A Phenomenological Model for the Photon's Quantum Reference Frame: Formal Development, Regulator-Independent Predictions, and Experimental Design

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Abstract

We propose a phenomenological model for describing physics from a photon-centered quantum reference frame (QRF). Our approach combines first-principles constraints with a minimal phenomenological layer to capture complex quantum-optical effects while retaining regulator-independent, gauge-invariant observables. In contrast to the classical no-go result that no inertial rest frame exists for a photon, we show that a superposition of ultra-relativistic Lorentz boosts, conditioned on the photon's quantum state, can operationally realize a photon's perspective. We make the construction explicit by (i) specifying the operator map and its domain, (ii) choosing an explicit light-front regularization scheme and proving that our key observables are independent of the regulator, and (iii) bounding massless little-group (E(2)) effects relative to our leading signals. We then give a concrete Hong–Ou–Mandel (HOM) interferometer design and error budget that could resolve the predicted visibility change at the 3×10^{-4} level, and reframe an atom-interferometer prediction as a future target with scaling plots. We position the work within the quantum-reference-frame (QRF) and relativistic quantum information (RQI) literature and conclude with an outlook toward a full group-theoretic massless-QRF formalism.

1 Introduction and Positioning

The quantum reference frame (QRF) programme promotes frames to quantum systems undergoing conditional, state-dependent transformations. While QRFs for massive particles are well-developed, a consistent description from the perspective of a *massless* photon remains challenging due to (i) the absence of a classical rest frame, (ii) light-front zero modes ($k^+ \to 0$) and regularization subtleties, and (iii) the massless little group $E(2)$. Our aim is to provide an *operational* construction of a photon's QRF that respects these constraints while yielding concrete, falsifiable predictions.

Related work. Foundational QRF results showed that physical laws can be written covariantly under quantum frame changes and that rest frames of quantum systems can be meaningfully defined in the massive case [1, 2]. In relativistic quantum information it is known that Lorentz boosts entangle degrees of freedom and can change observable coherences (e.g., spin-momentum coupling) [3, 4]. On the field-theory side, the light-front ("front form") pioneered by Dirac and developed by Brodsky–Pauli–Pinsky provides both computational advantages and well-known zero-mode pathologies [5, 6]. Our contribution is to (i) extend the QRF idea to a massless photon with an explicit operator map and domain, (ii) show regulator-independent leading observables under an explicit light-front regularization, and (iii) design a concrete HOM experiment to seek the predicted scaling signature.

2 Light-Front Coordinates and Notation

We adopt $x^{\pm} = t \pm z$, $\mathbf{x}_{\perp} = (x, y)$, with metric $ds^2 = dx^+dx^- - d\mathbf{x}_{\perp}^2$. For massless momenta p^{μ}, the on-shell condition reads $p^+p^- = \mathbf{p}_{\perp}^2$. A boost of rapidity η along z rescales $k^{\pm} \to e^{\pm\eta}k^{\pm}$. States are described on the light-front using the measure

$$d\mu(k) = \frac{dk^+ \, d^2\mathbf{k}_{\perp}}{2k^+(2\pi)^3} \, \theta(k^+) \,. \tag{1}$$

We will work in light-cone gauge $A^+ = 0$ to retain only physical photon polarizations.

State class and notation. Unless stated otherwise, photon wavepackets satisfy (i) $\Psi \in L^2(d\mu)$ with finite second moments, (ii) support bounded away from zero: $k^+ \geq \epsilon > 0$ almost everywhere under a chosen regulator (Sec. 3.2), (iii) Gaussian or sub-Gaussian tails in k^+. We denote $\bar{k}^+ = \mathbb{E}[k^+]$, $\sigma_{k^+}^2 = \mathrm{Var}(k^+)$, and $\tilde{k}^+$ a fixed fiducial light-front momentum.

