6-1

$$\int_0^x \frac{\pi - t}{2}\,dt = \int_0^x \sum_{n=1}^{\infty} \frac{\sin(nt)}{n}\,dt$$

$$\left[\frac{\pi}{2}t - \frac{t^2}{\boxed{4}}\right]_0^x = \sum_{n=1}^{\infty}\int_0^x \frac{1}{n}\sin(nt)\,dt$$

$$\frac{\pi}{2}x - \frac{x^2}{\boxed{4}} = \sum_{n=1}^{\infty} \frac{1}{n}\left[-\frac{\boxed{\cos(nt)}}{n}\right]_0^x$$

$$\frac{\pi}{2}x - \frac{x^2}{\boxed{4}} = \sum_{n=1}^{\infty} \frac{1-\boxed{\cos(nx)}}{n^2}$$

$$= \sum_{n=1}^{\infty}\frac{1}{n^2} - \sum_{n=1}^{\infty}\frac{\boxed{\cos(nx)}}{n^2}$$

6-2

$$\sum_{n=1}^{\infty}\frac{\cos(nx)}{n^2} = \frac{\pi^2}{6} - \frac{\pi}{2}x + \frac{x^2}{\boxed{4}} = \frac{3x^2 - 6\pi x + 2\pi^2}{12}$$

ここで、$x = \boxed{\frac{\pi}{2}}$ とおくと、

$$\frac{-1}{2^2} + \frac{1}{4^2} + \frac{-1}{6^2} + \cdots\cdots = \frac{3\left(\boxed{\frac{\pi}{2}}\right)^2 - 6\pi\left(\boxed{\frac{\pi}{2}}\right) + 2\pi^2}{12} = -\frac{\pi^2}{\boxed{48}}$$

$$-\left[\frac{1}{4\cdot 1^2} - \frac{1}{4\cdot 2^2} + \frac{1}{4\cdot 3^2} + \cdots\cdots\right] = -\frac{\pi^2}{\boxed{48}}$$

$$\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} + \cdots\cdots = \frac{\pi^2}{12}$$