

# Application of the Poisson Distribution

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## Introduction

The Poisson distribution is one of three discrete distributions, Binomial, Poisson, and Hypergeometric, that use integers as random variables. The Fourth Quarter 2000 issue of the RAC Journal included a START sheet on the use of Binomial Distribution to determine the reliability of a one-shot device. The sheet resulted in the development of a Binomial Calculator that is currently available on the RAC web site for general use <<http://rac.iitri.org/Toolbox/OneShotCalc.htm>>.

A Poisson Calculator has also been developed and placed on the RAC web site (<[http://rac.iitri.org/Toolbox/Rac\\_poisson.html](http://rac.iitri.org/Toolbox/Rac_poisson.html)>). This calculator is based on the algorithms that were developed by Mr. Leonard Stascavage, while acting as Failure Analysis and Reliability Lab Manager for the Naval Surface Warfare Center (NSWC) in Silver Springs, MD. Mr. Stascavage developed the algorithms to overcome the limited computing power of computers and to solve various Reliability and Maintainability problems associated with the Naval Weapons Systems Programs at NSWC. These algorithms were found to be suitable for web applications. RAC wishes to thank Mr. Stascavage for his generosity in providing the foundation for this tool.

## The Poisson Distribution

Simeon-Denis Poisson, 1781–1840, a mathematician was known for his work in the area of definite integrals, electrical theory and probability. In 1837, Poisson derived his distribution to approximate the Binomial Distribution when the probability of occurrence (p) is small. A use for this distribution was not found until 1898; when an individual named Bortkiewicz was tasked by the Prussian Army to investigate accidental deaths of soldiers attributed to being kicked by horses. The initial application of the Poisson distribution to

determine the number of deaths attributed to horsed kicks in the Prussian Army led to its use in analyzing accidental deaths, service requirements, errors over time, and its use in reliability engineering.

The Poisson equation for predicting the probability of a specific number of defects or failures (r) in time (t) is:

$$P(r) = \frac{(\lambda t)^r e^{-\lambda t}}{r!} \quad (1)$$

where:

- r = number of failures in time (t)
- $\lambda$  = failure rate per hour
- t = time expressed in hours
- P(r) = probability of getting exactly r failures in time t

To calculate the probability of k or fewer failures occurring in time t, the probability of each failure occurring must be summed as shown in Equation (2).

$$P(r \leq k) = \sum_{r=0}^k P(r) \quad (2)$$

The Confidence Level (CL) that the population has a failure rate ( $\lambda$ ) based on  $r \leq k$  failures occurring in time (t) is:

$$CL = 1 - P(r \leq k) \quad (3)$$

For example, assume that the population of a component has a failure rate of 121.7 failures per one million hours. The component is expected to operate 43,800 hours and only 2 failures are expected to occur. Using Equation (1), calculate the probability of exactly 0, 1, and 2 failures, as shown in the equations that follow.

$$P(0) = \frac{(121.7/10^6 \times 43,800)^0 e^{-121.7/10^6 \times 43,800}}{0!} = 0.0048$$

$$P(1) = \frac{(121.7/10^6 \times 43,800)^1 e^{-121.7/10^6 \times 43,800}}{1!} = 0.0256$$

$$P(2) = \frac{(121.7/10^6 \times 43,800)^2 e^{-121.7/10^6 \times 43,800}}{2!} = 0.0682$$

Using Equation (2), the probability of two or fewer failures is the sum of these probabilities. That is:

$$P(r \leq k) = 0.0048 + 0.0256 + 0.0682 = 0.0986$$

For this example, Equation (3) determines that the Confidence Level is approximately 90% that the failure rate of the population is 121.7 failures per one million hours.

$$CL = 1 - P(r \leq k) = 1 - 0.0986 = 0.9014 \text{ or } 90.14\%$$

## Sparing Analysis

Another application for the Poisson distribution is determining the number of spare line replacement units (LRU) that should be initially available to ensure a preselected probability that a spare is available. Frederick J. O'Neal of Bell Laboratories developed such a sparing equation in the 1970s for electronic systems. In his 1989 RAMS article, Al Myrick modified this equation to assure availability for a desired confidence level.

$$CL \leq \sum_{k=0}^S \frac{(n\lambda R)^k e^{-(n\lambda R)}}{k!} \quad (4)$$

where:

- n = number of LRU in service
- $\lambda$  = failure rate per hour
- R = repair time in hours
- CL = Confidence Level
- S = minimum number of spares required