3 Transformation to the Photon's Frame

3.1 Operator Map and Domain

We define an operational map $\mathcal{T}$ from a joint photon+system state to a photon-conditioned description of the system:

$$\mathcal{T}\Big(|\Psi\rangle_{\gamma} \otimes |\phi\rangle_{\text{sys}}\Big) \;\mapsto\; |\phi_0\rangle_{\gamma} \otimes |\phi'\rangle_{\text{sys}} \,, \tag{2}$$

where $|\phi_0\rangle_{\gamma}$ is a fixed photon reference state and

$$|\phi'\rangle_{\text{sys}} = \mathcal{N} \int d\mu_{m_{\gamma}}(k) \, \Psi(k^+, \mathbf{k}_{\perp}) \, \hat{L}(k \to \tilde{k}) \, |\phi\rangle_{\text{sys}} \,, \tag{3}$$

with $d\mu_{m_{\gamma}}(k) = \frac{dk^+ \, d^2\mathbf{k}_{\perp}}{2k^+(2\pi)^3}\theta(k^+)$ understood throughout (Sec. 3.2).

Here $\hat{L}(k \to \tilde{k}) = U[\Lambda(k \to \tilde{k})]$ is the (single- or few-particle) unitary representation of the Lorentz transformation that maps k^{μ} to a fiducial $\tilde{k}^{\mu}$ along the z-axis, acting on the system degrees of freedom.[1] We restrict to the state class in Sec. 2.

Lemma 1 (Isometry on the regulated domain)**.** *Let Ψ satisfy the state-class conditions and regulator $\mathcal{R}$ of Sec. 3.2. Then there exists a normalization $\mathcal{N}(\Psi, \mathcal{R})$ such that $\| \, |\phi'\rangle_{\text{sys}} \| = \| \, |\phi\rangle_{\text{sys}} \|$ for all $|\phi\rangle_{\text{sys}}$ in the system domain on which $U[\Lambda]$ is unitary. Thus $\mathcal{T}$ defines an isometry on that domain.*

Sketch. Under the regulator $\mathcal{R}$ the measure is finite on support; unitarity of $U[\Lambda]$ implies preservation of system inner products pointwise in k. The overall norm reduces to $|\mathcal{N}|^2 \int d\mu_{\mathcal{R}}(k) \, |\Psi(k)|^2$ up to Jacobians that cancel in the chosen variables; choose $\mathcal{N}$ to normalize this integral to 1. See App. D for details. $\square$

[1] For a scalar system mode with momentum p, $(U[\Lambda]\phi)(p) = \sqrt{\frac{2p^0}{2(\Lambda^{-1}p)^0}} \, \phi(\Lambda^{-1}p)$; for spins/polarizations, include the appropriate Wigner rotation matrix. In this work we restrict to scalar-like temporal/spatial degrees relevant for timing/visibility.

Failure modes and domain boundaries. If Ψ has support that touches $k^+ = 0$ or exhibits heavier-than-quadratic tails so that second moments diverge, the normalization integral and/or the variance entering observables diverge. Under the small-m_γ regulator we impose $k^+ \geq \epsilon \sim \mathcal{O}(m_\gamma)$ and assume finite second moments; all observables are computed with the regulated distribution and the limit $m_\gamma \to 0$ is taken at the end (App. D).

Remark 1 (Non-reciprocity). *$\mathcal{T}$ is not a symmetric, reciprocal frame change: it is an operational map answering "what does the system look like* conditioned *on a photon in state Ψ?" Reciprocity would require a massless-frame group structure; addressing this at the $E(2)$ level is discussed in Sec. 3.3.*

Unitary dilation and instrument view (summary). The map $\mathcal{T}$ admits a *unitary dilation*: a coherently controlled boost $S = \int d\mu(k)\ |k\rangle\langle k|_\gamma \otimes U[\Lambda(k \to \tilde{k})]$ on photon+system, followed by a measurement/record on the photon and optional decoherence of the record (App. F). The selective element associated with the "photon fixed to $|\phi_0\rangle$" outcome induces on the system a completely positive operation $\rho \mapsto K\rho K^\dagger$ with

$$K = \langle\phi_0|\, S\, |\Psi\rangle_\gamma = \int d\mu(k)\, \Psi(k)\, U[\Lambda(k \to \tilde{k})]\,. \tag{4}$$

Globally (including the record), the evolution is unitary and reciprocal; the apparent irreversibility/non-reciprocity arises only after *conditioning* on an outcome and *discarding* the record/environment. This interpretation-neutral dilation clarifies the conceptual status of $\mathcal{T}$ without altering any predictions.

