IEM-5013

Project Presentation

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Outline

- Problem Statement
- Data Overview
- Solution Approach and Model Formulation
- Results and Analysis
- Conclusion and Recommendations

Problem Statement

Maximizing Employee Satisfaction in Restaurant Work Scheduling

Problem Background

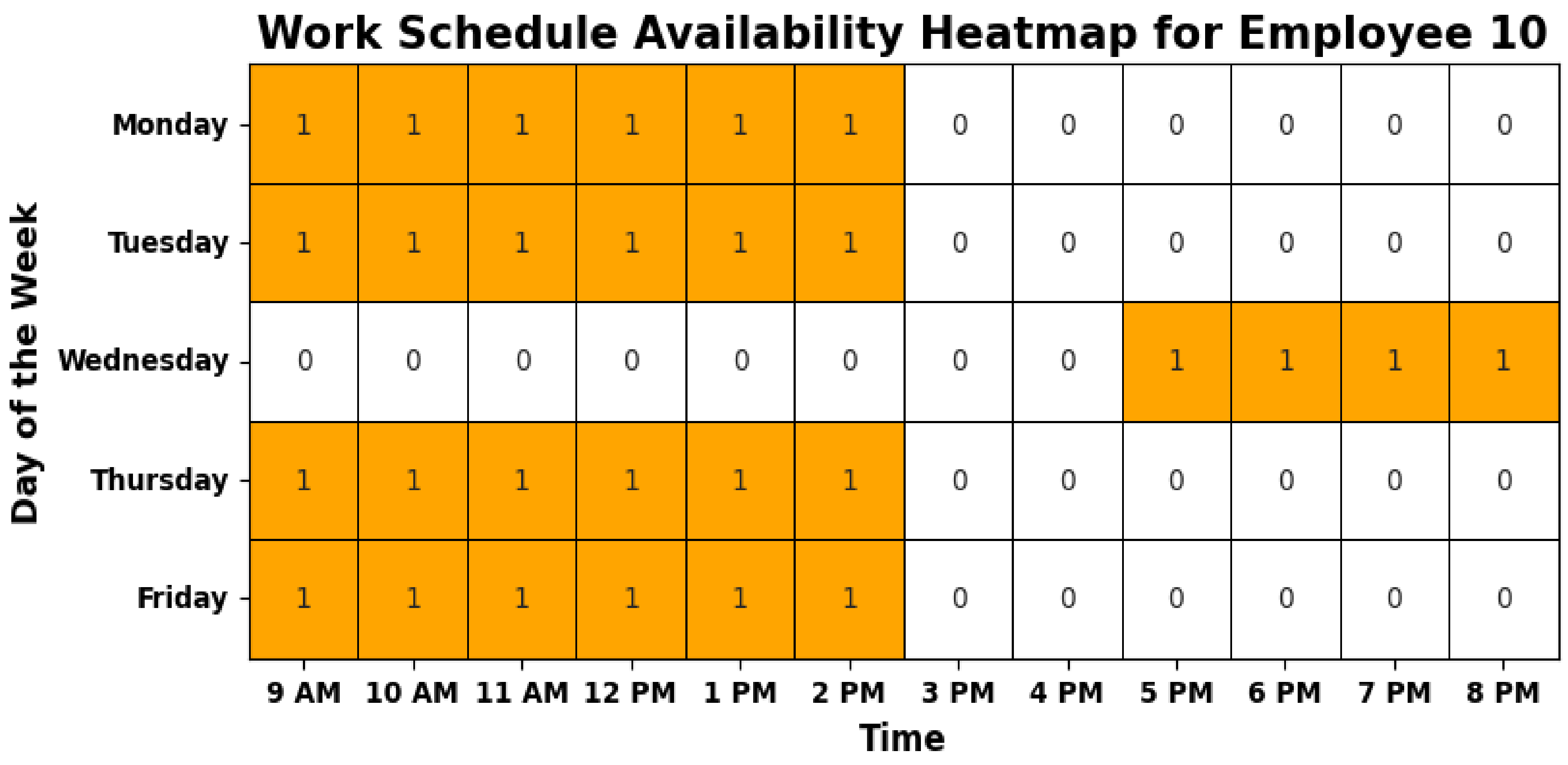
A small, family-owned, fast-casual restaurant seeks to use optimization methods to construct its weekly work schedule. The schedule is subject to requirements for role staffing and employees' availability as well as their desired number of working hours. The restaurant operates **Monday to Friday** from **11:00 AM to 8:00 PM**, with additional staffing needed **2 hours before opening** for preparation and **1 hour after closing** for cleanup, for a total of **12 hours** per day.

Key Points:

