Assumptions and equations used in ”Design Fire Creation kit”

The goal is to make a creation kit that make generating Design Fires easy. The Design Fire is split into three phases. A Growth Phase, a steady phase and a decay phase. Each phase uses difference equations that is derived in the following.

Nomenclature:

Term	Description	Unit
t	Time	s
t_d	Time duration	s
q	Rate of Heat Release	kW
E	Total energy released	MJ
α	Growth rate factor / Decay rate factor	kW/s^2
x_t	The end time for the current result	s
x_{t-1}	The end time for the result of the previous phase	s
y_q	The end effect for the current result	kW
y_{q-1}	The end effect for the result of the previous phase	kW

Growth phase:

The equation used to describe the growth phase is normally $q = \alpha \cdot t^2$. This equation only works with the assumption that a fire is starting. In other words, that the initial conditions is $(x, y) = (0, 0)$. To account for the initial conditions can change, one can add the end result coordinates of the previous phase. Hence a generic equation for the growth phase $q = y_{q-1} + \alpha \cdot (t - x_{t-1})^2$.

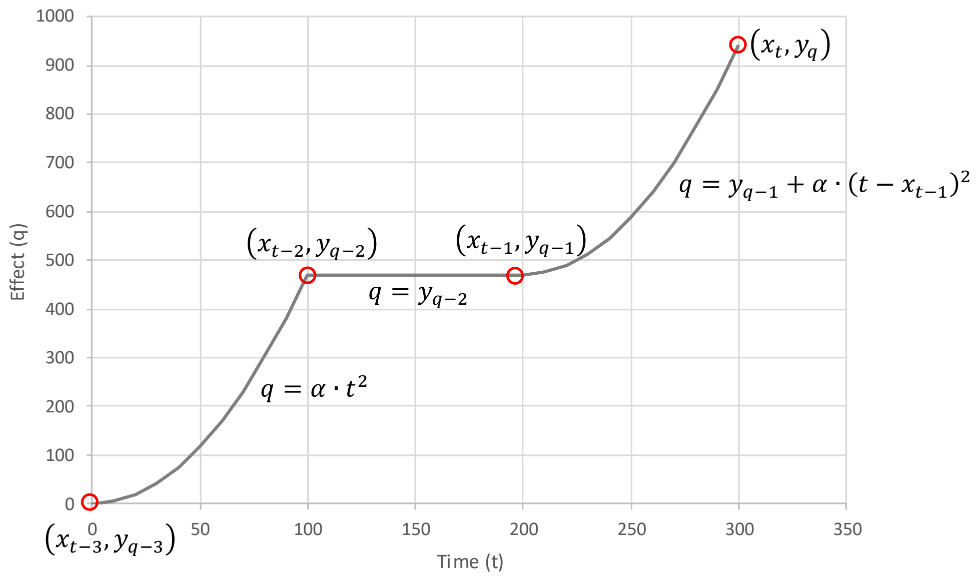


Figure 1 Shows the growth phase and steady phase

There is made three options to generate a growth phase:

Option	Description	Known parameters	Calculated parameters
1	One knows the growth rate factor and the time duration of this phase.	α, t_d	$q, y_{q-1}, y_q,$ x_{t-1}, x_t
2	One knows at what effect the phase shall end and the time duration of this phase.	y_q, t_d	$q, y_{q-1}, \alpha,$ $x_{t-1}, x_t,$
3	One knows at what effect the phase shall end and the growth rate factor.	y_q, α	$q, y_{q-1}, t_d,$ x_{t-1}, x_t

It's possible in the future that a fourth and fifth options will be added. The two extra options derive from the first and second option, by exchanging the time duration to the time at which the phase ends. It simplifies the growth phase for the user, but behind the scene a lot of validation needs to be put in place.

Equations used in this phase

$$q = y_{q-1} + \alpha \cdot (t - x_{t-1})^2$$

To calculate the total energy released during this phase, $t = x_t$ and $x_t - x_{t-1} = t_d$

$$E = \int y_{q-1} + \alpha \cdot (t_d)^2 \, dt \overset{E}{\Rightarrow}$$

$$E = \frac{1}{3} \cdot \alpha \cdot t_d^3 + y_{q-1} \cdot t_d$$

$$x_t = x_{t-1} + t_d$$

$$y_q = y_{q-1} + \alpha \cdot t_d^2$$

To find α in option 2, $t = x_t$ and $q = y_q$. Because $x_t - x_{t-1} = t_d$ the generic equation for growth phase becomes.

$$y_q = y_{q-1} + \alpha \cdot (t_d)^2 \overset{\alpha}{\rightarrow}$$

$$\alpha = \frac{y_q - y_{q-1}}{t_d^2}$$

To find t_d in option 3, the same prerequisites used when finding α will be applied here as well.

$$y_q = y_{q-1} + \alpha \cdot (t_d)^2 \overset{t_d}{\rightarrow}$$

$$t_d = \sqrt{\frac{y_q - y_{q-1}}{\alpha}}$$

Steady phase:

The equation used to describe the steady phase is simply $q = y_{q-1}$ for a time duration t_d. The terms is illustrated in figure 1.

