A Phenomenological Model for the Photon's Quantum Reference Frame: Formal Development, Regulator-Independent Predictions, and Experimental Design

(Author names omitted for preprint)

August 8, 2025

Abstract

We propose a phenomenological model for describing physics from a photon-centered quantum reference frame (QRF). Our approach combines first-principles constraints with a minimal phenomenological layer to capture complex quantum-optical effects while retaining regulator-independent, gauge-invariant observables. In contrast to the classical no-go result that no inertial rest frame exists for a photon, we show that a superposition of ultra-relativistic Lorentz boosts, conditioned on the photon's quantum state, can operationally realize a photon's perspective. We make the construction explicit by (i) specifying the operator map and its domain via a completely positive (CP) instrument (with an associated unitary dilation), (ii) choosing an explicit light-front regularization scheme and proving that key observables are regulator-independent (dominated-convergence with off-diagonal control), and (iii) bounding massless little-group (E(2)) effects relative to the leading signal, including a worked Gaussian example. We then give a concrete Hong–Ou–Mandel (HOM) interferometer design and error budget that could resolve the predicted visibility change at the 3×10^{-4} level, and reframe an atom-interferometer prediction as a future target with scaling plots. We position the work within the quantum-reference-frame (QRF) and relativistic quantum information (RQI) literature and conclude with an outlook toward a full group-theoretic massless-QRF formalism.

1 Introduction and Positioning

The quantum reference frame (QRF) programme promotes frames to quantum systems undergoing conditional, state-dependent transformations. While QRFs for massive particles are well-developed, a consistent description from the perspective of a *massless* photon remains challenging due to (i) the absence of a classical rest frame, (ii) light-front zero modes ($k^+ \to 0$) and regularization subtleties, and (iii) the massless little group $E(2)$. Our aim is to provide an *operational* construction of a photon's QRF that respects these constraints while yielding concrete, falsifiable predictions.

Related work. Foundational QRF results showed that physical laws can be written covariantly under quantum frame changes and that rest frames of quantum systems can be meaningfully defined in the massive case [1, 2]. In relativistic quantum information it is known that Lorentz boosts entangle degrees of freedom and can change observable coherences (e.g., spin-momentum coupling) [3, 4]. On the field-theory side, the light-front ("front form") pioneered by Dirac and developed by Brodsky–Pauli–Pinsky provides both computational advantages and well-known zero-mode pathologies [5, 6]. Our contribution is to (i) extend the QRF idea to a massless photon with an explicit operator map and domain, (ii) show regulator-independent leading observables under an explicit light-front regularization, and (iii) design a concrete HOM experiment to seek the predicted scaling signature.

2 Light-Front Coordinates and Notation

We adopt $x^{\pm} = t \pm z$, $\mathbf{x}_\perp = (x, y)$, with metric $ds^2 = dx^+dx^- - d\mathbf{x}_\perp^2$. For massless momenta p^μ, the on-shell condition reads $p^+p^- = \mathbf{p}_\perp^2$. A boost of rapidity η along z rescales $k^\pm \to e^{\pm\eta}k^\pm$. States are described on the light-front using the measure

$$d\mu(k) = \frac{dk^+\, d^2\mathbf{k}_\perp}{2k^+(2\pi)^3}\,\theta(k^+)\,. \tag{1}$$

We work in light-cone gauge $A^+ = 0$ to retain only physical photon polarizations. Intensity observables (e.g., HOM visibilities) are gauge invariant; gauge-like phases induced by null translations cancel in intensities (see App. E and G).

Definition 1 (State class)**.** *Photon wavepackets satisfy: (i) $\Psi \in L^2(d\mu)$ with finite second moments; (ii) support bounded away from zero: there exists a fixed $\epsilon_0 > 0$* independent of the regulator *such that $k^+ \geq \epsilon_0$ a.e.; (iii) sub-Gaussian tails in k^+, implying $\Psi \in L^1 \cap L^2$ and hence $\|\Psi\|_1 < \infty$.*

3 Transformation to the Photon's Frame

3.1 Operator Map and Domain (CP-instrument presentation)

We implement the photon-QRF as a *selective* CP operation on the system induced by a coherently controlled boost on photon+system and subsequent conditioning on a fixed photon reference state (see App. F for the dilation).