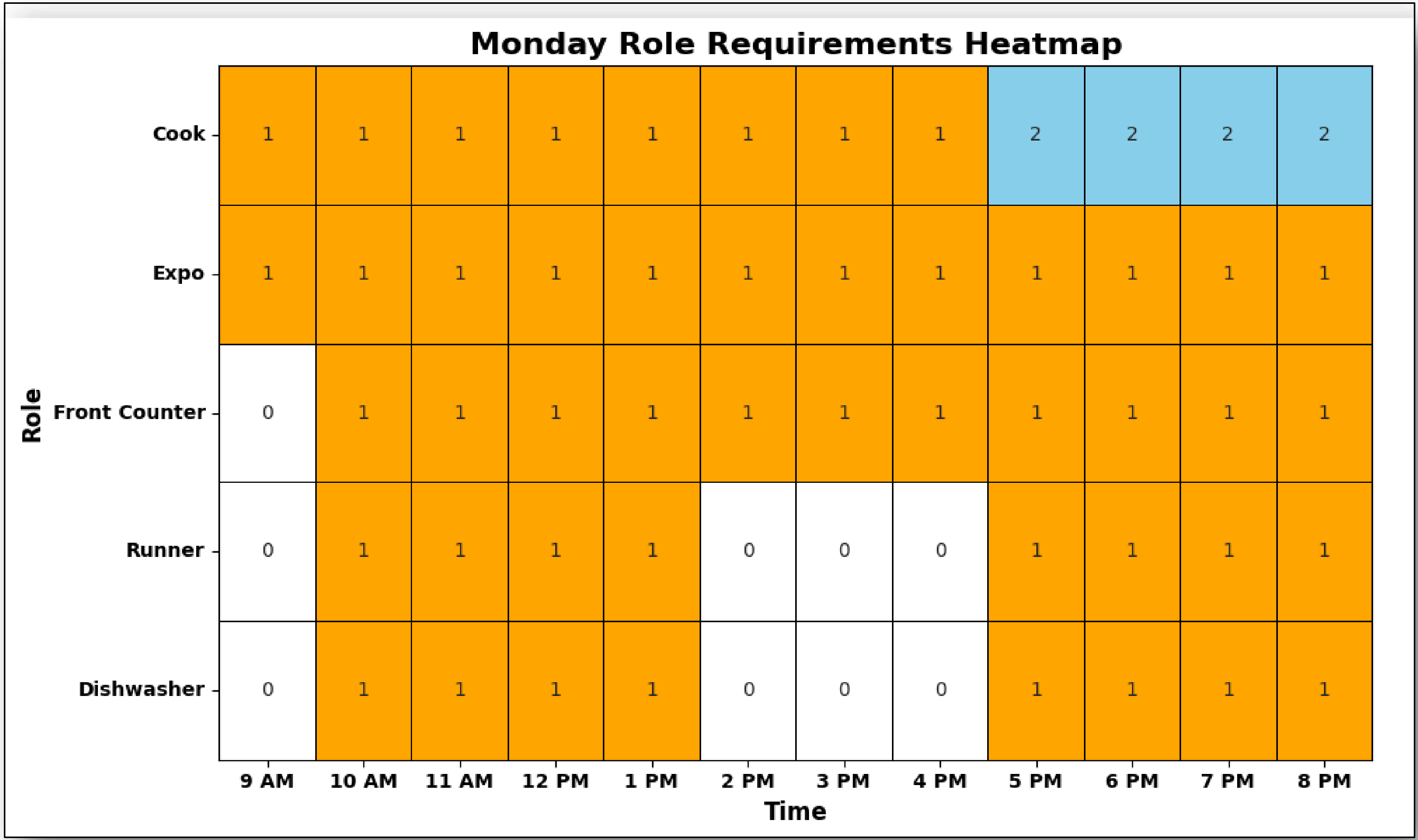
- The restaurant needs to fill **5 main roles**: **Cook**, **Expo**, **Front Counter**, **Runner**, and **Busser**.
- The number of positions to fill for a given role may vary by the time of day and day of the week.
- Employees can qualify for multiple roles and have unique availability schedules.
- Each employee specifies their preferred working hours and weekly availability.
- Shifts are a minimum of **4 hours**, and **overtime is not permitted**.

The goal is to create a schedule that minimizes deviations from employees' desired number of working hours while meeting all role requirements.

Data Overview



Data Overview



Initial Formulation:

The goal is to minimize the total deviation from the desired working hours for all employees.

Solution Approach and Formulation

$$\text{Minimize} \sum_{e} \delta_e$$

$$\delta_e \geq \sum_{r,h,d} x_{e,r,h,d} - W_{\max,e} \quad \forall e.$$

$$\delta_e \geq 0 \quad \forall e.$$

$$\delta_e \geq W_{\min,e} - \sum_{r,h,d} x_{e,r,h,d} \quad \forall e.$$

$$\sum_{e} x_{e,r,h,d} = n_{r,h,d} \quad \forall r, h, d.$$

$$\sum_{r} x_{e,r,h,d} \leq A_{e,h,d} \quad \forall e, h, d.$$

$$\sum_{h,d} x_{e,r,h,d} \leq 40 R_{e,r} \quad \forall e, r.$$

$$x_{e,r,h,d} \in \{0, 1\}, \quad \delta_e \geq 0 \quad integral \quad \forall e, r, h, d.$$

Solution Approach and Formulation

Decision Variables

- δ_e:
  - An integer variable.
  - Represents the absolute deviation between the hours worked by employee e and their desired total hours.
- $x_{e,r,h,d}$:
  - A binary variable.
  - Indicates whether employee e is scheduled for role r during hour h on day d.

Parameters

- $W_{\max,e}$ and $W_{\min,e}$: Represent the maximum and minimum hours employee e desires to work in a week.
- $n_{r,h,d}$: The number of positions in role r that need to be filled in hour h on day d.
- $A_{e,h,d}$: A binary parameter indicating whether employee e is available to work during hour h on day d.
- $R_{e,r}$: A binary parameter indicating whether employee e is qualified to work in role r.

Constraints

1. **Deviation Constraints:**

   - Ensures that the deviation captures cases where an employee works more hours than their maximum desired hours:

$$\delta_e \geq \sum_{r,h,d} x_{e,r,h,d} - W_{\max,e} \quad \forall e.$$

   - Ensures that the deviation captures cases where an employee works fewer hours than their minimum desired hours:

$$\delta_e \geq W_{\min,e} - \sum_{r,h,d} x_{e,r,h,d} \quad \forall e.$$

2. **Role Fulfillment:**

   - Ensures that the required number of positions are filled for each role, hour, and day:

$$\sum_{e} x_{e,r,h,d} = n_{r,h,d} \quad \forall r, h, d.$$

3. **Employee Availability:**

   - Ensures employees are only scheduled during hours they are available:

$$\sum_{r} x_{e,r,h,d} \leq A_{e,h,d} \quad \forall e, h, d.$$

4. **Qualification and Workload:**

   - Limits the total work hours for an employee in a specific role based on their qualification:

