Supplemental Materials for "A shifted Wald decomposition of the numerical size-congruity effect: Support for a late-interaction account"

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In these supplemental materials, I present details on the Bayesian hypothesis tests performed in the main paper, providing specific definitions and relevant R code. Then for each test, I provide two supplements: (1) a plot of the prior and posterior for effect size δ, giving a visual representation of the Bayes factor computation; and (2) a robustness check, showing the effect of prior choice on the resulting Bayes factor.

Definitions and R-code

The Bayesian t-tests described in this paper were performed using the `ttestBF` function from the `BayesFactor` package in R (Morey & Rouder, 2012). The `ttestBF` function implements the *JZS Bayes factor* computation for t-tests originally described in Rouder et al. (2009). Recall that the Bayes factor B_{01} represents the factor by which the prior odds for hypothesis $\mathcal{H}_0$ over hypothesis $\mathcal{H}_1$ are updated after observing data D. That is,

$$\frac{p(\mathcal{H}_0 \mid D)}{p(\mathcal{H}_1 \mid D)} = B_{01} \times \frac{p(\mathcal{H}_0)}{p(\mathcal{H}_1)}.$$

Computationally, Bayes Theorem implies that B_{01} is equal to the ratio of *marginal likelihoods* M_0/M_1, where

$$M_i = \int_{\boldsymbol{\theta}\in\Theta_i} f_i(\boldsymbol{\theta} \mid D)\pi_i(\boldsymbol{\theta})d\boldsymbol{\theta},$$

where $i = 0, 1$, Θ_i represents the space of parameters $\boldsymbol{\theta}$ under hypothesis $\mathcal{H}_i$, f_i denotes the likelihood function hypothesis $\mathcal{H}_i$, given parameters $\boldsymbol{\theta}$ and data D, and π_i represents the prior distribution of parameters $\boldsymbol{\theta}$ under hypothesis $\mathcal{H}_i$. In the case of a single-sample t-test, the likelihood f_i is simply the familiar normal density with parameters μ and σ. However, the Bayesian analyst still must choose priors π_i for each of these parameters.

Rouder et al. (2009) showed that the computation of the marginal likelihoods M_i (and, by implication, the Bayes factor B_{01}) becomes relatively straightforward with a few

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simple assumptions. First, instead of placing priors on the mean μ, we can instead place priors on the *effect size* $\delta = \mu/\sigma$. One benefit of doing so is that the null hypothesis $\mathcal{H}_0$ may be defined as $\delta = 0$. Then, Rouder et al. (2009) recommending placing a Cauchy prior on δ (following Jeffreys, 1961) and an inverse-chi-square prior σ_δ^2 (following Zellner and Siow, 1980). As a result, the formula derived by Rouder et al. (2009) is referred to as the JZS Bayes factor (named so for Jeffreys, Zellner, and Siow).

The JZS Bayes factor provides the user with a simple default Bayesian t-test. However, the Bayes factor is sensitive to the choice of prior, one should take care to be explicit with this choice. With the `ttestBF` function, one may easily choose between three default priors, parameterized as the scale (or width) r of the Cauchy prior on effect size δ. In this context, setting the prior scale to r means 50% of the effect sizes would be expected to be between $-r$ and $+r$. The user may specify this scale via an `rscale` argument in the function. Three convenient pre-defined scales are:

- "medium", corresponding to $r = \sqrt{2}/2$.
- "wide", corresponding to $r = 1$
- "ultrawide", corresponding to $r = \sqrt{2}$

By default, the `ttestBF` function uses the "medium" prior, which is equivalent to starting with a prior belief that effect sizes are distributed as a Cauchy distribution with scale $r = \sqrt{2}/2 \approx 0.707$. However, the choice of prior is subjective, so it is imperative that a complete Bayesian analysis should also include an analysis of the sensitivity of the analysis to prior choice.

In the following sections, I present the details of each Bayesian t-test that was performed in the original paper. Specifically, I define the appropriate null and alternative hypotheses, followed by a sensitivity analyses that shows how the obtained JZS Bayes factor depends on Cauchy prior scale r.

