

Project 2 Sorting Report

Using `std::chrono::high_resolution_clock::now()` I timed each algorithm after supplying each input file via `std::ifstream` and these are the results I obtained below. To calculate average, the times for each input were averaged to the nearest millisecond. Under each input, you will see the correct median listed which was used to verify if each sort returned the correct median. Lastly, worst case quick select input produces an input of 20k elements which was also fed to each algorithm to sort fully or halfway to find the median three times and the average of those three times are also available on the last table. I was timing on MacBook Air, M2, 2022, Apple Silicon M2 chip, 8 GB memory and results may vary across devices.

Input sizes for input 1, 2 and 3 are 1k (1,000)

| Algorithm | Input 1 (50492874) | Input 2 (19250688) | Input 3 (70244369) | Average (rounded to nearest ms) |
|------------------|-----------------------|-----------------------|-----------------------|------------------------------------|
| HalSelectionSort | 10ms | 6ms | 10ms | 9ms |
| StandardSort | 0ms | 0ms | 0ms | 0ms |
| MergeSort | 2ms | 1ms | 2ms | 2ms |
| InPlaceMergeSort | 1ms | 0ms | 1ms | 1ms |
| HalfHeapSort | 0ms | 0ms | 0ms | 0ms |
| QuickSelect | 0ms | 0ms | 0ms | 0ms |
| MedianOfMedians | 0ms | 0ms | 0ms | 0ms |

Input sizes for input 4, 5 and 6 are 31k (31,623)

| Algorithm | Input 4 (50173306) | Input 5 (18637175) | Input 6 (70984972) | Average (rounded to nearest ms) |
|------------------|-----------------------|-----------------------|-----------------------|------------------------------------|
| HalSelectionSort | 3445ms | 3447ms | 3488ms | 3460ms |
| StandardSort | 1ms | 1ms | 1ms | 1ms |
| MergeSort | 29ms | 30ms | 29ms | 29ms |
| InPlaceMergeSort | 14ms | 14ms | 15ms | 14ms |
| HalfHeapSort | 3ms | 3ms | 4ms | 3ms |
| QuickSelect | 0ms | 0ms | 0ms | 0ms |
| MedianOfMedians | 0ms | 0ms | 0ms | 0ms |

Input sizes for input 7, 8 and 9 are 1M (1,000,000)

| Algorithm | Input 7 (49971079) | Input 8 (18675104) | Input 9 (70722421) | Average (rounded to nearest ms) |
|------------------|-----------------------|-----------------------|-----------------------|------------------------------------|
| HalSelectionSort | Input too big | Input too big | Input too big | N/A |
| StandardSort | 48ms | 53ms | 49ms | 50ms |
| MergeSort | 1043ms | 1041ms | 1047ms | 1044ms |
| InPlaceMergeSort | 545ms | 545ms | 548ms | 546ms |
| HalfHeapSort | 157ms | 159ms | 159ms | 158ms |
| QuickSelect | 30ms | 27ms | 29ms | 20ms |
| MedianOfMedians | 0ms | 0ms | 0ms | 0ms |

Inputs produced by the worst case quickselect input and are of size 20k (20,000)

| Algorithm | Trial 1 - 20k | Trial 2 - 20k | Trial 3 - 20k | Average |
|------------------|---------------|---------------|---------------|---------|
| HalSelectionSort | 1156ms | 1152ms | 1155ms | 1ms |
| StandardSort | 0ms | 0ms | 0ms | 0ms |
| MergeSort | 15ms | 15ms | 16ms | 0ms |
| InPlaceMergeSort | 6ms | 6ms | 6ms | 0ms |
| HalfHeapSort | 1ms | 1ms | 1ms | 0ms |
| QuickSelect | 358ms | 356ms | 356ms | 0ms |
| MedianOfMedians | 0ms | 0ms | 0ms | 0ms |

Algorithmic Analysis:

- HalSelectionSort ($O(n^2)$ comparisons): This brute-force method iterates through the list, finding the minimum element and swapping it with the first. While conceptually simple, its performance suffers for larger datasets.
- StandardSort: As expected, the built-in function shines with its efficient implementation and consistently fast performance across all input sizes.
- Merge Sort and InPlaceMerge Sort ($O(n \log n)$ comparisons): These divide-and-conquer algorithms recursively split the list, sort each sub-list, and then merge them. Their time complexity explains the steady performance increase with input size.
- HalfHeapSort ($O(n \log n)$ comparisons): This algorithm builds a heap from the first half of the list, then inserts and adjusts the heap for the remaining elements. Its performance is comparable to Merge Sort, demonstrating the versatility of heap-based sorting.
- QuickSelect and MedianOfMedians ($O(n)$ expected time, $O(n^2)$ worst-case): These probabilistic methods randomly select a pivot element and partition the list around it.

While their average performance is excellent, the worst-case scenario can be significantly slower, as seen with the worst-case QuickSelect input.

Observations and Discussion:

- The results mostly align with the expected time complexities. StandardSort, Merge Sort, InPlaceMerge Sort, HalfHeapSort, QuickSelect, and MedianOfMedians all outperform HalSelectionSort, as predicted.
- QuickSelect and MedianOfMedians shine with their $O(n)$ expected time, but their worst-case scenarios are significantly slower. This highlights the importance of choosing the right algorithm for the specific needs of your application.
- It's safe to say that algorithms like Merge Sort and Heap Sort can save significant time on large datasets by stopping the sort once the median is found.

Comparing the Methods:

- Worst-case performance: StandardSort, Merge Sort, and InPlaceMerge Sort are the go-to choices with their guaranteed $O(n \log n)$ complexity.
- Average performance: QuickSelect and MedianOfMedians excel, but remember their worst-case is horrendous.
- Large inputs: StandardSort remains efficient, while Merge Sort and InPlaceMerge Sort offer good performance. QuickSelect and MedianOfMedians can be great choices if early termination is utilized effectively.

Interesting and Unexpected Results:

- The small difference between StandardSort, Merge Sort, and InPlaceMerge Sort for larger inputs was surprising. It suggests that the overhead of Merge Sort and InPlaceMerge Sort might be minimal in this specific implementation.
- The consistent good performance of HalfHeapSort was noteworthy. While often overshadowed by other algorithms, it proved to be a strong contender in this analysis.