

✓ Polygon filling

① Edge table

② Active edge table

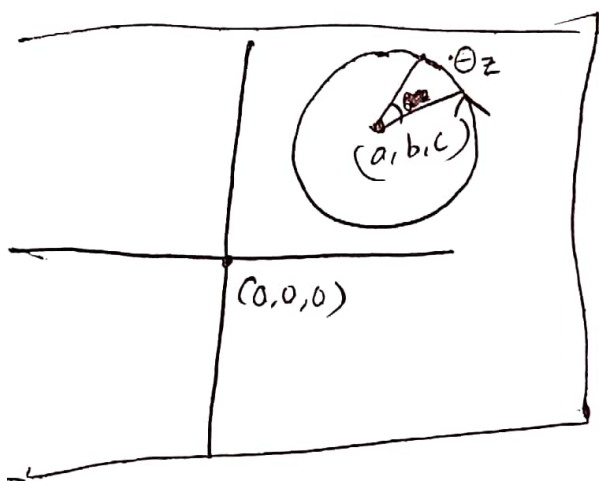
✓ Polygon clipping

① 4 rules of Cohen-Sutherland

② clip one boundary & show output (co-ordinate)

Graphics

Co-ordinate Transformation / Modeling Transformation



$$x_1 = x - a$$

$$y_1 = y - b$$

$$z_1 = z - c$$

①

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & a \\ 0 & 1 & 0 & b \\ 0 & 0 & 1 & c \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

3

$$\begin{bmatrix} \cos\theta_z & -\sin\theta_z & 0 & 0 \\ \sin\theta_z & \cos\theta_z & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

2

$$\begin{bmatrix} 1 & 0 & 0 & -a \\ 0 & 1 & 0 & -b \\ 0 & 0 & 1 & -c \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

1

$$\begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

$$x_2 = (x-a) \cos \theta_z - (y-b) \sin \theta_z$$

$$y_2 = (x-a) \sin \theta_z + (y-b) \cos \theta_z$$

$$z_2 = z - c$$

(2)

$$x' = (x-a) \cos \theta_z - (y-b) \sin \theta_z + a$$

$$y' = (x-a) \sin \theta_z + (y-b) \cos \theta_z + b$$

$$z' = z - c + c$$

(3)

$$x' = x \cos \theta_z - y \sin \theta_z + a(1 - \cos \theta_z) + b \sin \theta_z$$

$$y' = x \sin \theta_z - y \cos \theta_z + b(1 - \cos \theta_z) - a \sin \theta_z$$

$$z' = z$$

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos \theta_z & -\sin \theta_z & 0 & a(1 - \cos \theta_z) + b \sin \theta_z \\ \sin \theta_z & \cos \theta_z & 0 & b(1 - \cos \theta_z) - a \sin \theta_z \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

→ multiplication → could require 5 matrix if θ_x & θ_y also changed. Order of multiplication