$$\sum_{h,d} x_{e,r,h,d} \leq 40 R_{e,r} \quad \forall e, r.$$

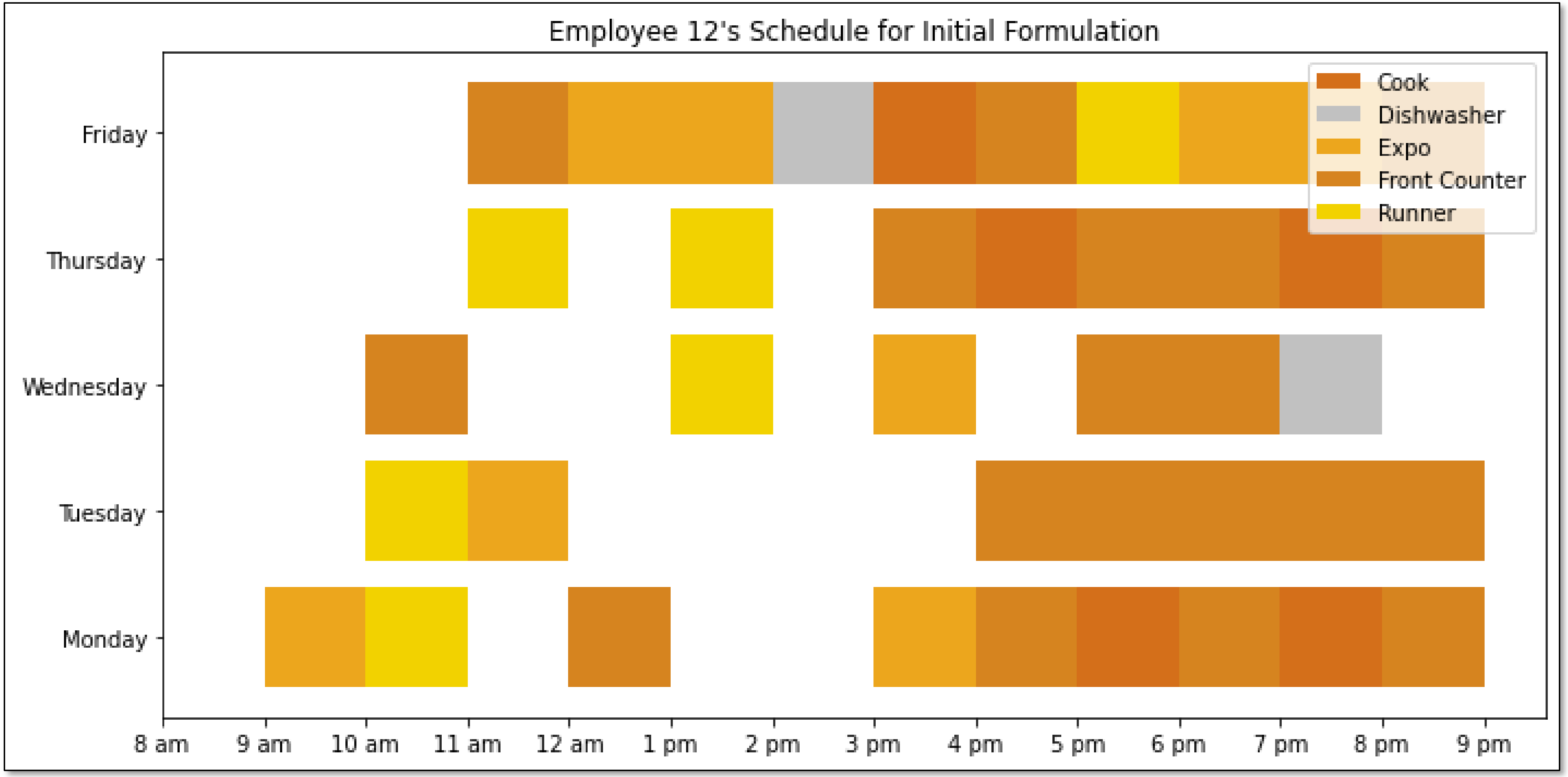
5. **Binary Variables:**

   - Ensures that the decision variables are binary for scheduling ($x_{e,r,h,d}$) and non-negative integers for deviation (δ_e):

$$x_{e,r,h,d} \in \{0,1\}, \quad \delta_e \geq 0 \quad \forall e, r, h, d.$$

Solution Approach and Formulation

Solution Approach and Formulation



Improved Formulation:

The goal remains the same: To minimize the total deviation from the desired working hours for all employees.

Solution Approach and Formulation

$$\text{Minimize } \sum_{e} \delta_e$$

$$\delta_e \geq \sum_{r,h,d} x_{e,r,h,d} - W_{\max,e} \quad \forall e.$$

$$\delta_e \geq 0 \quad \forall e.$$

$$\delta_e \geq W_{\min,e} - \sum_{r,h,d} x_{e,r,h,d} \quad \forall e.$$

$$\sum_{e} x_{e,r,h,d} = n_{r,h,d} \quad \forall r,h,d.$$

$$\sum_{r} x_{e,r,h,d} \leq A_{e,h,d} \quad \forall e,h,d.$$

$$\sum_{h,d} x_{e,r,h,d} \leq 40R_{e,r} \quad \forall e,r.$$

$$x_{e,r,h,d} + x_{e,r,h+1,d} + x_{e,r,h+2,d} + x_{e,r,h+3,d} \geq 4y_{e,r,h,d} \quad \forall e,r,h \in [1,9],d.$$

$$y_{e,r,h,d} - x_{e,r,h,d} \leq 0 \quad \forall e,r,h \in [1,9],d.$$

$$y_{e,r,h+1,d} + x_{e,r,h,d} + x_{e,r,h+1,d} \leq 2 \quad \forall e,r,h \in [1,9],d.$$

$$y_{e,r,h+1,d} + x_{e,r,h,d} - x_{e,r,h+1,d} \geq 0 \quad \forall e,r,h \in [1,9],d.$$

$$y_{e,r,h,d} = 0 \quad \forall e,r,h \in [10,12],d.$$

$$x_{e,r,0,d} = 0 \quad \forall e,r,d.$$

$$x_{e,r,h,d}, y_{e,r,h,d} \in \{0,1\}, \quad \delta_e \geq 0 \quad \forall e,r,h,d.$$

Decision Variable Added

- $y_{e,r,h,d}$:
  - A binary variable.
  - Indicates whether employee e starts working in role r at hour h on day d.

Constraints Added

1. **Minimum Shift Length:**
   - Ensures that each shift is at least 4 consecutive hours:

$$x_{e,r,h,d} + x_{e,r,h+1,d} + x_{e,r,h+2,d} + x_{e,r,h+3,d} \geq 4y_{e,r,h,d} \quad \forall e, r, h \in [1, 9], d.$$

2. **Shift Start Identification:**
   - Ensures that $y_{e,r,h,d}$ is only set to 1 if $x_{e,r,h,d}$ is 1:

$$y_{e,r,h,d} - x_{e,r,h,d} \leq 0 \quad \forall e, r, h \in [1, 9], d.$$

3. **Non-Overlapping Shifts:**
   - Ensures shifts only start after a period of no work:

$$y_{e,r,h+1,d} + x_{e,r,h,d} + x_{e,r,h+1,d} \leq 2 \quad \forall e, r, h \in [1, 9], d.$$

   - Ensures that a new shift cannot start immediately after a current hour of work:

$$y_{e,r,h+1,d} + x_{e,r,h,d} - x_{e,r,h+1,d} \geq 0 \quad \forall e, r, h \in [1, 9], d.$$

