演習

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\,dx$$

$$= \frac{1}{\pi}\left(\int_{-\pi}^{0} 1\cdot dx + \int_{0}^{\pi} 0\cdot dx\right)$$

$$= \frac{1}{\pi}\left(0-(-\pi)\right)$$

$$= \boxed{1}$$

$$a_k = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos kx\,dx$$

$$= \frac{1}{\pi}\left(\int_{-\pi}^{0} 1\cdot\cos kx\,dx + \int_{0}^{\pi} 0\cdot\cos kx\,dx\right)$$

$$= \frac{1}{\pi}\left[\frac{1}{k}\sin kx\right]_{-\pi}^{0}$$

$$= \boxed{0}$$

$$b_k = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin kx\,dx$$

$$= \frac{1}{\pi}\left(\int_{-\pi}^{0} 1\cdot\sin kx\,dx + \int_{0}^{\pi} 0\cdot\sin kx\,dx\right)$$

$$= \frac{1}{\pi}\left[-\frac{1}{k}\cos kx\right]_{-\pi}^{0}$$

$$= \begin{cases} -\dfrac{2}{\pi k} & (k\text{が奇数}) \\ 0 & (k\text{が偶数}) \end{cases}$$

| $k$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|---|
| $a_k$ | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $b_k$ | | $-\frac{2}{\pi}$ | 0 | $-\frac{2}{3\pi}$ | 0 | $-\frac{2}{5\pi}$ | 0 | $-\frac{2}{7\pi}$ |