VILNIUS UNIVERSITY FACULTY OF MATHEMATICS AND INFORMATICS DEPARTMENT OF MATHEMATICAL COMPUTER SCIENCE

Evolutionary optimisation algorithms: application to the Optimal Image Collage Composition problem

Course work

Author: Bioinformatikos 4 kurso studentas

Kazimieras Jasaitis

Supervisor: Lekt. Irus Grinis

TABLE OF CONTENTS

IN	TRODUCTION	3
1.	EVOLUTIONARY OPTIMISATION ALGORITHMS 1.1. Ant Colony Optimization 1.1.1. Behaviour of Ant Colonies 1.1.2. Ant Colony Optimization Algorithm 1.1.3. Application to the Traveling Salesman Problem 1.2. Genetic Algorithms	5 5 6 6 6
	1.2.1. Application to the Traveling Salesman Problem	7
2.	PARTICLE SWARM OPTIMISATION	8
	2.1. Metaphor	
	2.2. Unique architectural aspects	9
	2.3. Parameters	
	2.3.1. inertia weight	
	2.3.2. cognitive weight	
	2.3.3. social weight	
	2.4. Key mechanisms	
	2.4.1. cognitive velocity	
	2.4.2. global velocity	
	2.5. Well established applications	11
3.	HARMONY SEARCH OPTIMISATION	12
	3.1. Metaphor	
	3.2. Unique Architectural Aspects	
	3.3. Parameters	
	3.3.1. Harmony Memory Size	
	3.3.2. Memory Consideration Rate	
	3.3.3. Pitch Adjustment Rate	
	3.3.4. Pitch Bandwidth	
	3.4. Key Mechanisms	13
	3.4.1. Fine-Tuning of Solutions	13
	3.4.2. Exploration of New Solutions	13
	3.5. Well-Established Applications	14
1	EVOLUTIONARY ALGORITHMS FOR OPTIMAL IMAGE COLLAGE COMPOSITION	
	PROBLEM	15
	4.1. Relation to 2D bin packing problem	
	4.2. Input	
	4.3. Output	
	4.4. Fitness Function	
	4.4.1. Resizing	
	4.4.2. Overlapping.	
	4.4.3. Boundaries	
	4.4.4. Uncovered Area	
	4.5. Solution using PSO	
	4.5.1. Hyper-parameter values	
	4.5.2. Initialization	
	4.5.3. Execution	
	4.6. Solution using HSO	

4.6.1. Hyper-parameter values	20
4.6.2. Initialization	
4.6.3. Execution	21
4.7. Performance evaluation	21
4.7.1. Unit test generator	22
4.7.2. Testing	22
4.7.2.1. What to expect	
4.7.2.2. <i>n</i> = 2	23
4.7.2.3. $n = 3$	24
4.7.2.4. $n = 4$	24
4.7.2.5. $n = 5$	26
RESULTS AND CONCLUSIONS	27
LITERATURE	29
EVOLUTIONARY OPTIMISATION ALGORITHMS: PARTICLE SWARM OPTIMISATION ALOGORITHM KAZIMIERAS JASAITIS	
ABSTRACT	30
EVOLIUCINIAI OPTIMIZAVIMO ALGORITMAI: DALELIŲ SPIEČIAUS OPTIMIZACIJOS ALGORITMAS KAZIMIERAS JASAITIS	
SANTRAUKA	31
ADDITIONAL FILES	32
ADDITIONAL FILES	3/

Introduction

In contemporary society, technology permeates every aspect of life, with computational devices ubiquitously performing tasks with high precision. Generally, computations in computers are deterministic, yielding predictable outcomes by following a preordained set of logical steps. This implies that for the same input, the output is invariably identical and is produced by what is termed as deterministic algorithms [AB09].

However, there are computationally intensive problems where deterministic algorithms are insufficient. One of the classifications of these problems is the NP (nondeterministic polynomial) problems. According to [Gol10], NP problems encompass decision problems where, if the answer is affirmative, a solution can be verified in polynomial time. The concept of nondeterminism is crucial for understanding NP problems. Imagine a computational model capable of making multiple choices at each step, exploring various possibilities. This, in essence, defines nondeterminism. In the context of NP problems, a nondeterministic algorithm can hypothetically guess a solution and then verify it within polynomial time. Nevertheless, the physical computers available are fundamentally deterministic [AB09; Gol10].

An alternative approach to solving complex optimization problems is through evolutionary algorithms. One such algorithm is the Particle Swarm Optimization (PSO), which has been widely adopted in applications like antenna design, biomedical applications, image processing, and robotics among others [Pol08; Sim13].

Collage creation is a representative task that seems trivial but escalates in complexity, particularly when numerous images of varying dimensions are to be placed on a canvas with predefined dimensions, with constraints like maintaining relative sizes, avoiding cropping, overlaps, or leaving any space unoccupied. This task necessitates the optimal and proportional resizing and positioning of images, which is computationally demanding and classified as an NP-Hard problem.

The problem of optimally placing images on a canvas is analogous to the 2D bin packing problem. The 2D bin packing problem entails fitting a set of 2D objects into a larger 2D container, making it pertinent to image placement where images are to be fitted on a canvas. Efficient solutions to the 2D bin packing problem can, therefore, be adapted for collage creation [WQZ15].

Particle Swarm Optimization (PSO) algorithm has been utilized to tackle the challenge of collage creation. The PSO algorithm exploits a population of candidate solutions that traverse the search space to locate an optimal or near-optimal solution for the collage creation problem, thereby providing an efficient and practical approach [Pol08; WQZ15].

Objectives:

- 1. Provide a detailed analysis of the Particle Swarm and Harmony Search Optimization algorithms, elucidating their underlying principles and mechanics.
- 2. Develop a unit test generator that segments an image into n pieces, where n is an integer ranging from 2 to 5, to facilitate the evaluation of the PSO algorithm.

- 3. Implement the Particle Swarm and Harmony Search Optimization algorithms and apply them to generate solutions for placing images on a canvas of dimensions $30px \times 30px$.
- 4. Execute the generated tests to evaluate the performance of the algorithms with varying numbers of image pieces.
- 5. Perform a critical analysis of the algorithms' performances, drawing insightful comparisons based on the empirical results obtained.
- 6. Present the results of the performance analysis in a manner that is insightful and facilitates comparison between the two algorithms.
- 7. Draw conclusions based on the experimental results to determine the relative suitability of the Particle Swarm and Harmony Search Optimization Algorithms for solving the image placement problem.
- 8. Propose potential enhancements and future directions for the application of these algorithms in similar or related problem domains.

