1) subset relation on the power set of set S

→ Reflexive, anti-symmetric, transitive

relation $\leq$ on $\mathbb{R}$

→ Anti-symmetric, transitive, reflexive

relation $<$ on $\mathbb{Z}$

→ Anti-symmetric, transitive

relation shared a class with on the set of students @ hunter where two students share a class if there is a class they are both enrolled in this semester

→ Reflexive, symmetric

relation given by $\{(a,c),(a,f),(a,h),(b,h),(c,f)$ $(c,h),(d,h),(e,h),(f,h),(g,h)\}$

→ Anti-symmetric, transitive

relation R on $\mathbb{N}$ where $(a,b) \in R$ means $a \mid b$

→ Reflexive, anti-symmetric, transitive

relation R on $\mathbb{N}$ where $(x,y) \in R$ means $x < y+2$

→ Reflexive

2) $\mathbb{N} \times \mathbb{N}$ (set of ordered pairs of positive integers)

$(a,b) \equiv (c,d) \Leftrightarrow ab = cd$

prove equivalence relation

prove any integer $n \in \mathbb{N}$, exists classes of size n

same thing as saying theres integers with n divisors

are there infinetly many classes of equivalence of size 1

of size 2

Reflective: $(a,b) \in \mathbb{N} \times \mathbb{N}$

$ab = ab$

$(a,b) \equiv (a,b)$ ✓

Symmetric: $(a,b) \wedge (c,d) \in \mathbb{N} \times \mathbb{N}$

$(a,b) \equiv (c,d)$

$\rightarrow ab = cd$

$cd = ab$

$\rightarrow (c,d) \equiv (a,b)$ ✓

Transitive: $(a,b) \in \mathbb{N} \times \mathbb{N}$

$(c,d) \in \mathbb{N} \times \mathbb{N}$

$(e,f) \in \mathbb{N} \times \mathbb{N}$

$(a,b) \equiv (c,d)$

$(c,d) \equiv (e,f)$

$\rightarrow ab = cd$

$cd = ef$

$\rightarrow ab = ef$

$\rightarrow (a,b) \equiv (e,f)$ ✓

$\rightarrow$ equivalence relation

$n \in \mathbb{N}$

$[(1, 2^{n-1})] =$

$\{(1, 2^{n-1}), (2^1, 2^{n-2}) \ldots$

$(2^{n-1}, 1)\}$ size n

if $n = 1 \rightarrow ab$ 1 divisor

$ab = 1 = [(a,b)] = [(1,1)]$

$\rightarrow \infty$ many equivalence classes of size 1

---

prime $p \in \mathbb{N}$

$\rightarrow$ equivalence class $[(1,p)]$

$= \{(1,p), (p,1)\}$ size 2

$\exists$ infinetly many primes

$\rightarrow \infty$ many equivalence classes of size 2

3) Every non-empty subset of $\mathbb{N}$ (finite/infinite) has a minimum. Find total order relation $\prec$ on $\mathbb{Z}$ such that non-empty subset of $\mathbb{Z}$ has a minimum under the $\prec$ relation

$$x \prec y \longleftrightarrow |x| \leq |y| \wedge (|x| = |b| \leftrightarrow a < b)$$

4) Symmetric relation $\sim$ that satisfies $\forall x, y, z$

$x \sim y \Rightarrow (x \sim z \vee z \sim y)$. If $\sim$ is non reflexive for every x, what can we say about the relation $\nsim$? (compliment $\sim$)

$\forall x, y, z,\ x \sim y \rightarrow (x \sim y \vee z \sim y)$

- non reflective

$\rightarrow \forall x, y, z,\ x \nsim x,\ y \nsim y,\ z \nsim z$

$\rightarrow$ $\nsim$ is reflective

- if $x \nsim y \rightarrow y \nsim x$

$\rightarrow$ $\nsim$ is symmetric

- if $x \nsim y$ and $y \nsim z \rightarrow x \nsim z$

$\rightarrow$ $\nsim$ is transitive

As a result, $\nsim$ is an equivalence relation

5) Fermats theorem to find 2^{124} mod 127

2^{124} (mod 127)

$(2^{62})^2$ mod 127

$((2^{31})^2)^2$ mod 127

$((2\cdot 2^{30})^2)^2$ mod 127

$((2(2^{15})^2)^2)^2$ mod 127

$((2\cdot(2\cdot 2^{14})^2)^2)^2$ mod 127

$((2\cdot(2\cdot(2^7)^2)^2)^2)^2$ mod 127 *

$((2\cdot(2\cdot(1)^2)^2)^2)^2$ mod 127

$((2\cdot(2)^2)^2)^2$ mod 127

$((8)^2)^2$ mod 127

$(64)^2$ mod 127

$(2^6)^2$ mod 127

(2^{12}) mod 127

$2^5\cdot 2^7$ mod 127 *

$2^5\cdot 1$

32

2^7 mod 127 = 1