Math 2300 Project

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I have 1 of the 2 problems needed almost done. I just need to find the basis for problem 5 and i'll be done with this one. I plan on doing problem 6 for the second problem.

Problem 5 Power Method Eigenvale of Matrix.

I made a matrix M1 and used the eigen command to find the values so I knew what to look for.

```
> M1 <- matrix(c(6,-4,1,-4,6,-1,1,-1,11), ncol = 3, byrow =T)
> print(M1)

     [,1] [,2] [,3]
[1,]    6   -4    1
[2,]   -4    6   -1
[3,]    1   -1   11

> eigen((M1))

eigen() decomposition
$values
[1] 12  9  2

$vectors
           [,1]       [,2]          [,3]
[1,]  0.4082483 -0.5773503 -7.071068e-01
[2,] -0.4082483  0.5773503 -7.071068e-01
[3,]  0.8164966  0.5773503  3.053113e-16
```

```
>
>
```

Then I multiplied M1 to a [1,0,0] vector to find the initial eigenvalue. Then I divided that vector by 6. Then I multiplied that matrix by M1 and I did that sequence all the way untill I got a value of 12 which is the highest eigenvalue.

```
> v1 <- c(1,0,0)
> print(v1)

[1] 1 0 0

> V1 <- v1 %*% t(M1)
> print(V1)

     [,1] [,2] [,3]
[1,]    6   -4    1

> ev1 <- V1/6
> print(ev1)

     [,1]       [,2]      [,3]
[1,]    1 -0.6666667 0.1666667

> V2 <- ev1 %*% t(M1)
> print(V2)

         [,1]      [,2] [,3]
[1,] 8.833333 -8.166667  3.5

> ev2 <- V2/ 8.833333
> print(ev2)

     [,1]       [,2]      [,3]
[1,]    1 -0.9245283 0.3962264

> V3 <- ev2 %*% t(M1)
> print(V3)
```

```
          [,1]      [,2]     [,3]
[1,] 10.09434 -9.943397 6.283019

> ev3 <- V3/10.09434
> print(ev3)

     [,1]       [,2]      [,3]
[1,]    1 -0.9850467 0.6224299

> V4 <- ev3 %*% t(M1)
> print(V4)

          [,1]      [,2]     [,3]
[1,] 10.56262 -10.53271 8.831776

> ev4 <- V4/10.56262
> print(ev4)

          [,1]       [,2]      [,3]
[1,] 0.9999997 -0.9971683 0.8361349

> V5 <- ev4 %*% t(M1)
> print(V5)

          [,1]      [,2]     [,3]
[1,] 10.82481 -10.81914 11.19465

> ev5 <- V5/ 10.82481
> print(ev5)

          [,1]       [,2]     [,3]
[1,] 0.9999997 -0.9994766 1.034166

> V6 <- ev5 %*% t(M1)
> print(V6)

          [,1]      [,2]    [,3]
[1,] 11.03207 -11.03102 13.3753

> ev6 <- V6/11.03207
> print(ev6)
```

```
     [,1]       [,2]     [,3]
[1,]    1 -0.9999052 1.212402
```

```
> V7 <- ev6 %*% t(M1)
> print(V7)
```

```
         [,1]      [,2]     [,3]
[1,] 11.21202 -11.21183 15.33633
```

```
> ev7 <- V7/11.21202
> print(ev7)
```

```
     [,1]       [,2]     [,3]
[1,]    1 -0.9999834 1.367847
```

```
> V8 <- ev7 %*% t(M1)
> print(V8)
```

```
         [,1]      [,2]    [,3]
[1,] 11.36778 -11.36775 17.0463
```

```
> ev8 <- V8/11.36778
> print(ev8)
```

```
     [,1]       [,2]     [,3]
[1,]    1 -0.9999972 1.499528
```

```
> V9 <- ev8 %*% t(M1)
> print(V9)
```

```
         [,1]      [,2]    [,3]