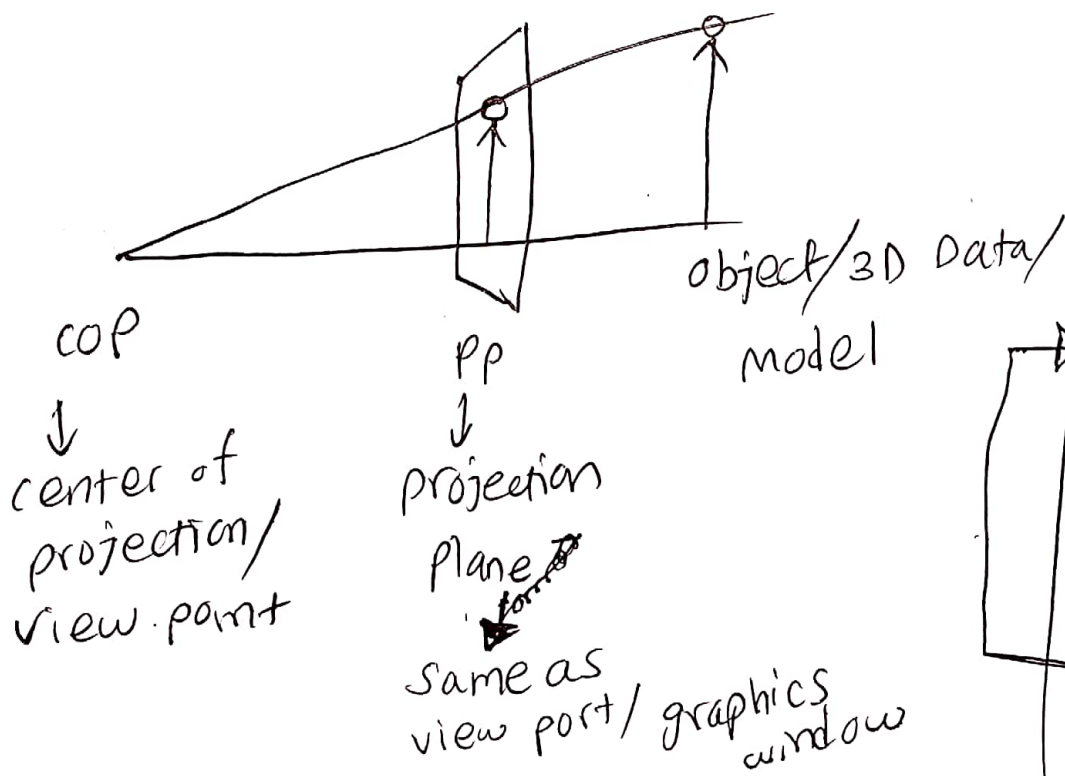
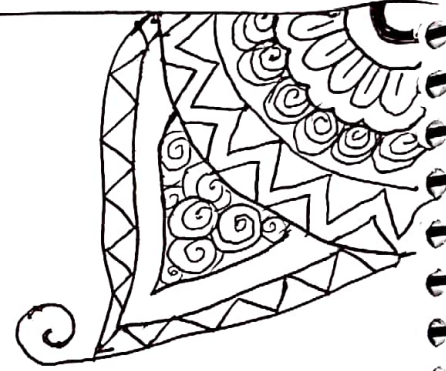
will be \rightarrow The rotation that occurs first will be on the right.

Quiz $\rightarrow \theta_x / \theta_y / \theta_z$ $f(x, y, z)$ will be given find (x', y', z')

For $\theta_y \rightarrow y' = y$, x & z will change
 $\theta_x \rightarrow x' = x$, y & z will change

Viewing in 3D

Projection



usually center is taken as COP / COP Plane.

3 centers, they must be integrated.
So \rightarrow

Projection scale unit depends on object. If object is in mm/km projection scale is in mm/km.

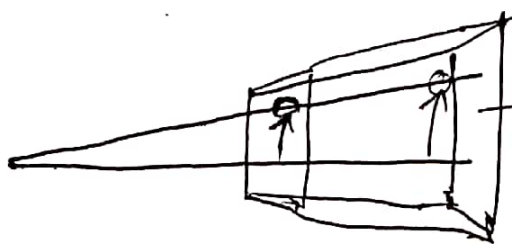
Display depends on pixel values.

So, values are multiplied by a factor to display the things ~~of~~ projection plane to screen.

Original projection plane can be defined in two way: ?? (google)

- ① Glortho (projection plane surface)
 - ② Glwindow (window size)
- } un-sure
-

View-volume: Left, Right, Top, Bottom, Near, Far, Limit of window display.



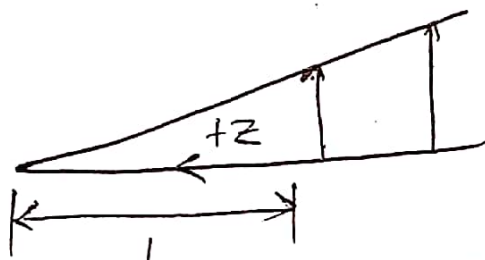
view volume

Glortho → just parallel
(google)

Projection Classification (old)

Display system is 2D. ^{3D} obj. needs to be projected to this 2D screen so that the shape is recognized, by adjusting colors & shades.

OpenGL \rightarrow view angle \times distance

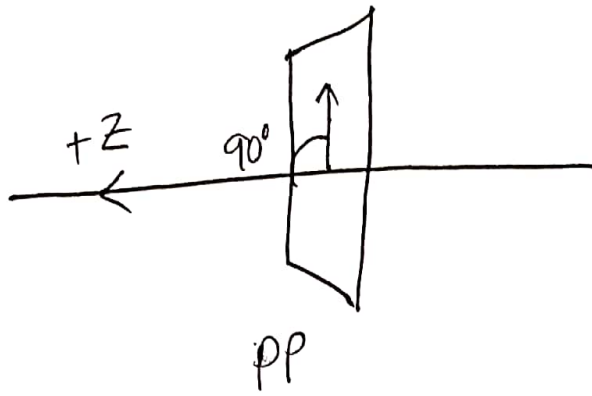


$d \rightarrow$ Distance between COP & PP

* Based on 'd'

- ① Parallel projection: 'd' infinite
- ② Perspective projection: 'd' finite

Nothing is truly parallel. Technically everything is perspective projection. Absolute ~~an~~ infinity doesn't exist. Infinity is relative.



