前期中間試験解答

1. $V(X) = E((X - E(X))^2) = E(X^2 - 2XE(X) + E^2(X))$

$= E(X^2) - 2E(X)E(X) + E^2(X)$

$= E(X^2) - E^2(X)$

2. (a) $-\log_2 \frac{1}{52} = \log_2 13 \times 4 = 2 + \log_2 13 \simeq 5.7$ bit

(b) $-\log_2 \frac{1}{4} = 2$

(c) $-\log_2 \frac{1}{13} = \log_2 13 = 3.7$

(d) $2 + 3.7 = 5.7$ bit となり (a) の値と同じ

3

$H(X,Y) = -\sum_{i,j} P(x_i, y_j) \log P(x_i, y_j)$

$= -0.6 \log 0.6 - 0.1 \log 0.1 - 0.05 \log 0.05 - 0.25 \log 0.25$

$\simeq 1.49$ bit $(1.03$ nat$)$

$H(X) = -\sum_i P(x_i) \log P(x_i) = -0.7 \log 0.7 - 0.3 \log 0.3 \simeq 0.88$ bit $(0.61$ nat$)$

$H(Y) = -0.65 \log 0.65 - 0.35 \log 0.35 \simeq 0.93$ bit

$H(X|Y) = H(X,Y) - H(Y) \simeq 0.56$ bit

$I(X,Y) = H(X) - H(X|Y) = 0.32$ bit

4. Aさんと Bさんの KLダイバージェンスは.

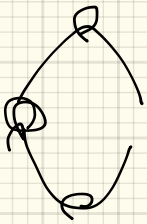
$D_A = -(0.7\log 0.8 + 0.3\log 0.2) + 0.7\log 0.7 + 0.3\log 0.3 \simeq 0.028$

$D_B = -(0.7\log 0.6 + 0.3\log 0.4) + 0.7\log 0.7 + 0.3\log 0.3 \simeq 0.021$

となる. よって Bさんの予想の方が合っているといえる.

5. (a)

$H(p) = -(p\log p + (1-p)\log(1-p))$



$p=0$ or 1 のとき $H(0)=H(1)=0$

$p=\frac{1}{2}$ のとき $H(\frac{1}{2})=1$

よって極値は最大値となる

(b) $H'(p) = -(\log p + 1 - \log(1-p) - 1)$

$= \log\frac{1-p}{p}$

極値を求める

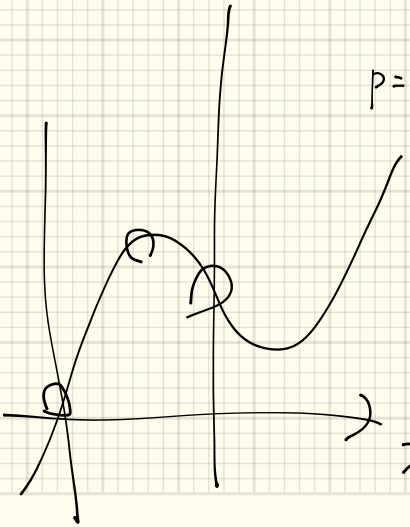
$\log\frac{1-p}{p} = 0$

$\frac{1-p}{p} = 1$

$2p = 1$

$p = \frac{1}{2}$

$p=\frac{1}{2}$ のとき $H(p)$ は最大値になる



6 (a) 相互情報量

(b) (a) $H(B|A)$ (b) $H(A,B)$

7 (a) $H(A) = -6 \cdot \frac{1}{6} \log_2 \frac{1}{6} = \log_2 6 = \log_2 3 + 1 = 2.6$ bit

$H(B) = -\frac{1}{3}\log_2 \frac{1}{3} - \frac{2}{3}\log_2 \frac{2}{3} = \frac{1}{3}\log_2 3 + \frac{2}{3}\log_2 3 - \frac{2}{3}$

$= \log_2 3 - \frac{2}{3} = 0.92$ bit

(b) $H(A|b_1) = -2 \times \frac{1}{2} \times \log_2 \frac{1}{2} = 1$

前期期末試験

1. (1)

$$(u_1, u_2, u_3, u_4)\begin{pmatrix} \frac{3}{4} & 0 & \frac{1}{4} & 0 \\ \frac{2}{3} & 0 & \frac{1}{3} & 0 \\ 0 & \frac{1}{3} & 0 & \frac{2}{3} \\ 0 & \frac{1}{4} & 0 & \frac{3}{4} \end{pmatrix} = (u_1, u_2, u_3, u_4)$$

