$$f(x) = \begin{cases} e^{-x} & (x \geq 0) \\ 0 & (x < 0) \end{cases}$$

1. $I = \displaystyle\int_{-\infty}^{\infty} f(x)\, dx = \int_{0}^{\infty} f(x)\, dx$

$$= \boxed{\left[-e^{-x} \right]_{0}^{\infty} \quad (< \infty)}$$

2.

$$F(\omega) = \int_{-\infty}^{\infty} f(x)\, e^{-i\omega x}\, dx$$

$$= \int_{0}^{\infty} e^{-x} \cdot e^{-i\omega x}\, dx$$

$$= \int_{0}^{\infty} e^{-(i\omega + 1)x}\, dx$$

$$= \left[-\frac{e^{-(i\omega + 1)x}}{i\omega + 1} \right]_{0}^{\infty}$$

$$= 0 - \left(-\frac{1}{i\omega + 1} \right)$$

$$= \boxed{\frac{1}{i\omega + 1}}$$