1) $i = 1$

$d = 2$

$n = 127$

$a_n = a_0 + d \cdot n$

$a_n = 1 + 2 \cdot n$

$127 = 1 + (2 \cdot n)$

$\frac{126}{2} = n = 63$

$$\sum_{i=0}^{63} 1+2i = \underbrace{\sum_{i=0}^{63} i}_{64} + \underbrace{2\sum_{i=0}^{63} i}_{4032}$$

$$64 + 4032$$

$$\boxed{4096}$$

2) $\prod$ notation $= \prod_{i=0}^{n-1} (1 - xy^i)$

$n-1$ → last term

$i=0$ → starting

$Pock_{x,y}(0)$ means $n=0$

So when $n=0$,

$$Pock_{x,y}(n) = \prod_{i=0}^{n-1} (1 - xy^i)$$

$$Pock_{x,y}(0) = \prod_{i=0}^{0-1} (1 - xy^i)$$

$$Pock_{x,y}(0) = \prod_{i=0}^{-1} (1 - xy^i)$$

-1 → upper bound

$i=0$ → lower bound

Since upper bound < lower bound,

$$\boxed{Pock_{x,y}(0) = \underline{1}}$$

3) $a_n = a_{n-1} + s$

$a_i - a_j = s(i-j)$

$a_1 - a_n = s(1-n)$

$-a_n = s(1-n) + a_1$

$a_n = -s(1+n) + a_1$

$$\boxed{\frac{a_n - a_1}{s} + 1} = n$$

4) $S = \sum_{i=1}^{n} a_i$

$a_i - a_j = s(i-j)$

$a_i - a_1 = s(i-1)$ $*j=1$

$a_i = s(i-1) + a_1$

$S = \sum_{i=1}^{n} a_i = \sum_{i=1}^{n} s(i-1) + a_1$

$S = \sum_{i=1}^{n} a_i$

$S = \sum_{i=1}^{n} s(i-1) + a_1$

$S = \sum_{i=1}^{n} si - s + a_1$

$S = s\sum_{i=1}^{n} i - s\sum_{i=1}^{n} + \sum_{i=1}^{n} a_1$

$S = s(\sum_{i=1}^{n} i - \sum_{i=1}^{n} 1) + \sum_{i=1}^{n} a_1$

$$\boxed{S = s\left(\frac{n(n-1)}{2}\right) + a_1(n)}$$

5) 5 numbers exactly so $\boxed{5!}$

# of permutations =

$n! = 1 \times 2 \times 3 \times 4 \times \ldots \times n$

6) $\boxed{51.}$ because there are 5 starting points where each point can only be used once.

7) _ _ Z

Both places have 25 options because the same letter can't be used more than once.

$n = 25$, you're choosing twice so

$\binom{n}{2}$ n choose 2

$$\frac{n(n+1)}{2} = \frac{24(24+1)}{2} = \boxed{300 \text{ ways}}$$

8) If order does not matter you dont need to divide by two so 600 ways

The last possible name is _ X _ Y Z

X is 24^{th} in the alphabet

Y is 25^{th} in the alphabet

$$24 \cdot 25 = \boxed{600}$$

9a) 8 rows, 8 colums, $n = 8$

$$\frac{n(n-1)}{\cancel{2}} \cdot \cancel{2}n = n^2(n-1)$$

$$8^2(8-1) = \boxed{448}$$

9b) First square → 64 choices

Second square → 32 choices (different color)

Divide by 2 for overcounting

$64 \cdot 32 \div 2 = \boxed{1024}$

9c) Snake 1 → 64 first sq, 32 second sq

Snake 2 → 62 first sq, 31 second sq

[snake 1 took first 2]

Overcount so divide 2 each snake + 2 overall → $2 \cdot 2 \cdot 2 = 8$

$$\frac{64 \cdot 32 \cdot 62 \cdot 31}{8} = \boxed{492032}$$