④ Based on the orientation of PP:

① Orthographic Projection: PP is perfectly perpendicular to Z-axis.

② Oblique projection: Not perpendicular to Z-axis.

perspective → perspective & orthographic
 orthographic → parallel & orthographic
 oblique → parallel & oblique

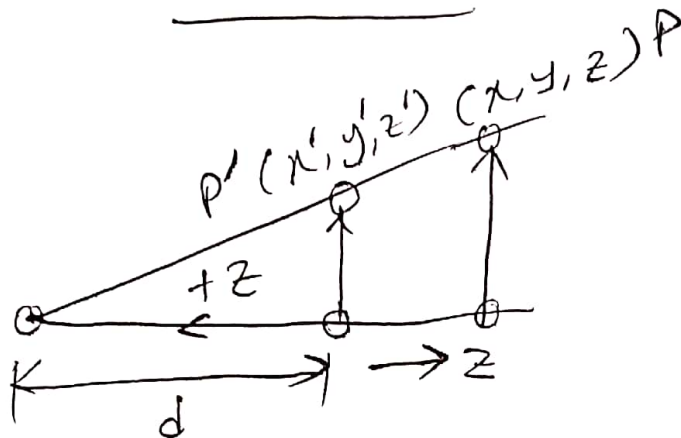
↑ terms used these days.
 there's no parallel separately

Derivation of Simple Projection

Matrix →

Derivation of Simple Projection

Matrix



Let,

P is projected on P' and d is given.

Distance \rightarrow vector. ?? Displacement.

$d \rightarrow$ displacement from cop to PP to cop

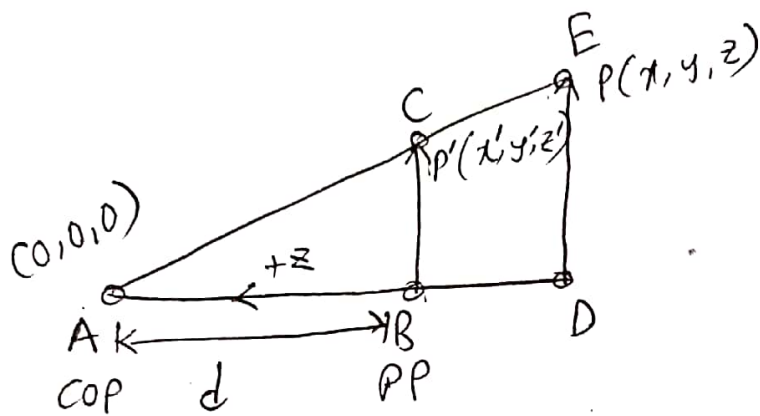
$d (+ve) \rightarrow bc$ along z

$z \rightarrow$ displacement from 0 to that point in z axis.

$z (-ve) \rightarrow bc$ not along z .

Graphics

Derivation of simple Projection Matrices



$$DE = y$$

$$BC = y'$$

origin A, $AD = -z$
 $AB = -z'$

origin B, $BD = -z$
 $AB =$

$$BD = -200$$

$$AB = 300$$

$$\frac{BC}{AB} = \frac{DE}{AD} \quad \text{--- (1) } [\triangle ABC \text{ \& } \triangle ADE \text{ similar}]$$

① Origin of COP :

$$A(0,0,0) \quad B(0,0,-d)$$

~~$BA = d$~~ ~~$AD = -z$~~ ~~$BA = d$~~ ~~Displacement from projection plane with respect to projection plane~~

$$BA = d$$

$$AD = -z$$

d is the distance from projection plane to COP.

