stands on a fixed platform which also measures the apparent weight of the hourglass. The sand is at rest in the upper chamber when, at time $t = 0$, a tiny disturbance causes the sand to start running through. The sand comes to rest in the lower chamber after a time $t = \tau$. Find the time average of the apparent weight of the hourglass over the time interval $[0, \tau]$. [The apparent weight of the hourglass is however *not constant* in time. One can advance an argument that, when the sand is steadily running through, the apparent weight of the hourglass *exceeds* the real weight!]

2.

A rocket of initial mass M, of which $M - m$ is fuel, burns its fuel at a constant rate in time τ and ejects the exhaust gases with constant speed u. The rocket starts from rest and moves vertically under uniform gravity. Show that the maximum speed acheived by the rocket is $u \ln \gamma - g\tau$ and that its height at burnout is

$$u\tau\left(1 - \frac{\ln \gamma}{\gamma - 1}\right) - \tfrac{1}{2} g\tau^2,$$

where $\gamma = M/m$. [Assume that the thrust is such that the rocket takes off immediately.]

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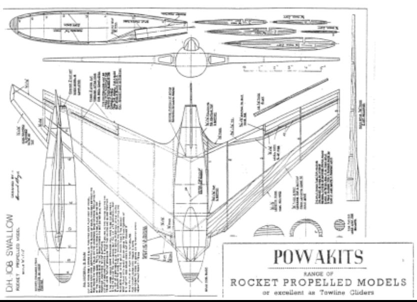

3.

In first stage of the Saturn V rocket, the initial mass was 2.8×10^6 kg, of which 2.1×10^6 kg was fuel. The fuel was burned at a constant rate over 150 s and the exhaust speed was $2,600\ \mathrm{m\,s^{-1}}$. Use the results of the last problem to find the speed and height of the Saturn V at first stage burnout. [Take g to be constant at $9.8\ \mathrm{m\,s^{-2}}$ and neglect air resistance.]

4.

A rocket of initial mass M, of which $M - m$ is fuel, burns its fuel at a constant rate k and ejects the exhaust gases with constant speed u. The rocket starts from rest and moves through a medium that exerts the resistance force $-\epsilon k v$, where v is the forward velocity of the rocket, and ϵ is a small positive constant. Gravity is absent. Find the maximum speed V achieved by the rocket. Deduce a two term approximation for V, valid when ϵ is small.

5.

Two-Stage Rocket (Apply Algebra)

A two-stage rocket has a first stage of initial mass M_1, of which $(1-\eta)M_1$ is fuel, a second stage of initial mass M_2, of which $(1-\eta)M_2$ is fuel, and an inert payload of mass m_0. In each stage, the exhaust gases are ejected with the same speed u. The rocket is initially at rest in free space. The first stage is fired and, on completion, the first stage carcass (of mass ηM_1) is discarded. The second stage is then fired. Find an expression for the final speed V of the rocket and deduce that V will be maximised when the mass ratio $\alpha = M_2/(M_1 + M_2)$ satisfies the equation

$$\alpha^2 + 2\beta\alpha - \beta = 0,$$

where $\beta = m_0/(M_1 + M_2)$. [Messy algebra.]

Show that, when β is small, the optimum value of α is approximatelely $\beta^{1/2}$ and the maximum velocity reached is approximately $2u \ln \gamma$, where $\gamma = 1/\eta$.