Bayes factor for median RT

Bayes factor for standard deviation

Bayes factor for drift rate γ

Bayes factor for response threshold α

Bayes factor for nondecision time θ

TODO - fix point null positions in posterior densities. Probably need to use `logspline` package

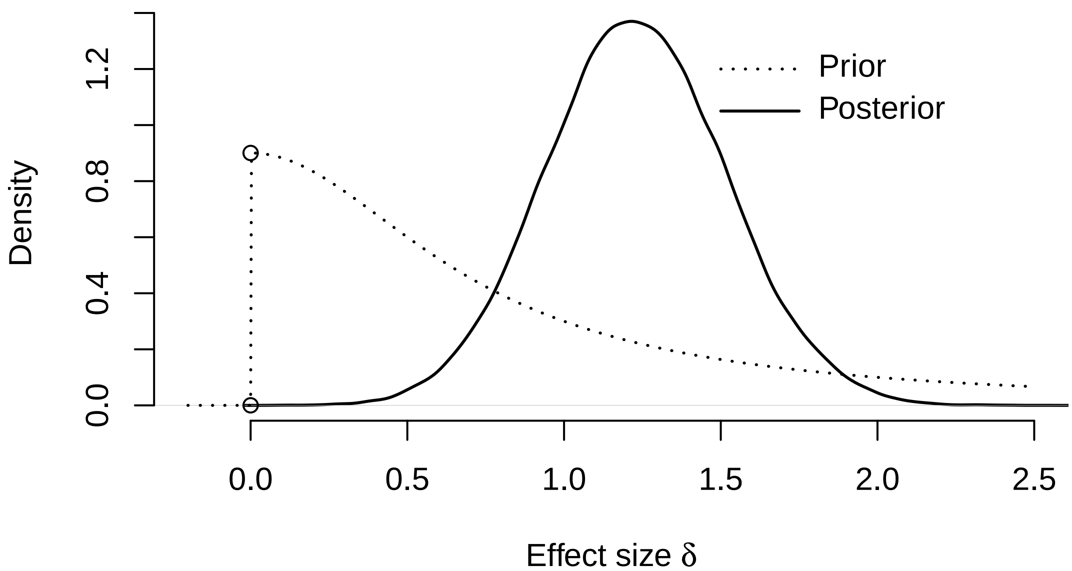

Figure 1. Prior and posterior for effect size. Points on the plot represent the density of the point null in both the prior and posterior.

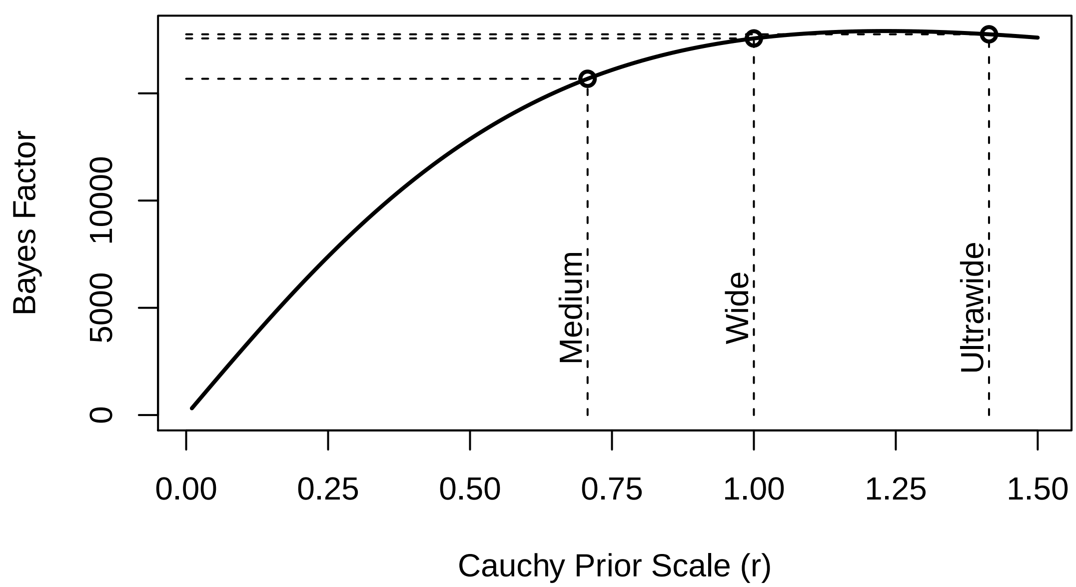

Figure 2. Robustness check for Bayes factor in favor of congruity effect on median RTs.