Define the Kraus operator

$$K[\Psi] \;=\; \int d\mu_{m_\gamma}(k)\,\Psi(k^+,\mathbf{k}_\perp)\;U[\Lambda(k\to\tilde{k})]\,, \tag{2}$$

where $U[\Lambda(k\to\tilde{k})]$ is the (single- or few-particle) unitary representation of the Lorentz transformation that maps k^μ to a fiducial $\tilde{k}^\mu$ along z.[1] *Mathematical convention.* The operator integral in (2) is taken as a *Bochner integral* in the strong operator topology; the map $k \mapsto U[\Lambda(k\to\tilde{k})]$ is strongly continuous on the regulated support.

The selective update for an input ρ_S is

$$\rho_S \;\mapsto\; \Phi(\rho_S) \;=\; \frac{K[\Psi]\,\rho_S\,K[\Psi]^\dagger}{\mathrm{Tr}\big[K[\Psi]\,\rho_S\,K[\Psi]^\dagger\big]}\,. \tag{3}$$

Lemma 1 (Boundedness and non-isometry)**.** *On the regulated domain, $K[\Psi]$ is bounded and $\|K[\Psi]\| \leq \|\Psi\|_1$, so for any $|\phi\rangle$, $\|K[\Psi]\,|\phi\rangle\,\| \leq \|\Psi\|_1\,\|\,|\phi\rangle\,\|$. Equality requires $U[\Lambda(k\to\tilde{k})]\,|\phi\rangle$ to be collinear in k a.e. Thus unless $U[\Lambda(k\to\tilde{k})]$ is k-independent on the support of Ψ, $K[\Psi]$ is a strict contraction on a dense set and not an isometry.*

[1] For a scalar system mode with momentum p, $(U[\Lambda]\phi)(p) = \sqrt{\frac{2p^0}{2(\Lambda^{-1}p)^0}}\,\phi(\Lambda^{-1}p)$; for spins/polarizations, include the appropriate Wigner rotation. In this work we restrict to scalar-like temporal/spatial degrees relevant for timing/visibility.

Remark 1 (Non-reciprocity)**.** *The selective map* (3) *is generically non-invertible (no CPTP left-inverse) unless* U_k *is constant; see Prop. 1 in App. F. Reciprocity at the global level holds in the unitary dilation before conditioning and discarding the record.*

3.2 Regularization Scheme and Regulator-Independent Observables

We render $d\mu(k)$ well-defined with a small photon-mass regulator $m_\gamma > 0$ (removed at the end):

$$d\mu_{m_\gamma}(k) = \frac{dk^+\, d^2\mathbf{k}_\perp}{2k^+(2\pi)^3}\,\theta(k^+), \qquad k^- = \frac{\mathbf{k}_\perp^2 + m_\gamma^2}{2k^+}\,. \tag{4}$$

Crucial assumption for dominated convergence: we impose the fixed cutoff from Def. 1, $k^+ \geq \epsilon_0 > 0$.

Theorem 1 (Regulator independence for moment-ratio observables)**.** *Let* Ψ *satisfy Def. 1. Then, as* $m_\gamma \to 0$,

$$\frac{\mathrm{Var}_{m_\gamma}(k^+)}{\left(\mathbb{E}_{m_\gamma}[k^+]\right)^2} \;\longrightarrow\; \frac{\mathrm{Var}(k^+)}{\left(\bar{k}^+\right)^2}\,, \tag{5}$$

with convergence justified by dominated convergence on the fixed support, using sub-Gaussian domination. Any intensity observable whose dependence enters through $\mathrm{Var}(\eta)$ *(e.g., HOM visibility) inherits this regulator independence. Off-diagonals of* $K^\dagger K$ *are controlled by Lemma 3 in App. D.*

Lemma 2 (Small-spread mapping)**.** *For* $\eta = \ln\!\left(k^+/\tilde{k}^+\right)$ *and small fractional spread* $\sigma_{k^+}/\bar{k}^+ \ll 1$,

$$\mathrm{Var}(\eta) = \frac{\mathrm{Var}(k^+)}{\left(\bar{k}^+\right)^2} + O\!\left(\left(\frac{\sigma_{k^+}}{\bar{k}^+}\right)^3\right). \tag{6}$$

