



Figure 26.6, continued (f) The residual network at the last **while** loop test. It has no augmenting paths, and the flow f shown in (e) is therefore a maximum flow. The value of the maximum flow found is 23.

The Edmonds-Karp algorithm

We can improve the bound on FORD-FULKERSON by finding the augmenting path p in line 3 with a breadth-first search. That is, we choose the augmenting path as a *shortest* path from s to t in the residual network, where each edge has unit distance (weight). We call the Ford-Fulkerson method so implemented the **Edmonds-Karp algorithm**. We now prove that the Edmonds-Karp algorithm runs in $O(VE^2)$ time.

The analysis depends on the distances to vertices in the residual network G_f . The following lemma uses the notation $\delta_f(u, v)$ for the shortest-path distance from u to v in G_f , where each edge has unit distance.

Lemma 26.7

If the Edmonds-Karp algorithm is run on a flow network $G = (V, E)$ with source s and sink t , then for all vertices $v \in V - \{s, t\}$, the shortest-path distance $\delta_f(s, v)$ in the residual network G_f increases monotonically with each flow augmentation.