Now,

$$\frac{BC}{BA} = \frac{DE}{AD}$$

$$\Rightarrow \frac{y'}{d} = \frac{y}{-z} \quad \text{--- (ii)}$$

$$\Rightarrow y' = \frac{y}{-z/d} \quad \text{--- (iii)}$$

$$\frac{x'}{d} = \frac{x}{-z}$$

$$\Rightarrow x' = \frac{x}{-z/d}$$

$$z' = \frac{z}{-z/d} \quad [\text{similarly}]$$

$$\Rightarrow z' = -d$$

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -1/d & 0 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

$$(x, y, z, -z/d) \rightarrow$$

② Origin on the PP :

$$A(0,0,d) \quad B(0,0,0) \quad D(0,0,-z)$$

$$BD = -z$$

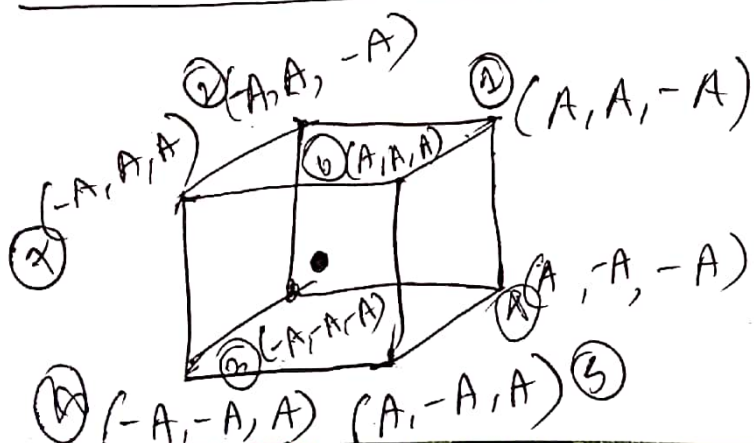
$$AB = d$$

$$AD = d - z$$

Now,

$$\left. \begin{aligned} \frac{y'}{d} &= \frac{y}{d-z} \\ \Rightarrow y' &= \frac{y}{1-z/d} \end{aligned} \right| \left. \begin{aligned} x' &= \frac{x}{1-z/d} \\ z' &= \frac{z}{1-z/d} \end{aligned} \right| z' = 0$$

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -1/d & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$



$$\begin{aligned} P2D[8] &\rightarrow x' \neq y' \\ &\quad z \text{ const} \\ P3D[8] &\rightarrow x' \neq y' \neq z' \end{aligned}$$

Now, for topmost point,

$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -\frac{1}{400} & 0 \end{bmatrix} \cdot \begin{bmatrix} -200 \\ -200 \\ -900 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} -200 \\ -200 \\ -900 \\ 9/4 \end{bmatrix}$$