There is made one option to generate a steady phase:

Option	Description	Known parameters	Calculated parameters
1	One knows the time duration of this phase.	t_d	$q, y_{q-1}, y_q, x_{t-1}, x_t$

It’s possible in the future that a second options will be added. Where one knows at which time the steady phase shall end. It simplifies the steady phase for the user, but behind the scene a lot of validation needs to be put in place.

<u>Equations used in this phase</u>

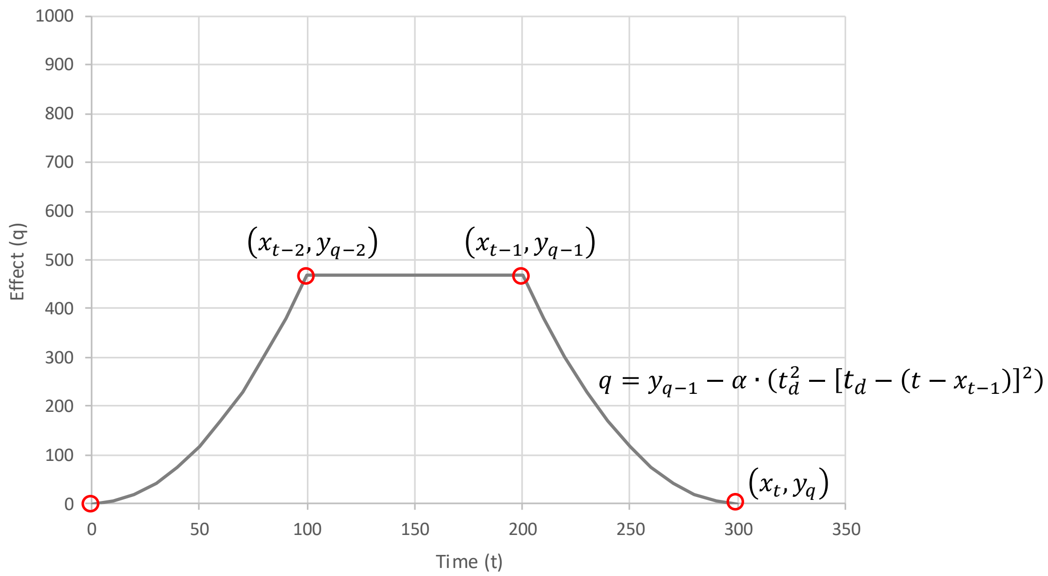
$$y_q = q = y_{q-1}$$

$$E = \int y_{q-1}\, dt = y_{q-1} \cdot t_d$$

$$x_t = x_{t-1} + t_d$$

Decay phase:

It's often assumed that the decay phase can be described as the growth phase but with negative growth rate factor. The same assumption is made for the options in this design fire creation kit. The equation for the decay phase can be found by displacing the growth phase equation $q = y_{q-1} + \alpha \cdot (t - x_{t-1})^2$ in the x-direction by $x_{t-1} - x_{t-2}$ and reflecting it on $y = x_{t-1}$.



Figur 2 Shows the growth phase, steady phase and decay phase

There is made three options to generate a decay phase:

Option	Description	Known parameters	Calculated parameters
1	One knows the growth rate factor and the time duration of this phase.	α, t_d	$q, y_{q-1}, y_q, x_{t-1}, x_t$
2	One knows at what effect the phase shall end and the time duration of this phase.	y_q, t_d	$q, y_{q-1}, \alpha, x_{t-1}, x_t,$
3	One knows at what effect the phase shall end and the growth rate factor.	y_q, α	$q, y_{q-1}, t_d, x_{t-1}, x_t$

It's possible in the future that a fourth and fifth options will be added. The two extra options derive from the first and second option, by exchanging the time duration to the time at which the phase ends. It simplifies the decay phase for the user, but behind the scene a lot of validation needs to be put in place.

Equations used in this phase

$q = y_{q-1} - \alpha \cdot (t_d^2 - [t_d - (t - x_{t-1})]^2)$

To calculate the total energy released during this phase, $t = x_t$ and $x_t - x_{t-1} = t_d$

$E = \int y_{q-1} - \alpha \cdot (t_d^2 - [t_d - (t_d)]^2)\, dt \overset{E}{\Rightarrow}$

$E = \frac{1}{3} \cdot \alpha \cdot t_d^3 + y_{q-1} \cdot t_d$

$x_t = x_{t-1} + t_d$

$y_q = y_{q-1} - \alpha \cdot t_d^2$

To find α in option 2, $t = x_t$ and $q = y_q$. Because $x_t - x_{t-1} = t_d$ the generic equation for growth phase becomes.

$y_q = y_{q-1} - \alpha \cdot (t_d^2 - [t_d - (t_d)]^2) \xrightarrow{\alpha}$

$\alpha = abs\left[\frac{y_q - y_{q-1}}{t_d^2}\right]$

To find t_d in option 3, the same prerequisites used when finding α will be applied here as well.

$y_q = y_{q-1} - \alpha \cdot (t_d^2 - [t_d - (t_d)]^2) \xrightarrow{t_d}$

$t_d = abs\left[\sqrt{\frac{y_q - y_{q-1}}{\alpha}}\right]$