1. Evolutionary optimisation algorithms

Evolutionary optimization algorithms represent a significant advancement in the realm of computational problem-solving, particularly for NP-complete problems. These algorithms stand out due to their population-based methodology, which contrasts with traditional single-solution approaches. By operating on a multitude of potential solutions simultaneously, evolutionary algorithms enhance the probability of discovering optimal solutions that might otherwise necessitate an exhaustive search process.

The conceptual foundation of these algorithms is deeply rooted in natural phenomena, employing metaphors inspired by biological and ecological systems to elucidate their operating principles. This approach not only makes the algorithms more relatable but also facilitates an intuitive understanding of their mechanisms.

A critical aspect of evolutionary algorithms is their reliance on stochastic processes. This inherent randomness is tempered by a set of parameters that serve to modulate the algorithm's exploratory behavior. These parameters are instrumental in directing the search towards viable regions of the solution space, thus imposing a structured yet flexible boundary on the otherwise chaotic exploration.

Furthermore, the notion of dimensionality plays a pivotal role in the functioning of these algorithms. In this context, the solution space can be envisioned as an N-dimensional continuum, where each iteration allows the constituent 'particles'—representing individual solutions—to assume positions within this space. The positioning of these particles, governed by their respective parameters, exemplifies the N-dimensional nature of each solution. This multidimensional perspective is crucial for understanding how evolutionary algorithms navigate and probe the vast expanse of potential solutions.

1.1. Ant Colony Optimization

Ants are small insects, which individually possess limited capabilities. However, when they operate collectively within colonies, which sometimes consist of millions of ants, they exhibit highly sophisticated behaviors and can accomplish complex tasks. This success is attributed to their collective intelligence and ability to adapt to various environments. [DS04].

1.1.1. Behaviour of Ant Colonies

One of the primary communication mechanisms among ants is through pheromones, which are chemicals they secrete. For instance, when ants forage for food, they leave a pheromone trail along their path. Other ants, equipped with antennas sensitive to these chemicals, follow these pheromone trails to locate food sources. As more ants traverse this path, additional pheromones are deposited, reinforcing the trail. Consequently, the shortest path to the food source is often highlighted due to the concentration of pheromones. Over time, as the pheromones on older trails evaporate, the ants adapt by finding and reinforcing new optimal paths to new food sources. [DS04].

1.1.2. Ant Colony Optimization Algorithm

Inspired by the pheromone-depositing behavior of ants, the Ant Colony Optimization (ACO) algorithm was developed. ACO is a population-based metaheuristic that has been effectively applied to solve combinatorial optimization problems. Contrary to Evolutionary Algorithms (EAs), ACO does not involve direct exchange of solution information among candidate solutions. Instead, it simulates the process of ants depositing pheromones on the paths they traverse and subsequent generations of simulated ants using this information to make probabilistic choices in exploring the search space [Sim13].

1.1.3. Application to the Traveling Salesman Problem

The Traveling Salesman Problem (TSP) is a classic optimization problem. In the Selective Traveling Salesman Problem variant, which can also be referred to as the Orienteering Problem, a weighted graph with profits associated with vertices is given. The objective is to find a simple circuit with the maximum total profit, subject to the constraint that the circuit's length does not exceed a predetermined bound [LM90].

ACO can be effectively employed to solve the TSP. In ACO, a population of simulated ants is initially positioned. Each ant selects a city to travel to based on probabilistic rules that account for the attractiveness of the path (often influenced by distance) and the intensity of the pheromone trail. As ants traverse between cities, they deposit pheromones, reinforcing paths. Over time, pheromones on less favorable paths evaporate, making them less attractive, while the paths more frequently taken become more attractive due to increased pheromone deposits. Through iterative processing, the algorithm converges towards an optimal or near-optimal solution. [GR04]

1.2. Genetic Algorithms

Genetic Algorithms (GAs) are among the earliest, most well-known, and widely used Evolutionary Algorithms (EAs). GAs are inspired by the process of natural selection and are employed to solve optimization problems. However, it is pertinent to note that GAs encompass a much broader class of systems than mere function optimizers [Sim13].

Natural selection exhibits certain fundamental characteristics, which include:

- 1. A biological system comprises a population of individuals, many of whom have the ability to reproduce.
- 2. The individuals have a finite lifespan.
- 3. Variation exists within the population.
- 4. Survival capabilities are positively correlated with reproductive abilities.

GAs simulate these features. For an optimization problem, a population of candidate solutions, termed individuals, is created. The individuals with better solutions have higher chances of

reproduction, while those with poorer solutions have lower chances. Over generations, as parents produce offspring and are subsequently replaced by them, the population becomes more adapted. Occasionally, highly fit individuals evolve, providing near-optimal solutions.

1.2.1. Application to the Traveling Salesman Problem

Though the Traveling Salesman Problem (TSP) is often solved more efficiently using the Ant Colony Optimization algorithm, Genetic Algorithms can also be devised for this purpose. The GA approach involves the following steps:

- 1. Initialize essential parameters such as city locations, maximum number of generations, population size, crossover probability, and mutation probability.
- 2. Generate an initial random population of chromosomes and evaluate the fitness of each chromosome.
- 3. Transform the population into a new generation using three genetic operators: selection, crossover, and mutation. The selection operator chooses parent chromosomes, which then produce offspring through crossover and/or mutation.
- 4. Retain a higher proportion of characteristics from the fitter members of the previous generation, facilitating the propagation and mixing of advantageous traits.
- 5. Produce a new set of chromosomes equal in size to the initial population.
- 6. Repeat the transformation process until the population converges to an optimal solution or other stopping criteria are met.

The process ensures that with each successive generation, the population evolves towards a more optimal solution, generally culminating when a significant proportion of the population possesses the same optimal chromosome [RG11].

2. Particle Swarm Optimisation

Most EAs tend to be comparatively static as they simulate the evolution of potential solutions across generations without explicitly modeling the dynamism of their movement through the search space.

