

宿題8

$$f(x) = |x| \quad [-\pi \leq x \leq \pi]$$

この関数の周期Tは $T = 2\pi$ である。

$$\begin{aligned} C_k &= \frac{1}{2\pi} \int_{-\pi}^{\pi} |x| e^{-ikx} dx = \frac{1}{2\pi} \left(\int_{-\pi}^0 -x e^{-ikx} dx + \int_0^{\pi} x e^{-ikx} dx \right) \\ &= \frac{1}{2\pi} \left(\int_0^{\pi} x e^{ikx} dx + \int_0^{\pi} x e^{-ikx} dx \right) = \frac{1}{2\pi} \int_0^{\pi} x (e^{ikx} + e^{-ikx}) dx \\ &= \frac{1}{\pi} \int_0^{\pi} x \cdot \cos(kx) dx = \frac{1}{\pi k} \left([x \sin(kx)]_0^{\pi} - \int_0^{\pi} \sin(kx) dx \right) \\ &= \frac{1}{\pi k} \left([0 - 0] - \left[\frac{-\cos(kx)}{k} \right]_0^{\pi} \right) = \frac{1}{\pi k^2} \left\{ (-1)^k - 1 \right\} \\ &= \begin{cases} \boxed{-\frac{2}{\pi k^2}} & (k \text{ が奇数}) \\ \boxed{0} & (k \text{ が偶数}) \end{cases} \end{aligned}$$

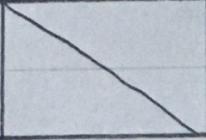
$$C_1 = -\frac{2}{\pi}, \quad C_2 = 0, \quad C_3 = -\frac{2}{9\pi}, \quad C_4 = 0, \quad C_5 = -\frac{2}{25\pi}$$

$$C_0 = \frac{1}{2\pi} \left(\int_{-\pi}^0 -x dx + \int_0^{\pi} x dx \right) = \frac{1}{2\pi} \left(\left[-\frac{1}{2}x^2 \right]_{-\pi}^0 + \left[\frac{1}{2}x^2 \right]_0^{\pi} \right)$$

$$= \frac{1}{2\pi} \left(-\left(-\frac{1}{2}\pi^2 \right) + \frac{1}{2}\pi^2 \right)$$

$$= \boxed{\frac{\pi}{2}}$$

$$\therefore a_0 = \pi$$

k	0	1	2	3	4	5
c_k	$\frac{\pi}{2}$	$-\frac{2}{\pi}$	0	$-\frac{2}{9\pi}$	0	$-\frac{2}{25}\pi$
a_k	π	$-\frac{4}{\pi}$	0	$-\frac{4}{9\pi}$	0	$-\frac{4}{25}\pi$
b_k		0	0	0	0	0