1) • 4 balls
   • 10 containers
   • one container must have atleast 2 balls

→ Every container has at most 1 ball:

Out of 10 containers, choose 4 to place balls in → $\binom{10}{4}$

$$\binom{10}{4} = \frac{n!}{k!(n-k)!} = \frac{10!}{4!(10-4)!} = \frac{10\cdot 9\cdot 8\cdot 7\cdot \cancel{6!}}{4!\,\cancel{6!}} = \frac{10\cdot \overset{3}{\cancel{9}}\cdot \cancel{8}\cdot 7}{\cancel{4}\cdot\cancel{3}\cdot\cancel{2}\cdot 1} = 210$$

→ 4 identical balls into 10 different containers:

k < n since its n choose k so n is 10, k is 4

$$\binom{n+k-1}{n-1} = \binom{10-1+4}{10-1} = \binom{13}{9} = \frac{n!}{k!(n-k)!} = \frac{13!}{9!(13-9)!}$$

$$= \frac{13\cdot 12\cdot 11\cdot 10\cdot \cancel{9!}}{\cancel{9!}\,(4!)} = 715$$

→ Subtracting 715 and 210 gives us $\boxed{505}$

2) • 10 candies

• 2 identical

• one after breakfast, lunch, dinner

→ Question said order is important. If all candies are different, there is no repetition. So we use $\frac{n!}{(n-k)!}$. You're choosing 3 times so $k = 3$.

If two candies out of the 10 are the same, theres 9 candies. $1\ 2\ 3\ 4\ 5\ 6\ 7\ 8\ \underbrace{9}$ cause these two are the same

so $n = 9$: $\frac{9!}{(9-3)!} = \frac{9 \cdot 8 \cdot 7 \cdot \cancel{6!}}{\cancel{6!}} = 504$

→ Again, order matters. If 2 candies are identical they repeat so repetition matters. So we use $\binom{n}{k}$. If you eat two of the same, the third candy you eat will come from this pile $\underbrace{1\ 2\ 3\ 4\ 5\ 6\ 7\ 8}_{8}\ \underbrace{9}$ so $n = 8$

and you are choosing 1 so $k = 1$: $\binom{n}{k} = \binom{8}{1} = \frac{n!}{k!(n-k)!}$

$= \frac{8 \cdot \cancel{7!}}{1! \cdot \cancel{7!}} = 8$. But you can eat the different candy first before the identical 2, after or between so three different ways to do one thing so $8 \times 3 = 24$

→ Adding 504 and 24 gives us $\boxed{528}$

3) $x + y + z + w = 15$

$x \geq 3$, $y > -2$, $z \geq 1$, $w > -3$

$\downarrow$ $\qquad$ $\downarrow$ $\qquad$ $\downarrow$ $\qquad$ $\downarrow$

$x' + 3$ $\qquad$ $y' - 1$ $\qquad$ $z' + 1$ $\qquad$ $w' - 2$

* $\geq$: replace with a + or add because $x+3$ satisfies $x \geq 3$
* $>$: add 1 to the number because $y-1$ satisfies $y > -2$

Now replace variables with new ones:

$$x' + 3 + y' - 1 + z' + 1 + w' - 2 = 15$$
$$x' + y' + z' + w' + 1 = 15$$
$$x' + y' + z' + w' = 14$$

# of variables = n ; total = k

4 = n ; 14 = k

$$\binom{n + k - 1}{n - 1}$$

$$\binom{4 + 14 - 1}{4 - 1}$$

$$\binom{17}{3}$$

$$\binom{n}{k} = \frac{n!}{k!\,(n-k)!} = \frac{17 \cdot 16 \cdot 15 \cdot 14!}{3!\,(17-3)!} = \frac{17 \cdot \cancel{16}^{8} \cdot \cancel{15}^{5} \cdot \cancel{14!}}{\cancel{3} \cdot \cancel{2} \cdot 1\,(\cancel{14!})} = \boxed{680}$$