$\frac{3}{4}u_1 + \frac{2}{3}u_2 = u_1$

$\frac{1}{3}u_3 + \frac{1}{4}u_4 = u_2 \rightarrow 4u_3 + 3u_4 = 12u_2$

$\frac{1}{4}u_1 + \frac{1}{3}u_2 = u_3$

$\frac{2}{3}u_3 + \frac{3}{4}u_4 = u_4$

$u_1 + u_2 + u_3 + u_4 = 1$

①より $3u_1 = 8u_2$

④より $3u_4 = 8u_3$

②', ④'より $\frac{3}{2}u_4 + 3u_4 = 12u_2$

$9u_4 = 24u_2$

よって $u_1 = u_4$

$u_2 = u_3$

$2u_1 + \frac{6}{8}u_1 = 1$ $u_2 = u_3 =$

$u_1 = u_4 = \frac{8}{22}, \frac{3}{22}$

(2) $2 \times \frac{1}{2}\log 2 = 1$ bit

$$-2 \times \frac{8}{22} \times \left(\frac{3}{4}\log\frac{3}{4} + \frac{1}{4}\log\frac{1}{4}\right)$$
$$-2 \times \frac{3}{22} \times \left(\frac{2}{3}\log\frac{2}{3} + \frac{1}{3}\log\frac{1}{3}\right)$$
$$= 0.84 \text{ bit}$$

(3) $1 - 0.84 = 0.16$

2 (a)

$P(Y=1) = 0.5a + (1-a) = 1 - 0.5a$

$P(Y=0) = 0.5a$

$H(Y) = -0.5a \log 0.5a - (1-0.5a) \log (1-0.5a)$

$= -x \log x - (1-x) \log (1-x)$ (令 $0.5a = x$)

(b) $P(Y=1|X=0) = 0.5$, $P(Y=1|X=1) = 1$

$P(Y=0|X=0) = 0.5$, $P(Y=0|X=1) = 0$

$H(Y|X) = -(0.5a \log 0.5 + 0.5a \log 0.5)$

$= a = 2x$

(c) $I(X,Y) = H(Y) - H(Y|X)$

$= -x \log x - (1-x) \log (1-x) - 2x$

$\frac{dI}{dx} = -1 - \log x + 1 + \log (1-x) - 2$

$= \log \frac{1-x}{x} - 2 = 0$

$\frac{1-x}{x} = 4$

$x = \frac{1}{5}$

$a = \frac{2}{5}$

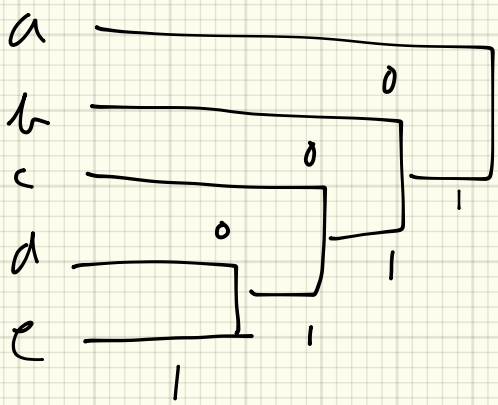
$\frac{d}{dx} \log (1-x)$

$= \frac{1}{1-x} \times \frac{d}{dx}(1-x)$

$= \frac{-1}{1-x}$

(d) $\frac{1}{5} \log 5 + \frac{4}{5} \log \frac{5}{4} - \frac{2}{5} = \log 5 - \frac{4}{5} \log 4 - \frac{2}{5} = 0.32$ bit

3. (a)



(b) a: 0, b: 10, c: 110, d: 1110, e: 1111

(c) $2^{-1}+2^{-2}+2^{-3}+2^{-4}\times 2 = \frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{8} = \frac{2+1+1}{4} = 1$

よってクラフトの不等式が成り立つ

(d) $1\times\frac{1}{2}+2\times\frac{1}{4}+3\times\frac{1}{8}+4\times\frac{1}{16}\times 2 = \frac{1}{2}+\frac{1}{2}+\frac{3}{8}+\frac{4}{8} = \frac{4+4+3+4}{8}$

$= \frac{15}{8}$

4. (a) 3 : (1, 1, 1)

4 : (1, 1, 2), (1, 2, 1), (2, 1, 1)

5 : (1, 2, 2), (2, 1, 2), (2, 2, 1)

6 : (2, 2, 2)

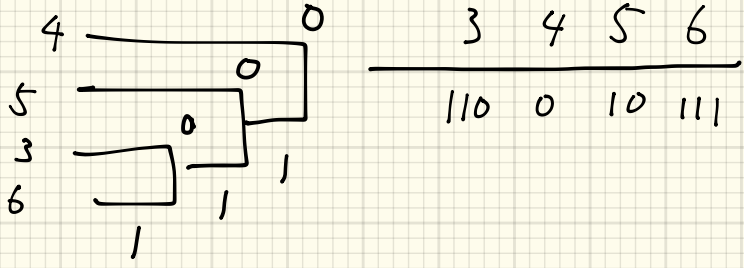
$P(3)=\frac{1}{8}$, $P(4)=\frac{3}{8}$, $P(5)=\frac{3}{8}$, $P(6)=\frac{1}{8}$

(b) $H=\frac{1}{8}\log 8+\frac{3}{8}(\log 8-\log 3)\times 2+\frac{1}{8}\log 8$

$=\frac{3}{4}+\frac{9}{4}-\frac{3}{4}\log 3=3-\frac{3}{4}\log 3$

(c) 2

(d)



(e) $\frac{3}{8}\times 2+\frac{3}{8}\times 2+\frac{3}{8}\times 1=\frac{6+6+3}{8}=\frac{15}{8}$