8_ 演習

$$C_n = \frac{1}{T}\int_0^T \cos 2\omega t \cdot e^{-i\frac{2\pi}{T}nt}\,dt$$

$$= \frac{\omega}{2\pi}\int_0^T \cos 2\omega t \cdot e^{-in\omega t}\,dt$$

$$= \frac{\omega}{2\pi}\int_0^T \frac{e^{i2\omega t} + e^{-i2\omega t}}{2} \times e^{-in\omega t}\,dt$$

$$= \frac{\omega}{4\pi}\int_0^T \left(e^{i(2-n)\omega t} + e^{-i(2+n)\omega t}\right)dt$$

$n=2$のとき、

$$\boxed{\begin{aligned} C_2 &= \frac{\omega}{4\pi}\int_0^T \left(1 + e^{-i4\omega t}\right)dt \\ &= \frac{\omega}{4\pi}\left[t + \frac{e^{-i4\omega t}}{-i4\omega}\right]_0^T \\ &= \frac{\omega}{4\pi}\left(T + \frac{e^{i8\pi}}{-i4\omega} - \frac{1}{-i4\omega}\right) \\ &= \frac{\omega T}{4\pi} \\ &= \frac{1}{2} \end{aligned}}$$

$$C_{-2} = \frac{w}{4\pi}\int_0^T \left(e^{i4wt} + 1\right) dt$$

$$= \frac{w}{4\pi}\left[\frac{e^{i4wt}}{i4w} + t\right]_0^T$$

$$= \frac{w}{4\pi}\left[\frac{e^{i8\pi}}{i4w} - \frac{1}{i4w} + T\right]$$

$$= \frac{wT}{4\pi} = \frac{1}{2}$$

$n \neq \pm 2$ のとき、

$$C_{n \neq \pm 2} = \frac{w}{4\pi}\int_0^T \left(e^{i(2-n)wt} + e^{-i(2+n)wt}\right) dt$$

$$= \frac{w}{4\pi}\left(\left[\frac{e^{i(2-n)wt}}{i(2-n)w}\right]_0^T + \left[\frac{e^{-i(2+n)wt}}{-i(2+n)w}\right]_0^T\right)$$

$$= \frac{1}{4wi}\left(\frac{e^{i(2-n)2\pi} - 1}{2-n} - \frac{e^{-i(2+n)2\pi} - 1}{2+n}\right)$$

$$= \frac{1}{4iw}\left(\frac{\cos\{2\pi(2-n)\} + i\sin\{2\pi(2-n)\} - 1}{2-n} - \frac{\cos\{2\pi(2+n)\} - i\sin\{2\pi(2+n)\} - 1}{2+n}\right)$$

$$= \frac{1}{4iw}\left(\frac{1+0-1}{2-n} - \frac{1-0-1}{2+n}\right)$$

$$= 0$$