3.3 E(2) Little Group: Bound on Neglected Terms

Composing non-collinear boosts induces Wigner rotations and null translations in the massless little group $E(2)$. For small transverse spreads and rapidities relevant here,

$$|\theta_W(k)| \leq C\,\frac{\|\mathbf{k}_\perp\|^2}{(k^+)^2} \qquad (C = \mathcal{O}(1)). \tag{7}$$

Mini-derivation. For small boosts $\theta_\perp$ transverse to $\hat{z}$, the Lorentz algebra gives $[K_i, K_j] = -\,\epsilon_{ijk}J_k$ so a product of two tilts yields a rotation R_k of angle $\frac{1}{2}\,\theta_i\theta_j$ about $\hat{z}$; isotropically $|\theta_W| \leq \frac{1}{2}\|\theta_\perp\|^2$. For a lightlike momentum, $\|\theta_\perp\| \sim \|\mathbf{k}_\perp\|/(k^+)$ [7, 8]. Hence the stated bound. In HOM, coincidence probabilities are intensities; null-translation phases cancel to first order, so the visibility correction satisfies

$$|\Delta V_{E(2)}| \leq C'\,\frac{\mathbb{E}[\|\mathbf{k}_\perp\|^4]}{\left(\bar{k}^+\right)^4} \ll \alpha_{\mathrm{HOM}}\left(\frac{\sigma_{k^+}}{\bar{k}^+}\right)^2\,. \tag{8}$$

With $\theta_{\mathrm{rms}} \sim$ mrad the resulting correction is $\ll 10^{-6}$ in visibility units, well below our leading effect (App. E). Polarization Wigner rotations share this scaling and are likewise $\ll 10^{-6}$.

4 Transformed Observables and State-Dependent Spacetime

Observables are evaluated on the selectively updated state ρ'_S. The entanglement between k^+ and system coordinates induces a state-dependent blur of classical events. For a world-line $z(t)$ probed semiclassically and small rapidities, $z' \approx z(t) - t\hat{\eta}$ with $\hat{\eta} = \ln\left(\hat{k^+}/\tilde{k}^+\right)$, so

$$(\Delta z')^2 = \mathrm{Var}(z(t) - t\hat{\eta}) \approx t^2 \mathrm{Var}(\hat{\eta}) \approx t^2 \left(\frac{\sigma_{k^+}}{\tilde{k}^+}\right)^2, \tag{9}$$

see App. A.

5 Experimental Predictions and Designs

5.1 Hong–Ou–Mandel (HOM) Interferometry

The photon-QRF transform induces an effective timing jitter on one interferometer arm (or a controlled fraction thereof), reducing visibility. For small spreads,

$$\Delta V_{\mathrm{QRF}} = \alpha_{\mathrm{HOM}} \left(\frac{\sigma_{k^+}}{\bar{k}^+}\right)^2, \qquad \alpha_{\mathrm{HOM}} \in [1/8, 1/2]\,. \tag{10}$$

Two explicit edge cases

Define $\Delta\tau = \kappa\,\tau_c\,\eta$ (geometry-dependent coupling κ), and let $c \in \{1, 1/\sqrt{2}\}$ encode single-arm vs. symmetric split of the relative delay. Linearizing $V = V_0 \mathbb{E}[C(\Delta\tau)]$ with $C''(0) = -1/\tau_c^2$ gives $\Delta V = (\kappa^2 c^2/2)\,\mathrm{Var}(\eta)$. Using Lemma 2:

$$\text{(i) Single-arm, full coupling: } \kappa = 1,\ c = 1 \ \Rightarrow\ \alpha_{\mathrm{HOM}} = \tfrac{1}{2}.$$
$$\text{(ii) Symmetric split, partial coupling: } \kappa = \tfrac{1}{\sqrt{2}},\ c = \tfrac{1}{\sqrt{2}} \ \Rightarrow\ \alpha_{\mathrm{HOM}} = \tfrac{1}{8}.$$

These establish the advertised range with clean, physically transparent cases.

Concrete design, control, and error budget

We propose a fiber-based HOM interferometer using spectrally factorable SPDC photon pairs at $\lambda \in [810,\ 1550]$ nm. One photon serves as the QRF probe; the partner is interfered on a 50:50 beamsplitter. Tunable bandwidth filtering changes $\sigma_{k^+}/\bar{k}^+$ while holding other parameters fixed. A *control slope* is obtained by filtering both arms identically; the expected slope is $0 \pm \delta$.