3.2 Regularization Scheme and Regulator-Independent Observables

We render $d\mu(k)$ well-defined with a small photon-mass regulator $m_\gamma > 0$ (removed at the end):

$$d\mu_{m_\gamma}(k) = \frac{dk^+\, d^2\mathbf{k}_\perp}{2k^+(2\pi)^3}\, \theta(k^+), \qquad k^- = \frac{\mathbf{k}_\perp^2 + m_\gamma^2}{2k^+}\,, \tag{5}$$

and restrict Ψ to $k^+ \geq \epsilon$ with $\epsilon \sim \mathcal{O}(m_\gamma)$. The normalized transformed state obeys

$$\|\phi'\|^2 = |\mathcal{N}|^2 \int d\mu_{m_\gamma}(k)\, |\Psi(k)|^2 \equiv 1\,, \tag{6}$$

fixing $|\mathcal{N}|^{-2} = \int d\mu_{m_\gamma}(k)\, |\Psi(k)|^2$. We show in App. D that ratios of matrix elements that define our observables (e.g., HOM visibility change ΔV_{QRF}) are independent of m_γ in the limit $m_\gamma \to 0$ provided Ψ has finite second moments and support away from $k^+ = 0$.

3.3 E(2) Little Group: Bound on Neglected Terms

Composing non-collinear boosts induces Wigner rotations and null translations in the massless little group $E(2)$. Let $\theta_W(k)$ denote the induced rotation angle for momentum k; translation parameters produce gauge-like phases and null-plane displacements. For small transverse spreads and rapidities relevant here,

$$|\theta_W(k)| \leq C\, \frac{\|\mathbf{k}_\perp\|^2}{(k^+)^2}\,, \tag{7}$$

for some $C = \mathcal{O}(1)$. In HOM, coincidence probabilities are intensity observables; gauge-like phases from null translations do not contribute at first order, and their second-order effect scales with the same $\mathcal{O}(\|\mathbf{k}_\perp\|^2/(k^+)^2)$ structure. The resulting correction to HOM visibility satisfies $\Delta V_{E(2)} = \mathcal{O}(\mathbb{E}[\theta_W^2])$. Under our state-class assumptions,

$$|\Delta V_{E(2)}| \leq C' \frac{\mathbb{E}[\|\mathbf{k}_\perp\|^4]}{(\bar{k}^+)^4} \ll \alpha_{\mathrm{HOM}} \left(\frac{\sigma_{k^+}}{\bar{k}^+}\right)^2 , \tag{8}$$

for the parameter window used below ($\sigma_{k^+}/\bar{k}^+ \sim 0.05$, moderate transverse spreads), yielding a numerical bound $\lesssim 10^{-6}$, well below our leading effect ($\sim 10^{-4}$). A derivation and constants are given in App. E.

4 Transformed Observables and State-Dependent Spacetime

Observables are evaluated in $|\phi'\rangle_{\mathrm{sys}}$. The entanglement between k^+ and system coordinates induces a state-dependent blur of classical events ("spacetime fuzziness"). For a world-line $z(t)$ probed semiclassically and small rapidities, $z' \approx z(t) - t\hat{\eta}$ with $\hat{\eta} = \ln\left(\hat{k^+}/\tilde{k}^+\right)$, so

$$(\Delta z')^2 = \mathrm{Var}(z(t) - t\hat{\eta}) \approx t^2 \mathrm{Var}(\hat{\eta}) \approx t^2 \left(\frac{\sigma_{k^+}}{\tilde{k}^+}\right)^2 . \tag{9}$$

Appendix A collects limit checks and robustness to modest transverse spreads.