4. **Dummy Hours:**
   - Ensures that no work is scheduled in dummy hours:

$$y_{e,r,h,d} = 0 \quad \forall e, r, h \in [10, 12], d.$$

$$x_{e,r,0,d} = 0 \quad \forall e, r, d.$$

5. **Binary Variables:**
   - Ensures the decision variables are binary or integers:

$$x_{e,r,h,d}, y_{e,r,h,d} \in \{0, 1\}, \quad \delta_e \geq 0 \quad \forall e, r, h, d.$$

Improved Formulation

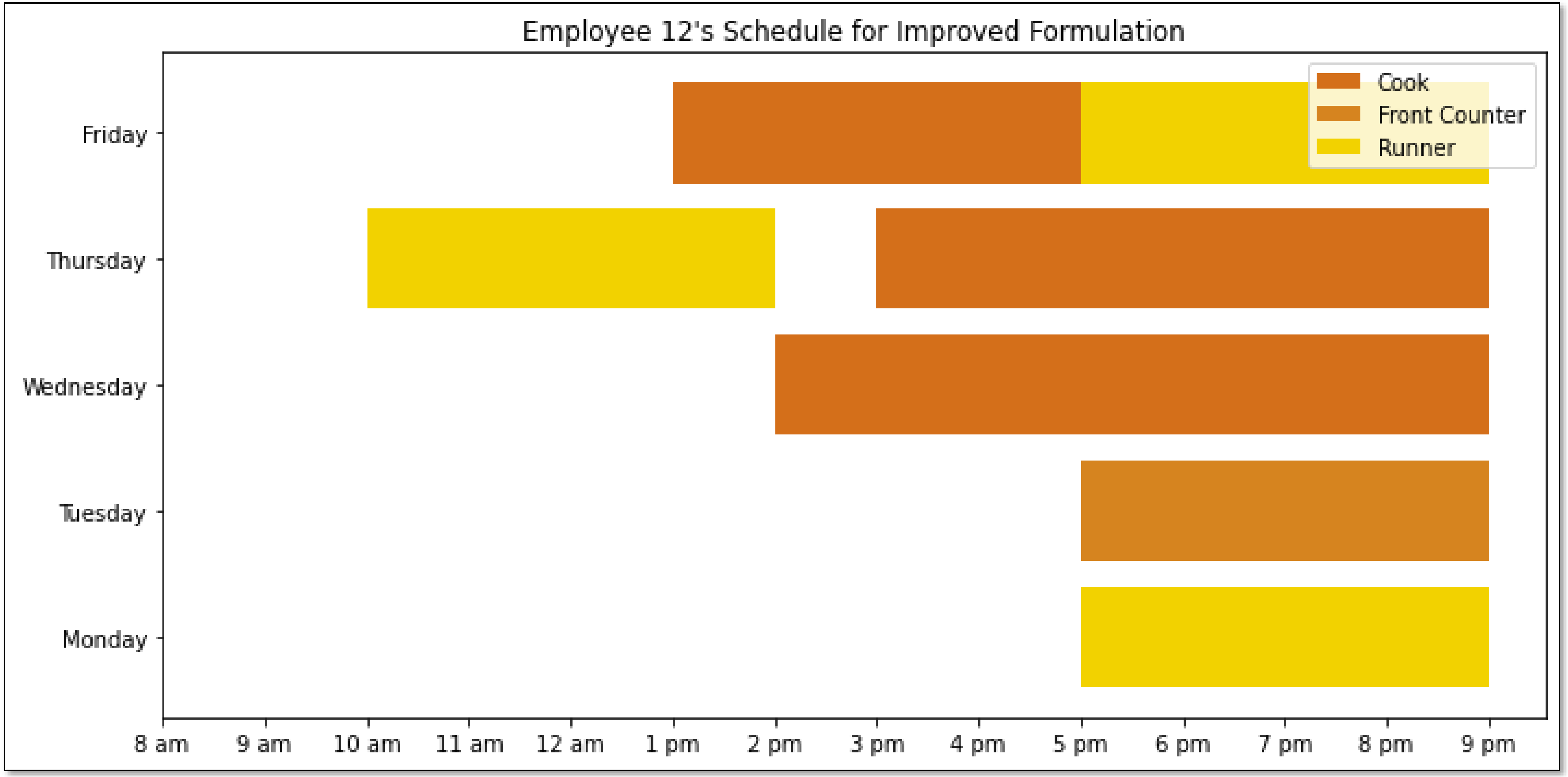
A new decision variable and constraints are added compared to the initial formulation.

Why Is It Improved?

Improved Formulation vs. Initial Formulation

- **The initial formulation** focuses on minimizing the total deviation of employees' scheduled hours from their desired hours to best meet employee work-hour preferences.
- **The improved formulation** adds a shift start variable and constraints to control minimum shift length and rules for shift starting to address issues in the initial model.
- **The initial formulation** provides a basic scheduling framework, then the **improved formulation** incorporates additional constraints to improve the practicality of the scheduling.

Solution Approach and Formulation



Variant Formulation:

The goal is to minimize the working hours of managers to provide more flexibility for handling absences or emergencies.

Solution Approach and Formulation

$$\text{Minimize} \sum_{e=11}^{12} x_{e,r,h,d}$$

$$\sum_{r,h,d} x_{e,r,h,d} - W_{\max,e} \leq 0 \quad \forall e$$

$$W_{\min,e} - \sum_{r,h,d} x_{e,r,h,d} \leq 0 \quad \forall e$$

$$\sum_{e} x_{e,r,h,d} = n_{r,h,d} \quad \forall r,h,d$$

$$\sum_{r} x_{e,r,h,d} \leq A_{e,h,d} \quad \forall e,h,d$$

$$\sum_{h,d} x_{e,r,h,d} \leq 40R_{e,r} \quad \forall e,r$$

$$x_{e,r,h,d} + x_{e,r,h+1,d} + x_{e,r,h+2,d} + x_{e,r,h+3,d} \geq 4y_{e,r,h,d} \quad \forall e,r,h \in [1,9],d.$$

$$y_{e,r,h,d} - x_{e,r,h,d} \leq 0 \quad \forall e,r,h \in [1,9],d.$$

$$y_{e,r,h+1,d} + x_{e,r,h,d} + x_{e,r,h+1,d} \leq 2 \quad \forall e,r,h \in [1,9],d.$$

$$y_{e,r,h+1,d} + x_{e,r,h,d} - x_{e,r,h+1,d} \geq 0 \quad \forall e,r,h \in [1,9],d.$$