- **Source:** CW-pumped type-II SPDC with programmable optical filters; target fractional bandwidths $\Delta\nu/\nu \in \{1\%,\ 3\%,\ 5\%,\ 8\%\}$.
- **Detectors:** SNSPDs with system jitter ≤ 20 ps, dark counts < 100 s^{-1}, efficiency $> 80\%$.
- **Paths:** Fiber delay lines with active stabilization; path-length noise $\lesssim 20$ nm/$\sqrt{\mathrm{Hz}}$.
- **Calibration:** Extract α_{HOM} by varying $\sigma_{k^+}/\bar{k}^+$ and fitting ΔV vs. $(\sigma_{k^+}/\bar{k}^+)^2$; confirm zero-slope control. With four equally spaced $x = (\sigma_{k^+}/\bar{k}^+)^2$ points and homoscedastic errors σ_V, the slope s.d. is $\sigma_m = \sigma_V/\sqrt{S_{xx}}$ with $S_{xx} = \sum(x_i - \bar{x})^2$.

Table 1: HOM visibility error budget (targets to resolve $\Delta V \sim 3 \times 10^{-4}$).

Mechanism	Symbol	Target contribution	Mitigation / calibration
Spectral distinguishability	$1 - V_{\mathrm{spec}}$	$< 1.0 \times 10^{-4}$	Factorable SPDC; symmetric filters; verify via joint spectral intensity.
Spatial/mode mismatch	$1 - V_{\mathrm{mode}}$	$< 1.0 \times 10^{-4}$	Single-mode fiber; active alignment; coupling fringes.
Detector timing jitter	$1 - V_{\mathrm{jitter}}$	$< 0.5 \times 10^{-4}$	SNSPDs ≤ 20 ps; deconvolution; keep $\tau_c \gg \sigma_{\mathrm{det}}$.
Path-length noise	$1 - V_{\mathrm{path}}$	$< 0.5 \times 10^{-4}$	Piezo stabilization; enclosures; reference laser.
Background/dark counts	$1 - V_{\mathrm{bg}}$	$< 0.2 \times 10^{-4}$	Gated detection; subtract accidentals; high heralding efficiency.
QRF signal (tunable)	ΔV_{QRF}	$\approx 3.1 \times 10^{-4}$ @ 5%	Extract from slope in ΔV vs. $(\sigma_{k^+}/\bar{k}^+)^2$.

Error budget and sensitivity. Table 1 specifies ordinary visibility-degrading mechanisms and target bounds so the QRF term remains resolvable. For shot-noise-limited counting, $\delta V \sim 10^{-4}$ requires $N_{\mathrm{eff}} \sim 10^8$ coincidences. At 50 kHz this is $\sim$ 30–60 minutes per bandwidth setting, enabling a slope fit with $> 3\sigma$ sensitivity to $\Delta V \approx 3 \times 10^{-4}$.

6 Discussion, Outlook, and Reproducibility

Summary. We gave an explicit operational map as a CP instrument with a unitary dilation, a regulator choice with regulator-independent observables (including off-diagonal control), a bound on massless little-group effects (with null translations accounted for), and a concrete HOM design with a control slope. The formalism is non-reciprocal after conditioning, consistent with the lightlike nature of the photon.

Outlook. A next step is a perspective-neutral or induced-representation approach that treats $E(2)$ exactly and clarifies reciprocity for lightlike frames. Another is a full field-theoretic treatment including polarization at the same order as timing effects, and exploration of multi-photon reference frames.

Reproducibility. A minimal code package (symbolic + numeric) can reproduce ΔV_{QRF} from the chosen regulator, fit α_{HOM} vs. bandwidth, and generate the atom-interferometer scaling plots. A step-by-step recipe is given in App. H.

A Coordinate Uncertainty: Limits and Robustness

Consider a classical event with lab coordinates $(t, z(t))$. With $\hat{\eta} = \ln\left(\hat{k^+}/\tilde{k}^+\right)$ and small rapidities, $z' \approx z(t) - t\hat{\eta}$, hence $(\Delta z')^2 = \mathrm{Var}(z(t) - t\hat{\eta}) = t^2 \mathrm{Var}(\hat{\eta})$. For $\hat{\eta} \approx (\hat{k^+} - \tilde{k}^+)/\tilde{k}^+$, $\mathrm{Var}(\hat{\eta}) \approx (\sigma_{k^+}/\tilde{k}^+)^2$, giving Eq. (9).