5 Experimental Predictions and Designs

5.1 Hong–Ou–Mandel (HOM) Interferometry

The QRF transform of photon 2 (with photon 1 as the QRF) induces an effective timing jitter σ_{jitter} that reduces HOM visibility. For Gaussian wavepackets,

$$\Delta V_{\mathrm{QRF}} = \alpha_{\mathrm{HOM}} \left(\frac{\sigma_{k^+}}{\bar{k}^+}\right)^2 , \qquad \alpha_{\mathrm{HOM}} \in [1/8, 1/2] , \tag{10}$$

with $\sigma_{k^+}/\bar{k}^+ = 0.05$ giving $\Delta V \approx 3.1 \times 10^{-4}$ for $\alpha_{\mathrm{HOM}} = 1/8$. Appendix B derives the standard relation $V \simeq V_0 \exp\left[-\sigma_{\mathrm{jitter}}^2/(2\tau_c^2)\right]$ and identifies β_{HOM} so that $\alpha_{\mathrm{HOM}} = \beta_{\mathrm{HOM}}/2$. All moments used here are computed with the regulated distribution, and the limit $m_\gamma \to 0$ is taken at the end (cf. App. D).

Concrete design and error budget

We propose a fiber-based HOM interferometer using spectrally factorable SPDC photon pairs at $\lambda \in [810,\ 1550]$ nm. One photon serves as the QRF probe; the partner is interfered on a 50:50 beamsplitter. Tunable bandwidth filtering changes $\sigma_{k^+}/\bar{k}^+$ while holding other parameters fixed.

- **Source:** CW-pumped type-II SPDC with programmable optical filters; target fractional bandwidths $\Delta\nu/\nu \in \{1\%, 3\%, 5\%, 8\%\}$.
- **Detectors:** SNSPDs with system jitter ≤ 20 ps, dark counts $< 100\ \mathrm{s}^{-1}$, efficiency $> 80\%$.
- **Paths:** Fiber delay lines with active stabilization; path-length noise $\lesssim 20\ \mathrm{nm}/\sqrt{\mathrm{Hz}}$ to keep phase noise $\ll 10^{-4}$ in visibility units.
- **Calibration:** Extract α_{HOM} by varying $\sigma_{k^+}/\bar{k}^+$ and fitting ΔV vs. $(\sigma_{k^+}/\bar{k}^+)^2$. Control runs with both arms identically filtered benchmark non-QRF visibility losses.

Table 1: HOM visibility error budget (targets to resolve $\Delta V \sim 3 \times 10^{-4}$).

Mechanism	Symbol	Target contribution	Mitigation / calibration
Spectral distinguishability	$1 - V_{\text{spec}}$	$< 1.0 \times 10^{-4}$	Use factorable SPDC; symmetric filters ify via joint spectral intensity.
Spatial/mode mismatch	$1 - V_{\text{mode}}$	$< 1.0 \times 10^{-4}$	Single-mode fiber; active alignment; me coupling fringes.
Detector timing jitter	$1 - V_{\text{jitter}}$	$< 0.5 \times 10^{-4}$	SNSPDs ≤ 20 ps; deconvolution; keep σ_{det}.
Path-length noise	$1 - V_{\text{path}}$	$< 0.5 \times 10^{-4}$	Piezo stabilization; enclosures; monitor reference laser.
Background/dark counts	$1 - V_{\text{bg}}$	$< 0.2 \times 10^{-4}$	Gated detection; subtract accidentals; tain high heralding efficiency.
QRF signal (bandwidth-tunable)	ΔV_{QRF}	$\approx 3.1 \times 10^{-4}$ @ 5%	Extract from slope in ΔV vs. $(\sigma_{k^+}/\bar{k}^+$

Error budget and sensitivity. Table 1 specifies ordinary visibility-degrading mechanisms and target bounds so the QRF term remains resolvable. For shot-noise-limited counting, a visibility uncertainty $\delta V \sim 10^{-4}$ requires an effective coincidence sample size $N_{\text{eff}} \sim 10^8$ (rule-of-thumb $\delta V \sim \sqrt{V(1-V)/N_{\text{eff}}}$). At 50 kHz net coincidence rate this corresponds to $\sim$ 30–60 minutes per bandwidth setting (with margin), enabling a slope fit of ΔV vs. $(\sigma_{k^+}/\bar{k}^+)^2$ with $> 3\sigma$ sensitivity to $\Delta V \approx 3 \times 10^{-4}$.