In PSO, as an individual progresses through the search space, it possesses inertia, thereby tending to sustain its velocity. Nonetheless, the velocity is subject to alteration owing to two principal factors:

- 1. The individual retains memory of its optimum position historically and has an inclination to modify its velocity in an effort to revert to that position. This is akin to human propensities to reminisce about the halcyon days and endeavor to rekindle past experiences. Within PSO, as the entity navigates the search space, its position undergoes a transformation from one generation to another. However, it keeps track of its performance through prior generations and recollects the coordinates within the search space where it achieved its peak performance.
- 2. Each particle is aware of the most favorable positions held by its neighbours during the current generation. This requires that all particles communicate their respective performances.

These dual facets randomly modulate an individual's velocity and bear resemblance to human social dynamics. There are instances when an individual may exhibit a higher degree of resolve, thus not being substantially swayed by its peers. Conversely, at times, an individual may be more influenced by nostalgia, thus being significantly impacted by past achievements.

2.1. Metaphor

In Particle Swarm Optimizations (PSOs), which take cues from the swarming behavior of birds and schools of fish, numerous basic units known as particles are introduced into the parameter space of a specific problem or function. Each of these particles assesses the quality or fitness of its current position. Subsequently, every particle decides its path through the parameter space by integrating elements of its own historical fitness data with that of one or more fellow particles. This path is navigated at a speed that is dictated by the positions and the evaluated fitness levels of the other particles, coupled with some random variations. The particles with which a given particle can engage are termed its social neighborhood. The aggregate of these social neighborhoods across all particles constitutes the social network of the PSO.

The particle swarm algorithm imitates human social behavior. —James Kennedy and Russell Eberhart [Kennedy and Eberhart, 2001]

Particle swarm optimization (PSO) draws its inspiration from the realization that clusters of entities collaborate to enhance both their joint and individual performance in a given task. The fundamentals of PSO are evidently manifested in the behavior of animals and humans alike. In our endeavors to better our performance in a particular task, we modify our strategies relying on certain elementary concepts. [Sim13]

2.2. Unique architectural aspects

In Particle Swarm Optimization (PSO), the configuration concerning how each particle is influenced by its neighboring particles is referred to as the swarm's topology. The topology encompasses the arrangement of neighbors that exert influence on a given particle. In certain cases, the neighborhood surrounding each particle evolves with each generation, rendering the topology dynamic. When the neighborhood is localized and does not incorporate the entire swarm, it is referred to as a local or *lbest* topology.

There are numerous methodologies to establish the neighborhood of each particle. For instance, neighborhoods can be predefined at the onset of the algorithm, creating static topologies that remain constant across generations. Alternatively, if the optimization process reaches stagnation, neighborhoods can be randomly redefined at that juncture. In an extreme scenario, a single neighborhood encompassing the entire swarm can be employed, implying that each particle is influenced by the entire swarm. This is termed the global or *gbest* topology and is historically significant as it represents the original topology employed during the development of PSO. It continues to be widely utilized. [Sim13]

Apart from the global topology, other common topologies include the ring topology, where each particle is connected to two other particles; the cluster topology, where particles are highly connected within their clusters with sparse connections to other clusters; the wheel topology, in which a central particle is connected to all others while the others are solely connected to the central particle; and the square or von Neumann topology, where each particle is connected to four neighbors, forming a toroidal structure as connections wrap from top to bottom and left to right. [Sim13]

The choice of topology in the social network of a PSO is crucial as it plays a significant role in dictating the behavior and performance of the algorithm, sometimes being as influential as the velocity update equation. There have been numerous variants of PSO and a plethora of topologies that have been developed and evaluated. PSO, with its varied topologies, has been successfully applied across a diverse range of applications. It has demonstrated particular effectiveness in multimodal problems and in scenarios where no specialized method exists or where specialized methods yield unsatisfactory results. [Sim13]

2.3. Parameters

As any other evolutionary algorithm the PSO defines certain parameters, some of which are unique to this optimisation algorithm. These parameters can influence the behaviour of the algorithm, exploration and exploitation rates. [Eng07]

2.3.1. inertia weight

Inertia weight is a factor that contributes to the momentum of the particles in PSO. Essentially, it determines the extent to which a particle's previous velocity influences its current velocity. It is the component that allows the particle to sustain its directional course through the search space.

A high inertia weight encourages exploration by permitting a broader search, which may help in escaping local minima. Conversely, a low inertia weight tends to facilitate exploitation, enabling the particle to conduct a more localized search. This is essential for refining solutions in the vicinity of the perceived optimum. Balancing the inertia weight is vital for ensuring an effective trade-off between exploration and exploitation. [Eng07]

2.3.2. cognitive weight

Cognitive weight, often denoted by a constant, reflects the significance given to the individual experiences and memory of particles. It represents how much a particle's velocity is influenced by its personal best position encountered thus far. When a particle moves through the search space, it keeps track of the best position it has ever achieved. The cognitive weight determines how strongly the particle is drawn towards this personal best position. A higher cognitive weight prompts the particle to favor its historical experiences, steering it back towards regions of the search space where it has previously found promising solutions. [Eng07]

2.3.3. social weight

Social weight, also commonly referred to as global weight and often represented by another constant, signifies the importance accorded to the collective knowledge of the swarm. It quantifies the degree to which a particle's velocity is influenced by the best position known to the entire swarm. Essentially, it reflects how much a particle is attracted towards the global best position found by any member of the swarm. A high global weight encourages cooperative behavior among particles, driving them towards regions of the search space deemed promising by their peers. [Eng07]

2.4. Key mechanisms

In Particle Swarm Optimization, velocity is a fundamental attribute that dictates the movement of particles through the search space. It is a vectorial quantity, dynamically adjusted in each iteration, that influences how a particle approaches potential solutions. The calculation of velocity encompasses various components, including inertia, cognitive velocity, and global velocity, which collectively balance the particle's momentum, individual experiences, and social information. The judicious modulation of velocity is essential for efficiently navigating the search space, striking a balance between exploration and exploitation in pursuit of optimal solutions. [Eng07]

2.4.1. cognitive velocity

Cognitive velocity represents the component of a particle's velocity that is influenced by its own historical best position. This aspect reflects the tendency of a particle to gravitate towards the most favorable position it has encountered in the past. The cognitive velocity is calculated as the product of the cognitive weight (often denoted by a constant) and the difference between the particle's personal best position and its current position. This component essentially quantifies the

pull towards the particle's historical achievements, guiding it based on its individual experiences. [Eng07]

2.4.2. global velocity

Global velocity, on the other hand, corresponds to the component of a particle's velocity that is influenced by the best position known to the entire swarm, or the global best position. This component reflects the inclination of the particle to move towards the most promising area found by any member of the swarm. The global velocity is computed as the product of the global weight (also often represented by a constant) and the difference between the swarm's global best position and the particle's current position. This component essentially quantifies the pull towards collective knowledge and achievements within the swarm. [Eng07]

2.5. Well established applications

The PSO algorithm can be implemented in solving a very broad range of problems. An analysis of the Publications on the Applications of Particle Swarm Optimisation details a broad range of applications. The most prevalent of which were in antenna design, biomedical applications, and modeling.

