Mathematical Formulation & Methodology

1. Notation and Sets

- Markets $m \in$ {ERCOT, MISO, CAISO}.
- Technology by market: $tech(m) \in$ {Wind, Solar}.
- Planning years $Y = \{2026,..., 2030\}$.
- Months $i \in \{1,...,12\}$.
- Peak flag $p \in$ {peak, offpeak} (Boolean equivalent $pk \in \{1, 0\}$).
- Discount rate (WACC) $r = 0.07$.
- DA/RT weights for DMB product: $w_{DA} = 0.8$, $w_{RT} = 0.2$.
- Number of Monte Carlo simulations $S = 5000$.

We simulate on a monthly calendar at two sub-periods per month (peak and off-peak). Let T index those sub-periods across all months in Y, so $|T| = 2 \times 12 \times |Y| = 120$.

2. Data Normalization (Hourly Historicals → Tidy Table)

From the raw hourly dataset (one Excel sheet per market), we construct:

- Timestamp t from either a "Timestamp/Datetime" column or (Date, HE) pair, with hour ending $HE \in \{1,...,24\}$ mapped to start of hour ($HE - 1$).
- Peak/off-peak flag using market rules:

```
■ Mon-Fri, HE ∈ {7,22} | m=ERCOT
Peak(m, HE, DoW) = ■ Mon-Fri, HE ∈ [8,23] | m=MISO
■ Mon-Sat, HE ∈ [7,22] | m=CAISO
```

- Numeric cleaning: currency symbols/commas removed; parentheses (-) treated as negatives for prices; generation negatives are clipped to 0:

$$Gen\_MWh(t) \leftarrow max\{0, Gen\_MWh(t)\}.$$

- Compute columns at the hour:

$$DA\_Hub(t), RT\_Hub(t), DA\_Busbar(t), RT\_Busbar(t).$$

3. Forward Curve Ingestion (Monthly Peak/Off-Peak)

Each forward file yields monthly hub forwards:

$$F^{hub}_{y,i,p} \ (\$/MWh).$$

Dates are parsed from a "Time/Date/Month/Period" column; "Peak" and "Off Peak" columns are cleaned similarly to prices above.

4. Seasonal Means and Driver Pools (Historical DA/RT Shocks and Basis)

Define Boolean peak indicator $1_{peak}(t)$ and month i = month(t). Compute seasonal means by (month, peak) on the historical hub prices:

$$\mu^{DA}_{i,p} = E[DA\_Hub(t) \mid month = i, peak = p], \quad \mu^{RT}_{i,p} = E[RT\_Hub(t) \mid month = i, peak = p].$$

Form multiplicative hub shocks and node-hub basis (at the same hour):

$$\varepsilon^{DA}_{i,p}(t) = DA\_Hub(t) / \mu^{DA}_{i,p}, \quad \varepsilon^{RT}_{i,p}(t) = RT\_Hub(t) / \mu^{RT}_{i,p}$$

$$B^{DA}(t) = DA\_Busbar(t) - DA\_Hub(t), \quad B^{RT}(t) = RT\_Busbar(t) - RT\_Hub(t).$$

For each (month i, peak flag p) we build a bootstrap pool of rows

$$P_{i,p} = \{(Gen\_MWh(t), \varepsilon^{DA}(t), \varepsilon^{RT}(t), B^{DA}(t), B^{RT}(t))\}.$$

5. Monthly Expected Generation Forecast with Degradation

From historical positive-generation hours, aggregate by (Month, Peak/Off-peak) the mean hourly output $g_{i,p}$ (MW) and the count of hours $H_{i,p}$. Let Y_0 be the latest historical year in the dataset for that market. With annual degradation rate d_{tech} (Wind 0.7%, Solar 0.5%):

$$G_{y,i,p} = g_{i,p} \times (1 - d_{tech})^{(y-Y_0)} \ (MW), \quad E_{y,i,p} = G_{y,i,p} \times H_{i,p} \ (MWh).$$

This yields a monthly volume plan $E_{y,i,p}$ for all years and peak buckets.

6. Price Anchoring to Forwards and Stochastic Reconstruction

We create a 2-subperiod monthly calendar over the horizon: $T = \{(y, i, p)\}$.