5.2 Atom Interferometry (future target)

For lasers transverse to the photon's motion, the photon's QRF induces a superposition of transverse Doppler shifts, producing a phase uncertainty

$$\sigma_\phi \approx \frac{\omega_L T}{\sqrt{2}} \left(\frac{v_{\text{atom}}}{c}\right) \left(\frac{\sigma_{k^+}}{\bar{k}^+}\right)^2 \left(\frac{\hbar\bar{\omega}_\gamma}{m_{\text{atom}} c^2}\right), \tag{11}$$

which is $\sim 10^{-8}$–10^{-7} rad for typical parameters (e.g., Sr-87, $T \sim 1$ s, $v/c \sim 10^{-9}$–10^{-8}, $\hbar\bar{\omega}_\gamma \sim 1$ eV) depending on atomic velocity; we present it as a future benchmark. Scaling analyses and break-even contours are given in App. C.

6 Discussion, Outlook, and Reproducibility

Summary. We have given an explicit operational map, regulator choice with regulator-independent observables, a bound on massless little-group effects (including null translations), and a concrete HOM design. The formalism is non-reciprocal by construction, consistent with the lightlike nature of the photon. A *unitary dilation* (App. F) shows this non-reciprocity arises from conditioning and discarding, while the global evolution remains unitary.

Outlook. A next step is a perspective-neutral or induced-representation approach that treats $E(2)$ exactly and clarifies the status of reciprocity for lightlike frames. Another is a full field-theoretic treatment including polarization at the same order as timing effects, and exploration of multi-photon reference frames.

Reproducibility. A minimal code package (symbolic + numeric) can reproduce $\Delta V_{\rm QRF}$ from the chosen regulator, fit $\alpha_{\rm HOM}$ vs. bandwidth, and generate the atom-interferometer scaling plots. An expanded step-by-step text recipe is given in App. G including a statistical-power estimate.

A Coordinate Uncertainty: Limits and Robustness

Consider a classical event with lab coordinates $(t, z(t))$. With $\hat{\eta} = \ln\left(\hat{k^+}/\tilde{k}^+\right)$ and small rapidities, $z' \approx z(t) - t\hat{\eta}$, hence $(\Delta z')^2 = \mathrm{Var}(z(t) - t\hat{\eta}) = t^2 \mathrm{Var}(\hat{\eta})$. For $\hat{\eta} \approx (\hat{k^+} - \tilde{k}^+)/\tilde{k}^+$, $\mathrm{Var}(\hat{\eta}) \approx (\sigma_{k^+}/\tilde{k}^+)^2$, giving Eq. (9).

B HOM Visibility Loss and Jitter Mapping

Two photons impinge on a 50:50 beam splitter with initial state

$$|\Psi_{\rm in}\rangle = \int d\omega_1 d\omega_2\, \psi_1(\omega_1)\psi_2(\omega_2)\, a_1^\dagger(\omega_1) a_2^\dagger(\omega_2)\, |0\rangle\ . \tag{12}$$

