1) Pigs can fly => Proffesor Saad loves us

$\underbrace{\text{Pigs can fly}}_{\text{hypothesis}} \Rightarrow \underbrace{\text{Proffesor Saad loves us}}_{\text{conclusion}}$

- The proposition is true because a false hypothesis gives a true conditional statement. According to the truth table, if the first statement is false (pigs can't fly), the proposition is always true. Pigs can't fly so hypothesis is false. According to the truth table, if hypothesis is false, the proposition is true regardless of Q.

- This is not a proof of P because its dependent on the hypothesis. Since P is false, $P \Rightarrow Q$ is true but the hypothesis being false does not have anything to do with if the conclusion is true or false. It's not a proof of P itself because it is dependent on whether pigs can fly is true.

if something false (pigs can fly) implies anything (true or false) it will always be true.

P	Q	$P \Rightarrow Q$	
F	F	T	→ P is false but outcome is true
T	F	F	
F	T	T	→ P is false but outcome is true
T	T	T	

2) $\wedge$ = and, $\vee$ = or, $\Rightarrow$ = implies, $\neg$ = not

$0 \wedge P$ = false, one 0 in and means its false

$1 \wedge P$ = neither, can be true or false

$0 \vee P$ = neither, can be true or false

$1 \vee P$ = true, one 1 in or means its true

$0 \Rightarrow P$ = true, false implies anything is true

$1 \Rightarrow P$ = neither, true imples anything is neither

$P \vee \neg P$ = true, one 1 in or means its true

$P \wedge \neg P$ = false, both need to be true for and

$P \Rightarrow \neg P$ = neither, true/false implies is neither

* AND = $\wedge$ = only true when both P Q are true
* OR = $\vee$ = only false when both P Q are false
* IMPLIES = $\Rightarrow$ = only false when P is true and Q is false

3) $(P \Rightarrow Q) \vee (Q \Rightarrow P)$

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P Q	$P \Rightarrow Q$	$Q \Rightarrow P$	$(P \Rightarrow Q) \vee (Q \Rightarrow P)$
0 0	1	1	1
0 1	1	0	1
1 0	0	1	1
1 1	1	1	1

All trues so its true

- $(P \Rightarrow Q) \vee (Q \Rightarrow P)$

$(\neg P \vee Q) \vee (\neg Q \vee P)$

$\neg P \vee Q \vee \neg Q \vee P$

$\underbrace{\neg P \vee P}_{1} \vee \underbrace{Q \vee \neg Q}_{1}$

$1 \vee 1$

1

OR

$\neg P \vee (Q \vee \neg Q) \vee P$

$(\neg P \vee 1) \vee P$

$1 \vee P$

1

both ways you get 1

P or Q → Q or P

P and Q → Q and P

$P \Rightarrow Q = (\neg P) \vee Q$

$x \Rightarrow y = (\neg x) \vee y$

$ab = ba \Rightarrow$ commutative

$(ab)c = a(bc) \Rightarrow$ associative

$\vee$ = and $\wedge$ = or

$\neg$ = not

in associative property you move brackets not the variables and is only when its all or or ands

4) • n is multiple of 3

• $n = 3k$, not multiple of 3 if $n = 3k+1$ or $3k+2$
where $k \in \mathbb{Z}$

$\forall n \in \mathbb{Z}$ (n^2 is multiple of 3 $\Rightarrow$ n is multiple of 3)

Contrapositive = double flip

$P \Rightarrow Q \overset{\text{turns}}{\underset{\text{into}}{}} \neg Q \Rightarrow \neg P$

n is not a multiple of 3 $\Rightarrow$ n^2 is not multiple of 3

$(n = 3k+1) \vee (n = 3k+2)$ | $(n^2 = 3k+1) \vee (n^2 = 3k+2$

choose either

$n = 3k+1$

$n^2 = 9k^2 + 6k + 1$

$n^2 = 3(3k^2 + 2k) + 1$

This follows $3k+1$ form $\underbrace{3}_{3}\underbrace{(3k^2+2k)}_{k}\underbrace{+1}_{+1}$

$k \in \mathbb{Z}$

$n = 3k+2$

$n^2 = 9k^2 + 12k + 4$

$n^2 = 3(3k^2 + 4k) + 4$

$n^2 = 3(3k^2 + 4k) + 3 + 1$

$n^2 = 3(3k^2 + 4k + 3) + 1$

This form follows $3k+1$

$\underbrace{3}_{3}\underbrace{(3k^2+4k+3)}_{k}\underbrace{+1}_{+1}$

$\boxed{\text{In both instances, } n^2 \text{ is not a multiple of 3 so the given statement } [\forall n \in \mathbb{Z}, (n^2 \text{ multiple of } 3 \Rightarrow n \text{ is multiple of } 3)] \text{ is true as proven by its direct contrapositive.}}$

