

Negligible Mass.

$$\begin{aligned}
 PE_{top} + KE_{top} &= PE_x + KE_x \\
 0 + 0 &= -mgx + \frac{1}{2}mv^2 \\
 \frac{1}{2}v^2 &= gx \\
 v &= \sqrt{2gx}
 \end{aligned}$$

$$\frac{dv}{dx} = \sqrt{2g} \cdot \frac{1}{2} \cdot x^{-\frac{1}{2}} = \sqrt{\frac{g}{2x}}$$

$$a = \frac{dv}{dx} \cdot \frac{dx}{dt} = \frac{dv}{dx} \cdot v = \sqrt{\frac{g}{2x}} \cdot \sqrt{2gx} = g$$

Rope Hangs down.

$$\begin{aligned}
 PE_{top} + KE_{top} &= PE_x + KE_x \\
 -\rho \frac{L}{2} \cdot g \cdot \frac{L}{4} - \rho \frac{L}{2} \cdot g \cdot \frac{L}{4} + 0 &= -mgx - \rho \left(\frac{L+x}{2}\right) \cdot g \cdot \left(\frac{L+x}{4}\right) - \rho \left(\frac{L-x}{2}\right) \cdot g \cdot \left(\frac{L+x}{4}\right) \\
 &\quad + \frac{1}{2}mv^2 + \frac{1}{2}\rho \left(\frac{L-x}{2}\right)v^2 \\
 -\rho g \frac{L^2}{4} &= -mgx - \rho g \left[\frac{L^2+2xL+x^2}{8} + \frac{L^2+2xL-3x^2}{8} \right] + \frac{1}{2}mv^2 + \frac{1}{2}\rho \left(\frac{L-x}{2}\right)v^2
 \end{aligned}$$

$$-\rho g L^2 = -4mgx - \frac{1}{2}\rho g (2L^2 + 4xL - 2x^2) + (2m + \rho(L-x))v^2$$

$$(2m + \rho(L-x))v^2 = 4mgx + \rho g (L^2 + 2xL - x^2) - \rho g L^2$$

$$(2m + \rho(L-x))v^2 = 4mgx + \rho g x (2L - x)$$

$$v = \sqrt{\frac{4mgx + \rho g x (2L - x)}{2m + \rho(L-x)}}$$

$$\frac{dv}{dx} = \frac{g(\rho^2 x^2 - (2L\rho^2 + 4m\rho)x + 2\rho^2 L^2 + 8Lm\rho + 8m^2)}{2(\rho x - L) - 2m)^2 \sqrt{4mgx + \rho g x (2L - x)}}$$

$$a = \frac{dv}{dx} \cdot v = \frac{g(\rho^2 x^2 - 2L\rho^2 x - 4m\rho x + 2L^2\rho^2 + 8mL\rho + 8m^2)}{2(\rho x - L) - 2m)^2}$$

Rope starts at top.

$$\begin{aligned} PE_{\text{top}} + KE_{\text{top}} &= PE_x + KE_x \\ 0 + 0 &= -mgx - \rho x \cdot g \cdot \frac{x}{2} + \frac{1}{2}mv^2 + \frac{1}{2}\rho x \left(\frac{v}{2}\right)^2. \end{aligned}$$

$$(m + \frac{1}{2}\rho x)v^2 = 2mgx + \rho gx^2$$

$$(4m + \rho x)v^2 = 4gx(2m + \rho x)$$

$$v = \sqrt{\frac{4gx(2m + \rho x)}{4m + \rho x}}$$

$$\frac{dv}{dx} = \frac{2g(\rho^2 x^2 + 8m\rho x + 8m^2)}{(\rho x + 4m)^2 \sqrt{\frac{4gx(2m + \rho x)}{4m + \rho x}}}$$

$$a = \frac{dv}{dx} \cdot v = \frac{2g(\rho^2 x^2 + 8m\rho x + 8m^2)}{(\rho x + 4m)^2}$$