宿題8

$$f(x) = |x| \quad [-\pi \leq x \leq \pi]$$

この関数の周期Tは、$T = 2\pi$ である。

$$c_k = \frac{1}{2\pi}\int_{-\pi}^{\pi} |x| e^{-ikx} dx = \frac{1}{2\pi}\left(\int_{-\pi}^{0} -x e^{-ikx} dx + \int_{0}^{\pi} x e^{-ikx} dx\right)$$

$$= \frac{1}{2\pi}\left(\int_{0}^{\pi} x e^{ikx} dx + \int_{0}^{\pi} x e^{-ikx} dx\right) = \frac{1}{2\pi}\int_{0}^{\pi} x\left(e^{ikx} + e^{-ikx}\right) dx$$

$$= \frac{1}{\pi}\int_{0}^{\pi} x \cdot \cos(kx)\, dx = \frac{1}{\pi k}\left([x \sin(kx)]_{0}^{\pi} - \int_{0}^{\pi} \sin kx\, dx\right)$$

$$= \frac{1}{\pi k}\left([0-0] - \left[\frac{-\cos(kx)}{k}\right]_{0}^{\pi}\right) = \frac{1}{\pi k^2}\left\{(-1)^k - 1\right\}$$

$$= \begin{cases} \boxed{-\frac{2}{\pi k^2} \quad (k\text{が奇数}) \\ 0 \quad (k\text{が偶数})} \end{cases}$$

$$c_1 = -\frac{2}{\pi},\ c_2 = 0,\ c_3 = -\frac{2}{9\pi},\ c_4 = 0,\ c_5 = -\frac{2}{25\pi}$$

$$c_0 = \frac{1}{2\pi}\left(\int_{-\pi}^{0} -x\, dx + \int_{0}^{\pi} x\, dx\right) = \frac{1}{2\pi}\left(\left[-\frac{1}{2}x^2\right]_{-\pi}^{0} + \left[\frac{1}{2}x^2\right]_{0}^{\pi}\right)$$

$$= \frac{1}{2\pi}\left(-\left(-\frac{1}{2}\pi^2\right) + \frac{1}{2}\pi^2\right)$$

$$= \boxed{\frac{\pi}{2}}$$

$$\therefore a_0 = \pi.$$

k	0	1	2	3	4	5
c_k	$\frac{\pi}{2}$	$-\frac{2}{\pi}$	0	$-\frac{2}{9\pi}$	0	$-\frac{2}{25}\pi$
a_k	π	$-\frac{4}{\pi}$	0	$-\frac{4}{9\pi}$	0	$-\frac{4}{25}\pi$
b_k		0	0	0	0	0