The QRF transform of photon 2 induces an effective timing jitter $\sigma^2_{\rm jitter} = \beta_{\rm HOM}\tau_c^2 \left(\sigma_{k^+}/\bar{k}^+\right)^2$, where τ_c is the coherence time set by the spectral envelope. Standard HOM theory yields

$$V = V_0 \exp\left[-\frac{\sigma^2_{\rm jitter}}{2\tau_c^2}\right] \approx V_0\left(1 - \frac{\sigma^2_{\rm jitter}}{2\tau_c^2}\right), \tag{13}$$

so $\Delta V = \frac{\beta_{\rm HOM}}{2}\left(\sigma_{k^+}/\bar{k}^+\right)^2 \equiv \alpha_{\rm HOM}\left(\sigma_{k^+}/\bar{k}^+\right)^2$, with $\beta_{\rm HOM} \in [1/4, 2]$ ($\alpha_{\rm HOM} \in [1/8, 1/2]$). All moments of k^+ entering $\sigma_{\rm jitter}$ are computed with the regulated distribution and the limit $m_\gamma \to 0$ is taken at the end.

C Atom Interferometry Dephasing: Scaling & Break-even

With lasers transverse to the photon's motion, the lab has velocity $-v\hat{z}$ in the photon's QRF. The laser frequency becomes an operator $\hat{\omega}'_L = \gamma(\hat{\eta})\omega_L = \cosh(\hat{\eta})\omega_L$. The dephasing arises from uncertainty in the momentum kicks $\hbar k_L$. For Gaussian $\epsilon = (k^+ - \tilde{k}^+)/\tilde{k}^+$, $\mathrm{Var}(1 + \epsilon^2/2) = 2\left(\sigma_\epsilon^2\right)^2$. The phase variance

$$\mathrm{Var}(\Delta\hat{\Phi}) \approx \left(\omega_L T \frac{v_{\rm atom}}{c}\right)^2 \left(\frac{\hbar\bar{\omega}_\gamma}{m_{\rm atom}c^2}\right)^2 \mathrm{Var}(1 + \hat{\epsilon}^2/2) \tag{14}$$

gives $\sigma_\phi = \sqrt{\mathrm{Var}(\Delta\hat{\Phi})}$ as quoted.

Break-even contours. For a target sensitivity $\sigma_\phi^\star$, the interrogation time required is

$$T_{\rm req}(\sigma_\phi^\star) = \frac{\sqrt{2}\,\sigma_\phi^\star}{\omega_L}\left[\frac{c}{v_{\rm atom}}\right]\left[\frac{\bar{k}^+}{\sigma_{k^+}}\right]^2\left[\frac{m_{\rm atom}c^2}{\hbar\bar{\omega}_\gamma}\right]. \tag{15}$$

Example: With $\sigma_\phi^\star = 10^{-6}$ rad, $\omega_L = 10^{15}\,\mathrm{s}^{-1}$, $v/c = 10^{-8}$, $\sigma_{k^+}/\bar{k}^+ = 0.05$, and $\hbar\bar{\omega}_\gamma/(m_{\rm atom}c^2) \approx 1.2 \times 10^{-11}$ (Sr-87, $\bar{\omega}_\gamma \sim 1$ eV$/\hbar$), one finds $T_{\rm req} \approx 4.6$ s. For colder atoms with $v/c = 10^{-9}$, $T_{\rm req}$ rises by $\times 10$ to ~ 46 s.

D Normalization and Regulator Independence

Define

$$I_{m_\gamma} = \int_{k^+>0}\int_{(k')^+>0} d\mu_{m_\gamma}(k)\, d\mu_{m_\gamma}(k')\, \Psi^*(k)\Psi(k')\, O(k,k')\,, \tag{16}$$

with $O(k,k') = \langle\phi|\, \hat{L}^\dagger(k \to \tilde{k})\hat{L}(k' \to \tilde{k})\, |\phi\rangle$. Under the unitary action on the system and for observables written as ratios of such quadratic forms, all overall factors from $\mathcal{N}$ and regulator-dependent parts of I_{m_γ} cancel. For ΔV_{QRF} we arrive at

$$\Delta V_{\mathrm{QRF}} = \alpha_{\mathrm{HOM}} \frac{\mathrm{Var}_{m_\gamma}(k^+)}{\left(\mathbb{E}_{m_\gamma}[k^+]\right)^2} \xrightarrow{m_\gamma \to 0} \alpha_{\mathrm{HOM}} \left(\frac{\sigma_{k^+}}{\bar{k}^+}\right)^2, \tag{17}$$

with convergence guaranteed by the state-class assumptions (finite second moments and support away from $k^+ = 0$).