* $\leq$ just take the # on the side of $\leq$ and do addition but negate variable like $x \leq 3 \rightarrow -x + 3$
* $<$ just add 1 to # on the side of $<$ then put before as $y < 2 \xrightarrow{2+1=3} y - 3$

] probably won't be given as variable has to be ≥ 0

4) • 14 men, 9 women, 23 chairs, no 2 women next to each

place men first = . 1 . 2 . 3 . 4 . 5 . 6 . 7 . 8 . 9 . 10 . 11 . 12 . 13 . 14 .

dots are women and the amount of dots are 15 but you have 9 women so $\binom{15}{9}$

$\underset{M_1}{\_\_}\ W\ \underset{M_2}{\_\_}\ W\ \underset{M_3}{\_\_}\ W\ \underset{M_4}{\_\_}\ W\ \underset{M_5}{\_\_}\ W\ \underset{M_6}{\_\_}\ W\ \underset{M_7}{\_\_}\ W\ \underset{M_8}{\_\_}\ W\ \underset{M_9}{\_\_}\ W\ \underset{M_{10}}{\_\_}$

$M_1 + M_2 + M_3 + M_4 + M_5 + M_6 + M_7 + M_8 + M_9 + M_{10} = \underset{\text{\# of total men}}{14}$

$$\left.\begin{aligned} M_1 &\geq 0 \rightarrow Y_1 \\ M_2 &\geq 1 \rightarrow Y_2 + 1 \\ M_3 &\geq 1 \rightarrow Y_3 + 1 \\ &\vdots \qquad\quad \vdots \\ M_{10} &\geq 0 \rightarrow Y_{10} \end{aligned}\right\} 14$$

* M's denote men position and M_1 / M_{10} can be zero because they are at the end. M_2 to M_9 must have at least 1 man because otherwise you'd have two women next to one another

$\rightarrow Y_1 + \underline{Y_2 + 1} + \underline{Y_3 + 1} + \underline{Y_4 + 1} \ldots + Y_{10} = 14$

You add 1 8 times cause Y_1 and Y_{10} don't have +1 so

$Y_1 + Y_2 + Y_3 + Y_4 + \ldots + Y_{10} + 8 = 14$

$Y_1 + Y_2 + Y_3 + Y_4 + \ldots + Y_{10} = 14 - 8 = 6 \rightarrow k$

* n is total num of variables so 10 cause its from Y_1 to Y_{10}

$\rightarrow \binom{n+k-1}{n-1}$

$\binom{10+6-1}{10-1}$

$\binom{15}{9}$

$\rightarrow$ 14 men can be placed in 14! ways

$\rightarrow$ 9 women can be placed in 9! ways

$\rightarrow \boxed{\binom{15}{9} \cdot 14! \cdot 9!}$

5) coefficient of x^3y^2 in $(x+y+2)^{10}$

$y+2 = z$

$(x + \underbrace{y+2}_{z})^{10} = (x+z)^{10}$

$x^3 \cdot z^7$ * you need x to have power 3 so you split

$x^3 \cdot (y+2)^7$ * sub y+2 back in

$x^3 \cdot (y^2 \cdot 2^5)$ * you need y to have power 5 so you split

$x^3 \cdot y^2 \cdot 2^5$ * notice how all the exponents add upto 10

Out of 10 total exponents, you are choosing 3 out of 10 for x, 2 out of 7 for y because you chose 3 out of 10 already so 10-3 means you got 7 options left to choose from 2 has power 5 cause out of 10, already chose 5 and $2^5 = 32$ so