B HOM Visibility Loss, Jitter Mapping, and α_{HOM} Range

Let $C(\tau)$ be the normalized field autocorrelation of the (smooth, even) spectral envelope, with $C(0) = 1$ and $C''(0) = -\tau_c^{-2}$ defining τ_c. For small random delays $\Delta\tau$ with variance $\sigma^2_{\Delta\tau}$,

$$V = V_0\,\mathbb{E}[C(\Delta\tau)] \;\approx\; V_0\left(1 - \frac{\sigma^2_{\Delta\tau}}{2\tau_c^2}\right), \tag{11}$$

so $\Delta V = \frac{\sigma^2_{\Delta\tau}}{2\tau_c^2}$. If $\Delta\tau = \kappa\,\tau_c\,\eta$ and the delay is split symmetrically between arms by a factor $c \in \{1, 1/\sqrt{2}\}$, then

$$\Delta V = \frac{\kappa^2 c^2}{2}\,\mathrm{Var}(\eta) = \alpha_{\mathrm{HOM}}\left(\frac{\sigma_{k^+}}{\bar{k}^+}\right)^2 \tag{12}$$

using Lemma 2. The edge cases in Sec. 5.1 follow immediately.

C Atom Interferometry Dephasing: Scaling & Break-even

With lasers transverse to the photon's motion, the lab has velocity $-v\hat{z}$ in the photon's QRF. The laser frequency becomes an operator $\hat{\omega}'_L = \gamma(\hat{\eta})\omega_L = \cosh(\hat{\eta})\omega_L$. The dephasing arises from uncertainty in the momentum kicks $\hbar k_L$. For Gaussian $\epsilon = (k^+ - \tilde{k}^+)/\tilde{k}^+$, $\mathrm{Var}(1 + \epsilon^2/2) = 2(\sigma^2_\epsilon)^2$. The phase variance

$$\mathrm{Var}(\Delta\hat{\Phi}) \approx \left(\omega_L T \frac{v_{\mathrm{atom}}}{c}\right)^2 \left(\frac{\hbar\bar{\omega}_\gamma}{m_{\mathrm{atom}}c^2}\right)^2 \mathrm{Var}(1 + \hat{\epsilon}^2/2) \tag{13}$$

gives $\sigma_\phi = \sqrt{\mathrm{Var}(\Delta\hat{\Phi})}$ as quoted.

D Normalization, Off-diagonal Control, and Regulator Independence

Define

$$I_{m_\gamma}[O] \;=\; \iint d\mu_{m_\gamma}(k)\, d\mu_{m_\gamma}(k')\, \Psi^*(k)\Psi(k')\; \langle\phi|\,\hat{L}^\dagger(k\to\tilde{k})\, O\, \hat{L}(k'\to\tilde{k})\,|\phi\rangle\;, \tag{14}$$

with $O = \mathbb{I}$ for norms and suitable system operators for observables. Writing $\eta = \ln\!\left(k^+/\tilde{k}^+\right)$ and expanding $U^\dagger_{k'}U_k = \exp[-i(\eta - \eta')G]$ for a Hermitian generator G, we obtain for small spreads

$$\langle\phi|\,U^\dagger_{k'}U_k\,|\phi\rangle \;=\; 1 - \frac{(\eta-\eta')^2}{2}\,\mathrm{Var}_\phi(G) \;+\; R_3, \tag{15}$$

with the remainder controlled by *moments*:

$$|R_3| \;\le\; \frac{1}{6}\,\mathbb{E}_\phi(|G|^3)\,\mathbb{E}\Big[|\eta - \eta'|^3\Big]\,. \tag{16}$$