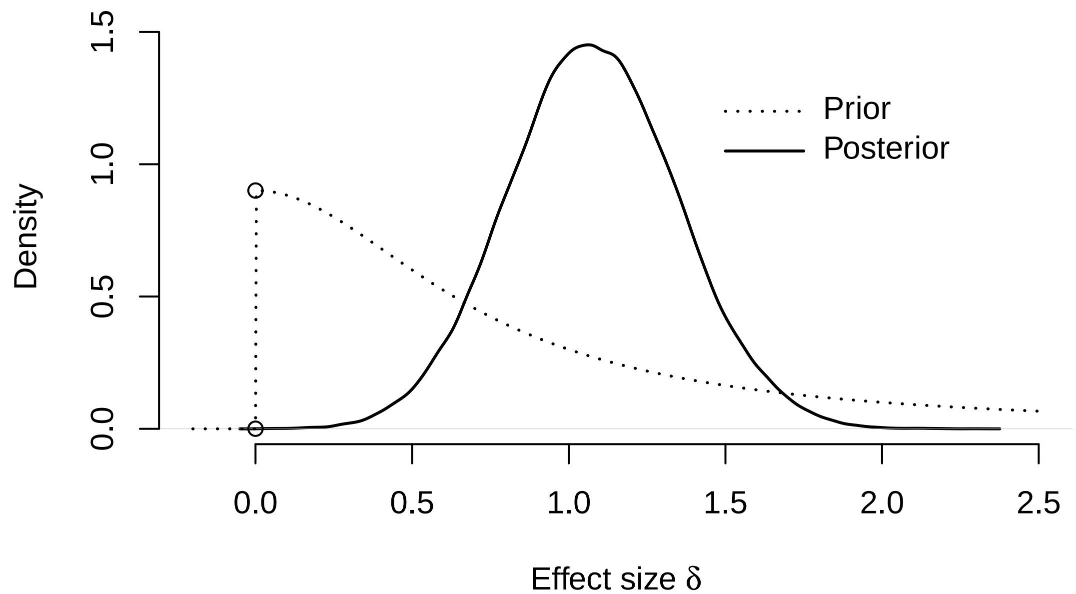

Figure 3. Prior and posterior for effect size on standard deviation. Points on the plot represent the density of the point null in both the prior and posterior.

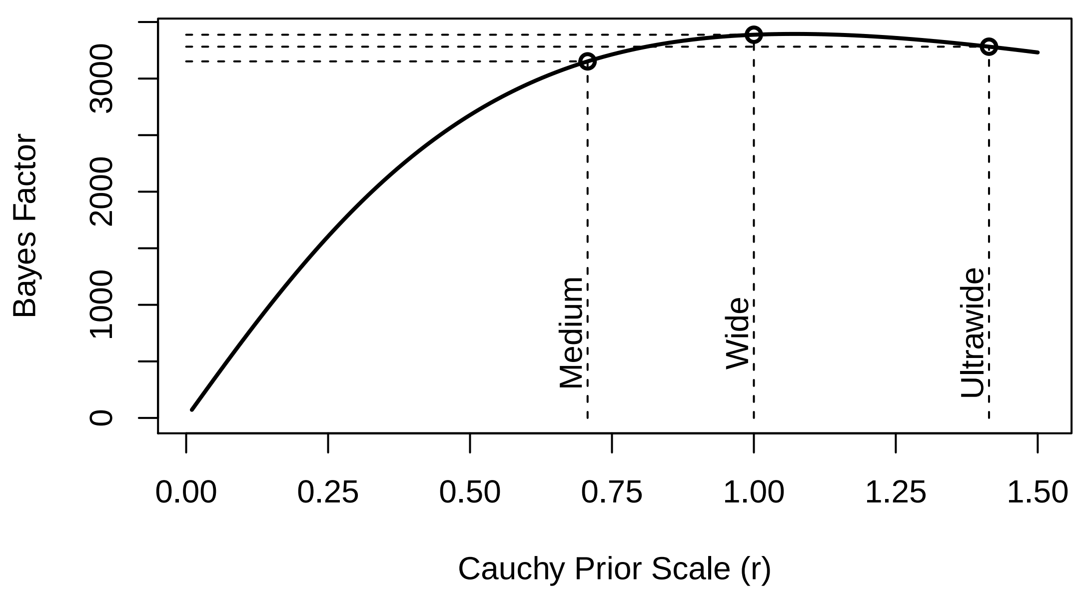

Figure 4. Robustness check for Bayes factor in favor of congruity effect on standard deviations.