LAB

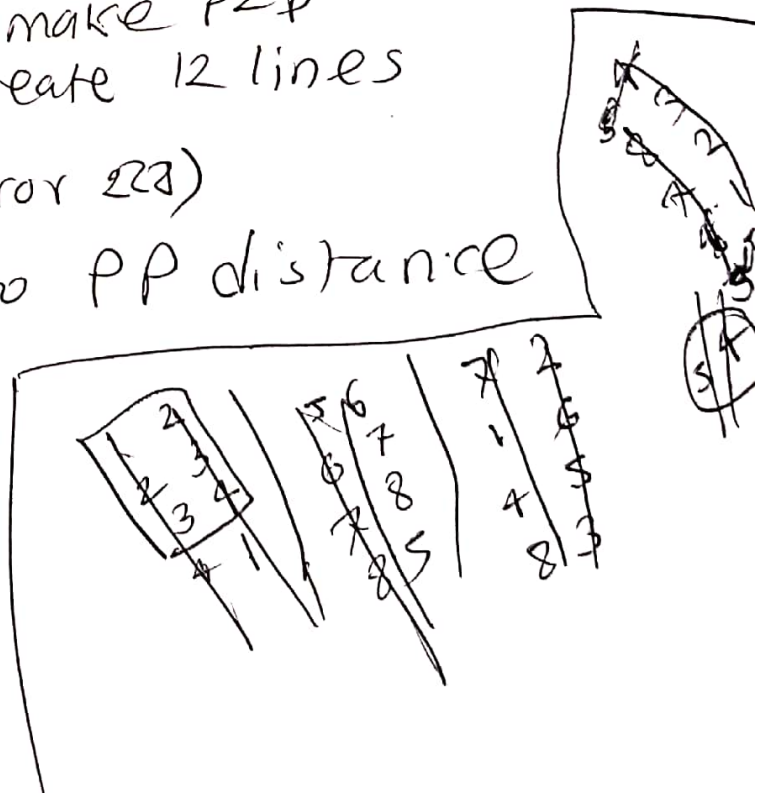
12 lines

- 1/ P3D \rightarrow P2D array
- 2/ use drawline to draw 12 lines

- 3/ Makecube(A) \rightarrow make P3D
make P2P
create 12 lines

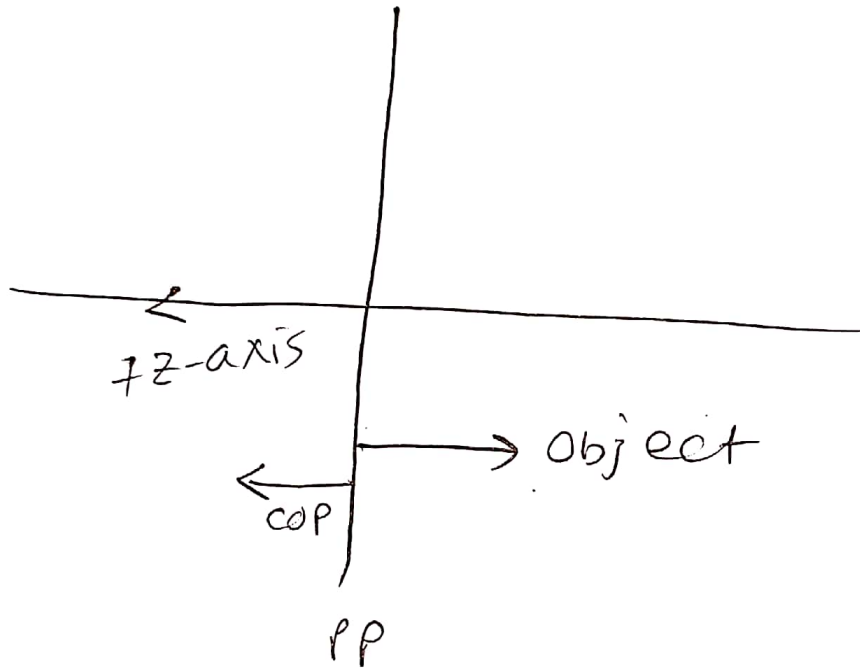
z \rightarrow never 0 (if 0 error 228)