Lemma 3 (Off-diagonal bound with explicit constant)**.** *Let Ψ satisfy Def. 1 and let the system inputs obey $\mathrm{Var}_\phi(G) < \infty$ and $\mathbb{E}_\phi(|G|^3) < \infty$. Then*

$$\Big|\, I_{m_\gamma}[\mathbb{I}] - \|\Psi\|^2_{m_\gamma,2} \,\Big| \;\le\; \underbrace{\tfrac{1}{2}\,\mathrm{Var}_\phi(G)}_{C_G}\,\mathbb{E}_{m_\gamma}[(\eta-\eta')^2] \;+\; \frac{1}{6}\,\mathbb{E}_\phi(|G|^3)\,\mathbb{E}_{m_\gamma}[|\eta-\eta'|^3], \tag{17}$$

where $\|\Psi\|^2_{m_\gamma,2} = \int|\Psi|^2\,d\mu_{m_\gamma}$. By sub-Gaussian tails, $\mathbb{E}[|\eta - \eta'|^3] = o(\mathrm{Var}(\eta))$, hence the off-diagonal contribution is $O(\mathrm{Var}(\eta))$.

Ratios of quadratic forms defining our intensity observables reduce to combinations of $\mathbb{E}_{m_\gamma}[k^+]$ and $\mathrm{Var}_{m_\gamma}(k^+)$ up to $O(\mathrm{Var}(\eta))$ corrections that vanish in the small-spread regime used experimentally. Dominated convergence on the fixed support yields

$$\Delta V_{\mathrm{QRF}} = \alpha_{\mathrm{HOM}} \frac{\mathrm{Var}_{m_\gamma}(k^+)}{\left(\mathbb{E}_{m_\gamma}[k^+]\right)^2} \xrightarrow{m_\gamma \to 0} \alpha_{\mathrm{HOM}} \left(\frac{\sigma_{k^+}}{\bar{k}^+}\right)^2 . \tag{18}$$

E Bound on E(2) Corrections (rotations + translations) with a Gaussian example

Parametrize the composed boost as a longitudinal rapidity plus a small transverse tilt $\theta_\perp$. From the algebra $[K_i, K_j] = -\epsilon_{ijk} J_k$, two tilts generate a rotation $|\theta_W| \le \frac{1}{2}\|\theta_\perp\|^2$ about $\hat{z}$, and $\|\theta_\perp\| \sim \|\mathbf{k}_\perp\|/(k^+)$ for massless momenta [7, 8]. Null translations produce gauge-like phases and null-plane displacements; in intensity-based HOM observables first-order contributions cancel and the leading effect appears at second order with the same parametric scaling. Expanding the visibility to second order yields

$$|\Delta V_{E(2)}| \ \le\ c_3\, \mathbb{E}[\theta_W^2] \ \le\ c_3\, \mathbb{E}\left[\frac{\|\mathbf{k}_\perp\|^4}{(k^+)^4}\right]. \tag{19}$$

Gaussian example. Let $\mathbf{k}_\perp$ be zero-mean Gaussian with rms angular spread θ_{rms} (so that $\mathbb{E}[\|\mathbf{k}_\perp\|^2]/(\bar{k}^+)^2 \approx \theta_{\mathrm{rms}}^2$) and k^+ narrowly distributed about $\bar{k}^+$. Then $\mathbb{E}[\|\mathbf{k}_\perp\|^4]/(\bar{k}^+)^4 \approx 2\,\theta_{\mathrm{rms}}^4$, whence $|\Delta V_{E(2)}| \lesssim 2\,\theta_{\mathrm{rms}}^4 \ll 10^{-6}$ for $\theta_{\mathrm{rms}} \sim \mathrm{mrad}$.

F Unitary Dilation, Instrument Structure, and Reciprocity Deficit

Define the isometry on photon+system

$$S \ =\ \int d\mu(k)\ |k\rangle\langle k|_\gamma \otimes U[\Lambda(k \to \tilde{k})], \qquad S\big(|\Psi\rangle_\gamma \otimes |\phi\rangle_S\big) = \int d\mu(k)\, \Psi(k)\ |k\rangle_\gamma \otimes U_k\, |\phi\rangle_S . \tag{20}$$

Let $M = \sum_m M_m \otimes |m\rangle\langle 0|_R$ be a pre-measurement on the photon coupled to a record R, with $\sum_m M_m^\dagger M_m = \mathbb{I}_\gamma$. The selective operation on the system for outcome m is

$$\Phi_m(\rho_S) \ =\ \frac{\mathrm{Tr}_\gamma\Big[\, M_m\, S(\rho_\gamma \otimes \rho_S) S^\dagger M_m^\dagger\, \Big]}{p_m}, \qquad p_m = \mathrm{Tr}\Big[\, M_m\, S(\rho_\gamma \otimes \rho_S) S^\dagger M_m^\dagger\, \Big], \tag{21}$$

which is CP and trace-nonincreasing; $\sum_m p_m \Phi_m$ is CPTP. Choosing the Kraus density $M_{\phi_0} = \int d\mu(k)\ |\phi_0\rangle\langle k|$ yields the single-Kraus operation $\rho \mapsto K\rho K^\dagger$ with K in (2).