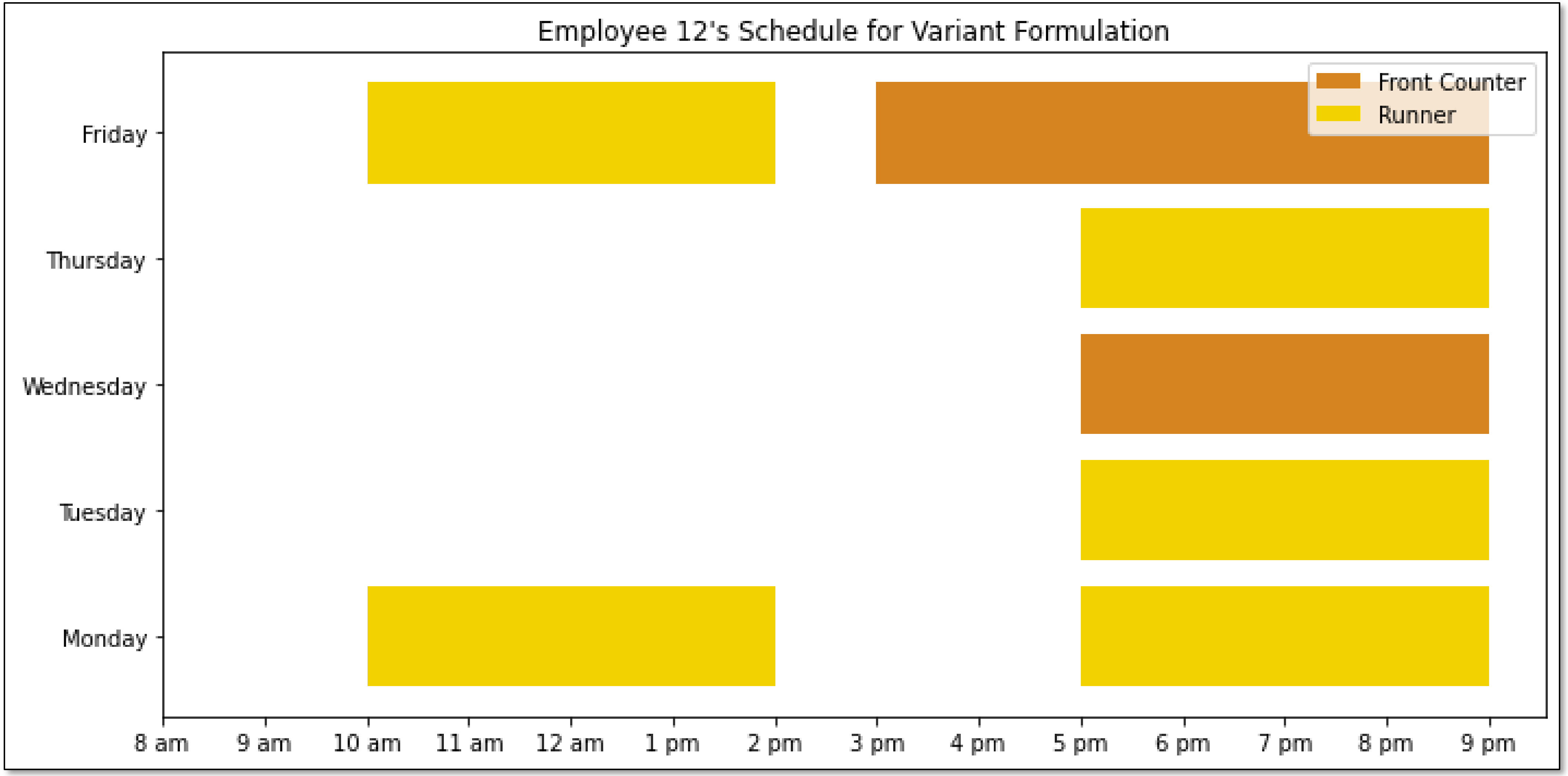
$$y_{e,r,h-1,d} - x_{e,r,h,d} \geq 0 \quad \forall e,r,h \in [10,12],d.$$

$$y_{e,r,h,d} = 0 \quad \forall e,r,h \in [10,12],d.$$

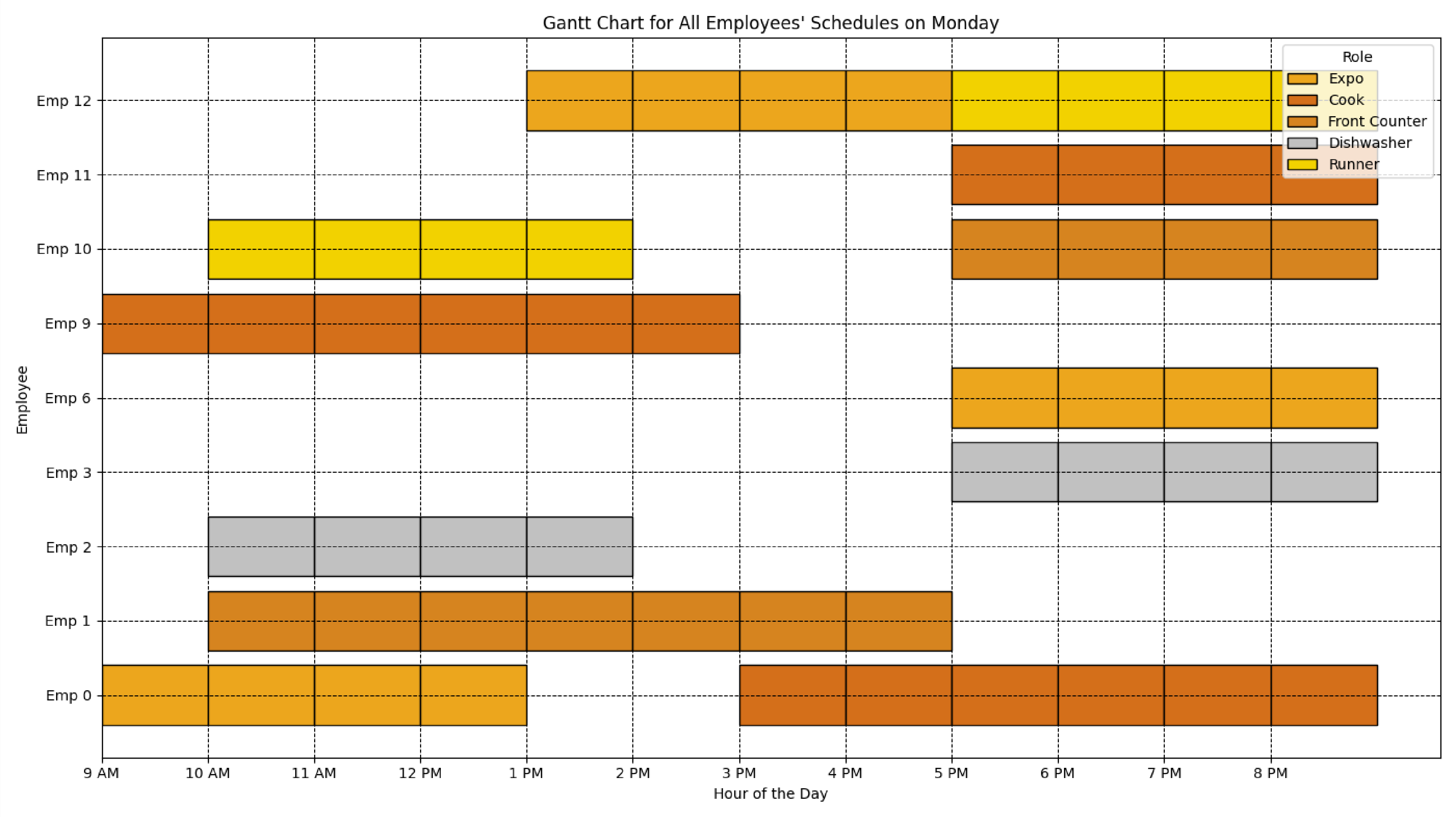
$$x_{e,r,0,d} = 0 \quad \forall e,r,d.$$