For each $t = (y,i,p) \in T$:

1. Draw one bootstrap row $(\varepsilon^{DA}, \varepsilon^{RT}, B^{DA}, B^{RT}) \sim P_{i,p}$

2. Determine hub anchors:

$M^{DA}_{y,i,p} = F^{hub}_{y,i,p}$ if forward exists; $\mu^{DA}_{i,p}$ otherwise
$M^{RT}_{y,i,p} = F^{hub}_{y,i,p}$ if forward exists; $\mu^{RT}_{i,p}$ otherwise

3. Construct simulated prices:

$$P^{DA}_{hub,t} = M^{DA}_{y,i,p} \times \varepsilon^{DA}$$
$$P^{RT}_{hub,t} = M^{RT}_{y,i,p} \times \varepsilon^{RT}$$
$$P^{DA}_{bus,t} = P^{DA}_{hub,t} + B^{DA}$$
$$P^{RT}_{bus,t} = P^{RT}_{hub,t} + B^{RT}$$

4. DMB blended prices (per subperiod):

$$P^{DMB}_{hub,t} = w_{DA} \times P^{DA}_{hub,t} + w_{RT} \times P^{RT}_{hub,t}$$
$$P^{DMB}_{bus,t} = w_{DA} \times P^{DA}_{bus,t} + w_{RT} \times P^{RT}_{bus,t}$$

7. "No-Take When Negative" Energy Rule (Optional, Enabled by Default)

Define effective energy taken for a product $q \in$ {DA_hub, RT_hub, DA_bus, RT_bus, DMB_hub, DMB_bus}:

$E^{(q)}_{y,i,p}$ = 0 if no_take_negative = True and $P^{(q)}_t < 0$; $E_{y,i,p}$ otherwise

Interpretation: when the (product-specific) price for the period is negative, the model "doesn't take" generation for that product (sets MWh to zero). Prices are kept negative; only energy credited to that product is zeroed.

8. Discounting and PV Denominators

Let subperiod index $k = 1,...,|T|$ run in calendar order (month by month, peak then off-peak). The model uses monthly discount factors:

$$DF_k = (1 + r)^{-k/12}.$$

For each product q, PV revenue in one simulation is:

$$PVRev^{(q)} = \Sigma_k P^{(q)}_k \times E^{(q)}_k \times DF_k.$$

9. PV Energy Denominator Policies

PV energy denominator has two policies; the code uses the inclusive physical generation by default:

- **Inclusive (default):**

$$PVE = \Sigma_k E_k \times DF_k.$$

- **Exclusive (optional):**

$$PVE = \Sigma_k (max\ E^{(q)}_k) \times DF_k$$

(i.e., only "taken" MWh contribute to the denominator).

10. Unit PV Capture Prices (Risk-Indifference Metric)

For each product q and simulation s, compute the unit PV price:

$$Z^{(s)} = PVRev^{(q)} / PVE\ (\$/MWh).$$

Across the S simulations, report P50/P75/P90 as:

$$Quantile^{\alpha} = Quantile(\{Z^{(s)}\}_{s=1...S}, \alpha), \alpha \in \{0.50, 0.75, 0.90\}.$$

Products reported:

- RT at Busbar/Hub, DA at Busbar/Hub.
- DMB(DA80/RT20) at Busbar and Hub.

11. Volatility/Driver Diagnostics (Descriptive Stats)

From hourly historicals:

- **Spreads:**

$$Spread_{bus}(t) = DA\_Busbar(t) - RT\_Busbar(t), \quad Spread_{hub}(t) = DA\_Hub(t) - RT\_Hub(t).$$

- **Basis:**

$$Basis_{RT}(t) = RT\_Busbar(t) - RT\_Hub(t), \quad Basis_{DA}(t) = DA\_Busbar(t) - DA\_Hub(t).$$

- **Daily peak/trough (by hub):**

$\Delta^{RT}_{d} = max_{day}(RT\_Hub) - min_{day}(RT\_Hub), \quad \Delta^{DA}_{d} = max_{day}(DA\_Hub) - min_{day}(DA\_Hub).$

- **Correlations:**

$$\rho(Gen, RT\_Hub), \rho(Gen, DA\_Hub),$$

computed on hourly pairs where both are present.

For each metric the report shows mean, p10, p50, p90, std where meaningful.