$x^3 \cdot y^2 \cdot 2^5$

$\binom{10}{3} \cdot \binom{7}{2} \cdot 32$ $\binom{n}{k} = \frac{n!}{k!(n-k)!}$

$$\frac{10!}{3!(10-3)!} \cdot \frac{7!}{2!(7-2)!} \cdot 32$$

$$\frac{10 \cdot \overset{3}{\cancel{9}} \cdot \overset{4}{\cancel{8}} \cdot \cancel{7} \cdot \cancel{6} \cdot \cancel{5} \cdot \cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot \cancel{1}}{\underset{1}{\cancel{3}} \cdot \underset{1}{\cancel{2}} \cdot 1 \cdot \cancel{7} \cdot \cancel{6} \cdot \cancel{5} \cdot \cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot \cancel{1}} \cdot \frac{7 \cdot \overset{3}{\cancel{6}} \cdot \cancel{5} \cdot \cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot \cancel{1}}{\underset{1}{\cancel{2}} \cdot 1 \cdot \cancel{5} \cdot \cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot \cancel{1}} \cdot 32$$

$$120 \cdot 21 \cdot 32 = \boxed{80640}$$

6) $\binom{100}{0}\left(\frac{1}{3}\right)^{100} 6^0 + \binom{100}{1}\left(\frac{1}{3}\right)^{101} 6^1 + \dots + \binom{100}{100}\left(\frac{1}{3}\right)^{200} 6^{100}$

Turn this into general summation:

$$\sum_{k=0}^{100} \binom{100}{k}\left(\frac{1}{3}\right)^{100+k} 6^k$$

$$\sum_{k=0}^{100} \binom{100}{k}\left(3^{-1}\right)^{100+k} 6^k$$

$$\sum_{k=0}^{100} \binom{100}{k}\left(3^{-100-k}\right) 6^k$$

* 3^{-100} same as $\frac{3^{100}}{3^{200}}$

$$\sum_{k=0}^{100} \binom{100}{k}\left(\frac{3^{100-k}}{3^{200}}\right) 6^k$$

* $\frac{3^{100-k}}{3^{200}}$ same as $\frac{1}{3^{200}}$ which is a number so bring that to the front

$$\frac{1}{3^{200}} \sum_{k=0}^{100} \binom{100}{k}\left(3^{100-k}\right)\left(6^k\right)$$

* $\underbrace{(x+y)(x+y)\dots(x+y)}_{n}$

$= \boxed{?}\ x^{n-k} y^k$

The form $\boxed{?}\ x^{n-k} y^k$ is same as $3^{100-k} 6^k$

so $x = 3$, $y = 6$. We know $\underbrace{(x+y)(x+y)\dots(x+y)}$ and summation goes to 100 so $n = 100$

$$\frac{1}{3^{200}} \cdot (3+6)^{100}$$

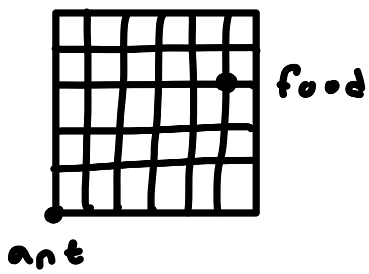
$$\frac{1}{3^{200}} \cdot 9^{100}$$

$$\frac{1}{3^{200}} \cdot (3^2)^{100}$$

$$\frac{1}{3^{200}} \cdot 3^{200} = \boxed{1}$$

so its $(x+y)^n$ so $(3+6)^{100}$

7a)



Total steps = 8

Horizontal distance to food = 5

Vertical distance to food = 3

Anagrams formula $= \dfrac{\text{total thing}!}{\text{individual thing}! \cdot \text{individual thing}! \dots}$

$$= \frac{8!}{5!\,3!}$$

$$= \frac{8 \cdot 7 \cdot \cancel{6} \cdot \cancel{5!}}{\cancel{5!} \cdot \cancel{3} \cdot \cancel{2} \cdot 1}$$

$$= \boxed{56 \text{ paths}}$$

b) Total amount of moves have to add up to 8 and ant has to return to original position at the end. So stuff has to cancel out so down = up and right = left so

Combo 1	Combo 2	Combo 3	Combo 4	Combo 5
0 up	1 up	2 up	3 up	4 up
4 right	3 R	2 R	1 R	0 R
4 left	3 L	2 L	1 L	0 L
0 down	1 D	2 D	3 D	4 D

