演習9

$$f(t) = \begin{cases} E & 0 \leqq t < 2d \\ 0 & t < 0,\ t \geqq 2d \end{cases}$$

$$F(w) = \int_{-\infty}^{\infty} f(t)\, e^{-iwt}\, dt$$

$$= \int_{0}^{2d} E \cdot e^{-iwt}\, dt$$

$$= \frac{E}{-iw} \left[ e^{-iwt} \right]_{0}^{2d}$$

$$= -\frac{E}{iw} \left( e^{-i2wd} - 1 \right)$$

$$= -\frac{E}{iw} e^{-iwd} \left( e^{-iwd} - e^{+iwd} \right)$$

$$= 2E \cdot (2d - 0) \cdot \frac{\sin wd}{wd} \exp\left[ -iwd \right]$$

$$= 4Ed \cdot \frac{\sin wd}{wd} \cdot e^{-iwd}$$

$|G(w)| = Ed \left| \frac{\sin wd}{wd} \right|$ の零点は、

$\sin wd = 0$

をみたす周波数にある。つまり、

$w_0 d = \pm\pi, \pm 2\pi, \pm 3\pi \cdots\cdots$

$w_0 = \pm \frac{\pi}{d}, \pm \frac{2\pi}{d}, \pm \frac{3\pi}{d}$

$\Theta(w) = -wd.$

$\Theta_0\left(\frac{\pi}{d}\right) = -\pi\left(\frac{d+\beta}{d-\beta}\right)$

$= -\pi\left(\frac{2d}{2d}\right)$

$= -\pi$