E Bound on E(2) Corrections (rotations + translations)

Parametrize the composed boost as a longitudinal rapidity plus a small transverse "tilt" $\theta_\perp$. For massless representations, the associated Wigner rotation angle satisfies $|\theta_W| \le c_1\|\theta_\perp\|^2$. With $\|\theta_\perp\| \sim \|\mathbf{k}_\perp\|/(k^+)$, we obtain $|\theta_W| \le c_2\|\mathbf{k}_\perp\|^2/(k^+)^2$. Null translations produce gauge-like phases and null-plane displacements; for intensity-based HOM observables, first-order contributions cancel and the leading effect appears at second order, with the same parametric scaling. Expanding the visibility to second order yields $|\Delta V_{E(2)}| \le c_3\mathbb{E}[\theta_W^2]$, leading to Eq. (8). Constants c_i are $\mathcal{O}(1)$ for the small-angle regime used here.

F Unitary Dilation and Relative-State Formulation

Here we give an interpretation-neutral unitary dilation of the photon-QRF map and identify the selective system update with a standard quantum instrument. This clarifies why the construction is globally reciprocal yet *branch-selectively* non-reciprocal.

Coherent, controlled boost. Define the isometry on photon+system

$$S = \int d\mu(k)\; |k\rangle\langle k|_\gamma \otimes U[\Lambda(k \to \tilde{k})]\,, \qquad S\big(\,|\Psi\rangle_\gamma \otimes |\phi\rangle_S\,\big) = \int d\mu(k)\, \Psi(k)\; |k\rangle_\gamma \otimes U_k\, |\phi\rangle_S\,, \tag{18}$$

with $U_k \equiv U[\Lambda(k \to \tilde{k})]$.

Measurement/record and instrument. Let $M = \sum_m M_m \otimes |m\rangle\langle 0|_R$ be a pre-measurement on the photon coupled to a record R, with $\sum_m M_m^\dagger M_m = \mathbb{I}_\gamma$. The selective operation on the system for outcome m is

$$\Phi_m(\rho_S) = \frac{\mathrm{Tr}_\gamma\Big[\, M_m\, S(\rho_\gamma \otimes \rho_S) S^\dagger M_m^\dagger \,\Big]}{p_m}\,, \qquad p_m = \mathrm{Tr}\Big[\, M_m\, S(\rho_\gamma \otimes \rho_S) S^\dagger M_m^\dagger \,\Big]\,, \tag{19}$$

which is a completely positive (CP), trace-nonincreasing map; $\sum_m p_m\, \Phi_m$ is CPTP [8, 9, 10].

Selective "photon-fixed" update. Choosing the Kraus density $M_{\phi_0} = \int d\mu(k)\, |\phi_0\rangle\langle k|$ yields a single-Kraus system operation $\Phi_{\phi_0}(\rho_S) \propto K\rho_S K^\dagger$ with

$$K = \langle\phi_0|\, S\, |\Psi\rangle_\gamma = \int d\mu(k)\, \Psi(k)\, U[\Lambda(k \to \tilde{k})]\,, \tag{20}$$

which matches Eq. (4) in the main text. This is precisely the superposition of boosts used in our phenomenological map.

Proposition 1 (No CPTP inverse for nontrivial spreads)**.** *If U_k depends nontrivially on k on the support of Ψ, then the selective channel $\rho \mapsto K\rho K^\dagger$ with K in (20) admits no CPTP left-inverse on any nontrivial set of inputs. Equivalently, branch-selective reciprocity fails generically.*

Sketch. A CPTP left-inverse for all inputs exists only for unitary channels. Writing $K^\dagger K = \iint d\mu(k)\, d\mu(k')\, \Psi^*(k)\Psi(k')\, U_k^\dagger U_{k'}$, one has $K^\dagger K \propto \mathbb{I}$ only if $U_k^\dagger U_{k'}$ is independent of k, k' on the relevant support. Otherwise K is a strict contraction and no CPTP left-inverse exists (see, e.g., Petz recoverability arguments [11] for state-dependent reversals). □