$$x_{e,r,h,d}, y_{e,r,h,d} \in \{0,1\}, \quad \forall e,r,h,d.$$

Solution Approach and Formulation



Results and Analysis



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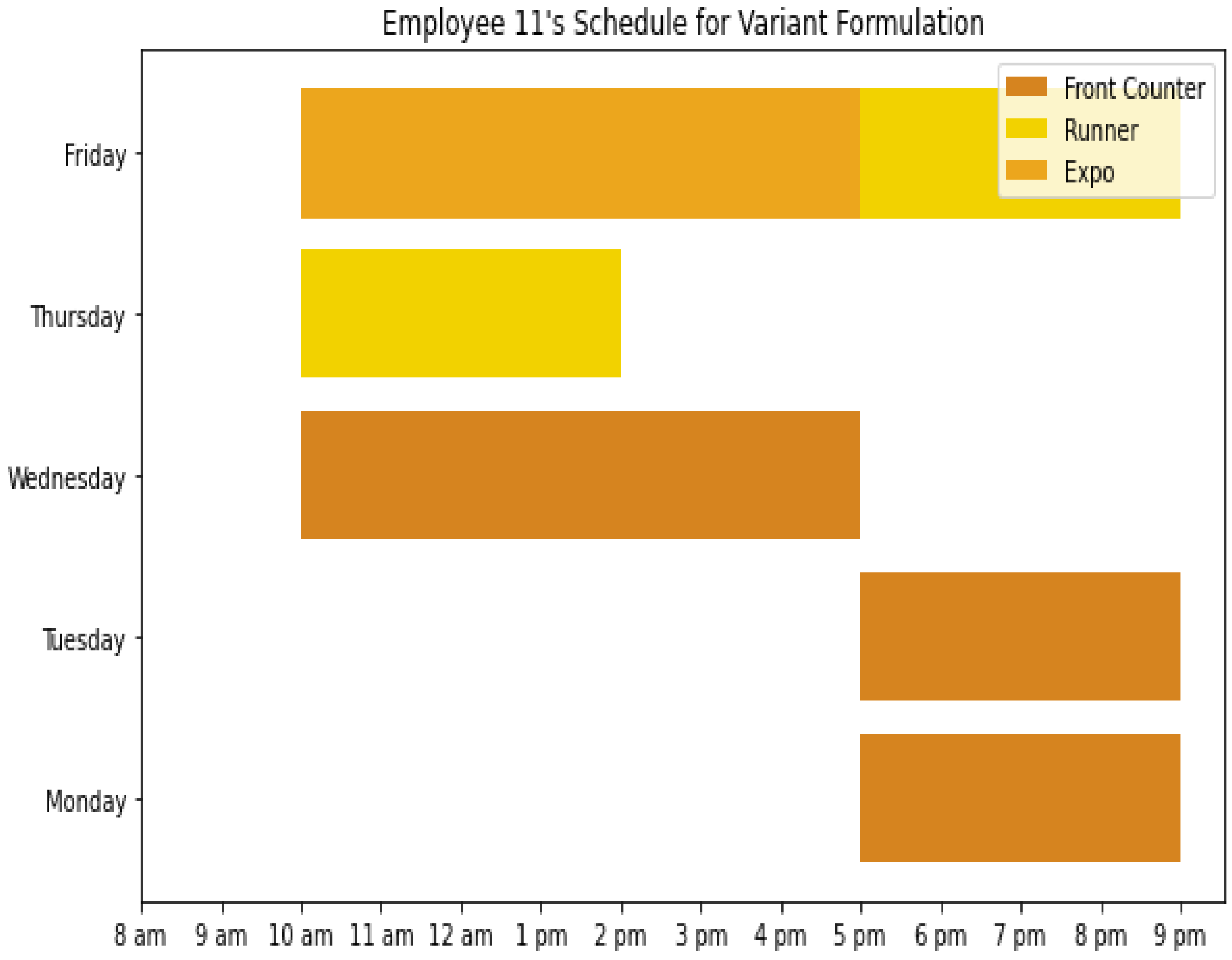

