3_演習解答

(1)
$$\int_{\alpha}^{2\pi+\alpha} \sin 2\theta \cos 3\theta d\theta = \frac{1}{2}\left[\int_{\alpha}^{2\pi+\alpha} \{\sin 5\theta + \sin(-\theta)\} d\theta\right]$$

$$= \frac{1}{2}\left[\int_{\alpha}^{2\pi+\alpha} \sin 5\theta d\theta - \int_{\alpha}^{2\pi+\alpha} \sin(\theta) d\theta\right] = \frac{1}{2}\left\{\left[\frac{-\cos 5\theta}{5}\right]_{\alpha}^{2\pi+\alpha} - [-\cos\theta]_{\alpha}^{2\pi+\alpha}\right\}$$

$$= -\frac{1}{2}\left\{\left[\frac{\cos(10\pi+5\alpha)-\cos(-5\alpha)}{5}\right] - [\cos(2\pi+\alpha)-\cos(-\alpha)]\right\}$$

$$= -\frac{1}{2}\left\{\left[\frac{\cos(5\alpha)-\cos(5\alpha)}{5}\right] - [\cos(\alpha)-\cos(\alpha)]\right\} = 0$$

(2)
$$\int_{\alpha}^{2\pi+\alpha} \sin 2\theta \sin 2\theta d\theta = \int_{\alpha}^{2\pi+\alpha} \{\sin 2\theta\}^2 d\theta = \int_{\alpha}^{2\pi+\alpha} \sin^2 2\theta d\theta$$

$$= \frac{1}{2}\left[\int_{\alpha}^{2\pi+\alpha} (1-\cos 4\theta) d\theta\right] = \frac{1}{2}\left\{[\theta]_{\alpha}^{2\pi+\alpha} - \left[\frac{\sin 4\theta}{4}\right]_{\alpha}^{2\pi+\alpha}\right\}$$

$$= \frac{1}{2}\left\{[(2\pi+\alpha)-\alpha] - \frac{1}{4}[\sin(8\pi+4\alpha)-\sin(4\alpha)]\right\} = \pi$$

3_宿題解答[1]

次の関数の基本周期はいくらか。

(1) $\sin 8\theta + \sin 6\theta$

解答　サイン関数の周期は2πであるから

$$\sin 8\theta = \sin\left(2\pi \frac{\theta}{\frac{2\pi}{8}}\right) = \sin\left(2\pi \frac{\theta}{T_1}\right), \quad \sin 6\theta = \sin\left(2\pi \frac{\theta}{\frac{2\pi}{6}}\right) = \sin\left(2\pi \frac{\theta}{T_2}\right)$$

$$T_1 = \frac{2\pi}{8}, \quad T_2 = \frac{2\pi}{6}$$

$$4T_1 = 4\frac{2\pi}{8} = \frac{2\pi}{2} = 3\frac{2\pi}{6} = 3T_2 = \pi$$

よって基本周期は π。

3_宿題解答[2]

次の関数の基本周期はいくらか。

(2) $\sin\theta\cos\theta$

$$\begin{array}{rl} & \sin(\alpha+\beta)=\sin\alpha\cos\beta+\cos\alpha\sin\beta \\ +) & \sin(\alpha-\beta)=\sin\alpha\cos\beta-\cos\alpha\sin\beta \\ \hline & \sin(\alpha+\beta)+\sin(\alpha-\beta)=2\sin\alpha\cos\beta \end{array}$$

ここで$\alpha=\beta$とおくと

$$\sin 2\alpha+\sin 0=2\sin\alpha\cdot\cos\alpha$$

$$\sin\alpha\cdot\cos\alpha=\frac{1}{2}\sin 2\alpha$$

右の公式より

$$\sin\theta\cos\theta=\frac{1}{2}\sin 2\theta$$

サイン関数の周期は2πなので

$$\sin\theta\cos\theta=\frac{1}{2}\sin 2\theta=\frac{1}{2}\sin\left(2\pi\frac{2\theta}{2\pi}\right)$$

$$=\frac{1}{2}\sin\left(2\pi\frac{\theta}{\frac{2\pi}{2}}\right)=\frac{1}{2}\sin\left(2\pi\frac{\theta}{\pi}\right)=\frac{1}{2}\sin\left(2\pi\frac{\theta}{T}\right)$$

よって、基本周期はπである。