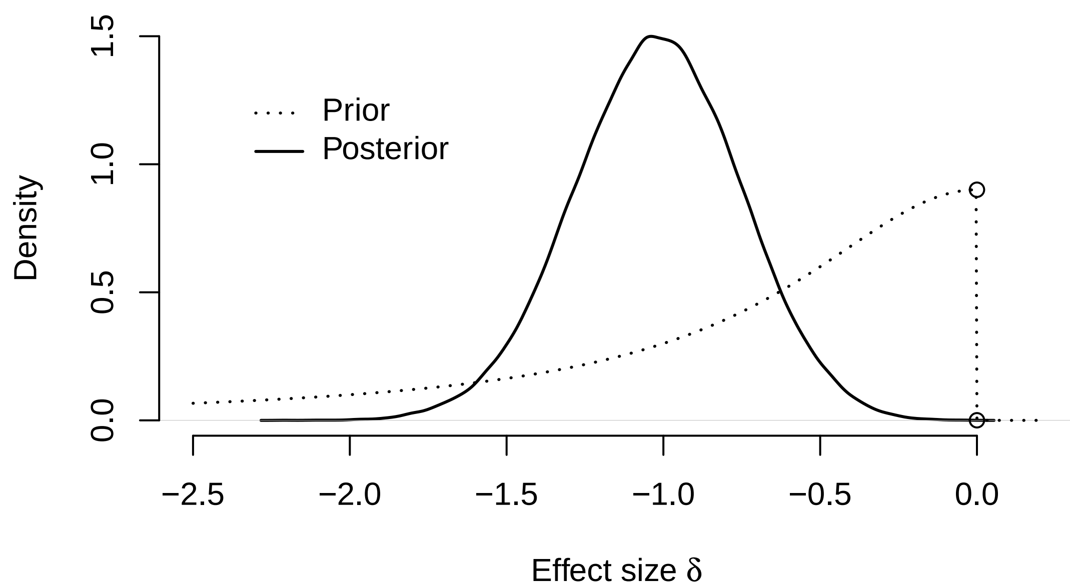

Figure 5. Prior and posterior for effect size on drift rate. Points on the plot represent the density of the point null in both the prior and posterior.

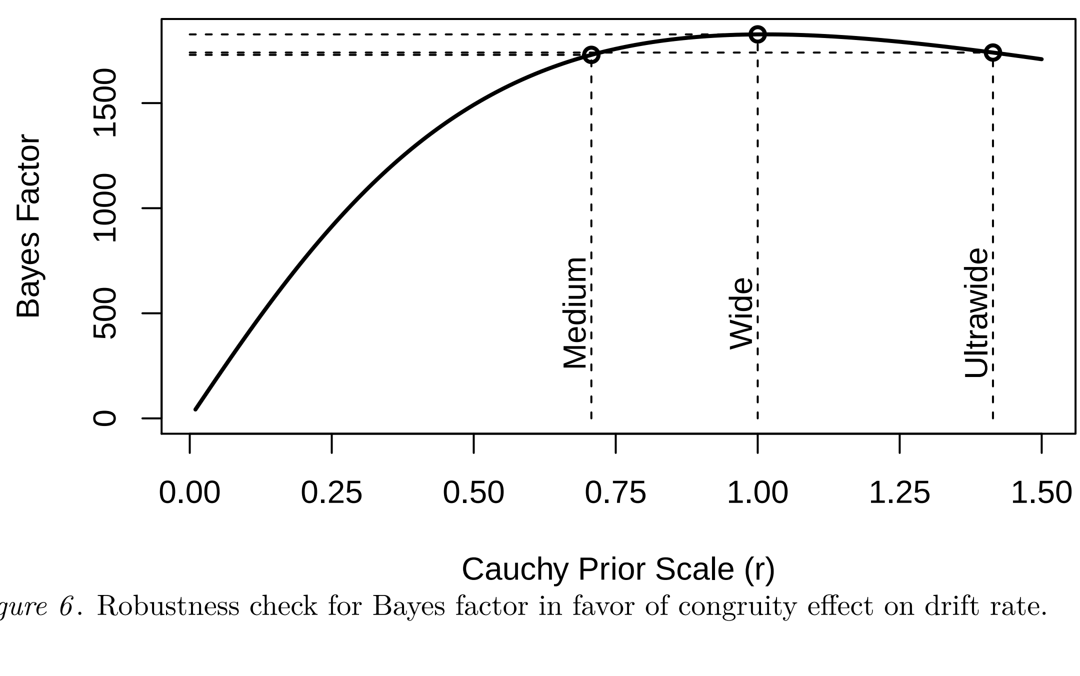

Figure 6. Robustness check for Bayes factor in favor of congruity effect on drift rate.