Antenna Design: The telecommunications industry heavily relies on antennas for the propagation of signals. The performance and efficiency of antennas are crucial for reliable communication. PSO algorithms help in optimizing various attributes of antenna structures. For instance, it can be used for fine-tuning phased array antennas to achieve desired beam direction and shape.

Biomedical Applications: The complexity and sensitivity of biomedical data and systems necessitate sophisticated optimization techniques. PSO algorithms have been embraced in the biomedical sector for a plethora of applications. For instance, PSO is employed in the optimization of radiotherapy planning, where the objective is to maximize the dosage to cancerous tissues while minimizing exposure to surrounding healthy tissues. Additionally, PSO algorithms have been used for gene clustering and the identification of transcription factor binding sites in DNA, which is significant in genomics research.

Modelling: The development of descriptive and predictive models is integral to understanding and solving complex problems across various domains. PSO algorithms are used in creating models that accurately represent phenomena or predict future outcomes. For example, in systems engineering, PSO is used for model order reduction which simplifies complex systems without significant loss of accuracy.

Apart from the aforementioned applications, Particle Swarm Optimization (PSO) algorithms are utilized across diverse fields. In image processing and robotics, they aid in tasks like image segmentation and path planning, respectively. Additionally, PSO finds applications in optimizing logistics operations in supply chain management and in financial market analyses for portfolio optimization.

3. Harmony Search Optimisation

The Harmony Search (HS) algorithm, emerging as a novel paradigm in the field of evolutionary algorithms (EAs), is inspired by the phenomenological process of musical improvisation. This optimization technique, conceptualized by Zong Woo Geem et al. in 2001, diverges from traditional evolutionary strategies by embodying the principles of jazz musicians' spontaneous and harmonious improvisation. Distinct from the Darwinian concepts of natural selection and genetic crossover predominant in most EAs, HS encapsulates a unique mechanism of global search and local refinement, drawing parallels with the creative and iterative process of achieving harmony in music.

3.1. Metaphor

The inception of the Harmony Search (HS) algorithm is attributed to Geem et al. [GKL01]. The algorithm is fundamentally inspired by the process of musical improvisation, wherein musicians strive to achieve harmonious melodies. This metaphorical framework is adeptly applied to the domain of optimization, where a similar pursuit for harmonious solutions occurs within a defined problem space.

3.2. Unique Architectural Aspects

According to Gao et al. [GGX⁺15], the Harmony Search algorithm is distinguished by its simplicity and efficiency. This unique architectural aspect is derived from the natural process of improvisation in music, wherein various combinations and pitches are experimented with to achieve aesthetic harmony.

3.3. Parameters

The effectiveness of the Harmony Search method is contingent upon several key parameters, as elucidated by Geem et al. [GKL01].

3.3.1. Harmony Memory Size

Harmony Memory Size refers to the number of solution vectors stored in the Harmony Memory. This parameter is crucial as it represents the size of the solution space that the algorithm considers for optimization. A larger Harmony Memory Size allows for a more diverse set of solutions, potentially leading to a better final solution, but it also increases computational complexity.

3.3.2. Memory Consideration Rate

The Memory Consideration Rate, as discussed by Geem et al. [GKL01], determines the probability with which the algorithm selects values from the existing solutions in the Harmony Memory as opposed to generating new random values. A higher rate typically encourages the

algorithm to explore the existing solution space more thoroughly, while a lower rate increases the potential for discovering new and diverse solutions.

3.3.3. Pitch Adjustment Rate

Pitch Adjustment Rate is a parameter that dictates the rate at which the algorithm makes small adjustments to the solution vectors. As described by Geem et al. [GKL01], this parameter is akin to fine-tuning the pitches in a musical composition, allowing for subtle adjustments that can lead to an improved solution.

3.3.4. Pitch Bandwidth

Pitch Bandwidth, as introduced by Gao et al. [GGX⁺15], controls the degree of variation allowed during the pitch adjustment process. This parameter can be viewed as defining the range or intensity of the adjustments made to the solution vectors. A wider bandwidth allows for more significant changes, potentially leading to faster convergence but also increasing the risk of skipping over the optimal solution.

3.4. Key Mechanisms

Geem et al. [GKL01] describe the primary mechanisms underpinning the Harmony Search (HS) algorithm. These mechanisms involve initializing the Harmony Memory, improvising new harmonies, and updating the Harmony Memory.

3.4.1. Fine-Tuning of Solutions

The fine-tuning of solutions in HS is primarily achieved through the Pitch Adjustment process. This mechanism is analogous to a musician slightly altering the pitch of notes to achieve a more harmonious melody. In the context of HS, each component of a solution vector is considered for potential adjustment with a certain probability, defined by the Pitch Adjustment Rate. The degree of adjustment is governed by the Pitch Bandwidth. This process allows HS to refine and enhance the quality of solutions stored in the Harmony Memory, leading to a gradual improvement in search results.

3.4.2. Exploration of New Solutions

Exploration in HS is facilitated by the Harmony Memory Consideration Rate and the random selection of new components. When improvising a new harmony, the algorithm decides for each component whether to choose from the existing solutions in the Harmony Memory or to generate a new random value. This decision is based on the Harmony Memory Consideration Rate. A lower rate increases the likelihood of introducing new random components, thereby enhancing exploration and the potential to escape local optima. This aspect of HS ensures that while the al-

gorithm refines solutions, it also maintains a healthy level of exploration, vital for effective search in complex optimization landscapes.

3.5. Well-Established Applications

The Harmony Search algorithm has found widespread application in various fields, a testament to its versatility and effectiveness. As noted by Gao et al. [GGX+15] and Geem et al. [GKL01], it has been successfully employed in function optimization, mechanical structure design, and pipe network optimization, among others.

