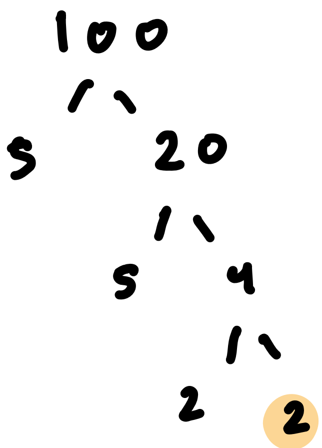
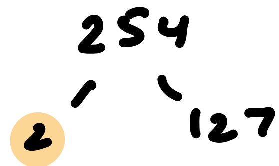
1) $100 = 2 \cdot 2 \cdot 5 \cdot 5 = 2^2 \cdot 5^2$

$254 = 2 \cdot 127 = 2^1 \cdot 127^1$

$GCD(100, 254) = 2$





2) $a_0, a_1, a_2 \dots, a_x, a_{x+1}$

$\downarrow$

GCD

254, 100, 54, 46, 8, 6, 2, 0

GCD(100, 254) = 2

$254 \div 100 \quad r = 54$

$100 \div 54 \quad r = 46$

$54 \div 46 \quad r = 8$

$46 \div 8 \quad r = 6$

$8 \div 6 \quad r = 2$

$6 \div 2 \quad r = 0$

3)

	254	100	54	46	8	6	2
x	1	0	1	-1	2	-11	13
y	0	1	-2	3	-5	28	33

$\gcd(254, 100) = 2 = 254(13) + 100(-33)$

$3302 + -3300$

2

4) $\gcd(a,b)=1 \Rightarrow \exists x,y \in \mathbb{Z} \quad ax+by=1$

$$c=c$$

$$\Rightarrow c(ax+by)=c$$

$$= cax+cby=c$$

$a|bc \Rightarrow bc=aq$

$cax+cby=c= cax+aqy=c$

$$= a(cx+qy)=c$$

$$(cx+qy)\in \mathbb{Z}$$

$$= a|c$$

– – – – – – –

$a|(bc) \rightarrow a|b \vee a|c$

if $\gcd(a,b)=1$ then $a \nmid b$ cause coprimes

so $a|bc \wedge \gcd(a,b)=1$ then $a|c$

5) $\gcd(F_{2023}, F_{2022})$

$\gcd(F_{2022} + F_{2021}, F_{2022})$

$\gcd(F_{2022} + F_{2021} \% F_{2022}, F_{2022})$

$\gcd(F_{2021}, F_{2022})$

$\vdots$

$\gcd(f_1, f_0)$

$\gcd(F_{2023}, F_{2022}) = \gcd(0, 1) = 1$

fibonacci numbers: $0, 1, 1, 2, 3, 5, 8, 13 \ldots$

$f_0\ f_1\ f_2\ f_3\ f_4\ f_5\ f_6\ f_7$

$F_{2023} \div F_{2022} \quad r = f_{2021}$

$F_{2022} \div f_{2021} \quad r = f_{2020}$

$f_{2021} \div f_{2020} \quad r = f_{2019}$

$f_{2020} \div f_{2019} \quad r = f_{2018}$

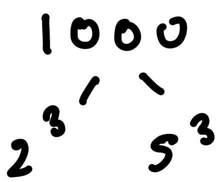
$\vdots$

$F_3 \div f_2 \quad r = 0$

$f_2 = 1$

$\gcd(F_{2023}, f_{2022}) = 1$

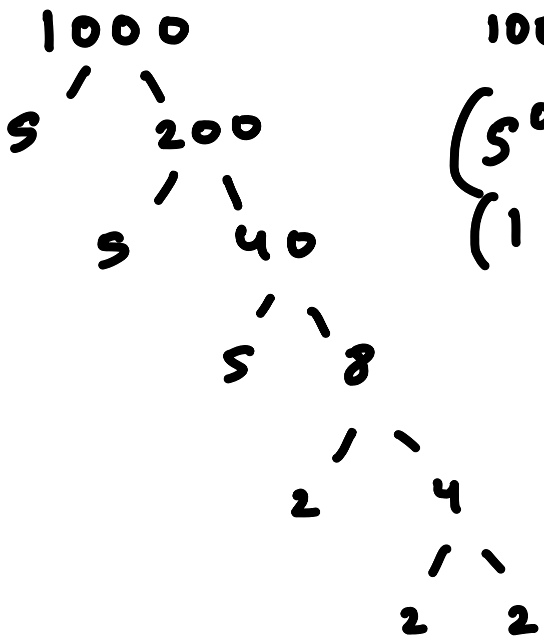
6)



