$$\frac{g(y)}{dy} = \int_0^\infty \frac{d}{dy}\left\{e^{-sy}\frac{\sin s}{s}\right\} ds$$

$$= \boxed{\lim_{a\to\infty}\int_0^a - e^{-sy}\sin s\, ds} = \int_0^\infty - e^{-sy}\sin s\, ds.$$

$$= \lim_{a\to\infty}\left(\left[e^{-sy}\cos s\right]_0^a - \int_0^a - y e^{-sy}\cos s\, ds\right)$$

$$= \lim_{a\to\infty}\left(e^{-ay}\cos a - 1 + \left[y e^{-sy}\sin s\right]_0^a + \int_0^a y^2 e^{-sy}\sin s\, ds\right)$$

$$= -1 + y^2\int_0^\infty e^{-sy}\sin s\, ds$$

$$\therefore\ (y^2+1)\int_0^\infty e^{-sy}\sin s\, ds = 1$$

$$\int_0^\infty e^{-sy}\sin s\, ds = \boxed{\frac{1}{y^2+1}}$$

よって.

$$g(\infty) = 0 = C - \tan^{-1}(\infty)$$

$$= C - \boxed{\frac{\pi}{2}}$$