5) $\forall n \in \mathbb{Z}$ (n^2 is multiple of 4 $\Rightarrow$ n is multiple of 4)

counter-example $\to$ find one example where its not true

If $n = 2$, $n^2 = 4$. 4 is a multiple of 4 but n (2) is not a multiple of 4. So the given statement [$\forall n \in \mathbb{Z}$, (n^2 is a mutiple of 4 $\Rightarrow$ n is a multiple of 4] is false as proven by counter-example.

6) $\sqrt{3} \notin \mathbb{Q}$

Prove by contradiction - one flip no negation

$\sqrt{3} \in \mathbb{Q} \Rightarrow \sqrt{3} = \frac{a}{b}$; $a, b \in \mathbb{Z}, b \neq 0$

a, b not reducible

$\Rightarrow (\sqrt{3})^2 = (\frac{a}{b})^2$

$\Rightarrow 3 = \frac{a^2}{b^2}$

$\Rightarrow 3b^2 = a^2$

a^2 is multiple of 3
a is multiple of 3
so $a = 3k, k \in \mathbb{Z}$

$\Rightarrow 3b^2 = (3k)^2$

$\Rightarrow 3b^2 = 9k^2$

$\Rightarrow b^2 = 3k^2$

b^2 is multiple of 3
b is multiple of 3

$$\boxed{\text{If a is multiple of 3 and b is multiple of 3 that means that } \frac{a}{b} \text{ is reducible.}}$$

If a is multiple of 3 and b is multiple of 3 that means that $\frac{a}{b}$ is reducible. This contradicts what we said earlier that $\frac{a}{b}$ is not reducible. Rational numbers are not reducible but if a and b are both multiples of 3, they can be reduced further. Thus, the statement [$\sqrt{3} \notin \mathbb{Q}$] is true as proven by contradiction.

7)

P	Q	R	$P \land Q$	$(P \land Q) \Rightarrow R$	$\neg Q$	$R \lor \neg Q$	$P \Rightarrow (R \lor \neg R)$
1	1	1	1	1	0	1	1
1	1	0	1	0	0	0	0
1	0	1	0	1	1	1	1
1	0	0	0	1	1	1	1
0	1	1	0	1	0	1	1
0	1	0	0	1	0	0	1
0	0	1	0	1	1	1	1
0	0	0	0	1	1	1	1

$(P \land Q) \Rightarrow R$ and $P \Rightarrow (R \lor \neg Q)$

P and Q must be true for $P \land Q$ to be a true hypothesis.

R has to be false to have a false conclusion.

$P \land Q$ can only be true when both P and Q are true so P = True, Q = True

R has to be false to make it true $\Rightarrow$ false so R = False

P must be true fo P to be a true hypothesis.

$R \lor \neg Q$ has to be false to have a false conclusion

P has to be true to make it true $\Rightarrow$ false so P = True.

$R \lor \neg Q$ has to be false and can only be false when R is false and Q is true be case in OR both have to be false to ouput false

For both these false instances, P is true, Q is true but R is false. You can get the same outcome (false) using the same values for P Q R (T T F). As a result, both these statments are equivalent

8) • $(P \vee Q) \Rightarrow R$
• $Q \vee R$
• $R \Rightarrow P$

You know all this is true (hypothesis)

contradiction - negate conclusion (P is true)

P is true contradiction is $\neg P$ is true
P is false
P is 0

$R \Rightarrow P$ ← prompt said this is true so R has
$R \Rightarrow 0$ to be false (0) for statement to be true

$Q \vee R$ ← prompt said this is true and if R is 0
$Q \vee 0$ then Q has to be true (1) for statement to be true because you need at least one true in OR

$P \vee Q \Rightarrow R$ ← prompt said this is true, we have values
$0 \vee 1 \Rightarrow 0$ for R (0) and Q (1). P should be true (1)
$\underbrace{1 \Rightarrow 0}$ but we contradicted it to be false (0)

This is false but the prompt said this should be true. As we only took logical steps, the only conclusion we can make is that our starting point (P is false) is flawed. Thus, P is true as proven by contradiction.