Why does this algorithm work?

Hint

We know: $N = \underbrace{\text{integer part}}_{n} \cdot \underbrace{\text{fraction part}}_{m}$

$N_{(r-1)c} = r^n - r^{-m} - N$

1) $M + N_{(r-1)c} = M + r^n - r^{-m} - N$

2) Whether we follow a) or b) depends on the presence of an e. a. c.

What is the magnitude of the e.a.c.? It's an 'overflow':

e.a.c $\underbrace{10 \ldots\ldots 0}_{n}$ $= r^n$ On **branch a)** there is an e. a. c., which means we have:

$\underbrace{M + r^n - r^{-m} - N}_{\text{from 1)}} \geq \underbrace{r^n}_{\text{e.a.c.}} \leftrightarrow M - N \geq r^{-m} \leftrightarrow$

r^{-m} = smallest positive number in our representation

$\leftrightarrow M - N > 0$ or $M > N$ It also means, that the case when

$M - N = 0$ will take branch b), which means that 0 will be expressed as -0 by this Alg.

Continue justifying the computations in the branches 2-a) and 2-b) as HW.

3-A Do the following conversion problems:

(a) Convert decimal 34.4375 to binary.

(b) Calculate the binary equivalent of 1/3 out to 8 places. Then convert from binary to decimal. How close is the result to 1/3?

(c) Convert the binary result in (b) into hexadecimal. Then convert the result to decimal. Is the answer the same?

a) 34.4375 to binary

34/2 = 17 rem 0
17/2 = 8.5 rem 1
8/2 = 4 rem 0
4/2 = 2 rem 0
2/2 = 1 rem 0
1/2 = 0 rem 1

Binary: 100010

.4375 · 2 = 0.875
.875 · 2 = 1.75
.75 · 2 = 1.5
.5 · 2 = 1

Binary = 0111

100010.0111

b) $\frac{1}{3}$ = 0.3333 · 2 = 0.666 → 0.01010101

2^{-1} 2^{-3} 2^{-5} 2^{-7}

0.666 · 2 = 1.332
0.332 · 2 = 0.664
0.664 · 2 = 1.328
0.328 · 2 = 0.656
0.656 · 2 = 1.312
0.312 · 2 = 0.624
0.624 · 2 = 1.248

0.6640625

c) Binary: 01010101 to Hexadecimal: 0.55

Hexadecimal: 0.33203125

3- B Determine the value of base x if $(211)_x = (152)_8$.

$(152)_8 = (1 \cdot 8^2) + (5 \cdot 8^1) + (2 \cdot 8^0) = 106$

$(211)_x = (2 \cdot x^2) + (1 \cdot x) + (1 \cdot 1) = 2x^2 + x + 1$

$2x^2 + x + 1 = 106$

$2x^2 + x + 1 - 106 = 0$

$2x^2 + x - 105 = 0$

$(2x + 15)(x - 7) = 0$

$x = -\frac{15}{2}, 7$ # $x > 0$ so $\boxed{7}$

3-C Noting that $3^2 = 9$, formulate a simple procedure for converting base-3 numbers directly to base-9. Use the procedure to convert $(2110201102220112)_3$ to base 9.

$(21\,10\,20\,11\,02\,22\,01\,12)_3$

$7\ \ 3\ \ 6\ 4\ \ 2\ \ 8\ \ 1\ \ 5$

$\text{Base}_9 = 73642815$

3-D The solutions to the quadratic equation

$x^2 - 11x + 22 = 0$

are $x = 3$ and $x = 6$.

Determine the base of the numbers in the equation.

$x^2 - 11x + 22 = 0$ $\quad x = 3$ $\quad x = 6$ $\quad$ base $= b$

$x = 3$

$3^2 - 3(1 \times B^1 + 1 \times B^0) + (2 \times B^1 + 2 \times B^0) = 0$

$9 - 3(B + 1) + (2B + 2) = 0$

$9 - 3B - 3 + 2B + 2 = 0$

$-B + 8 = 0$

$B = 8$

$x = 6$

$6^2 - 6(1 \times B^1 + 1 \times B^0) + (2 \times B^1 + 2 \times B^0) = 0$

$36 - 6(B + 1) + (2B + 2) = 0$

$36 - 6B - 6 + 2B + 2 = 0$

$32 - 4B = 0$

$-4B = -32$

$B = 8$

$B = 8$

3-E Convert the hexadecimal number 68BE to binary, and then convert it from binary to octal and then to base 32.

68BE

$(6 \cdot 16^3) + (8 \cdot 16^2) + (11 \cdot 16^1) + (14 \cdot 16^0)$

$24576 + 2048 + 176 + 14 = 26814$

26814/2 = 13407 rem 0
13407/2 = 6703.5 rem 1
6703/2 = 3351 rem 1
3351/2 = 1675.5 rem 1
1675/2 = 837.5 rem 1
837/2 = 418.5 rem 1
418/2 = 209 rem 0
209/2 = 104.5 rem 1
104/2 = 52 rem 0
52/2 = 26 rem 0
26/2 = 13 rem 0
13/2 = 6.5 rem 1
6/2 = 3 rem 0
3/2 = 1.5 rem 1
1/2 = 1 rem 1

110 1000 1011 1110
6 4 2 7 6

Octal: 64276

$Base_{32}$: 6SU