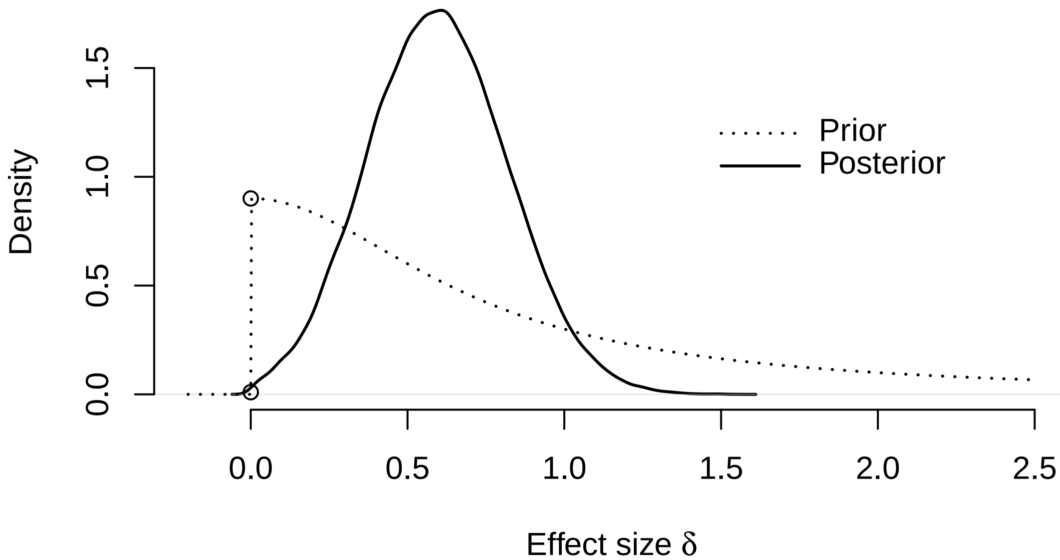

Figure 7. Prior and posterior for effect size on response threshold. Points on the plot represent the density of the point null in both the prior and posterior.

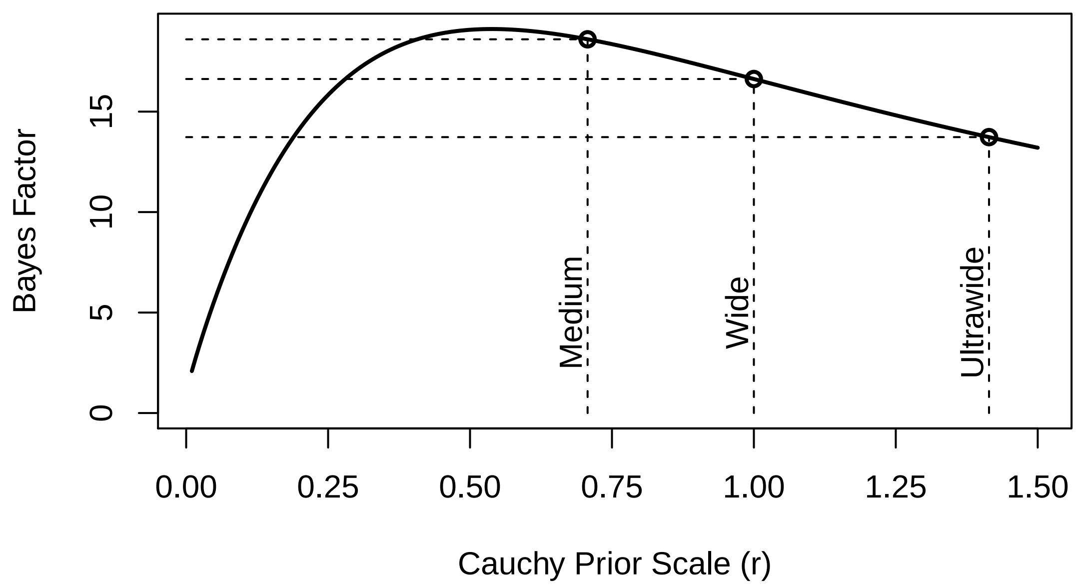

Figure 8. Robustness check for Bayes factor in favor of congruity effect on response threshold.

