宿 6-1

左辺は、

$$\int_0^u \sum_{n=1}^{\infty} \frac{\cos(nx)}{n^2}\,dx = \sum_{n=1}^{\infty} \frac{1}{n^2}\int_0^u \boxed{\cos}(nx)\,dx$$

$$= \sum_{n=1}^{\infty} \frac{1}{n^2}\left[\frac{\boxed{\sin}(nx)}{n}\right]_0^u$$

$$= \sum_{n=1}^{\infty} \frac{\boxed{\sin}(nu)}{n^3}$$

右辺は、

$$\int_0^u \frac{3x^2-6\pi x+2\pi^2}{12}\,dx = \left[\frac{x^3-3\pi x^2+2\pi^2 x}{12}\right]_0^u$$

$$= \boxed{\frac{u^3}{12} - \frac{\pi}{4}u^2 + \frac{\pi^2}{6}u}$$

宿 6-2 合わせて

$$\sum_{n=1}^{\infty} \frac{\sin(nu)}{n^3} = \frac{u^3}{12} - \frac{\pi u^2}{4} + \frac{\pi^2 u}{6}$$

ここで、$u = \boxed{\frac{\pi}{2}}$ とおくと、

左辺: $$\sum_{n=1}^{\infty} \frac{\sin(n\cdot\boxed{\frac{\pi}{2}})}{n} = \frac{1}{1^3} - \frac{1}{3^3} + \frac{1}{5^3} - \cdots\cdots$$

右辺: $$\frac{1}{12}\left(\boxed{\frac{\pi}{2}}\right)^3 - \frac{\pi}{4}\left(\boxed{\frac{\pi}{2}}\right)^2 + \frac{\pi^2}{6}\cdot\boxed{\frac{\pi}{2}} = \frac{\pi^3}{32}$$

よって、 $$\frac{1}{1^3} - \frac{1}{3^3} + \frac{1}{5^3} - \cdots\cdots = \frac{\pi^3}{32}$$

答 6-3

$$f(t) = \frac{\boxed{4}}{\pi}\left(\cos t - \frac{\cos 3t}{3} + \frac{\cos 5t}{5} - \cdots\cdots \right)$$

$$f^2(t) = \left(\frac{\boxed{4}}{\pi}\right)^2 \left[\cos t - \frac{\cos 3t}{3} + \frac{\cos 5t}{5} - \cdots\cdots \right]^2 = \boxed{1}$$

$$\int_0^{2\pi} \boxed{\cos(mt) \cdot \cos(nt)}\, dt = \begin{cases} 0 & m \neq n \\ \pi & m = n \end{cases}$$

答 6-4

$$\left(\frac{\boxed{4}}{\pi}\right)^2 \left[\pi + \frac{\pi}{3^2} + \frac{\pi}{5^2} + \frac{\pi}{7^2} + \cdots\cdots \right] = 2\pi$$

$$\therefore\ 1 + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} = \frac{\pi^2}{8}$$

$$\sum_{n=1}^{\infty} \frac{1}{n^2} = \boxed{\sum_{n=1}^{\infty} \frac{1}{(2n)^2}} + \sum_{n=1}^{\infty} \frac{1}{(2n-1)^2} = \boxed{\frac{\pi^2}{24}} + \frac{\pi^2}{8}$$

$$\therefore\ \sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$$