4. Evolutionary Algorithms for Optimal Image Collage Composition problem

Collage is a prevalent medium employed for the representation of information and the exhibition of a collection of images. Occasionally, the construction of a collage demands an artistic perspective to effectively arrange the given images in a manner that is aesthetically pleasing. However, frequently, the task becomes unnecessarily convoluted. While it might be relatively straightforward to find an optimal arrangement for a small number of images, the complexity increases substantially when the images have varying dimensions or do not completely fill the designated space. The task can be formulated as follows: Place a set of n images onto a canvas with predetermined dimensions, ensuring that the images are positioned such that there is no overlap or extension beyond the canvas boundaries. Furthermore, the images must be resized proportionally in relation to each other, and the arrangement should aim to minimize the amount of uncovered area on the canvas. It is worth noting that this task is computationally intractable and is classified as NP-Complete.

4.1. Relation to 2D bin packing problem

In the realm of optimization problems, the Two-Dimensional (2D) Packing Problem holds particular significance, especially in industries where efficient space utilization is crucial. The problem involves fitting a collection of 2D rectangular items into a confined area or container, with the objective to maximize the utilization of the available space. This requires careful arrangement of the items such that there is no overlap between them. The 2D Packing Problem finds its applications in various industries including manufacturing, logistics, and shipping. Due to the combinatorial nature of this problem, finding an optimal solution is computationally challenging, especially as the number of items increases. Therefore, various heuristic and meta-heuristic algorithms such as Particle Swarm Optimization, Genetic Algorithms, and Simulated Annealing are often employed to find approximate solutions within a reasonable time frame.

The image placement problem shares similarities with the classical 2D bin packing problem, which can be defined as follows: The task is to pack a collection of two-dimensional rectangles into the minimum number of larger rectangular bins, where each bin has a predetermined width and height. The primary objective is to arrange the rectangles efficiently within the bins, ensuring that each rectangle is entirely contained within a bin, with no overlap and without any rotation.

It is pertinent to recognize the image placement problem defined in this article as a variant of the 2D bin packing problem. The distinction here is that the focus is not on minimizing the quantity of bins but on strategically positioning and scaling a collection of images onto a sheet of paper. The sheet of paper could be analogous to a bin, and the images - the objects awaiting packing. Adhering to the constraints of the 2D bin packing problem, the images must be positioned within the sheet without overlap and must not exceed the boundaries of the paper.

However, the image placement problem introduces some intricacies to the 2D bin packing problem. One such complexity is the necessity to scale images. This adds an additional layer to the problem, which is absent in the conventional 2D bin packing problem. This task encompasses

the dual challenge of ascertaining the optimal position for each image coupled with identifying an appropriate scaling factor. Moreover, this project could extend beyond the confines of packing efficiency. Here, needs of digital designers need to be taken into account. From this, the need for consistency in image scaling, tolerance for overlapping and out of boundary solutions arises.

Therefore, though this project bears semblance to the 2D bin packing problem by focusing on the efficient allocation of 2D objects within a limited space, it incorporates additional elements. This suggests that while algorithms and solutions employed in classical 2D bin packing could provide a rudimentary framework, there is a necessity to adapt and augment these strategies to cater to the distinct nature of this project.

4.2. Input

The program accepts an array of image dimensions as input, offering flexibility for modifications and extensions. It is noteworthy that the program can be adapted to perform a scan of the specified folder to identify image files, specifically those with the file extensions .png, .jpg, and .jpeg¹. And subsequently, extract the dimensions of these images that would be used as input.

4.3. Output

The output of the program is systematically presented as a vector, which encapsulates attributes of each image: the image identifiers (IDs), their respective coordinates on the page, and the relative scaling factors. Notably, the coordinate system employed for image placement originates from the lower-left corner of the page, which serves as the reference point (0,0).

Each image's scale is expressed relative to its original dimensions, thereby preserving aspect ratios and ensuring consistency across varying image sizes. For instance, an image retaining its original size during the process will have a relative scale factor of 1. This approach allows for the precise and unaltered representation of images, respecting their inherent proportions while situating them appropriately within the defined coordinate space.

4.4. Fitness Function

The complexity inherent in the optimization problem necessitates a fitness function that judiciously considers multiple factors to attain the desired outcome. In an ideal scenario, a solution achieving a fitness score of zero is considered perfect. Nevertheless, the specific implementation of this optimization algorithm might lead to situations where certain combinations of inputs do not yield a perfect solution.

To enhance the algorithm's effectiveness, especially in cases where a perfect solution is elusive, it becomes essential to fine-tune the desired fitness value. This adjustment takes into account the unique demands of digital designers. Emphasis is placed on assigning more severe penalties

¹The file extensions considered are .png, .jpg, and .jpeg.

for solutions that result in overlaps, as opposed to those leading to uncovered areas. This approach involves the integration of several adjustment factors into the fitness function, each acting as a multiplier for its respective penalty factor:

- $Scaling\ Penalty\ Factor = 1$,
- *Boundary Penalty Factor* = 10,
- *Overlap Penalty Factor* = 5,
- *Uncovered Area Penalty Factor* = 5.

These penalty factors are calibrated to impose lesser severity for penalties related to the relative resizing of images. Importantly, these factors do not influence solutions in scenarios where achieving a perfect fitness score (zero) is feasible. They are designed to prioritize certain solutions over others, aligning with our conceptualization of what constitutes a superior collage composition.

4.4.1. Resizing

Moreover, solutions are subjected to penalties for substantial deviations in resizing. These penalties are computed in relative terms, as the algorithm aims to cover the plane with the provided images.

For instance, consider a scenario where the program is supplied with four images, each having dimensions of $25px \times 25px$. To fully cover the plane, the images must be scaled up by a factor of two. A desired solution in this case would have the following characteristics:

```
• Image1: x = 0; y = 0; scale = 2;
```

• Image2: x = 50; y = 0; scale = 2;

• Image3: x = 0; y = 50; scale = 2;

• Image4: x = 50; y = 50; scale = 2.

For such a solution to attain the perfect fitness score of zero, the scale penalty must be computed relative to each image. Essentially, scaling is a vital component of the solution, and to ensure that the scaling of images does not adversely impact the fitness score, the scale factor for all images must be identical.

4.4.2. Overlapping

The algorithm imposes penalties on solutions wherein the images overlap. The objective is to arrange all the images on a designated plane, such as a piece of paper, ensuring that they are all discernible and free of overlaps.

4.4.3. Boundaries

The instances where images extend beyond the boundaries are penalized analogously to the overlapping of images. Essentially, cases where an image extends beyond the designated boundaries can be conceptualized as overlapping, where the image in question is obstructed by the confines of the designated plane.