An example of how this equation can be used to determine the number of spares that must be available, taking into account the repair time, the failure rate of the LRU, number of units in serv-

ice, and a given level of confidence follows. Assume that 2,000 LRUs are in service. The failure rate is 121.7 failures per million hours and the mean time to repair a failure is 4 hours. The plan (hypothesis) is to have one spare unit available. We want to be 90% confident that one spare unit will be adequate. Using Equation (4), we can test the hypothesis that one spare will assure 90% confidence.

$$CL \leq \sum_{k=0}^S \frac{(n\lambda R)^k e^{-(n\lambda R)}}{k!}$$

$$0.90 \leq \sum_{k=0}^1 \frac{[(2000)(121.7/10^6)(4)]^k e^{-[(2000)(121.7/10^6)(4)]}}{k!}$$

$$0.90 \leq 0.3777 + 0.3677 = 0.7454 \text{ or } \sim 75\%$$

One would reject the hypothesis that one spare would provide an adequate number of spares with 90% confidence. If two spares were made available, then the Confidence Level would be 0.9245, and the hypothesis would be accepted.

## Poisson Calculator

The Poisson Calculator can be found on the RAC web site (<[http://rac.iitri.org/Toolbox/Rac\\_poisson.html](http://rac.iitri.org/Toolbox/Rac_poisson.html)>). The calculator has six tabs that permit the user to elect the desired output:

- Confidence
- Failure Rate
- Time Period
- Max. No. Failures
- Probability of Failure
- Sparing Analysis that can solve for confidence level of the minimum number of spares

The web site layout is shown in Figure 1. Figures 2 through 6 show the different outputs available and the required inputs in each case.

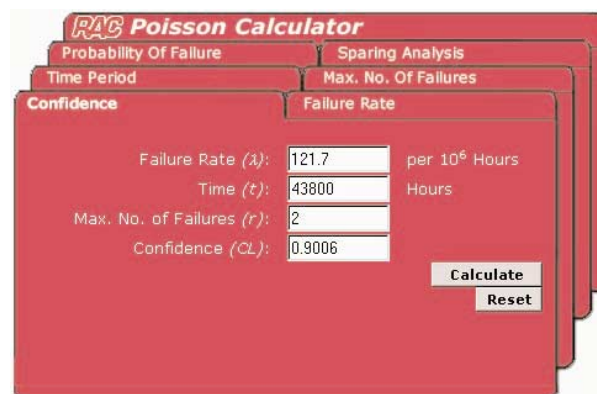


Figure 1. The Poisson Calculator showing the tab for Confidence Level. Assume that your product has been in the field for 43,800 hours and only 2 failures occurred. The product's failure rate was estimated to be 121.7 failures per million hours of operation. How confident would you be that the failure rate of the population was 121.7? Using the calculator as indicated, you would be 90.1% confident that your product has a failure rate of 121.7 per million operating hours.

**RAC Poisson Calculator**

Probability Of Failure | Sparing Analysis

Time Period | Max. No. Of Failures

Confidence | Failure Rate

Failure Rate ( $\lambda$ ): 3.3 per  $10^6$  Hours

Time ( $t$ ): 2500000 Hours

Max. No. of Failures ( $r$ ): 3

Confidence ( $CL$ ): 0.95

Calculate

Reset

Figure 2. The Poisson Calculator showing only the tab for failure rate calculations. Assume that your product has accumulated 2,500,000 hours of operation with 3 failures. The goal is to be 95% confident in establishing the product's failure rate. How many failures should one expect in one million hours? Using the calculator as indicated, you would be 95% confident that the population of product would have an average failure rate of 3.3 per million hours.

**RAC Poisson Calculator**

Confidence | Failure Rate

Probability Of Failure | Sparing Analysis

Time Period | Max. No. Of Failures

Failure Rate ( $\lambda$ ): 1.4469 per  $10^6$  Hours

Time ( $t$ ): 1312000 Hours

Max. No. of Failures ( $r$ ): 0

Confidence ( $CL$ ): 0.85

Calculate

Reset

Figure 3. The Poisson Calculator showing only the tab for time period calculations. The established failure rate is 1.4469 failures per million hours. Assume that no failures will be allowed. What period of time must be accumulated before you are 85% confident that the failure rate is 1.4469? Using the calculator as indicated, after 1,312,000 hours of operation without a failure, you would be 85% confident that the failure rate is 1.4469.