[1,] 11.49952 -11.49951 18.4948
```

```
> ev9 <- V9/11.49952
> print(ev9)
```

```
          [,1]       [,2]     [,3]
[1,] 0.9999998 -0.9999993 1.608311
```

```
> V10 <- ev9 %*% t(M1)
> print(V10)
```

```
          [,1]      [,2]     [,3]
[1,] 11.60831 -11.60831 19.69142

> ev10 <- V10/11.60831
> print(ev10)

          [,1]       [,2]     [,3]
[1,] 0.9999997 -0.9999996 1.696321

> V11 <- ev10 %*% t(M1)
> print(V11)

          [,1]      [,2]     [,3]
[1,] 11.69632 -11.69632 20.65953

> ev11 <- V11/11.69632
> print(ev11)

          [,1]       [,2]     [,3]
[1,] 0.9999998 -0.9999998 1.766327

> V12 <- ev11 %*% t(M1)
> print(V12)

          [,1]      [,2]    [,3]
[1,] 11.76632 -11.76632 21.4296

> ev12 <- V12/11.76632
> print(ev12)

     [,1] [,2]     [,3]
[1,]    1   -1 1.821266

> V13 <- ev12 %*% t(M1)
> print(V13)

          [,1]      [,2]     [,3]
[1,] 11.82127 -11.82127 22.03393

> ev13 <- V13/11.82127
> print(ev13)
```

```
     [,1] [,2]     [,3]
[1,]    1   -1 1.863922

> V14 <- ev13 %*% t(M1)
> print(V14)

         [,1]      [,2]     [,3]
[1,] 11.86392 -11.86392 22.50314

> ev14 <- V14/11.86392
> print(ev14)

     [,1] [,2]     [,3]
[1,]    1   -1 1.896771

> V15 <- ev14 %*% t(M1)
> print(V15)

         [,1]      [,2]     [,3]
[1,] 11.89677 -11.89677 22.86449

> ev15 <- V15/11.89677
> print(ev15)

     [,1] [,2]     [,3]
[1,]    1   -1 1.921907

> V16 <- ev15 %*% t(M1)
> print(V16)

         [,1]      [,2]     [,3]
[1,] 11.92191 -11.92191 23.14098

> ev16 <- V16/11.92191
> print(ev16)

     [,1] [,2]     [,3]
[1,]    1   -1 1.941046

> V17 <- ev16 %*% t(M1)
> print(V17)
```

```
          [,1]      [,2]     [,3]
[1,] 11.94105 -11.94105 23.35151

> ev17 <- V17/ 11.94105
> print(ev17)

           [,1]       [,2]     [,3]
[1,] 0.9999997 -0.9999997 1.955566

> V18 <- ev17 %*% t(M1)
> print(V18)

          [,1]      [,2]     [,3]
[1,] 11.95556 -11.95556 23.51122

> ev18 <- V18/11.95556
> print(ev18)

     [,1] [,2]     [,3]
[1,]    1   -1 1.966551

> V19 <- ev18 %*% t(M1)
> print(V19)

          [,1]      [,2]     [,3]
[1,] 11.96655 -11.96655 23.63206

> ev19 <- V19/11.96655
> print(ev19)

     [,1] [,2]     [,3]
[1,]    1   -1 1.974844

> V20 <- ev19 %*% t(M1)
> print(V20)

          [,1]      [,2]     [,3]
[1,] 11.97485 -11.97485 23.72328

> ev20 <- V20/11.97485
> print(ev20)
```

```
          [,1]       [,2]     [,3]
[1,] 0.9999997 -0.9999997 1.981092

> V21 <- ev20 %*% t(M1)
> print(V21)

         [,1]      [,2]     [,3]
[1,] 11.98109 -11.98109 23.79201

> ev21 <- V21/ 11.97485
> print(V21)

         [,1]      [,2]     [,3]
[1,] 11.98109 -11.98109 23.79201

> V22 <- ev21 %*% t(M1)
> print(ev1)

     [,1]       [,2]      [,3]
[1,]    1 -0.6666667 0.1666667

> ev22 <- V22/11.99204
> print(ev22)

     [,1] [,2]     [,3]
[1,]    1   -1 1.989335