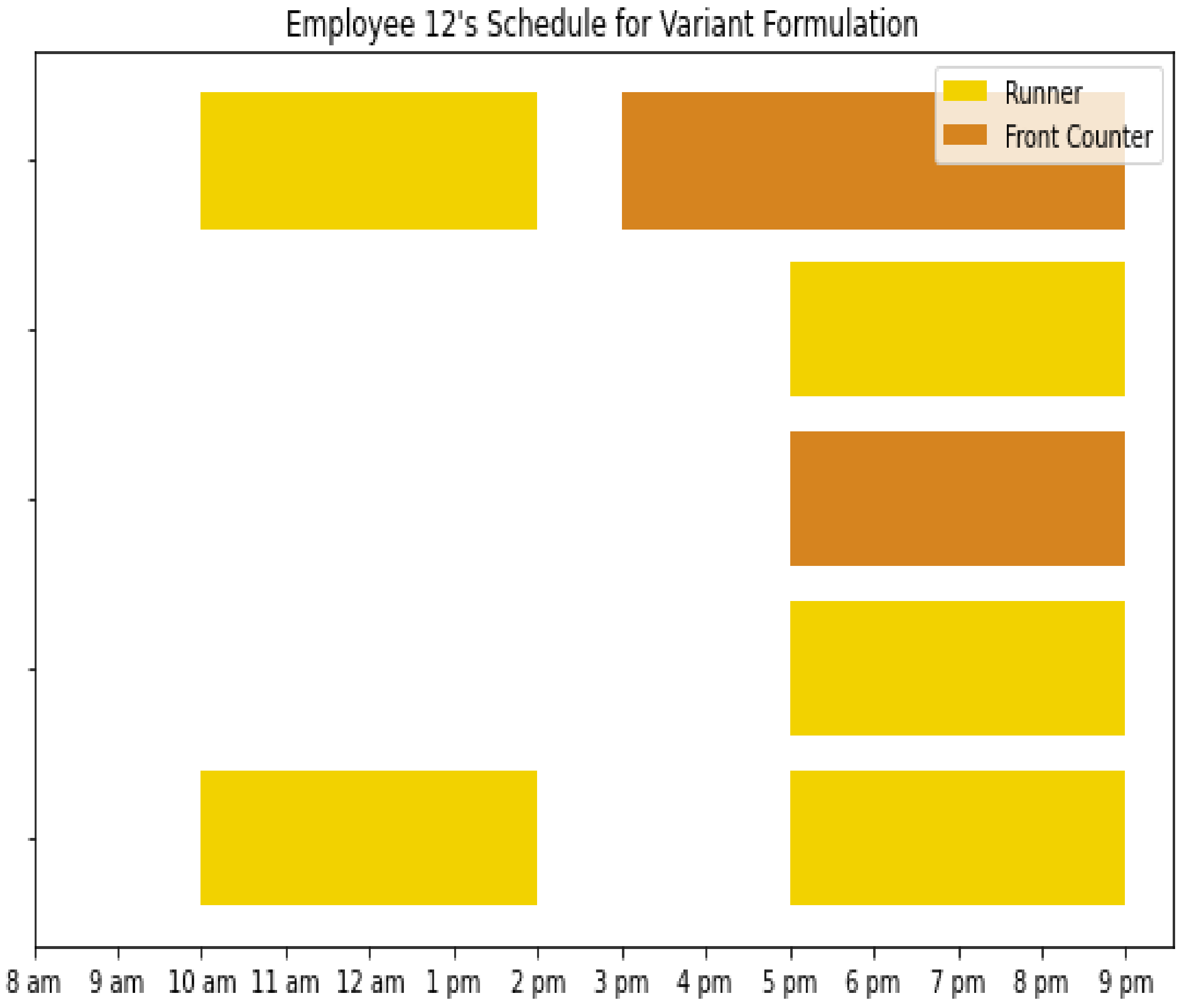

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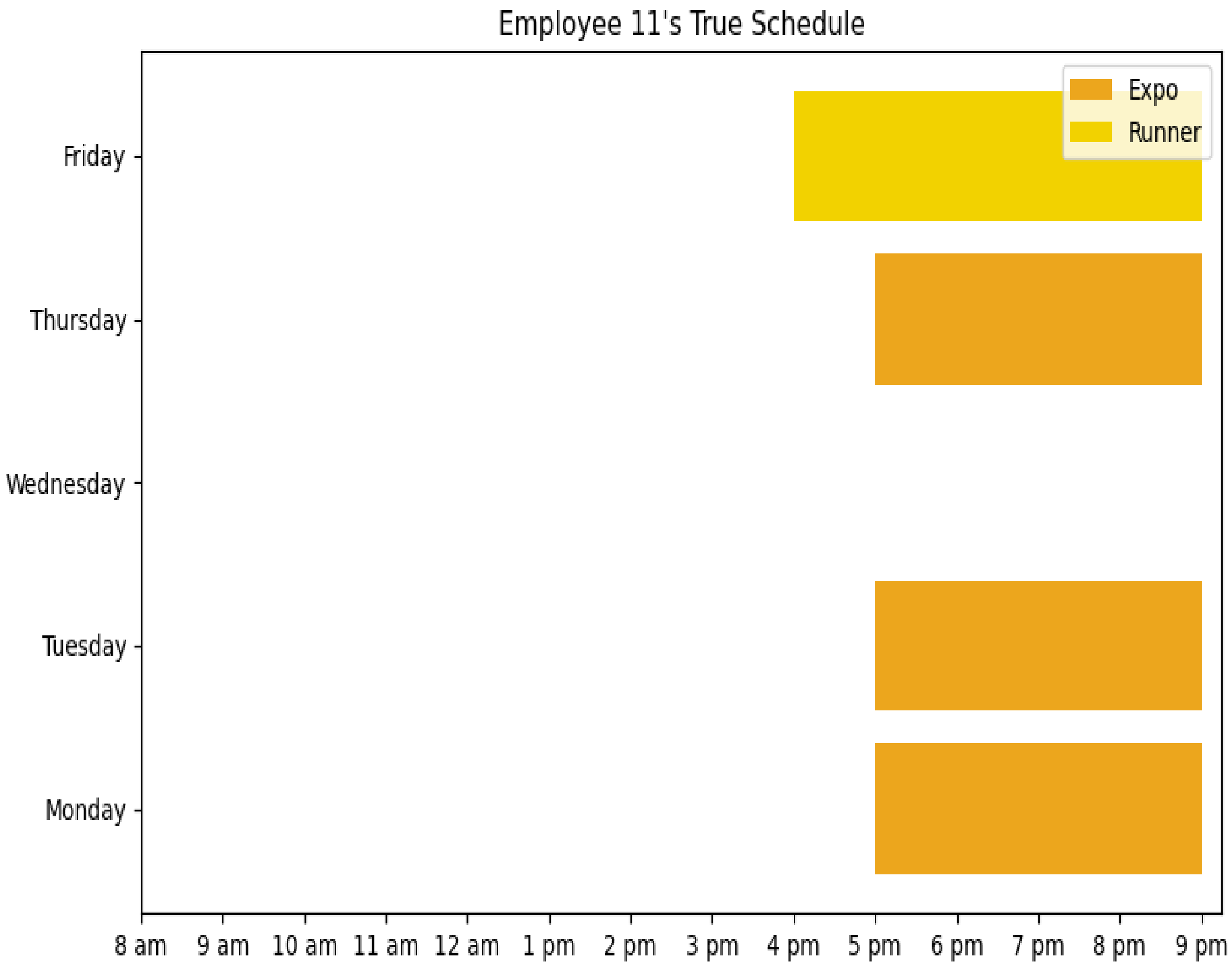