4.4.4. Uncovered Area

Certain combinations of inputs may not yield optimal solutions. In such scenarios, it is imperative to discriminate between solutions based on their desirability. In the context of image placement, preventing clipping is often prioritized as it can obfuscate crucial visual information and necessitate manual adjustments by the editor, rendering the automated solution ineffectual. Additionally, alternative artistic approaches, such as the incorporation of a background or the inclusion of supplementary images to fill the gaps, can be employed. Consequently, the fitness function is calibrated to favor solutions with uncovered areas over those with clipping or overlapping. This adaptation yields more pragmatic results in scenarios where optimal solutions are either unattainable or remain unidentified.

4.5. **Solution using PSO**

4.5.1. Hyper-parameter values

Best parameter values were found using exhaustive search. The algorithm was run 20 000 times, once for each different combination of parameters. It was run for 4 images that are of equal size and completely tile the canvas. Input:

• Canvas Size: 100 × 100

• Images: Four images, each with dimensions 50×50

The algorithm performed the best with the hyper-parameter values denoted in 1.

Table 1. Hyper-parameter ranking for PSO

Population Size |w| c1 | c2 | Fitness | Iterations | Particles

_						
110	0.7	1.5	1.5	0	28	3080
50	0.7	1.1	1.3	0	66	3300
70	0.5	1.5	1.7	0	55	3850
50	0.7	1.1	1.7	0	84	4200
170	0.3	1.5	1.7	0	27	4590
110	0.6	1.7	1.5	0	48	5280
100	0.5	1.9	1.5	0	53	5300
180	0.5	0.1	1.7	0	30	5400
80	0.6	1.9	1.3	0	68	5440
150	0.4	0.7	1.7	0	37	5550

The following values will be used:

1. Population Size (population_size): 110

2. Inertia Weight (w): 0.7

3. Cognitive Coefficient (c1): 1.5

4. Social Coefficient (c2): 1.5

4.5.2. Initialization

Velocity. Each particle is initialized with a velocity value randomly selected within the range [-1, 1].

Position Initialization of Particles. The initialization of each particle within the algorithm is a critical step that sets the foundation for subsequent optimization processes. Each particle is assigned a position and scale based on randomized values within predefined ranges. The *x*-coordinate of a particle is randomly selected within the range from 0 to the width of the paper. Similarly, the *y*-coordinate is chosen randomly from a range spanning from 0 to the paper's height. Additionally, each particle is initialized with a scale factor, which is also determined randomly but within a range defined by minimum and maximum scale factors.

Minimum and maximum scale factors. The minimum and maximum scale factors are calculated based on the dimensions of the paper and the sizes of the images to be placed. Specifically, the maximum scale factor is derived by dividing the larger dimension of the paper (either width or height) by the smallest dimension found among all images. Conversely, the minimum scale factor is calculated by considering the total length of images along the y-axis and the x-axis, and then determining the ratio of the paper's height to this total y-axis length, and the paper's width to the total x-axis length, respectively. The smaller of these two ratios is taken as the minimum scale factor.

This method of initializing particle positions and scale factors strategically places the program near the potentially viable solution space, enhancing the efficiency of the optimization process.

4.5.3. Execution

Velocity. As previously discussed, velocity plays a crucial role in determining the efficacy of a Particle Swarm Optimization (PSO) algorithm. There are five hyper-parameters that aid in the fine-tuning of the algorithm, three of which have a direct impact on velocity computation.

The new velocity for each particle is derived by combining the current velocity, cognitive velocity, and social velocity. The parameters r1 and r2 are incorporated to instill stochasticity into the velocity update equation, which fosters diverse exploration of the search space by the particles.

• Current Velocity: It is multiplied by an inertia weight (w) to contribute to the new velocity.

- Cognitive Velocity: It is computed by multiplying the cognitive scaling parameter (c1) by a random value $(r1^2)$ and the vector directed towards the particle's personal best position.
- Social Velocity: Similar to cognitive velocity, social velocity is computed by multiplying the social scaling parameter (c2) by a random value $(r2^3)$ and the vector directed towards the global best position.

Position. The position of each particle is updated by adding its current velocity to its existing position. This process iteratively modifies the positions of the particles in the search space.

4.6. Solution using HSO

4.6.1. Hyper-parameter values

For HSO, the best parameter values were found using exhaustive search. The algorithm was once again run $20\,000$ times, once for each different combination of parameters. It was run for 4 images that are of equal size and completely tile the canvas. Input:

• Canvas Size: 100 × 100

• Images: Four images, each with dimensions 50×50

The algorithm performed the best with the hyper-parameter values denoted in ??.

HM Size **MCDR PAR** PB **Fitness Iterations** 50 0.95 0.4 0.1 0 11398 180 1 0.1 0.6 0 11835 180 1 0.2 0.6 0 12782 110 1 0.2 0.7 0 12860 0 190 1 0.1 1 13522 120 1 0.2 0.9 0 15995 30 1 0.7 0 0.4 16787 1 0 80 0.3 17138 1 90 0.3 0 17709 90 0.9 0.2 1 0 18925

Table 2. Hyper-parameter ranking for PSO

- 1. Harmony Memory Size (*HM_size*): 50
- 2. Memory Consideration Rate (memory_consideration_rate): 0.95
- 3. Pitch Adjustment Rate (pitch_adjustment_rate): 0.4
- 4. Pitch Bandwidth (pitch_bandwidth): 0.1

 $^{^2}r1$ and r2 introduce stochasticity into the velocity update equation, enabling the particles to explore the search space more diversely.

 $^{^{3}}$ Refer to the footnote regarding r1.

4.6.2. Initialization

Harmony Memory. The Harmony Memory (HM) forms a fundamental component of the Harmony Search algorithm. It is initialized by populating it with a predefined number of harmonies. This number is determined by the hyper-parameter 'HM_size', which specifies the size of the Harmony Memory. The initialization of HM is a crucial step as it sets the initial solution space from which the algorithm begins its search process.

Harmonies. Each harmony within the Harmony Memory is initialized with specific values for each dimension. These dimensions correspond to the position and scale of images in the context of the problem being solved. The initialization process for the dimensions of each harmony mirrors that used in Particle Swarm Optimization (PSO). Specifically, the *x*-coordinate for each image is randomly chosen from a range between 0 and the paper width, while the *y*-coordinate is selected from a range between 0 and the paper height. Additionally, the scale for each image is initialized to values between the computed minimum and maximum scale factors. These scale factors are calculated as described in the section on *Minimum and Maximum Scale Factors*.