Anagrams formula $= \dfrac{\text{total thing}!}{\text{individual thing}! \cdot \text{individual thing}! \cdots}$

$$\frac{8!}{0!4!4!0!} \to 70 \qquad \frac{8!}{1!3!3!1!} \to 1120 \qquad \frac{8!}{2!2!2!2!} \to 2520 \qquad \frac{8!}{1!3!3!1!} \to 1120 \qquad \frac{8!}{0!4!4!0!} \to 70$$

Addition Rule

$\boxed{4900}$

c) <u>One to one using contrapositive:</u>

if $f(i_1, j_1) = f(i_2, j_2)$ then $i_1 = i_2$, $j_2 = j_2$

$f(i_1, j_1) = (i+j, i-j)$

$f(i_2, j_2) = (i_2+j_2\ i_2-j_2)$

$i_1 + j_1 = i_2 + j_2$

$i_1 - j_1 = i_2 - j_2$

$2i_1 = 2i_2$

$i_1 = i_2$

$i_1 - j_1 = i_2 - j_2$

$i_2 - j_1 = i_2 - j_2$

$-j_1 = j_2$

$j_1 = j_2$

Since $i_1 = i_2$ and $j_1 = j_2$

$f(i_1, j_1) = f(i_2, j_2)$

so its $\boxed{\text{one to one}}$

not <u>onto:</u>

$(k, l) = (i+j, i-j)$

$k + l = i+j + i - j$

$k + l = 2i$

K and l are even as $i \in \mathbb{Z}$

So $K=2$ is fine but $l=3$ is not because its odd so:

$k + l = 2i \rightarrow 2+3 = 2i \rightarrow 5 \div 2 = i \rightarrow 2.5 = i$ but $i \in \mathbb{Z}$

so its $\boxed{\text{not onto}}$

<u>$f(5,3)$:</u>

$f(i, j) = (i+j, i-j)$ $k+l$ = even

$5 + 3 = i+j + i-j$

$8 = 2i$

$i = 4$

$j = 1$

$\boxed{f(4,1) = (5,3)}$

d) 8 moves with at least one up and one right. She will make the up moves first then left then down then right

$U + L + D + R = 8$ $\quad$ $U \geq 1 \rightarrow U' + 1$ * at least 1 U

$R \geq 1 \rightarrow R' + 1$ * at least 1 R

$U' + 1 + L + D + R' + 1 = 8$

$U' + L + D + R' + 2 = 8$

$U' + L + D + R' = 6$

# of variables $= n$; total $= k$

$4 = n$, $6 = k$

$\binom{n+k-1}{n-1}$

$\binom{4+6-1}{4-1}$

$\binom{9}{3}$

$$\binom{n}{k} = \frac{n!}{k!(n-k)!} = \frac{9!}{3!(9-3)!} = \frac{\overset{3}{\cancel{9}} \cdot \overset{4}{\cancel{8}} \cdot 7 \cdot \cancel{6!}}{\cancel{3} \cdot \cancel{2} \cdot 1 (\cancel{6!})} = \boxed{84}$$

e) 8 moves so $n = 8$ and ant makes only right/up moves so out of 8 moves the ant makes all the way from 0 to 8 right moves so

$$\binom{8}{0} + \binom{8}{1} + \binom{8}{2} + \binom{8}{3} + \binom{8}{4} + \binom{8}{5} + \binom{8}{6} + \binom{8}{7} + \binom{8}{8}$$

For each turn ant has 2 options → dropping pheremone / → not dropping pheremone

So thats 2^n so now you have

$$\binom{8}{0} 2^0 + \binom{8}{1} 2^1 + \binom{8}{2} 2^2 + \binom{8}{3} 2^3 + \binom{8}{4} 2^4 \ldots \binom{8}{8} 2^8$$

$$\underbrace{(x+y)(x+y)\ldots(x+y)}_{n} = \boxed{?}\, x^{n-k} y^k$$

so in $(x+y)^n$, the term repeats 8 times so $n = 8$, 2 is constant so x is 2, y is 1 so

$$(x+y)^n \rightarrow (2+1)^8 \rightarrow \boxed{3^8}$$