

$$\begin{aligned}
 \frac{g(y)}{dy} &= \int_0^\infty \frac{d}{dt} \left\{ e^{-sy} \frac{\sin s}{s} \right\} ds \\
 &= \boxed{\lim_{a \rightarrow \infty} \int_0^a -e^{-sy} \sin s ds} = \int_0^\infty -e^{-sy} \sin s ds. \\
 &= \lim_{a \rightarrow \infty} \left([e^{-sy} \cos s]_0^a - \int_0^a -y e^{-sy} \cos s ds \right) \\
 &= \lim_{a \rightarrow \infty} \left(e^{-ay} \cos a - 1 + [y e^{-sy} \sin s]_0^a + \int_0^a y^2 e^{-sy} \sin s ds \right) \\
 &= -1 + y^2 \int_0^\infty e^{-sy} \sin s ds \\
 \therefore (y^2 + 1) \int_0^\infty e^{-sy} \sin s ds &= 1
 \end{aligned}$$

$$\int_0^\infty e^{-sy} \sin s ds = \boxed{\frac{1}{y^2 + 1}}$$

2. 2.

$$g(\infty) = 0 = C - \tan^{-1}(\infty)$$

$$\begin{aligned}
 &= C - \boxed{\frac{\pi}{2}}
 \end{aligned}$$