This initialization approach ensures that each harmony in the Harmony Memory represents a feasible solution within the problem's constraints, thus positioning the algorithm effectively within the solution space for subsequent optimization iterations.

4.6.3. Execution

Harmony Memory. During execution, the Harmony Memory (HM) plays a crucial role as the repository of potential solutions. As the algorithm iterates, the HM is dynamically updated with new harmonies that are improvised based on the existing ones. This process involves evaluating the fitness of each harmony in the HM and replacing less fit harmonies with better ones. The continuous update and refinement of the HM allow the algorithm to progress towards more optimal solutions over time. The effectiveness of the HM relies on its ability to balance the exploration of new solutions with the exploitation of already discovered promising solutions.

Pitch Adjustment. During execution, for each component of a harmony, there is a certain probability that a pitch adjustment will occur. This probability is governed by the Pitch Adjustment Rate. When a pitch adjustment is made, the value of the component is slightly altered within a range defined by the Pitch Bandwidth. This process is akin to a musician delicately adjusting the pitch of notes to improve the harmony. In the context of optimization, pitch adjustment allows the algorithm to explore the solution space near the current solution, enhancing the algorithm's ability to converge on a global optimum by refining the existing solutions.

4.7. Performance evaluation

Reverse Engineering Bench-marking will be used to evaluate performance of these algorithms. How well the program performs will be evaluated by giving it a problem that the solution is

known for. In this case, for the evaluation to be quantitative and more accurate it is important for the perfect solution (fitness of 0) to exist for given input. By providing the program with input that we know has at least one perfect solution we can perform bench-marking and objectively gauge the performance of the algorithm.

4.7.1. Unit test generator

A program cuts an image up into n pieces, where n is a natural number 1 > n > 8. It does so by selecting 1-2 random points on the image and drawing orthogonal cut lines on it. Then The program crops the original image n times thus generating n images that cover the whole plain, given the original image is of the same proportions, as the paper we are trying to cover.

4.7.2. Testing

10 tests were carried out with each of the values of n. The test generator cuts the given image into n pieces 10 times. To account for the test generator generating problems of different complexities, each test was repeated 10 times. Overall the algorithm was run 100 times for each value of n. First, tests were made for each value of n only changing the population size intuitively. The algorithm was set to stop after 1000 iterations without any improvement on the best global position, assuming at this point it has already converged. This approach allows for determining the convergence rate for the algorithm with each test and different values of n. In addition, if the algorithm does not converge in any of the 10 runs, the best found solution for each test is saved and can be applied or analysed. The image chosen is a semi-transparent visual in the .png format with the goal to effortlessly detect overlapping.



Fig. 1. Image used for test generation



Fig. 2. Solution where pieces overlap

For these tests a canvas of $30px \times 30px$ was chosen and the image provided for the tests is of the same dimensionality. At first, the program was run from the n of 2 to the n of 5.

4.7.2.1. What to expect

Since this testing approach provides tasks that have at least one perfect solution, the x and y values should be in the range of 0-30. For the first basic tests of the algorithm, the image provided to the test generator is of the same dimensions as the canvas. Therefore, the scale of each piece, assuming the perfect solution was found, should be exactly 1. Given the nature of evolutionary optimisation algorithms, the algorithms are not expected to converge 100% of the time.

4.7.2.2. n = 2

Since the tests provide solutions that cover the whole plain, with the n of, 2 pieces of one image is passed to the program and essentially it only has to place each of the images on the opposite sides of the paper. This, so far, is a trivial task for humans, so this is a rudimentary stage for the algorithm to successfully work, else chances are slim that it would provide any value or serve any useful purpose further down the line.



Fig. 3. Results for tests run when n=2

Particle swarm optimisation algorithm showed better results. It was able to find perfect solutions on the first run for all 5 tests. Harmony search optimisation algorithm took up to 6 runs to deliver a perfect solution.

4.7.2.3. n = 3

With the n of 3, the task is equally as trivial for a human to solve if a simple algorithm of putting each of the images in opposing corners is followed. 5 tests were run. The program was successful at finding the perfect solution for all of the tests. Once again, particle swarm optimisation algorithm performed better.



Fig. 4. Results for tests run when n=3

For 3 of the 5 tests PSO converged on the perfect solution in the first run. Other two took 4 and 10 runs. Harmony search was unable to find the perfect solution under 6 runs. In total it was run 113 times.

4.7.2.4. n = 4

When n is 4, the task remains trivial for a human as long as the same approach of putting each piece in one of the corners is followed. 5 tests were run. PSO was successful at finding the perfect solution for only 3 of the tests. HSO for only one. The program became less reliable at finding satisfactory solutions.

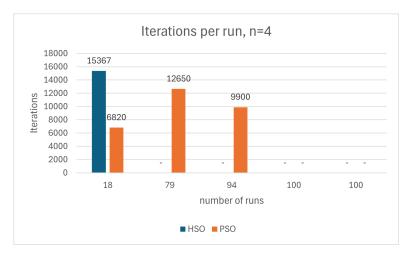


Fig. 5. Results for tests run when n = 4

The algorithm failed to find viable solutions for some generated tests. The test that generated worst fitness and its best found solution looks like this:

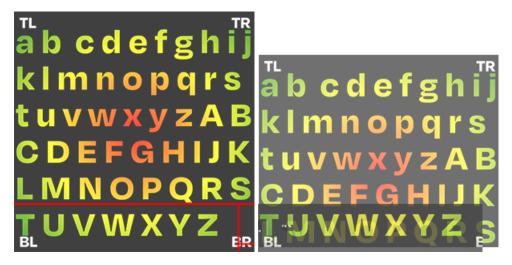


Fig. 6. Complicated test n = 4

On the other hand here is what images from a test that performed the best looks like:



Fig. 7. Easy test n = 4

4.7.2.5. n = 5

When n takes on a value of 5, the task becomes more complicated for humans. It now may require some rearrangement of the images to achieve the same result.

5 tests were run. The program was unsuccessful at finding the perfect solution any of tests. Particle swarm optimisation algorithm performed better.

