

Negligible Mass.

$$\begin{aligned} PE_{top} + KE_{top} &= PE_x + KE_x \\ 0 + 0 &= -mgx + \frac{1}{2}mv^2. \\ \frac{1}{2}v^2 &= gx \\ v &= \sqrt{2gx}. \end{aligned}$$

$$\frac{dv}{dx} = \sqrt{2g} \cdot \frac{1}{2} \cdot x^{-\frac{1}{2}} = \sqrt{\frac{g}{2x}}$$

$$a = \frac{dv}{dx} \cdot \frac{dx}{dt} = \frac{dv}{dx} \cdot v = \sqrt{\frac{g}{2x}} \cdot \sqrt{2gx} = g.$$

Rope hangs down.

$$\begin{aligned} PE_{top} + KE_{top} &= PE_x + KE_x. \\ -\rho \frac{L}{2} \cdot g \cdot \frac{L}{4} - \rho \frac{L}{2} \cdot g \cdot \frac{L}{4} + 0 &= -mgx - \rho \left(\frac{L+x}{2}\right) \cdot g \cdot \left(\frac{L+x}{4}\right) - \rho \left(\frac{L-x}{2}\right) \cdot g \cdot \left(\frac{L+3x}{4}\right) \\ &\quad + \frac{1}{2}mv^2 + \frac{1}{2}\rho \left(\frac{L-x}{2}\right)v^2. \\ -\rho g \frac{L^2}{4} &= -mgx - \rho g [L^2 + 2xL + x^2 + L^2 + 2xL - 3x^2] + \frac{1}{2}mv^2 + \frac{1}{2}\rho \left(\frac{L-x}{2}\right)v^2. \\ -\rho g L^2 &= -4mgx - \frac{1}{2}\rho g (2L^2 + 4xL - 2x^2) + (2m + \rho(L-x))v^2. \\ (2m + \rho(L-x))v^2 &= 4mgx + \rho g(L^2 + 2xL - x^2) - \rho g L^2. \end{aligned}$$

$$(2m + \rho(L-x))v^2 = 4mgx + \rho g x(2L - x)$$

$$v = \sqrt{\frac{4mgx + \rho g x(2L - x)}{2m + \rho(L - x)}}$$

$$\frac{dv}{dx} = g \frac{\rho^2 x^2 + (2L\rho^2 + 4m\rho)x + 3\rho^2 L^2 + 8Lm\rho + 8m^2}{2(x-L-2m)^2 \sqrt{\frac{4mgx + \rho g x(2L - x)}{2m + \rho(L - x)}}}$$

$$a = \frac{dv}{dx} \cdot v = g \frac{(\rho^2 x^2 - 2L\rho^2 x - 4m\rho x + 2L^2\rho^2 + 8mL\rho + 8m^2)}{2 \frac{1}{\rho} (x-L-2m)^2}.$$

Rope starts at top.

$$\begin{aligned} \text{PE}_{\text{top}} + \text{KE}_{\text{top}} &= \text{PE}_x + \text{KE}_x \\ 0 + 0 &= -mgx - px \cdot g \cdot \frac{x}{2} + \frac{1}{2}mv^2 + \frac{1}{2}px(\frac{v}{2})^2. \end{aligned}$$

$$(m + \frac{1}{2}px)v^2 = 2mgx + pgx^2$$

$$(4m + px)v^2 = 4gx(2m + px)$$

$$v = \sqrt{\frac{4gx(2m + px)}{(4m + px)}}$$

$$\frac{dv}{dx} = 2g \frac{(\rho^2x^2 + 8mpx + 8m^2)}{(px + 4m)^2} \sqrt{\frac{4gx(2m + px)}{4m + px}}$$

$$a = \frac{dv}{dx} \cdot v = 2g \frac{(\rho^2x^2 + 8mpx + 8m^2)}{(px + 4m)^2} \cdot \sqrt{\frac{4gx(2m + px)}{4m + px}}$$