Write all 7) - 12) properties for the algebra of the power set of a set $S \neq \emptyset$, namely: $(P(S), \cup, \cap, {}^{c}; \emptyset, S)$, where $P(S) = \{ X : X \subseteq S \}$,

if you haven't done this for today.

$(B, +, \cdot, ', 0, 1)$ → + = or and · = and

7) $\cap, \cup$ are idempotent → idempotent = element that is in a set and is unchanged in value when multiplied or operated on by itself

$x \cup x = x$

$x \cap x = x$

8) $x \cap 0 = 0$ → annulment law = there is a constant that when combined with a signal cancels out the signal

$x \cup 1 = 1$

9) $y \cap x \overset{+}{\cup} x = x$

$(y \overset{+}{\cup} x) \cap x = x$ (dual)

10) $(x + y) = x' y'$ → demorgans law = complement of product of all terms is equal to sum of complement of each term

$(xy)' = x' + y', \forall x, y \in B$

11) $(a + b) + c = a + (b + c)$ → associativity = 3 bool variables can be or'ed and and'ed right to left or left to right

$(a \cdot b) \cdot c = a \cdot (b \cdot c)$

12) ??

HW 6.2-assigned :

Prove all properties 8) – 12) using axioms 1) – 6) and the properties you just proved. **Attention**: For property 11), associativity, use truth tables on the variables and assume only the values of the 2-element B.A., namely B = {0, 1}, also called the switching algebra.

8) $x + 1 = 1$ or
$x \cdot 0 = 0$ and

$x + 1 \overset{(5)}{=} \overbrace{x + x}^{(7)\ x} + x' = x + x' \overset{(5)}{=} 1$
$x + x'$

annulment law

9) $x'' = x, \forall x \in B \longleftrightarrow (x')' = x$

We know $x + x' = 1$ Switch $x' + x = 1$
$x \; x' = 0$ x & x' $x' \; x = 0$

According to commutativeness, thes equations still hold true so compliment of x' is x

double complement

10) $(x+y) = x'y'$ $(xy)' = x'+y', \forall x,y \in B$

Use $x + x' = 1$ and $x x' = 0$ $x = x+y$ $x' = x'y'$

$(x+y)+(x'y') = 1$
$(x + x'y') + y = 1$

$(x+y) \cdot (x'y') = 0$
$(x \cdot x'y') + (y \cdot x'y') = 0$

demorgan's law

11) $(a+b)+c = a+(b+c)$
$(a \cdot b) \cdot c = a \cdot (b \cdot c)$

associativity

12) ??