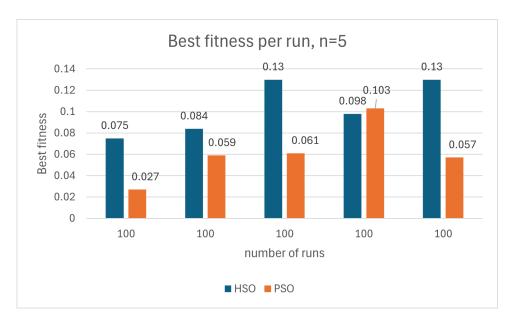


Fig. 8. Results for tests run when n = 5

Results and Conclusions

Results. This section presents the key findings derived from the application of Particle Swarm Optimization (PSO) and Harmony Search Optimization (HSO) algorithms to the image placement problem. The performance of these algorithms was evaluated based on their ability to find optimal solutions and the efficiency of their search process.

- The Particle Swarm Optimization algorithm demonstrated a higher success rate in identifying perfect solutions across various test runs. This underlines the effectiveness of PSO in navigating the solution space for the image placement problem.
- In instances where the Harmony Search Optimization algorithm was capable of finding perfect solutions, it achieved these results in a fewer number of iterations compared to PSO. This indicates a potential for greater efficiency in HSO under certain conditions.
- A notable limitation was observed in both algorithms concerning their capability to handle larger problem sets. Specifically, the algorithms failed to find perfect solutions when tasked with placing five or more images. This highlights a scalability challenge in the current implementation of these optimization techniques.

Conclusions.

- A comprehensive literature review was conducted, leading to the selection of Particle Swarm Optimization (PSO) and Harmony Search Optimization (HSO) for their effectiveness in complex optimization scenarios.
- The relevance of the 2D bin packing problem to the image placement problem was established, guiding the adoption of effective methodologies.
- PSO and HSO algorithms were implemented and rigorously tested for solving the image placement problem, with a focus on their application and performance evaluation.
- A test generator was developed, enabling the creation of diverse scenarios for a thorough assessment of both algorithms.
- The testing process provided critical insights into the performance of PSO and HSO, highlighting their strengths and areas for improvement in the context of the image placement problem.
- This research contributes significantly to the understanding of evolutionary optimization algorithms, particularly PSO, and opens up new avenues for future research and enhancements in the field of complex optimization problems.

Potential Enhancements.

- Conduct further experiments to identify optimal hyper-parameter values, enhancing algorithm performance. This includes faster convergence, more efficient iteration processes, and improved reliability in finding satisfactory solutions.
- Customize hyper-parameters more precisely according to specific problem attributes. For example, different parameter settings should be applied for scenarios with varying numbers of images, such as five or six images.
- Implement dynamic parameter adjustments for PSO, specifically in the computation of solution fitness and particle positioning across various dimensions.
- For HSO, consider additional strategies like re-initializing harmony memory when there is a significant deviation from the desired fitness level.
- Develop mechanisms to detect early convergence towards non-optimal solutions, enabling timely intervention and alternative solution approaches.

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Evolutionary optimisation algorithms: Particle Swarm Optimisation alogorithm

Kazimieras Jasaitis

Abstract

In contemporary society, technology has become integral to everyday life, with computers executing a plethora of tasks with deterministic precision. However, there exists a category of complex, computationally intensive problems for which deterministic algorithms are insufficient. One such problem is collage creation, which involves arranging multiple images on a fixed-size canvas without overlaps or unoccupied space, and is classified as an NP-Hard problem. This thesis explores the application of Particle Swarm Optimization (PSO) and Harmony Search Optimisation (HSO) an evolutionary optimization algorithms, in solving the Image Collage Composition problem. The PSO and HSO algorithms are analyzed in-depth, and their applications to the problem are investigated through the development of a unit test generator and the execution of performance tests. The unit test generator segments an image into n pieces, and the PSO algorithm is applied to find optimal or near-optimal placements of these pieces on a canvas. The performance of the PSO algorithm is evaluated and analyzed critically, considering various metrics. The study concludes with insights drawn from the results and recommendations for potential enhancements to the PSO algorithm for future research. This work contributes to the understanding of evolutionary optimization algorithms and presents a practical approach to solving complex optimization problems such as collage creation.

Keywords: Particle Swarm Optimisation, optimization, evolutionary methods, 2D bin packing problem.

Evoliuciniai optimizavimo algoritmai: Dalelių Spiečiaus Optimizacijos algoritmas

Kazimieras Jasaitis

Santrauka

Šiuolaikinėje visuomenėje technologijos tapo neatsiejama nuo kasdienio gyvenimo dalimi, kompiuteriai atlieka gausybę užduočių labai dideliu tikslumu. Tačiau yra kompleksiškų, daug skaičiavimų reikalaujančių problemų kategorija, kuriai deterministinių algoritmų nepakanka. Viena iš tokių problemų yra koliažo, iš kelių fiksuoto dydžio vaizdų, kūrimas. Tokia problema yra klasifikuojama kaip NP-Hard problema (sudėtinga nedeterministinė polinominė problema). Šis straipsnis tiria dalelių spiečiaus optimizavimo algoritmo, pritaikymą šiai problemai. Šis straipsnis tiria dalelių spiečių optimizavimo (PSO) ir Harmonijų paieškos (HSO) evoliucinių optimizavimo algoritmų, pritaikymą. Šiame darbe nuodugniai išanalizuotas dalelių spiečiaus optimizacijos ir Harmonijų paieškos optimizacijos algoritmai tiriami jų pritaikymai problemai, vykdomas programos testavimas, bei analizė. Testai buvo vykdomi suskirstant vaizdą į n dalių, algoritmas taikomas ieškant optimalaus šių dalių išdėstymo. Algoritmo veikimas vertinamas ir analizuojamas kritiškai, atsižvelgiant į įvairius rodiklius. Tyrimo pabaigoje pateikiamos įžvalgos, padarytos remiantis rezultatais, ir rekomendacijos dėl galimų algoritmo patobulinimų būsimiems tyrimams ar praktiniam pritaikymui. Šis darbas padeda suprasti evoliucinio optimizavimo algoritmus ir pateikia praktinį požiūrį į sudėtingų optimizavimo problemų, tokių kaip koliažo kūrimas, sprendimą.

Raktažodžiai: Dalelių Spiečiaus Optimizacijos algoritmas, optimizacija, evoliuciniai programavimo metodai, dvimatė pakavimo problema.

Additional files

Additional file no. 1

Program files.

https://github.com/KazimierasJasaitis/2023Kursinis