$(2^0 + 2^1 + 2^2 + 2^3)(5^0 + 5^1 + 5^2 + 5^3)$

$(15) \qquad (156)$

2340



$1000 = 5^3 2^3$

$(5^0 + 5^1 + 5^2 + 5^3)(2^0 + 2^1 + 2^2 + 2^3)$

$(1 + 5 + 25 + 125)(1 + 2 + 4 + 8)$

$156 \cdot 15$

2340

7) $1000 = 2^3 \cdot 5^2$

$2^0 = 5^0 = 1$	$5^1 = 5$	$5^2 = 25$	$5^3 = 125$
$2^1 = 2$	$2^1 \cdot 5^1$	$2^1 \cdot 5^2$	$2^1 \cdot 5^3$
$2^2 = 4$	$2^2 \cdot 5^1$	$2^2 \cdot 5^2$	$2^2 \cdot 5^3$
$2^3 = 8$	$2^3 \cdot 5^1$	$2^3 \cdot 5^2$	$2^3 \cdot 5^3$

2^1 = appens 4 times $\quad$ 5^1 = appens 4 times

2^2 = appens 4 times $\quad$ 5^2 = appens 4 times

2^3 = appens 4 times $\quad$ 5^3 = appens 4 times

$$(2^1)^4 (2^2)^4 (2^3)^4 (5^1)^4 (5^2)^4 (5^3)^4$$

$$2^4 \; 2^3 \; 2^{12} \; 5^4 \; 5^8 \; 5^{12}$$

$$2^{24} \; 5^{24}$$

$$10^{24}$$

8) $\gcd(a,b)=1$

coprime!

9) $\binom{2n}{n} = \frac{(2n)!}{n!(2n-n)!} = \frac{(2n)!}{n!\,n!} = \frac{(2n)!}{(n!)^2}$

$= \frac{1\cdot 2\cdot 3\ldots\cdot n\cdot\ldots\cdot 2n}{(1\cdot 2\cdot 3\ldots\cdot n)(1\cdot 2\cdot 3\ldots\cdot n)} = \frac{n+1\ldots 2n}{1\cdot 2\cdot 3\ldots\cdot n}$

$n < p < 2n$

p is factor of numerator

numerator is divisible by p

P is not a factor of denominator

Denominator is not divisible by P

P cannot be cancelled out by denominator

P divides $\binom{2n}{n}$

Problem

a) mod 127

x = # of jumps

$5x \equiv 1 \bmod 127$

$x = 5^{-1}$

$5 \cdot 5^{-1} \equiv 1 \bmod 127$

	127	5	2	1
x	1	0	doesnt matter	
y	0	1	-25	51

$x = 5^{-1} = 51$ jumps to land on point 1

b) $5(2x+y \equiv 4 \mod 17)$
$+ \quad 5x - 5y \equiv 9 \mod 17$

$\rightarrow$

$$\begin{array}{rl} & 10x + 5y \equiv 20 \mod 17 \\ + & 5x - 5y \equiv 9 \mod 17 \\ \hline & 15x \equiv 29 \mod 17 \end{array}$$

$15x \equiv 12 \mod 17$

$x \equiv 15^{-1} \, 12 \mod 17$

	17	15	2	1
x	1	0	doesn't matter	
Y	0	1	-1	8

$15^{-1} = 8$

$x \equiv 8(12) \mod 17$

$x \equiv 96 \mod 17$

$x \equiv 11$

$2x + y \equiv 4 \mod 17$

$2(11) + y \equiv 4 \mod 17$

$22 + y \equiv 4 \mod 17$

$y \equiv -18 \mod 17$

$y \equiv 16$