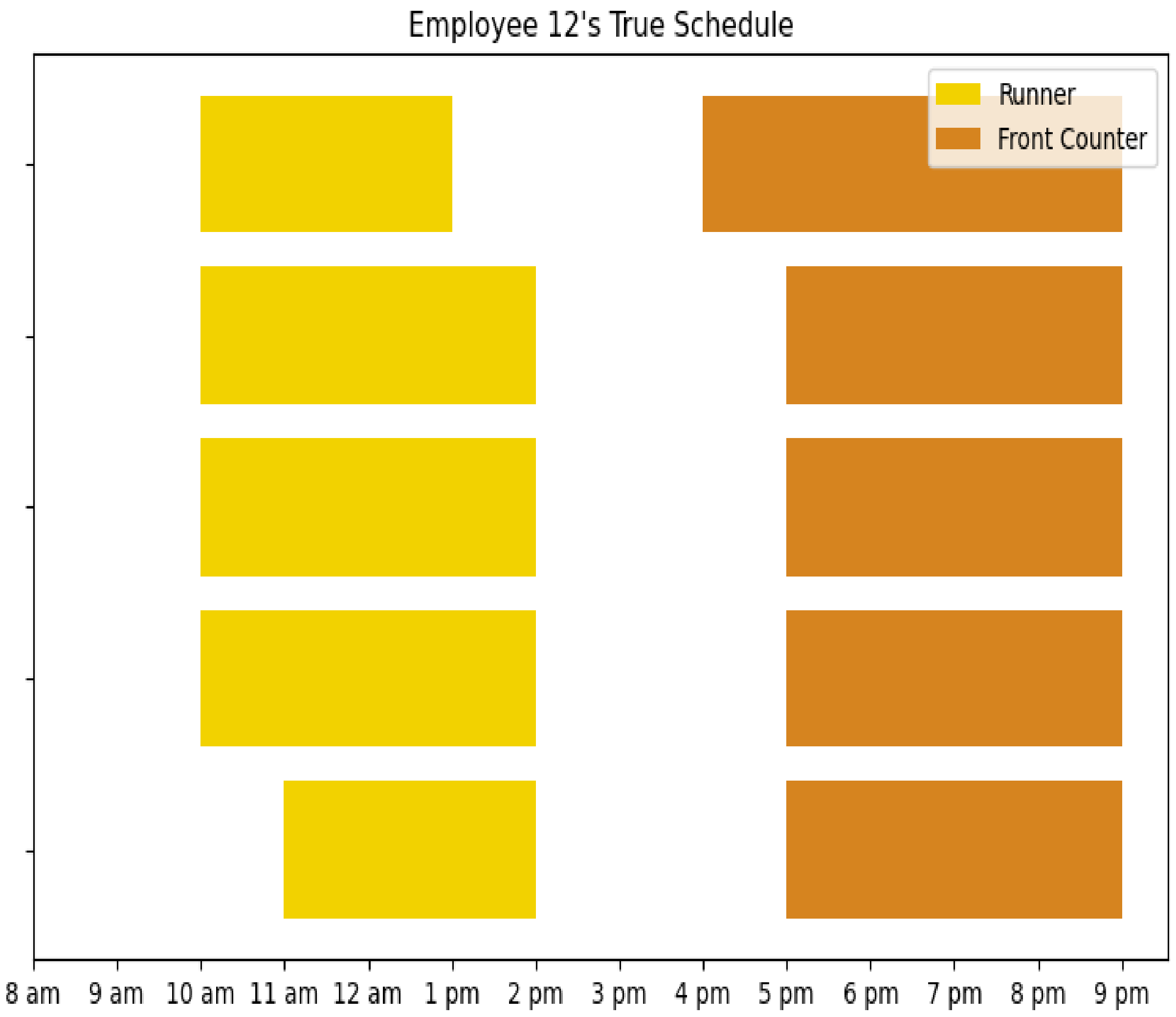

Results and Analysis

True Schedule							
Monday							
	Cook 1	Cook 2	Expo 1	Expo 2	Front Counter	Runner	Dishwasher
9:00 AM	1		9				
10:00 AM	1		9		10	Not covered!	2
11:00 AM	1		9		10	12	2
12:00 PM	1		9		10	12	2
1:00 PM	1		9		10	12	2
2:00 PM	1		0		10		
3:00 PM	1		0		10		
4:00 PM	1		0		10		
5:00 PM	3	0	11		12	Not covered!	6
6:00 PM	3	0	11		12	Not covered!	6
7:00 PM	3	0	11		12	Not covered!	6
8:00 PM	3	0	11		12	Not covered!	6

Results and Analysis

True Schedule Hours				
Employee	# hours scheduled	Min	Max	delta
0	35	40	40	5
1	38	30	40	0
2	12	12	15	0
3	20	15	20	0
4	4	4	4	0
5	4	4	4	0
6	12	12	16	0
7	8	8	12	0
8	8	8	10	0
9	19	20	30	1
10	35	30	40	0
11	17	0	40	0
12	39	0	40	0
13	4	0	4	0

- Employees' schedules are optimized to minimize deviations (δ) from their desired working hours.
- The model found it feasible to reduce δ to 0, and the optimized schedule achieves 100% role coverage.
- The true schedule has a total deviation of 6 hours from employee requirements.
- It also fails to satisfy role coverage requirements in numerous places across the week.

Conclusion and Recommendations

Benefits of the Approach

- **Employee Satisfaction**: Ensures availability and total hours requirements are met for all employees.
- **Operational Efficiency**: Guarantees that all role requirements are fulfilled without unnecessary resource allocation.
- **Managerial Flexibility**: Frees up owners to fill roles as needed (e.g. call-ins, emergencies).
- **Scalability**: Adapts to changes in employee availability, role requirements, and operational constraints.

Recommendations and Future Improvements

- ✓ Use the Gantt chart outputs to plan weekly employee scheduling.
- ✓ Use outputs to check if an interviewee's availability fits within the existing workforce.
- ✓ Regularly update the model inputs (availability, role requirements, etc.) to reflect real-world changes.
- ✓ Transform the model to have multiple weighted objectives such as employee skill level, wages for employees or roles, or schedule similarity across days or weeks.

The End!

Any Questions ??