> V23 <- ev22 %*% t(M1)
> print(V23)

         [,1]      [,2]     [,3]
[1,] 11.98934 -11.98934 23.88269

> ev23 <- V23/11.98934
> print(ev23)

          [,1]       [,2]     [,3]
[1,] 0.9999997 -0.9999997 1.991994

> V24 <- ev23 %*% t(M1)
> print(V24)
```

```
         [,1]      [,2]     [,3]
[1,] 11.99199 -11.99199 23.91193

> ev24 <- V24/11.99199
> print(ev24)

     [,1] [,2]     [,3]
[1,]    1   -1 1.993992

> V25 <- ev24 %*% t(M1)
> print(V25)

         [,1]      [,2]     [,3]
[1,] 11.99399 -11.99399 23.93391

> ev25 <- V25/ 11.99399
> print(ev25)

     [,1] [,2]     [,3]
[1,]    1   -1 1.995492

> V26 <- ev25 %*% t(M1)
> print(V26)

         [,1]      [,2]     [,3]
[1,] 11.99549 -11.99549 23.95041

> ev26 <- V26/11.99549
> print(ev26)

     [,1] [,2]     [,3]
[1,]    1   -1 1.996618

> V27 <- ev26 %*% t(M1)
> print(V27)

         [,1]      [,2]    [,3]
[1,] 11.99662 -11.99662 23.9628

> ev27 <- V27/11.99662
> print(ev27)
```

```
     [,1] [,2]     [,3]
[1,]    1   -1 1.997463

> V28 <- ev27 %*% t(M1)
> print(V28)

         [,1]      [,2]     [,3]
[1,] 11.99746 -11.99746 23.97209

> ev28 <- V28/11.99746
> print(ev28)

     [,1] [,2]     [,3]
[1,]    1   -1 1.998097

> V29 <- ev28 %*% t(M1)
> print(V29)

        [,1]     [,2]     [,3]
[1,] 11.9981 -11.9981 23.97907

> ev29 <- V29/11.9981
> print(ev29)

     [,1] [,2]     [,3]
[1,]    1   -1 1.998572

> V30 <- ev29 %*% t(M1)
> print(V30)

         [,1]      [,2]     [,3]
[1,] 11.99857 -11.99857 23.98429

> ev30 <- V30/11.99857
> print(ev30)

     [,1] [,2]     [,3]
[1,]    1   -1 1.998929

> V31 <- ev30 %*% t(M1)
> print(V31)
```

```
          [,1]      [,2]     [,3]
[1,] 11.99893 -11.99893 23.98822

> ev31 <- V31/11.99893
> print(ev31)

     [,1] [,2]     [,3]
[1,]    1   -1 1.999197

> V32 <- ev31 %*% t(M1)
> print(V32)

        [,1]     [,2]     [,3]
[1,] 11.9992 -11.9992 23.99117

> ev32 <- V32/11.9981
> print(ev32)

         [,1]      [,2]    [,3]
[1,] 1.000092 -1.000092 1.99958

> V33 <- ev32 %*% t(M1)
> print(V33)

        [,1]     [,2]     [,3]
[1,] 12.0005 -12.0005 23.99557
```

To find the basis for eigenvalue lambda=2 you need to start with the formula (A-lamdaI). A comes from the original matrix. Lambda is the eigenvalue, and I is the identity matrix. Then you need to row reduce the new matrix. Then you need to multiply the matrix by the vector [x,y,z]. When you do that you end up wiith x - y = 0 and z = 0. That turns into x=y and z=0. Plug that into an [x,y,z] vector and then let y=1 you will end up with a basis of [1,1,0] for eigenvalue 2.

```
> library("pracma")
> bs2m <- M1 - 2*diag(3)
> print(bs2m)
```

```
     [,1] [,2] [,3]
[1,]    4   -4    1
[2,]   -4    4   -1
[3,]    1   -1    9
```

```
> rr <- rref(bs2m)
> nm <- matrix(c("x","y","z"), ncol=1)
> z <- "x-y=0"
```