

宿題

$$\begin{aligned}
 a_0 &= \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) dx = \frac{1}{\pi} \int_{-\frac{\pi}{2}}^{\frac{3}{2}\pi} f(x) dx \\
 &= \frac{1}{\pi} \left(\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} 1 \cdot dx + \int_{\frac{\pi}{2}}^{\frac{3}{2}\pi} 0 \cdot dx \right) \\
 &= \frac{1}{\pi} [x]_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \\
 &= \boxed{1}
 \end{aligned}$$

$$\begin{aligned}
 a_k &= \frac{1}{\pi} \int_{-\frac{\pi}{2}}^{\frac{3}{2}\pi} f(x) \cos kx dx \\
 &= \frac{1}{\pi} \left(\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} 1 \cdot \cos kx dx + \int_{\frac{\pi}{2}}^{\frac{3}{2}\pi} 0 \cdot \cos kx dx \right) \\
 &= \frac{1}{\pi} \left[\frac{1}{k} \sin kx \right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \\
 &= \frac{1}{\pi k} \left(\sin \frac{\pi k}{2} - \sin \left(-\frac{\pi k}{2} \right) \right) \\
 &= \begin{cases} 0 & (k \text{ 为偶数}) \\ \frac{2}{\pi k} & (k = 4n+1) \\ -\frac{2}{\pi k} & (k = 4n+3) \end{cases} \quad (\text{对 } n = 0, 1, 2, \dots)
 \end{aligned}$$

$$\begin{aligned}
 b_k &= \frac{1}{\pi} \int_{-\frac{\pi}{2}}^{\frac{3}{2}\pi} f(x) \sin kx dx \\
 &= \frac{1}{\pi} \left(\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} 1 \cdot \sin kx dx + \int_{\frac{\pi}{2}}^{\frac{3}{2}\pi} 0 \cdot \sin kx dx \right) \\
 &= \frac{1}{\pi} \left[-\frac{1}{k} \cos kx \right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \\
 &= \boxed{0}
 \end{aligned}$$

k	0	1	2	3	4	5	6	7
a _k	1	$\frac{2}{\pi}$	0	$-\frac{2}{3\pi}$	0	$\frac{2}{5\pi}$	0	$-\frac{2}{7\pi}$
b _k	0	0	0	0	0	0	0	0