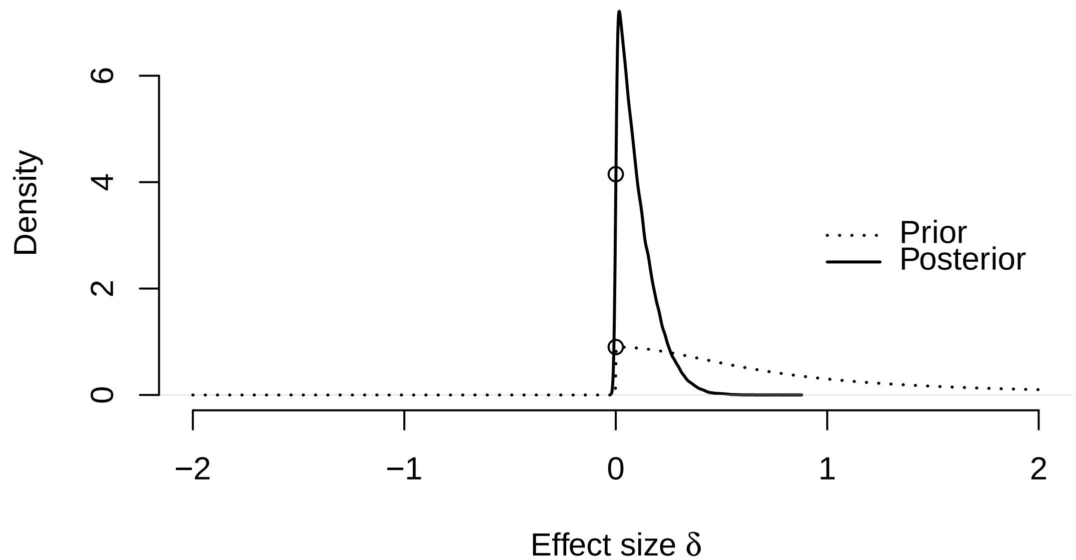

Figure 9. Prior and posterior for effect size on nondecision time. Points on the plot represent the density of the point null in both the prior and posterior.

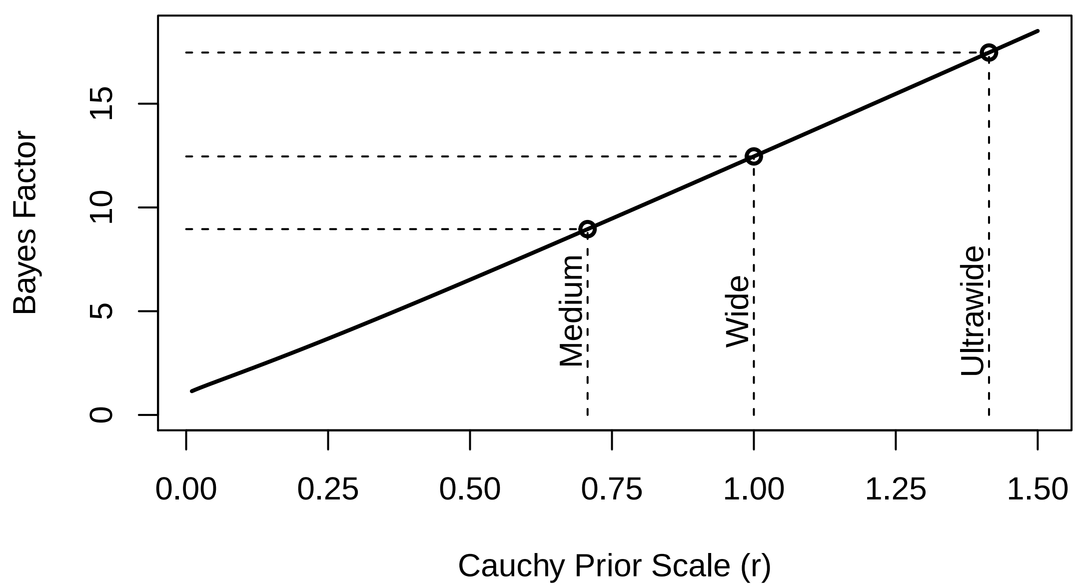

Figure 10. Robustness check for Bayes factor in favor of null effect of congruity effect on nondecision time

References