

宿題[1]

$$F(p) = \frac{p-1}{p^2+3p+2} = \frac{p-1}{(p+2)(p+1)} = \frac{a}{p+2} + \frac{b}{p+1}$$

$$\frac{p-1}{(p+2)(p+1)} = \frac{(a+b)p + a+2b}{(p+2)(p+1)}$$

$$\therefore a=3, b=-2$$

よって、

$$f(t) = \frac{1}{2\pi i} \int_{C_2} \left( \frac{3}{p+2} - \frac{2}{p+1} \right) e^{pt} dp$$

$$= \boxed{3e^{-2t} - 2e^{-t}}$$

$$X(p) = \int_0^{\infty} e^{-t} e^{-pt} dt = \int_0^{\infty} e^{-(1+p)t} dt$$

$$= \left[ -\frac{e^{-(1+p)t}}{(1+p)} \right]_0^{\infty}$$

$$= \frac{1}{1+p}$$

$$Y(p) = \int_0^{\infty} (2e^{-t} + e^{-2t}) e^{-pt} dt$$

$$= 2 \left[ -\frac{e^{-(1+p)t}}{1+p} \right]_0^{\infty} + \left[ -\frac{e^{-(2+p)t}}{2+p} \right]_0^{\infty}$$

$$= \frac{2}{1+p} + \frac{1}{2+p}$$

$$\therefore H(p) = \frac{\frac{2}{1+p} + \frac{1}{2+p}}{\frac{1}{1+p}} = 2 + \frac{1+p}{2+p} = \boxed{\frac{3p+5}{p+2}}$$

$$h(t) = \frac{1}{2\pi i} \int_{C_2} \frac{3p+5}{p+2} e^{pt} dp = \frac{1}{2\pi i} \int_{C_2} \left( 3 - \frac{1}{p+2} \right) e^{pt} dp$$

$$= \boxed{3\delta(t) - e^{-2t}}$$