L \rightarrow object center to PP distance



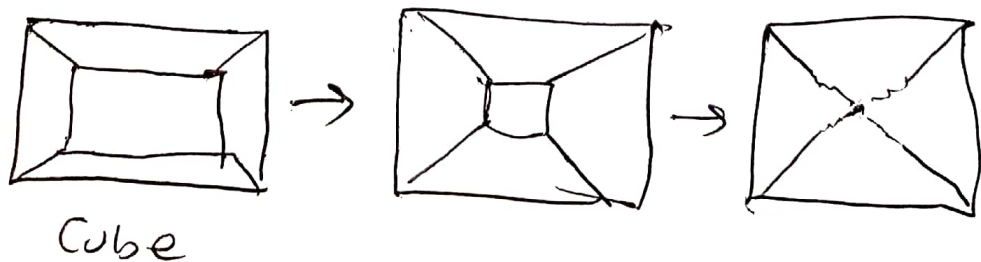
Graphics

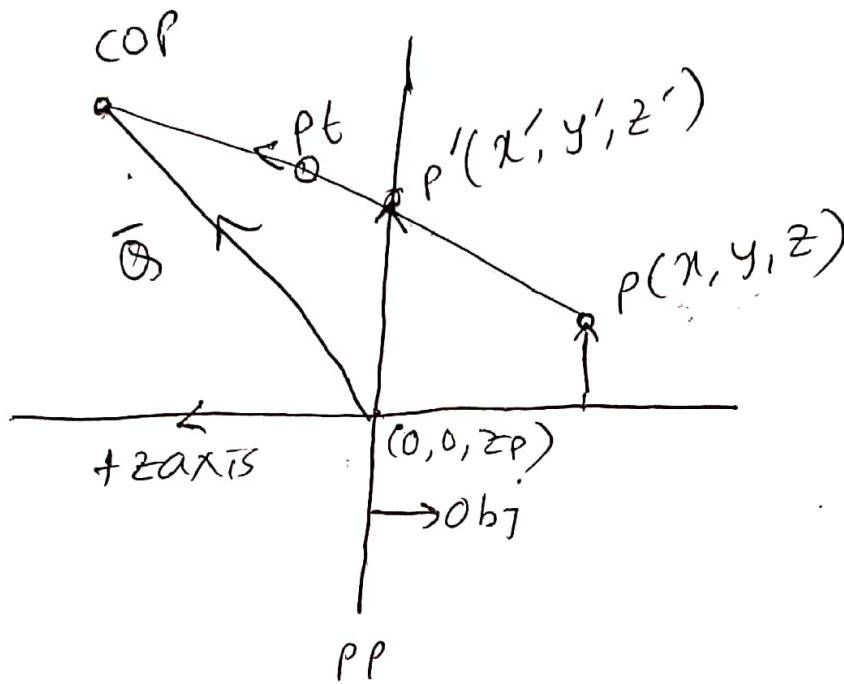
General purpose projection Matrix



If COP in z-axis \rightarrow single vanishing point

single vanishing point





$\bar{Q} = d$ if \bar{Q} is along the z -axis

$$COP = (0, 0, z_p) + \bar{Q} \quad \text{--- ①. a}$$

~~$$COP = Q \cdot X$$~~

$$COP \cdot x = Q \cdot x$$

$$COP \cdot y = Q \cdot y$$

$$COP \cdot z = z_p + Q \cdot z$$

①. b

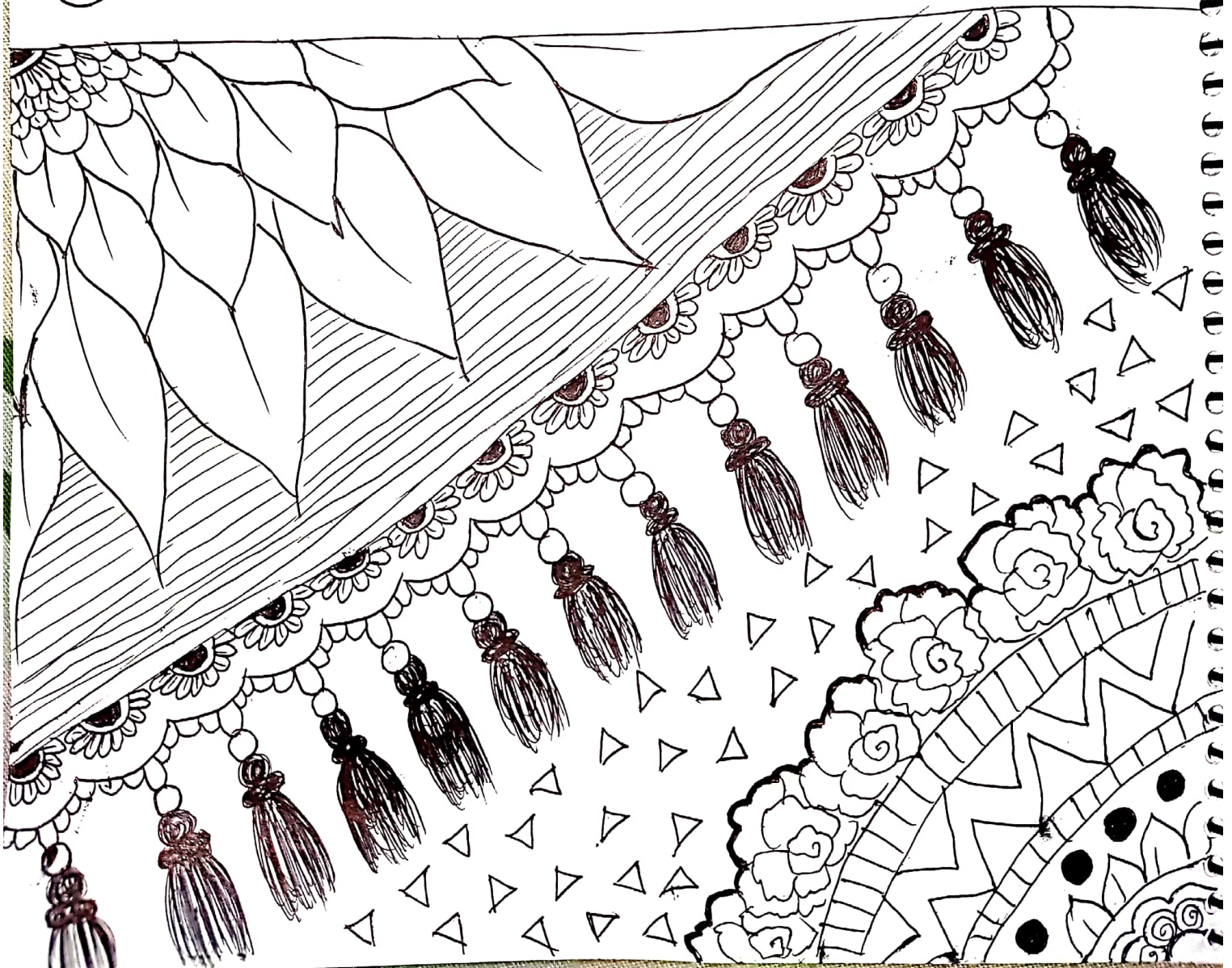
P_t is a point on the line segment $COP \xrightarrow{t_0} P$