**RAC Poisson Calculator**

Confidence | Failure Rate

Probability Of Failure | Sparing Analysis

Time Period | Max. No. Of Failures

Failure Rate ( $\lambda$ ): 2.56 per  $10^6$  Hours

Time ( $t$ ): 2500000 Hours

Max. No. of Failures ( $r$ ): 3

Confidence ( $CL$ ): 0.9

Calculate

Reset

Figure 4. The Poisson Calculator showing only the tab for max. failures calculations. A new product's failure rate was predicted to be 2.56 failures per million hours. The test plan was to accumulate 2,500,000 hours of operational time. What is the maximum number of failures allowed to be 90% confident that the failure rate is 2.56? Using the calculator as indicated, with 3 or fewer failures in 2,500,000 hours, you would be 90% confident that the failure rate is 2.56 per million hours.

**RAC Poisson Calculator**

Time Period | Max. No. Of Failures

Confidence | Failure Rate

Probability Of Failure | Sparing Analysis

Failure Rate ( $\lambda$ ): 0.04 per  $10^6$  Hours

Time ( $t$ ): 55000000 Hours

No. of Failures ( $r$ ): 2

Probability ( $r=k$ ) ( $P(r)$ ): 0.2681

Probability ( $r \leq k$ ) ( $P(r)$ ): 0.6227

Probability ( $r > k$ ) ( $P(r)$ ): 0.3773

Calculate

Reset

Figure 5. The Poisson Calculator showing only the tab for probability of failure calculations. The failure rate is estimated to be 0.04 failures per million hours. The product has accumulated 55,000,000 hours of operation time and 2 failures occurred. What is the probability of getting exactly 2 failures, 2 or fewer failures and more than 2 failures? Using the calculator as indicated, the probability of exactly 2 failures is 0.2681, the probability of 2 or fewer failures is 0.6227 and the probability of more than 2 failures is 0.3773.

Figure 6. The Poisson Calculator showing only the tab for sparing analysis. The plan is to sell 2,000 assemblies. The failure rate for these assemblies has been determined to be 121.7 failure per million hours. The time to repair an assembly is 4 hours. The desire is to provide only 1 spare assembly. The goal is to be 90% confident that 1 assembly will be adequate. Using the calculator as indicated, the resulting confidence is 0.7455. In order to be 90% confident, the number of spares must be increased.

## For Further Study

1. O'Connor, P.D.T., "Practical Reliability Engineering," John Wiley & Sons, 1995.
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3. Lipson, C. and Sheld, N.J., "Statistical Design and Analysis of Engineering Experiments," McGraw-Kill, 1973.
4. Reliability Analysis Center, "Practical Statistical Tools for Reliability Engineer," September 1999.
5. Myrick, A., "Sparing Analysis – A Multi-Use Planning Tool," Proceedings: Annual Reliability and Maintainability Symposium, 1989.
6. University of Utah Web Site: <<http://www.math.uah.edu/stat/poisson/poisson4.html>>.
7. University of Massachusetts's Web Site: <<http://www.umass.edu/wsp/acquiring/tales/poisson.html>>.

## About the Author

Edward R. Sherwin is a Senior Engineer with Alion Science and Technology. Over the past five years, he has worked on a variety of reliability improvement projects of "one-shot" Naval weapon systems and industry. Before joining Alion, and its predecessor IIT Research Institute (IITRI), he spent 25 years with Carrier Corporation, a division of United Technologies, was the Program Manager of Manufacturing & Process Technologies

for Carrier's worldwide manufacturing operations. He also served an adjunct professor at Syracuse University. Mr. Sherwin holds a B.S. in Industrial Engineering from the University of Dayton and a M.S. in Engineering Science from Pennsylvania State University. He is also a registered professional engineer.

## Other START Sheets Available

Many Selected Topics in Assurance Related Technologies (START) sheets have been published on subjects of interest in reliability, maintainability, quality, and supportability. START sheets are available on-line in their entirety at <<http://rac.alionscience.com/rac/jsp/start/startsheet.jsp>>.

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## About the Reliability Analysis Center

The Reliability Analysis Center is a world-wide focal point for efforts to improve the reliability, maintainability, supportability and quality of manufactured components and systems. To this end, RAC collects, analyzes, archives in computerized databases, and publishes data concerning the quality and reliability of equipments and systems, as well as the microcircuit, discrete semiconductor, electronics, and electromechanical and mechanical components that comprise them. RAC also evaluates and publishes information on engineering techniques and methods. Information is distributed through data compilations, application guides, data products and programs on computer media, public and private training courses, and consulting services. Alion, and its predecessor company IIT Research Institute, have operated the RAC continuously since its creation in 1968.