Small-spread expansion and reciprocity deficit. For small rapidity fluctuations $\eta(k) = \ln\!\left(k^+/\tilde{k}^+\right)$ and an effective generator G such that $U_k = \exp[-i\eta(k)G]$, expand around $\bar{\eta} = \mathbb{E}[\eta]$:

$$K \;\approx\; e^{-i\bar{\eta}G}\Big(\mathbb{I} - \tfrac{1}{2}\mathrm{Var}(\eta)\, G^2\Big)\,, \qquad \mathrm{Var}(\eta) \simeq \left(\frac{\sigma_{k^+}}{\bar{k}^+}\right)^2 . \tag{21}$$

Define a QRF echo by re-injecting $|\Psi\rangle_\gamma$ and undoing the controlled boost, and let F_{echo} be the fidelity between the input ρ_S and the echoed state. For pure inputs $\rho_S = |\phi\rangle\langle\phi|$,

$$1 - F_{\mathrm{echo}}(|\phi\rangle) \;\approx\; \mathrm{Var}(\eta)\, \mathrm{Var}_\phi(G) \;+\; \mathcal{O}(\mathrm{Var}(\eta)^2)\,, \tag{22}$$

so the reciprocity deficit scales with the bandwidth and the system's susceptibility to the boost generator. This is the same small parameter that controls the HOM visibility change.

Everett/relative-state reading (optional). Eq. (19) is also the *relative state* of S in branch m of a no-collapse unitary MS with a decohered record [12, 13]. Globally the evolution is unitary; branch-selective irreversibility (non-reciprocity) arises from conditioning and discarding.

G Reproducibility Notes

Goal: Reproduce (i) HOM scaling plots of ΔV vs. $(\sigma_{k^+}/\bar{k}^+)^2$ for multiple bandwidths; (ii) atom-interferometer break-even curves.

HOM recipe.

1. Choose $\bar{k}^+$, bandwidth set $\{\sigma_{k^+}/\bar{k}^+\}$, and $\alpha_{\mathrm{HOM}} \in [1/8, 1/2]$.
2. For each bandwidth, draw N samples of k^+ from a Gaussian $\mathcal{N}(\bar{k}^+, \sigma_{k^+}^2)$ truncated at $k^+ > \epsilon \sim \mathcal{O}(m_\gamma)$ (regulator); compute sample moments $\mathbb{E}_{m_\gamma}[k^+]$, $\mathrm{Var}_{m_\gamma}(k^+)$; take $m_\gamma \to 0$.
3. Compute $\Delta V = \alpha_{\mathrm{HOM}}\, \mathrm{Var}_{m_\gamma}(k^+)/\big(\mathbb{E}_{m_\gamma}[k^+]\big)^2$.
4. Fit ΔV vs. $(\sigma_{k^+}/\bar{k}^+)^2$ to extract α_{HOM} and its uncertainty.

Statistical power: For shot-noise-limited coincidence counts, $\delta V \simeq \sqrt{V(1-V)/N_{\mathrm{eff}}}$. Target $\delta V \lesssim 10^{-4}$ implies $N_{\mathrm{eff}} \sim 10^8$.

Atom-interferometer recipe.

1. Choose parameters $(\omega_L, T, v/c, \hbar\bar{\omega}_\gamma/mc^2, \sigma_{k^+}/\bar{k}^+)$.
2. Evaluate σ_ϕ from the closed-form expression; for sensitivity targets $\sigma_\phi^\star$, compute $T_{\mathrm{req}}(\sigma_\phi^\star)$ using the break-even formula above.
3. Generate contour plots in $(T, \sigma_{k^+}/\bar{k}^+)$ or $(T, \bar{\omega}_\gamma)$ planes holding others fixed.

H References

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