$$P_t = COP + t(P - COP) \quad \text{--- (11)}$$

$$\text{AOP } t = t_p, \quad P_t = P'$$

$$P' = COP + t_p(P - COP) \quad \text{--- (11).a}$$

(*) P & COP both are 3D points



$$x' = Q \cdot x + t_P (x - Q \cdot x)$$

$$y' = Q \cdot y + t_P (y - Q \cdot y)$$

$$z' = Q \cdot z_P + Q \cdot z + t_P (z - z_P - Q \cdot z)$$

(iii). b

From eq (iii). b,

$$z' = z_P + Q \cdot z + t_P (z - z_P - Q \cdot z)$$

$$\Rightarrow t_P = \frac{z' - z_P - Q \cdot z}{z - z_P - Q \cdot z} \quad \text{--- (iv)}$$

solving for x' ,

$$x' = Q \cdot x + \frac{Q \cdot z (x - Q \cdot x)}{z - z_P - Q \cdot z}$$

$$= \frac{z \cdot Q \cdot x - z_P \cdot Q \cdot x - \cancel{Q \cdot x \cdot Q \cdot z} - x \cdot Q \cdot z + \cancel{Q \cdot x \cdot Q \cdot z}}{z - z_P - Q \cdot z}$$

$$x' = \frac{x - z \frac{Q \cdot x}{Q \cdot z} + z_P \frac{Q \cdot x}{Q \cdot z}}{-\frac{z}{Q \cdot z} + 1 + \frac{z_P}{Q \cdot z}} \quad \text{--- (v)}$$

Similarly,

$$y' = \frac{y - z \frac{Q \cdot y}{Q \cdot z} + z_p \frac{Q \cdot y}{Q \cdot z}}{-\frac{z}{Q \cdot z} + 1 + \frac{z_p}{Q \cdot z}} \quad (v1)$$

$$z' = z_p$$

$$z' = z_p = \frac{z_p \left(-\frac{z}{Q \cdot z} + 1 + \frac{z_p}{Q \cdot z} \right)}{-\frac{z_p}{Q \cdot z} + 1 + \frac{z_p}{Q \cdot z}}$$

$$z' = \frac{-z \cdot \frac{z_p}{Q \cdot z} + z_p \left(1 + \frac{z_p}{Q \cdot z} \right)}{-z \cdot \frac{1}{Q \cdot z} + 1 + \frac{z_p}{Q \cdot z}} \quad (v11)$$

$$\begin{pmatrix} x' \\ y' \\ z' \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & -\frac{Q \cdot x}{Q \cdot z} & z_p \frac{Q \cdot x}{Q \cdot z} \\ 0 & 1 & -\frac{Q \cdot y}{Q \cdot z} & z_p \frac{Q \cdot y}{Q \cdot z} \\ 0 & 0 & -\frac{z_p}{Q \cdot z} & z_p \left(1 + \frac{z_p}{Q \cdot z} \right) \\ 0 & 0 & -\frac{1}{Q \cdot z} & 1 + \frac{z_p}{Q \cdot z} \end{pmatrix} \cdot \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$$

Origin at PP, $z_p = 0$

$$Q_x = 0$$

$$Q_y = 0$$

$$Q_z = d$$

Now,

$$\begin{vmatrix} x' \\ y' \\ z' \\ 1 \end{vmatrix} = \begin{vmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -1/d & 1 \end{vmatrix} \cdot \begin{vmatrix} x \\ y \\ z \\ 1 \end{vmatrix}$$

DS

Stratified class Distribution ***

Bootstrap

stat extra → Sunday 11:30 [online]

wednesday 2-3:20 [offline]