Laboratory surrogate for M_{ϕ_0}. Exact projectors on continuous spectra are idealizations; in optics one approximates M_{ϕ_0} by *mode matching + narrowband filtering* onto $|\phi_0\rangle$ (or via adaptive homodyne for CV envelopes), which realizes the same instrument to the accuracy needed for our visibility measurement.

Proposition 1 (No CPTP inverse for nontrivial spreads)**.** *If U_k depends nontrivially on k on the support of Ψ, the selective channel $\rho \mapsto K\rho K^\dagger$ admits no CPTP left-inverse on any nontrivial set of inputs. Equivalently, branch-selective reciprocity fails generically.*

Small-spread expansion and reciprocity deficit. For small rapidity fluctuations $\eta(k) = \ln\Big(k^+/\tilde{k}^+\Big)$ and generator G with $U_k = \exp[-i\eta(k)G]$, expand around $\bar{\eta} = \mathbb{E}[\eta]$:

$$K \;\approx\; e^{-i\bar{\eta}G}\Big(\mathbb{I} - \tfrac{1}{2}\mathrm{Var}(\eta)\,G^2\Big), \qquad \mathrm{Var}(\eta) \simeq \left(\frac{\sigma_{k^+}}{\bar{k}^+}\right)^2. \tag{22}$$

Define a QRF echo that inverts the dilation and reconditions; for pure inputs $|\phi\rangle$,

$$1 - F_{\mathrm{echo}}(|\phi\rangle) \;\approx\; \mathrm{Var}(\eta)\,\mathrm{Var}_\phi(G) \;+\; \mathcal{O}\big(\mathrm{Var}(\eta)^2\big). \tag{23}$$

G Polarization, Null Translations, and Gauge

We work in $A^+ = 0$ to keep only physical polarizations. In the HOM configuration (co-polarized, single spatial mode per arm), the coincidence probability depends on second-order field correlations $G^{(2)}$. A null translation in arm j contributes a phase $e^{i\phi_j}$ to the field operator $E_j^{(+)}$. Then

$$G^{(2)}(1,2) \propto \Big\langle E_1^{(-)} E_2^{(-)} E_2^{(+)} E_1^{(+)} \Big\rangle \;\mapsto\; e^{-i(\phi_1+\phi_2)} e^{+i(\phi_2+\phi_1)}\, G^{(2)}(1,2) = G^{(2)}(1,2), \tag{24}$$

so first-order null-translation phases cancel identically in intensities; residual effects arise only at second order and are included in the $E(2)$ bound.

H Reproducibility Notes

Goal: Reproduce (i) HOM scaling plots of ΔV vs. $(\sigma_{k^+}/\bar{k}^+)^2$ for multiple bandwidths; (ii) atom-interferometer break-even curves.

HOM recipe.

1. Choose $\bar{k}^+$, bandwidth set $\{\sigma_{k^+}/\bar{k}^+\}$, and $\alpha_{\mathrm{HOM}} \in [1/8, 1/2]$.
2. For each bandwidth, draw N samples of k^+ from a Gaussian $\mathcal{N}(\bar{k}^+, \sigma_{k^+}^2)$ truncated at $k^+ > \epsilon_0$ (fixed cutoff); compute sample moments $\mathbb{E}_{m_\gamma}[k^+]$, $\mathrm{Var}_{m_\gamma}(k^+)$; take $m_\gamma \to 0$.
3. Compute $\Delta V = \alpha_{\mathrm{HOM}}\,\mathrm{Var}_{m_\gamma}(k^+)/\big(\mathbb{E}_{m_\gamma}[k^+]\big)^2$.
4. Fit ΔV vs. $(\sigma_{k^+}/\bar{k}^+)^2$; include a zero-slope control.

I References

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