1) $(\{2,4,6\} \cup \{6,4\}) \cap \{4,6,8\}$

$\cup$ = union = take both sets, combine so no elements repeat

$\cap$ = intersection = take both sets, select the elements both sets have in common

$(\{2,4,6\} \cup \{6,4\}) \cap \{4,6,8\}$

$(\{2,4,6\}) \cap \{4,6,8\}$

$\boxed{\{4,6\}}$

2) $\{1,2,3\} \times \{0,1\}$

$\times$ = cross product = multiply sets together into ordered pairs

$\{1,2,3\} \times \{0,1\}$

$$\boxed{\{(1,0),(1,1),(2,0),(2,1),(3,0),(3,1)\}}$$

3) $P(S)$ is a powerset - a set of all subsets of S

if $|S| = n$, then $|P(S)| = 2^n$ [n = # of elements)

(size of)

* $P(\{x, y, z\})$ = 3 elements so $n = 3$ so

$|P(S)| = 2^n = 2^3 = 8$

$\downarrow$

all possible subsets = $\{\emptyset, \{x\}, \{y\}, \{z\}, \{x,y\}, \{y,z\}, \{x,z\}, \{x,y,z\}\}$

* $P(\{x, z\})$ = 2 elements so $n = 2$ so

$|P(S)| = 2^n = 2^2 = 4$

$\downarrow$

all possible subsets = $\{\emptyset, \{x\}, \{z\}, \{x,z\}\}$

* $P(\{x, y, z\} - P(\{x, z\}) =$

$$\{\cancel{\emptyset}, \{\cancel{x}\}, \{y\}, \cancel{\{z\}}, \{x,y\}, \{y,z\}, \{\cancel{x,z}\}, \{x,y,z\}\}$$

$$- \quad \{\cancel{\emptyset}, \{\cancel{x}\}, \cancel{\{z\}}, \cancel{\{x,z\}}\}$$

$$\boxed{\{\{y\}, \{x,y\}, \{y,z\}, \{x,y,z\}\}}$$

4) P( ) = powerset P
{P} = element P
Size = 2^n with n being # of elements

$P(P(\{P\}))$

$P(\{\emptyset, \{p\}\})$

$\boxed{\{\emptyset, \{\emptyset\}, \{\{p\}\}, \{\emptyset, \{p\}\}\}}$

* 1 element so $2^1 = 2$ elements in this set

* 2 elements so $2^2 = 4$ elements in this set

5) 3 digit positive integers

→ first is 100

→ last is 999

Set $X = \{100, 101, 102, \dots, 999\}$

$$\boxed{\{x \in \mathbb{N} \mid 99 < x < 1000\}}$$

6) $\emptyset = \{\emptyset\} \rightarrow \{\emptyset\}$ is an element, $\emptyset$ is not ✗

$|\emptyset| = 0 \rightarrow$ the size of $\emptyset$ is 0 ✓

$|P(\emptyset)| = 0 \rightarrow$ size of $P(\emptyset)$ is not 0 ✗

$\emptyset \in \{\} \rightarrow \emptyset$ is not an element of $\{\}$ ✗

$\boxed{|\emptyset| = 0 \text{ is true}}$

7) f is one to one:

Each person owns an unique umbrella.
Two people cant have the same umbrella, they must be different.

f is not onto:

Some umbrellas don't have any owners so some umbrellas are unused.

f is a bijection:

Every person owns one unique umbrella and all umbrellas have owners. Each umbrella is owned by exactly one person.

8) If two people share the same umbrella, then all rainy days are bad

two people exist = $\exists (p_1, p_2)$

share same umbrella = $f(p_1) = f(p_2)$

All = $\forall$

rainy days = $r \in R$

if - then = $\Rightarrow$

are bad = are bad

$$\boxed{\left(\exists (p_1, p_2 \in P), f(p_1) = f(p_2)\right) \Rightarrow \left(\forall r \in R, \text{are bad}\right)}$$

9) $\mathbb{N}$ = natural numbers = positive integers = 0, 1, 2...
$\mathbb{R}$ = real numbers = $-1/3$, 0, 0.50, $2/10$, $-30/100$...
$\mathbb{Q}^+$ = positive rational numbers = $\frac{3}{7}$, 5, 0.02...
$\mathbb{Z}$ = integers = -1, 0, 1, 50, -50, 1000...
One to one = each x maps to only one distinct y
Onto = every y-value has a x value that maps to it

- $f: \mathbb{N} \to \mathbb{R}$, $f(x) = \frac{1}{x}$

  It is one to one but not onto

- $g: \mathbb{N} \times \mathbb{N} \to \mathbb{Q}^+$, $g(x, y) = x/y$

  It is not one to one but is onto

- $h: \mathbb{Z} \to \mathbb{Z}$, $h(x) = x^2$

  It is not one to one neither onto

- $w: \mathbb{R} \to \mathbb{R}$, $w(x) = 3x + 1$

  It is one to one and onto so it is a bijection

10a)

So in 20 we choose 2 so $\binom{20}{2} = 190$
All pairs consisting of neighbours is (1,2)(3,4), (5,6)(7,8)(9,10)(11,12)(13,14)(15,16)(17,18)(19,20).
Which goes up to 19. So we subtract 19 from 190 which leaves us with $\boxed{171}$

Set A = all neighbouring pairs
Set B = all non-neighbouring pairs

Addition rule says if two sets are disjointed, You get the total number of elements by adding the two sets. If a pair is neighbouring it cant also be non-neighbouring hence the two sets A & B are disjointed. This is why we are able to use the addition rule.

10b) $d_1 + d_2 + \ldots + d_n = \sum_{i=1}^{n} d_i = 2 \times \# \text{edges}$

Each person is a vertex
Each edge is formed by two people

The first two vertices will have a degree of 18 so $2 \times 18 = 36$.

The rest of the vertices will have a degree of 17 so $18 \times 17 = 306$.

After adding $306 + 36$ we get 342 then we divide by 2 to get

$$\boxed{171 \ (\# \text{ of edges}, \binom{19}{2})}$$

10c) from the examples given:

$f(3,7) = (3,4,13)$

$f(12,14) = (12,2,6)$

Since $f(i,j) \neq f(m,n)$ each x has a distinctly unique y making it one to one

$$f(i,j) = (x,y,z) = (i, j-i, 20-j)$$
$$(x, 20-z-x, z)$$

i is a natural number, j is a natural number

The difference of a natural number subtracted by a natural number will be a natural number.

Since all possible outputs are natural numbers and the domain is all natural numbers, all codomains will be used up, hence its onto.

As it is both onto and one to one, it's a bijection.